Optimizing System Efficiency and Reliability: Integrating Semi-Markov Processes and Regenerative Point Techniques for Maintenance Strategies in Plate Manufacturing

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Abstract: - This paper offers a modeling methodology to evaluate similar systems' performance and optimize system effectiveness with customized maintenance plans. This strategy, which combines regeneration point techniques with semi-Markov processes, is used to assess a plate manufacturing company's system. Key insights are obtained using a thorough cost-benefit analysis that includes numerical analysis, graphical interpretations, and system efficacy indicators. The study clarifies the dynamics of the system's reliability under different repair rates by showing an inverse relationship between Mean Time to System Failures (MTSF) and failure rate. Additionally, the analysis clarifies how profit is affected by failure rate, highlighting the necessity of efficient maintenance plans from an economic standpoint. This research highlights the growing significance of availability and dependability in technology-driven sectors and emphasizes the need for dependable systems. This paper makes a substantial contribution to the knowledge already available in the field of reliability engineering by providing a thorough framework that combines regeneration point approaches and semi-Markov processes. The metrics that are obtained offer a comprehensive understanding of the behavior of the system, which in turn helps industry practitioners make well-informed decisions.

Key-Words: - Reliability analysis, system modeling, hot standby parallel system, switching, regenerative-point technique, semi-Markov process.

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1 Introduction

Quality and reliability of products have emerged as key drivers of competitive advantage in the current globalization and market economy. Controlling the cost of unreliability from different types of equipment and various process breakdowns, which waste money, is the financial issue of reliability for businesses.

The study's value is found in its careful analysis of a dependability model that is specially adapted to the operational dynamics of a plate manufacturing company. Through the examination of a system consisting of three similar units, each responsible for generating different types of plates, the study clarifies the complex relationship between production efficiency and system reliability in such industrial environments. The outline of a backup plan that manages the breakdown of any unit in a smooth and continuous manner to keep production going highlights how useful the suggested model is in real-world scenarios.

To meet society's growing demand, industries are increasingly incorporating automation into their manufacturing processes, and more sophisticated and complicated systems are under development. Researchers evaluated different reliability models to find optimizing factors that affect the system. [1], summarizes the reasoning behind and statistical methods used to examine some failure data acquired from activities carried out by both machines and people. [2], obtained a likelihood test-ratio and gave a formula(approx.) for the O.C. (operating characteristics) curve, towards the expected no. of failure and wait time before a decision is reached. [3], discussed failure time and parallel system availability with the repair. He used a cold standby unit and a repairperson on multiple vacations. [4], worked on the cost analysis of a single-server 2-unit system with a cold standby subject to degradation. [5], studied a system with two units of cold-standby systems with two repairpersons when one is occupied repairing a failed unit the assistant repairperson needs or doesn't need the instructions, according to a probabilistic analysis. [6], discussed systems with different maintenance techniques. In [7], formulated modeling of a 3-unit cold standby (induced draft fan) system operating at full/reduced capacity, [8], [9], [10] discussed multi-state machines that have similar failure causes and their reliability using fuzzy probability and Bayesian networks. [11], found a novel approach with a neural network as a base for reliability-centered maintenance. This method is formulated for false alarm detection and accuracy of 90%.

[12], discussed parallel system reliability modeling with maximum operation and repair durations. [13], studied dependability metrics for the performance evaluation of mobile communication systems to evaluate the performance of phone communications and their reliability measures. [14], discussed an analysis of the costs for two-unit warm standby models. They believed that an expert was only contacted when a regular repair person was unable to fix the problem within the allotted time. They had a regular repairperson and patience. [15], studied study of the costs and benefits of operating two of the three induced draught fans at cold standby rather than at decreased capacity using a semi-Markov process. [16], did the reliability of an evaluation of large-scale industries such as steel plant production of biscuits etc. with a machine as hot standby. [17], studied Reverse-osmosis and forward-osmosis integrated desalination network accessibility and dependability. The probabilities were estimated using fuzzy set theory and failure probabilities were calculated. [18], calculated and discussed reliability, availability, and maintainability for the wine packaging industry. [19], [20] proposed a theory of functional failures as a basis for making early predictions about the availability and reliability of hardware-software systems coupled. Also, [21] proposed an assessment of the reliability and availability of a photovoltaic power plant. In the present reliability model, A plate production company with three similar units is considered as the system under examination. There are two types of plates produced by the company: full plate and half plate. The first unit produces a full plate, the second unit produces a half plate, and the third unit is kept on hot standby to produce either type of plate. If one of the units fails, the remaining two can be used to complete the task. If the second unit fails, the dye will be replaced, ensuring that the production of both plates is not harmed. If the system is fully inoperable then it is considered failed.

