

New Two Parameter Integral Transform “MAHA Transform” and Its Applications in Real Life

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Abstract: - This paper proposes a novel integral transform with two parameters, designed to obtain exact solutions for higher-order linear ordinary differential equations with constant coefficients. The proposed transform, referred to as the Maha transform, showcases significant utility in solving difficult differential equations with high precision. Additionally, it finds valuable applications in nuclear physics and medical fields, demonstrating its versatility and broad effectiveness. By introducing this innovative method, the paper aims to substantially enhance problem-solving techniques in both theoretical and applied sciences. This research provides a powerful new tool for addressing challenging differential equations across various scientific and engineering disciplines comprehensively and effectively.

Key-Words: - MAHA transform, Inverse MAHA transform, Integral transform, Ordinary differential equations, Partial differential equations, Coefficients of higher orders, Laplace transform, Integral transform for two parameters.

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1 Introduction

The authors have derived their methods from the classical Laplace transform, considering it indispensable. This is due to the numerical simplicity inherent in the MAHA transform and its key properties, [1], [2], [3].

The MAHA transform is tailored for addressing ordinary and partial differential equations in the time domain. While traditional mathematical tools like Fourier technique, Laplace, Aboodh, Elzaki, Mohanad, Al-Zughair, Kamal, Mahgoub, and SEE transforms are commonly employed for solving differential equations, the MAHA transform, along with some of its essential properties, has become a valuable approach in this regard, [4],[5], [6], [7], [8], [9].

A novel integral transform, known as the MAHA transform, has been defined, specifically

characterized for its outstanding capability in handling functions within the set A.

$A = \{f(t) : \text{there exist } M, l_1, l_2 > 0.$

$$|f(t)| < Me^{l_1|t|}, \text{ if } t \in (-1)^i x[0, \infty)\} \quad (1)$$

For a function within the set A, the positive constant M should be a bounded number, and l_1, l_2 may be boundless. The MAHA fundamental transform, represented via the operator $M(\cdot)$, is defined by the essential data:

$$F(u, v) = M[f(t)] = (uv)^\beta \int_0^\infty f\left(\frac{t}{u}\right) e^{-\frac{t}{u}} dt \\ = F(u, v), u, v > 0, \beta \in Z, t \geq 0 \quad (2)$$

The parameters u and v in this essential transform are employed to compute the variable t, the argument of the function f. This indispensable

transform exhibits additional connections with the Laplace, Aboodh, and Mohanad transforms.

The purpose of this study is to demonstrate the significance of this intriguing novel transform and its efficiency in solving linear differential equations.

Through studying and researching integral transformations and their applications in life, we found that there are many integral transformations with two parameters, the most important of which, according to the year of publication, are ZZ transformations. Group Ramadan, Shehu, ZMA, Formable, Khalouta, KKAT, Rishi, SEA, Abaoub-Shkheam Quideen, Honaiber, [10], [11].

When $\beta = 0$ the Maha transform becomes Shehu transform.

When $\beta = -1$ then Maha transform becomes KKAT transform and Quideen transform.

In addition, the form of our transformation is the Maha integral transformation, which is a more general transformation than the previous transformations with two parameters.

Through this β , which belongs to the set of integers, we can choose the appropriate number for the application of physics, engineering, or any life application to convert the ordinary differential equation into an easy and simple algebraic equation, then by taking the inverse of this transform to obtain the exact solution.

And that β is the one who gave this transformation the generality and the preference by using it in important applications.

And that this β can be referred to as, for example, the number of samples taken in the application. There are many mathematical models in the linear modeling that depend on this β , for example, drug concentration problems, chemical problems, and others. Through

our study and our knowledge of many integral transforms with one parameter or two parameters or more, there is no preference between one transform and another, but there is an important matter, which is that each transform has a use for some applications through after converting differential equations, integral equation or systems into algebraic equations

or algebraic system. Conversion is made easier by simplifying that application or example, [12], [13].

It is worth noting that each integral transformation is not better than the other. Each integral transformation solves a specific real-life problem. For example, the Fourier transform solves the problems of vector transmission, heat, signal, and others.

For the purpose of verifying the advantages of this integral transformation, it is possible to compare

the examples in the paper with the results of references in [14], [15], [16].

The applications presented are only an explanation of the importance and role of the presence of the two parameters in the transform, and this is what distinguished this transform from others.