The study is an essential tool for developing research in reliability engineering and supporting the creation of focused maintenance plans because of its emphasis on real-world scenarios and systematic framework for evaluating reliability in the face of operational complexity. This study is important because it establishes a foundation for future research endeavors on the subject by bridging the gap between theoretical insights and practical necessities, so paving the way for greater operational resilience and efficiency in manufacturing organizations.

The article has been organized as follows: Section 2 delivers many terminologies that will help in evaluating the performance of various system measures. In Section 3, different measures of system effectiveness are calculated followed by profit analysis, and graphical interpretations in Section 4 are done. Finally, Section 5 states the conclusion of the article.

The following assumptions are taken under consideration for the model:

- Initially, all three units are fully operative.
- Every unit under consideration is identical.
- The rate of failure for all units is constant as they are identical.
- Preference is given to switching instead of repair.
- The repairperson is readily available for any unit needing repair.
- Failure time follows an exponential distribution.
- The system is considered perfect after each repair.
- When the system undergoes switching it can't fail.
- If all units get failed, the system is considered to be completely failed
- The unit cannot fail immediately after repair

This research aims to assess the system's availability and reliability.

- The findings demonstrate that, in comparison to other systems, the three-unit hot standby parallel system with one hot standby unit operating according to demand offers the highest dependability and availability.
- To increase system performance and decrease downtime, hot standby parallel

systems can be designed and optimized with the help of the study's findings.

2 Problem Formulation

2.1 Description of the Model

In this reliability model, a plate manufacturing firm's system is under consideration consisting of three identical units. The firm manufactures two types of plates- full plate and half plate. The first unit manufactures a full plate, the second unit manufactures a half plate, and the third unit is kept as hot standby manufacturing a full plate. If any of the units stops working or is failed, then the other two units can be used to fulfil the purpose. If the second unit fails, then the dye will be replaced so that production of both plates will remain unaffected. If all three units under consideration are inoperable, the system is considered failed. For this study, information was gathered about the failures.

2.2 Nomenclature

Table 1. Annotations used in the model

Notation	Meaning
Op_1	operative state for machine manufacturing plate 1.
Op_2	operative state for machine manufacturing plate 2.
F_w	failed unit awaiting repair.
F_r	The failed unit is being repaired.
F_R	unit is being repaired from its prior state.
S	switching of plates is taking place.
α	repair-rate
λ	Failure rate of the working unit.
g(□), G(□)	p.d.f and c.d.f of repair time of the unit under consideration.
$\bar{G}$(□)	survival function.
β_1	constant rate of allowed time for switching the plates.
H_S	hot standby.
©, Ⓢ	Laplace convolution function, Laplace-Stieltjes convolution function.
C_0	revenue takings per unit uptime when the system is at maximum efficiency.
C_1	revenue takings per unit of running time when the given system is operating with reduced capacity.
C_2	cost per unit time when the repairperson is already busy.
C_3	cost per unit of time when the system is not working or is down.
C_4	Payment per unit time given to the person performing repairs
AF_0	The probability that the system operates at full capacity under the condition that it is initially at state 0 at □ =0
AR_0	The probability that the system operates at reduced capacity under the condition that it is initially at state 0 at □ =0
B_0	The time when the repair man is busy

2.3 Data Summary

- Total number of failures that occurred in the plant in a year =9 failures

Failure per hour $= \frac{9}{365*24} = 0.00102$ failures per hour.

- Time required for changing a plate = 120 mins

Rate of change $= \frac{1}{2} = 0.5$ per hour.

- Total time taken for repair = 32 hrs.

Rate of repair for 9 failures $= \frac{9}{32} = 0.281$.

Table 2 states the values of the estimated repair/failure rate and time taken while switching from the maintenance data of the firm.

Table 2. Estimated values of rates for the system

S.no.	Rate(/hour)	Value(/hour)	
1	α, repair-rate	0.08	
2	β_1, the time allowed for switching plates	0.5	
3	λ, rate of failure	0.0017	

2.4 State Transition Diagram

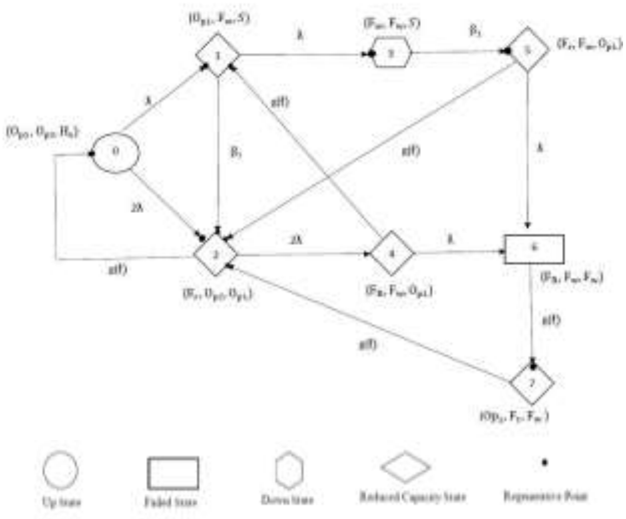

Fig. 1: State Transition Diagram

Figure 1 gives the state transition diagram covering all the possibilities of the system under consideration and a table has been formulated for better understanding of Figure 1. It also states that where the system is fully operational and from which states it underwent repair.

The rates of transition from S^i to S^j are shown in Table 3.

Table 3. System Transition Rates

$S^i \backslash S^j$	S^0	S^1	S^2	S^3	S^4	S^5	S^6	S^7
S^0	0	λ	2λ	0	0	0	0	0
S^1	0	0	β_1	λ	0	0	0	0
S^2	$g(\square)$	0	0	0	2λ	0	0	0
S^3	0	0	0	0	0	β_1	0	0
S^4	0	$g(\square)$	0	0	0	0	λ	0
S^5	0	0	$g(\square)$	0	0	0	λ	0
S^6	0	0	0	0	0	0	0	$g(\square)$
S^7	0	0	$g(\square)$	0	0	0	0	0

The rates of transition from S^i to S^j are shown in Table 3. Further it can be explained as the system can be in one of several states, each of which denotes a distinct functioning level. The upstate, or state 0, or S^0, denotes that the system is completely functional and capable. But there's also a chance that the system will have partial breakdowns, which would lower its capacity. States 1, 2, 4, 5, and 7 experience this. In state 1, there has been some system failure and switching has already occurred to fix the problem. State 2 denotes that while the system is functional, one of the units has failed and is undergoing repair. State 3, on the other hand, denotes a down state, meaning that the system is entirely broken and in need of repair. State 4 denotes a partially functional state, when one unit is undergoing repair while another is awaiting attention. Like this, in state 5, one unit is undergoing repair while the other is in waiting. With one unit undergoing repair and the other two units awaiting repair, State 6 indicates a total breakdown of the system. Finally, the system is only partially functional in state 7, with one unit awaiting repair and another undergoing repair currently. And the annotations that are used throughout the model are in Table 1.

The following measures of system effectiveness are calculated

- Mean Time to System Failure (M.T.S.F.).
- Mean sojourn time.
- Analysis of Availability for the system working at Full Capacity.
- Analysis of Availability when the system is working at Reduced Capacity.
- Busy period analysis of Repairperson.
- Downtime or time for which the system is down.

2.5 Transition Probabilities:

Transition probabilities are evaluated as follows:

$dQ_{01}(\square) = \lambda e^{-3\lambda\square} d\square$

$dQ_{02}(\square) = 2\lambda e^{-3\lambda\square} d\square$

$dQ_{12}(\square) = \beta_1 e^{-(\lambda+\beta_1)\square} d\square$

$dQ_{13}(\square) = \lambda e^{-(\lambda+\beta_1)\square} d\square$

$dQ_{20}(\square) = e^{-2\lambda\square} g(\square) d\square$

$dQ_{26}^{4}(\square) = (2\lambda e^{-2\lambda\square} © \lambda e^{-\lambda\square})\overline{G}(\square) d\square$

$dQ_{21}^{4}(\square) = (2\lambda e^{-2\lambda\square} © e^{-\lambda\square}) g(\square)\, d\square$

$dQ_{27}^{4,6}(\square) = (2\lambda e^{-2\lambda\square} © \lambda e^{-\lambda\square} © 1) g(\square)\, d\square$

$dQ_{35}(\square) = \beta_1 e^{-\beta_1\square} d\square$

$dQ_{52}(\square) = e^{-\lambda\square} g(\square) d\square$

$dQ_{56}(\square) = \lambda e^{-\lambda\square}\, \overline{G}(\square) d\square$

$dQ_{57}^{6}(\square) = (\lambda e^{-\lambda\square} © 1) g(\square)\, d\square$

p_{ij} transition probabilities are calculated as follows:

$p_{ij} = \lim_{s\to 0} q_{ij}^{*}(s)$ where $\frac{dQ_{ij}(\square)}{d\square} = q_{ij}(\square)$