2 Methodology

2.1 MAHA Integral Transform of Some Functions

Assuming the existence of the fundamental data (2) for any function $f(t)$, the sufficient data for the presence of the MAHA integral transform are that, for t greater than or equal to 0, $f(t)$ should be piecewise continuous and of exceptional order. Otherwise, the MAHA transform may or may not exist. In this segment In this segment we evaluate MAHA integral transform of fundamental capacities:

- 1) If $f(t) = k$, k is an arbitrary constant function, then via the definition we get:

$$M[k] = (uv)^\beta \int_0^\infty k e^{-\frac{t}{u}} dt = (uv)^\beta k \left[\frac{e^{-\frac{t}{u}}}{-\frac{1}{u}} \right]_0^\infty = ku^\beta v^{\beta+1} \quad (3)$$

- 2) If $f(t) = t$ then:

$$M[t] = (uv)^\beta \int_0^\infty \frac{t}{u} e^{-\frac{t}{u}} dt \quad (4)$$

Integration by parts method, we obtain:

$$M[t] = u^{\beta-1} v^{\beta+2} \quad (5)$$

So:

(i) $M[t^2] = 2u^{\beta-2} v^{\beta+3}$

(ii) $M[t^3] = 6u^{\beta-3} v^{\beta+4}$

(iii) In general, if n is a positive integer number, then $M[t^n] = u^{\beta-n} v^{\beta+n+1} n!$, and if $n > -1$ then $M[t^n] = u^{\beta-n} v^{\beta+n+1} \tau(n+1)$, where $\tau(\cdot)$ Gamma function.

- 3) If $f(t) = e^{-at}$, where a is an arbitrary constant number, so:

$$M[e^{-at}] = (uv)^\beta \int_0^\infty e^{-\frac{at}{u}} e^{-\frac{t}{u}} dt = (uv)^\beta \left[\frac{e^{-\left(\frac{a}{u} + \frac{1}{v}\right)t}}{-\left(\frac{a}{u} + \frac{1}{v}\right)} \right]_0^\infty = \frac{(uv)^{\beta+1}}{v+au}, \quad (6)$$

also

$$M[e^{at}] = \frac{(uv)^{\beta+1}}{u-av} \quad (7)$$

- 4) If $f(t) = \sin(at)$, where a is an arbitrary constant number, so:

$$M[\sin(at)] = (uv)^\beta \int_0^\infty \sin\left(a\frac{t}{u}\right)e^{-\frac{t}{v}} dt$$

$$= \frac{au^{\beta+1}v^{\beta+2}}{u^2+a^2v^2} \quad (8)$$

5) If $f(t) = \cos(at)$, where a is an arbitrary constant number, so:

$$M[\sin(at)] = (uv)^\beta \int_0^\infty \cos\left(a\frac{t}{u}\right)e^{-\frac{t}{v}} dt \quad (9)$$

After basic computations, we obtain:

$$M[\cos(at)] = \frac{u^{\beta+2}v^{\beta+1}}{u^2+a^2v^2} \quad (10)$$

6) If $f(t) = \sinh(at)$, where a is an arbitrary constant number, so:

$$M[\sinh(at)] = (uv)^\beta \int_0^\infty \sinh\left(a\frac{t}{u}\right)e^{-\frac{t}{v}} dt$$

$$= (uv)^\beta \int_0^\infty \frac{e^{\frac{at}{u}} - e^{-\frac{at}{u}}}{2} e^{-\frac{t}{v}} dt \quad (11)$$

After basic computations, we obtain

$$M[\sinh(at)] = (uv)^{\beta+1} \left(\frac{av}{u^2-a^2v^2}\right) \quad (12)$$

7) If $f(t) = \cosh(at)$, where a is an arbitrary constant number, then:

$$M[\cosh(at)] = \frac{u^{\beta+2}v^{\beta+1}}{u^2-a^2v^2} \quad (13)$$

8) Shifting property of MAHA integral transform

If MAHA integral transform of $f(t)$ is $F(u,v)$, so MAHA technique of $e^{at}f(t)$ is given by

$$\frac{uv^\beta}{[u(\frac{uv}{u-av})]^\beta} F\left(u, \frac{uv}{u-av}\right).$$

Proof

$$M[f(t)e^{at}] = (uv)^\beta \int_0^\infty f\left(\frac{t}{u}\right)e^{a\frac{t}{u}} e^{-\frac{t}{v}} dt$$

$$M[f(t)e^{at}] = (uv)^\beta \int_0^\infty f\left(\frac{t}{u}\right)e^{-t\left[\frac{1}{v}-\frac{a}{u}\right]} dt$$

$$M[f(t)e^{at}] = (uv)^\beta \int_0^\infty f\left(\frac{t}{u}\right)e^{-t\left[\frac{u-av}{uv}\right]} dt$$

$$M[f(t)e^{at}] = (uv)^\beta \frac{u(\frac{uv}{u-av})^\beta}{u(\frac{uv}{u-av})^\beta} \int_0^\infty f\left(\frac{t}{u}\right)e^{-t\left[\frac{u-av}{uv}\right]} dt$$

$$M[f(t)e^{at}] = \frac{(uv)^\beta}{u(\frac{uv}{u-av})^\beta} F\left(u, \frac{uv}{u-av}\right)$$