$p_{01} = \frac{1}{3}$

$p_{02} = \frac{2}{3}$

$p_{12} = \frac{\lambda}{\beta_1+\lambda}$

$p_{13} = \frac{\beta_1}{\beta_1+\lambda}$

$p_{35} = p_{72} = 1$

$p_{20} = g^{*}(2\lambda)$

$p_{26}^{4} = 1 - 2g^{*}(\lambda) + g^{*}(2\lambda)$

$p_{21}^{4} = 2g^{*}(\lambda) - 2g^{*}(2\lambda)$

$p_{27}^{4,6} = 1 - 2g^{*}(\lambda) + g^{*}(2\lambda)$

$p_{52} = g^{*}(\lambda)$

$p_{56} = 1 - g^{*}(\lambda)$

$p_{57}^{6} = 1 - g^{*}(\lambda)$

using the transition probabilities evaluated above, we can conclude that:

$p_{01} + p_{02} = 1$

$p_{12} + p_{13} = 1$

$p_{35} = p_{72} = 1$

$P_{20} + p_{26}^{4} + p_{21}^{4} = 1$

$p_{20} + p_{27}^{4,6} + p_{21}^{4} = 1$

$p_{52} + p_{56} = 1$

$p_{52} + p_{57}^{6} = 1$

2.6 Mean-Sojourn Times

Before relocating to another state, the average length of time spent in one state is known as the mean sojourn time (MST). When speaking of Markov chains, it is the anticipated amount of time that a state will remain in before changing into a new one.

$\mu_0 = \int_0^\infty e^{-3\lambda\square} d\square = \frac{1}{3\lambda}$

$\mu_1 = \int_0^\infty e^{-(\lambda+\beta_1)t} dt = \frac{1}{\lambda+\beta_1}$
$\mu_2 = \int_0^\infty e^{-2\lambda t}\, \overline{G}(t) dt = \frac{1-g^*(2\lambda)}{2\lambda}$
$\mu_3 = \int_0^\infty e^{-\beta_1 t} dt = \frac{1}{\beta_1}$
$\mu_5 = \int_0^\infty e^{-\lambda t}\, \overline{G}(t) dt = \frac{1-g^*(\lambda)}{\lambda}$
$\mu_7 = \int_0^\infty t g(t) dt = -g^{*'}(0)$

We can conclude that

$m_{01} + m_{02} = \mu_0$
$m_{12} + m_{13} = \mu_1$
$m_{20} + m_{26}^4 + m_{21}^4 = k_1$ (say)
$m_{20} + m_{27}^{4,6} + m_{21}^4 = k_2$ (say)
$m_{35} = \mu_3$
$m_{52} + m_{56} = \mu_5$
$m_{52} + m_{57}^6 = k$ (say)
$m_{72} = \mu_7$

3 Measures of System Effectiveness

3.1 M.T.S.F. (Mean Time to System Failure)

Mean time to system failure (MTSF) is a measurement of the anticipated interval between a physical or electronic system's commencement of operation and failure. It shows how long a system or component should typically last before failing. A reliability metric called MTSF is used to evaluate a system or component's operational reliability. The higher the MTSF, the more reliable the system is considered to be.

$\phi_i(t)$ is the c.d.f. for the first passage of the interval(time) from i[th] phase(state) taking the fully failed state as absorbing states following recursive relation for $\phi_I(t)$ as obtained

$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t) + Q_{02}(t) \circledS \phi_2(t)$

$\phi_1(t) = Q_{13}(t) \circledS \phi_3(t) + Q_{12}(t) \circledS \phi_2(t)$

$\phi_2(t) = Q_{20}(t) \circledS \phi_0(t) + Q_{26}^4(t) + Q_{21}^4(t) \circledS \phi_1(t)$

$\phi_3(t) = Q_{35}(t) \circledS \phi_5(t)$

$\phi_5(t) = Q_{52}(t) \circledS \phi_2(t) + Q_{56}(t)$

$\phi_7(t) = Q_{72}(t) \circledS \phi_2(t)$

Using 'Laplace-Stieltjes' Transform on either side of the equation and further on solving we get:

$\phi_0^{**}(s) = \frac{N_0(s)}{D_0(s)}$

M.T.S.F.= $\frac{D_0'(0) - N_0'(0)}{D_0(0)} = \frac{N_1}{D_1}$

$N_1 = \mu_0 (p_{20}p_{56}p_{13}p_{21}^4 + p_{26}^4 p_{13}) + \mu_1 (p_{20}p_{01} + p_{01}p_{21}^4 p_{26}^4) + k_1 (p_{01}p_{12} + p_{01}p_{52}p_{13} + p_{02}) + \mu_3 (p_{13}p_{52}p_{21}^4 + p_{20}p_{01}p_{52}p_{13} + p_{01}p_{56}p_{13} + p_{01}p_{52}p_{13}p_{26}^4 + p_{21}^4 p_{02}p_{56}p_{13}) + \mu_5 (p_{01}p_{13} + p_{02}p_{13}p_{21}^4)$
$D_1 = 1 - p_{21}^4 p_{12} - p_{21}^4 p_{52}p_{13} - p_{20}p_{01}p_{12} - p_{20}p_{01}p_{52}p_{13} - p_{20}p_{02}$

3.2 Availability

The ability to get and utilize a good, service, or resource when required is referred to as availability. It is a measurement of the proportion of time that a system, gadget, or service is up and running, error-free. When a system or service has high availability, it means that people can rely on it to be available when they need it.

3.2.1 Availability at Full Capacity (AF_0)

Let $AF_i(t)$ denote the probability that the system is in upstate at instant 't', provided that the system entered regenerative state 'i' at t = 0. After applying the Laplace transform to the equations obtained, we obtain the following recursive relations.

$AF_0(t) = M_0(t) + q_{01}(t) © AF_1(t) + q_{02}(t) © AF_2(t)$
$AF_1(t) = q_{13}(t) © AF_3(t) + q_{12}(t) © AF_2(t)$
$AF_2(t) = q_{20}(t) © AF_0(t) + q_{21}^4(t) © AF_1(t) + q_{27}^{4,6}(t) © AF_7(t)$
$AF_3(t) = q_{35}(t) © AF_5(t)$
$AF_5(t) = q_{52}(t) © AF_2(t) + q_{57}^6(t) © AF_7(t)$
$AF_7(t) = q_{72}(t) © AF_2(t)$

Where $M_0(t) = e^{-3\lambda t}$

Taking Laplace Transforms on both sides:

$AF_0^*(s) = \frac{N_2(s)}{D_2(s)}$

$AF_0 = \lim_{s\to 0} sAF_0^*(s) = \frac{N_2(0)}{D_2'(0)} = \frac{N_2}{D_2}$

where

$D_2 = k_2 + \mu_0 p_{20} + \mu_1 (p_{20}p_{01} + p_{21}^4) + \mu_3 (p_{20}p_{13}p_{01} + p_{13}p_{21}^4) + k(p_{20}p_{13}p_{01} + p_{13}p_{21}^4) + \mu_7 (p_{20}p_{13}p_{01}p_{57}^6 + p_{13}p_{21}^4 p_{57}^6 + p_{27}^{4,6})$
$N_2 = \mu_0 p_{20}$

3.2.2 Availability at Reduced Capacity (AR_0)

With the help of the probabilistic justifications, we get recursive relations for $AR_i(t)$:

$AR_0(t) = q_{01}(t) © AR_1(t) + q_{02}(t) © AR_2(t)$
$AR_1(t) = M_1(t) + q_{13}(t) © AR_3(t) + q_{12}(t) © AR_2(t)$
$AR_2(t) = M_2(t) + q_{20}(t) © AR_0(t) + q_{21}^4(t) © AR_1(t) + q_{27}^{4,6}(t) © AR_7(t)$
$AR_3(t) = q_{35}(t) © AR_5(t)$

$AR_5(t) = M_5(t) + q_{52}(t) © AR_2(t) + q_{57}^{6}(t) © AR_7(t)$
$AR_7(t) = M_7(t) + q_{72}(t) © AR_2(t)$
where, $M_1(t) = e^{-(\lambda+\beta_1)t}$
$M_2(t) = e^{-2\lambda t}\,\overline{G}(t)$
$M_5(t) = e^{-\lambda t}\,\overline{G}(t)$
$M_7(t) = \overline{G}(t)$
Taking Laplace Transforms on both sides:
$AR_0^*(s) = \frac{N_3(s)}{D_2(s)}$
$AR_0 = \lim_{s\to 0} sAR_0^*(s) = \frac{N_3(0)}{D_2'(0)} = \frac{N_3}{D_2}$
where, $N_3 = \mu_1 p_{01} p_{27}^{4,6} - \mu_2 p_{01} p_{13} p_{57}^{6} + \mu_5 p_{01} p_{13} p_{27}^{4,6} - \mu_7 (p_{02} p_{27}^{4,6} + p_{01} p_{13} p_{57}^{6} + p_{01} p_{13} p_{52} p_{27}^{4,6} + p_{01} p_{12} p_{27}^{4,6} + p_{02} p_{13} p_{57}^{6} p_{21}^{4})$

3.3 Busy Period of Repairperson (B_0)
Let $B_i(t)$ be the probability that a repairperson is busy with the system in the interval (0, t), given by $B = \lim_{t\to 0} B(t)$
The following recursive relations are obtained after applying the Laplace transform for $B_i(t)$ are:
$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t)$
$B_1(t) = W_1(t) + q_{13}(t) © B_3(t) + q_{12}(t) © B_2(t)$
$B_2(t) = W_2(t) + q_{20}(t) © B_0(t) + q_{21}^{4}(t) © B_1(t) + q_{27}^{4,6}(t) © B_7(t)$
$B_3(t) = W_3(t) + q_{35}(t) © B_5(t)$
$B_5(t) = W_5(t) + q_{52}(t) © B_2(t) + q_{57}^{6}(t) © B_7(t)$
$B_7(t) = W_7(t) + q_{72}(t) © B_2(t)$
where, $W_1(t) = e^{-(\lambda+\beta_1)t}$
$W_2(t) = 2\lambda e^{-2\lambda t}\,\overline{G}(t) + [\, 2\lambda\, e^{-2\lambda t} © \lambda e^{-\lambda t} © 1 \,]\, \overline{G}(t)$
$W_3(t) = e^{-\beta_1 t}$
$W_5(t) = W_7(t) = \overline{G}(t)$
Taking Laplace Transforms on both sides
$B_0^*(s) = \frac{N_4(s)}{D_2(s)}$
$B_0 = \lim_{s\to 0} sB_0^*(s) = \frac{N_4(0)}{D_2'(0)} = \frac{N_4}{D_2}$
where,
$N_4 = (\mu_7 + \mu_3)(p_{01} p_{13} p_{27}^{46}) + \mu_7 (-p_{02} p_{27}^{46} - p_{01} p_{12} p_{27}^{46} - p_{13} p_{01} p_{52} p_{27}^{46} - p_{01} p_{12} p_{57}^{6} - p_{02} p_{13} p_{57}^{6} p_{21}^{4}) + \mu_1 (p_{27}^{46} p_{01}) + \mu_2 (p_{57}^{6} p_{01} p_{13})$

3.4 Down Time of the System (DT_0)
Let us assume that the system entered regenerative state I at t=0. Then, the probability that the system is in down mode at instant t is given by
$DT_0 = \lim_{s\to 0} sDT_0^*(t)$

The recursive relations for downtime after applying Laplace transform are as follows: $DT_0(t)$:
$DT_0(t) = q_{01}(t) © DT_1(t) + q_{02}(t) © DT_2(t)$
$DT_1(t) = q_{13}(t) © DT_3(t) + q_{12}(t) © DT_2(t)$
$DT_2(t) = q_{20}(t) © DT_0(t) + q_{21}^{4}(t) © DT_1(t) + q_{27}^{4,6}(t) © DT_7(t)$
$DT_3(t) = W_3(t) + q_{35}(t) © DT_5(t)$
$DT_5(t) = q_{52}(t) © DT_2(t) + q_{57}^{6}(t) © DT_7(t)$
$DT_7(t) = q_{72}(t) © DT_2(t)$
where, $W_3(t) = e^{-\beta_1 t}$
using Laplace Transforms on either side:
$DT_0^*(s) = \frac{N_5(s)}{D_2(s)}$
$DT_0 = \lim_{s\to 0} sDT_0^*(s) = \frac{N_5(0)}{D_2'(0)} = \frac{N_5}{D_2}$
where, $N_5 = \mu_3\, p_{01} p_{13} p_{27}^{4,6}$

3.5 Profit Analysis
A financial analysis procedure known as "profit analysis" assesses the profitability of a specific good, service, or enterprise. To ascertain if an endeavor is profitable, revenue and expense analysis is required. It aids industries in locating areas where they may minimize expenditures, boost earnings, and save costs.
The profit function is derived as:
$P = C_0\,(AF_0) + C_1\,(AR_0) - C_2\,(B_0) - C_3\,(DT_0) - C_4$
where, $C_1 < C_0$

Utilizing values inferred from the acquired data, the values of various system effectiveness metrics are calculated. i.e $\beta_1 = 0.5$, $\alpha = 0.281$, $\lambda = 0.0017$, $C_0 = 2600$, $C_1 = 1600$, $C_2 = 500$, $C_3 = 4200$, $C_4 = 50$
From the particular cases and estimated values, we obtain (Table 4):

Table 4. Estimated values of measures of system effectiveness

S.No.	Measures of system effectiveness	
1	MTSF (in hrs.)	9491.22830403
2	AF_0	0.87748846
3	AR_0	0.00000290
4	B_0	0.00003386
5	DT_0	0.00000022
6	Profit	2231.45680 Rs.

3.6 Graphical Interpretations and Numerical Outcomes
For the computations, a specific instance is taken into consideration, and the distribution of time is assumed to be exponential. Consider

$g(t) = \alpha e^{-\alpha t}$ $\quad g_1^{*\prime}(0) = -\frac{1}{\alpha}$

Using the above values, graphical interpretation is plotted to study the behavior of different system measures

Figure 2 depicts the behavior of MTSF with ƛ(rate of failure) for various repair-rate (α) values. According to the provided graph, the MTSF drops as the rate of failure increases, and it is greater when the repair rate increases.

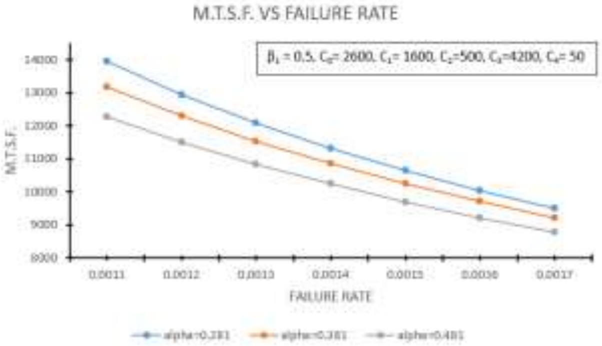

Fig. 2: MTSF vs Rate of failure(ƛ) for various values of Repair Rate(α).

Figure 3 illustrates the relationship between availability at maximum capacity and ƛ (rate of failure) for various values of α (repair rate). The availability at full capacity shows decrement as the rate of failure rises, while it shows increment for bigger numerical values of α (repair rate).

Fig. 3: Availability at Full Capacity vs ƛ (Rate of failure) for various values of α(Repair Rate).

Figure 4 represents the graph of availability at reduced capacity w.r.t. rate of failure (ƛ) from the given graph we can conclude that availability at reduced capacity shows increment as ƛ (rate of failure) is increased and is higher for greater values of α (repair rate).

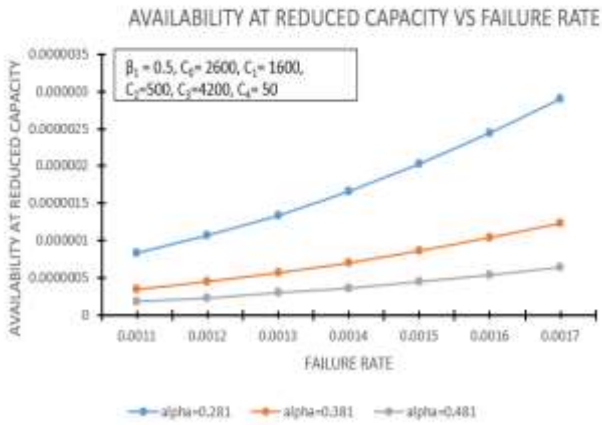

Fig. 4: Availability at Reduced Capacity vs ƛ (Rate of failure), using various values of α(rate of repair).

In Figure 5, the relationship between the generated revenue at maximum efficiency and the payoff (profit) is illustrated for different values of C_0. The graph depicts intercepts at three different repair rates: 0.281, 0.381, and 0.481, with corresponding coordinates (x, y) as follows: (54.7, 0.08488965), (53.05, 0.0848896), and (52.6, 0.0424448), respectively. Additionally, the system will not be profitable if the revenue falls below 54 Rs at a repair rate of 0.281, 53 Rs at a repair rate of 0.381, or 52 Rs at a repair rate of 0.481. Therefore, it can be concluded that the system's profitability is highly dependent on the generated revenue and the repair rate.

Table 5. Values of profit generated by the system for different values of repair rate

ƛ	Rate of repair=0.281	Rate of repair =0.381	Rate of repair=0.481
0.0011	2334.439	2384.764	2415.186
0.0012	2316.625	2370.787	2403.624
0.0013	2299.079	2356.972	2392.171
0.0014	2281.795	2343.316	2380.825
0.0015	2264.767	2329.815	2369.585
0.0016	2247.989	2316.467	2358.448
0.0017	2231.457	2303.269	2347.415

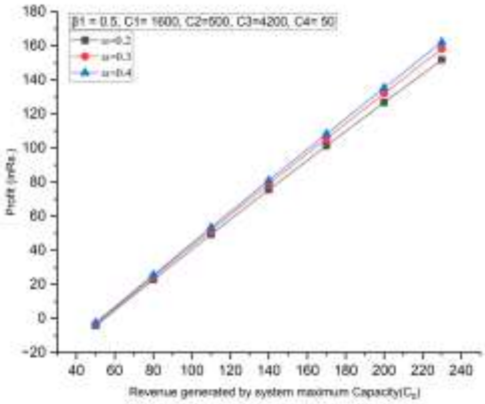

Fig. 5: Profit vs Revenue generated at maximum Capacity with various values of C_0.

Figure 6 illustrates the relationship between the profit generated and the failure rate (ƛ) for various values of the repair rate (α). This diagram illustrates the complex relationship between profit margins and failure rates (λ) over a range of repair rates (α). With the different values of repair rate, the graphs show that failure rate impacts the profit of the system with different kinds of maintenance practices. By studying the repair rates further the researchers will be able to see financial outcomes with respect to reliability do effective maintenance practices and find the most cost-effective way for the system to run at maximum efficiency.

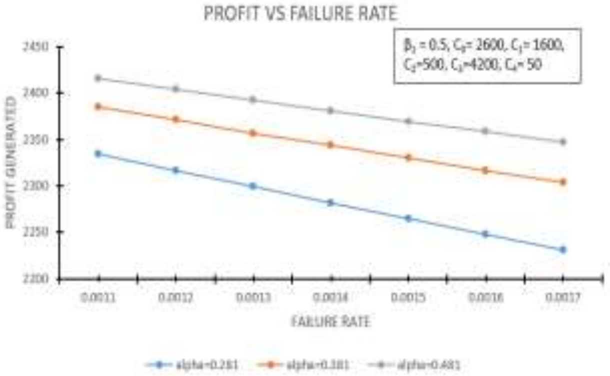

Fig. 6: Profit generated with respect to failure rate (ƛ) for different values of repair rate (α).

Figure 6 shows that with an increase in failure rate the profit generated by the system decreases and same can be seen in Table 5.

3.7 Applications of the Work

To find and analyze the difficulties in predicting the behavior of a system similar to this system under consideration, this study includes the factors of system effectiveness that can play a vital role in the type of maintenance strategy a firm can adopt by studying the different values calculated and plotted in graphs and tables.

This study further provides a platform for similar industries to assess their operating environment and apply the methods to predict the reliability indices.

The findings in this study about profit generation, system failure times, and the times when the system is down will help in reducing downtime in increasing profit. Armed with this knowledge, interested parties may implement tailored maintenance programs to optimize productivity and increase overall profitability. Moreover, the presented modeling technique has the potential for broader applications in relevant industrial situations. It offers a transferable framework for assessing dependability and developing maintenance plans, which may significantly impact manufacturing companies' operational performance and resilience.

4 Conclusion

In the present study, for prediction of the reliability of the system, a transition diagram is made using the information gathered and various system effectiveness metrics like MTSF, availability at full as well as reduced-capacity availability, busy-period in which repairperson is occupied, and downtime when the system is at complete shutdown are estimated.

For cost-benefit analysis, the various system effectiveness metrics are further used. With the help of numerical analysis, the various system effectiveness metrics were obtained followed by graphical interpretations. The following conclusions were made: MTSF decreased as the rate of failure increased. It was observed profit increases with the declining value of the rate of failure. Further, the relationship between profit and rate of failure was shown for different repair rates.

This work makes a significant scientific contribution by presenting a method for predicting system reliability that is specifically suited to plate production companies and similar industrial environments. By carefully combining several system effectiveness measurements, such as availability and MTSF, the research provides decision-makers with a thorough framework to optimize operational dynamics, which in turn helps with resource allocation and strategic planning. Furthermore, the developed modeling methodology exhibits potential for wider applicability in analogous industrial contexts, highlighting its importance in propelling concrete enhancements in operational performance and resilience in manufacturing organizations and furthering the field of reliability engineering.

4.1 Suggested Improvements of this Work

Several enhancements are proposed in order to resolve the stated constraints. First and foremost, a more thorough study that takes into account financial measures in addition to a wider range of variables including consumer effect and environmental sustainability should be conducted. The result can be further validated by assessment of sensitivity. Furthermore, adding new data sources and incorporating more sophisticated reliability modeling methods may enhance the analysis's accuracy and dependability. The application of this study in real-time will validate the findings further.

4.2 Future Directions

The technique described can be used in systems with similar configurations. Further research based on this study's findings might expand its conclusions by exploring additional facets of system reliability and maintenance optimization within the framework of plate manufacturing and related sectors. It may also be possible to determine the modeling methodology's broader applicability and enable operational improvements by examining how well it fits different industrial settings outside of the plate manufacturing industry. To increase the precision of maintenance strategies and enable more proactive approaches to system dependability management, predictive maintenance techniques, and real-time data analytics may be used.

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- Prawar carried out Investigation, Methodology, Software, Writing-Original draft, Software, Writing- review & editing
- Anjali Naithani carried out Investigation, Methodology, Software, Resources, Writing-review & editing, Supervision
- H. D. Arora carried out Writing-review & editing, Supervision, Validation.
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