Theorem (2.1)

(i) $M[f'(t)] = (uv)^\beta [-uf(0)] + \frac{u}{v} F(u,v)$

(ii) $M[f''(t)] = (uv)^\beta [-uf'(0) - \frac{u^2}{v} f(0)] + \frac{u^2}{v^2} F(u,v)$

(iii) $M[f'''(t)] = (uv)^\beta [-uf''(0) - \frac{u^2}{v} f'(0) - \frac{u^3}{v^2} f(0)] + \frac{u^3}{v^3} F(u,v)$

(iv) $M[f^{(4)}(t)] = (uv)^\beta [-uf'''(0) - \frac{u^2}{v} f''(0) - \frac{u^3}{v^2} f'(0) - \frac{u^4}{v^3} f(0)] + \frac{u^4}{v^4} F(u,v)$

(iv) $M[f^{(4)}(t)] = (uv)^\beta [-uf'''(0) - \frac{u^2}{v} f''(0) - \frac{u^3}{v^2} f'(0) - \frac{u^4}{v^3} f(0)] + \frac{u^4}{v^4} F(u,v)$

(iv) $M[f^{(m)}(t)] = (uv)^\beta [-uf^{(m-1)}(0) - \frac{u^2}{v} f^{(m-2)}(0) - \frac{u^3}{v^2} f^{(m-3)}(0) - \dots - \frac{u^m}{v^{m-1}} f(0)] + \frac{u^m}{v^m} F(u,v)$

(iv) $M[f^{(m)}(t)] = (uv)^\beta [-uf^{(m-1)}(0) - \frac{u^2}{v} f^{(m-2)}(0) - \frac{u^3}{v^2} f^{(m-3)}(0) - \dots - \frac{u^m}{v^{m-1}} f(0)] + \frac{u^m}{v^m} F(u,v)$

(iv) $M[f^{(m)}(t)] = (uv)^\beta [-uf^{(m-1)}(0) - \frac{u^2}{v} f^{(m-2)}(0) - \frac{u^3}{v^2} f^{(m-3)}(0) - \dots - \frac{u^m}{v^{m-1}} f(0)] + \frac{u^m}{v^m} F(u,v)$

Proof

(i) By definition we obtain:

$$M[f'(t)] = (uv)^\beta \int_0^\infty f'\left(\frac{t}{u}\right)e^{-\frac{t}{v}} dt$$

Integration by parts method, we have:

$$M[f'(t)] = (uv)^\beta [-uf(0)] + \frac{u}{v} F(u,v)$$

(ii) Also, by the definition, we obtain:

$$M[f''(t)] = (uv)^\beta \int_0^\infty f''\left(\frac{t}{u}\right)e^{-\frac{t}{v}} dt$$

Integration by parts method, we have:

$$M[f''(t)] = (uv)^\beta \left[-uf'(0) - \frac{u^2}{v} f(0)\right] + \frac{u^2}{v^2} F(u,v)$$

The proof of (iii) and (iv) is similar to (ii).

(iv) We can confirm by Mathematical Induction.

2.2 The Inverse of MAHA Integral Technique

In this part, we present the inverse of MAHA transform technique of basic functions:

(1) $M^{-1}[ku^\beta v^{\beta+1}] = k$

(2) $M^{-1}[u^{\beta-n} v^{\beta+n+1}] = \frac{t^n}{n!}$, where $n > 0$ integer number.

(3) $M^{-1}\left[\frac{(uv)^{\beta+1}}{v+au}\right] = e^{-at}$, where a is a constant number.

$$\begin{aligned} (4) M^{-1} \left[\frac{(uv)^{\beta+1}}{u-av} \right] &= e^{at} \\ (5) M^{-1} \left[\frac{au^{\beta+1}v^{\beta+2}}{u^2+a^2v^2} \right] &= \sin(at) \\ (6) M^{-1} \left[\frac{u^{\beta+2}v^{\beta+1}}{u^2+a^2v^2} \right] &= \cos(at) \\ (7) M^{-1} \left[\frac{au^{\beta+1}v^{\beta+2}}{u^2-a^2v^2} \right] &= \sinh(at) \\ (8) M^{-1} \left[\frac{u^{\beta+2}v^{\beta+1}}{u^2-a^2v^2} \right] &= \cosh(at) \end{aligned}$$

2.3 Application of MAHA Transform of Ordinary Differential

As mentioned in the introduction of this study, the MAHA essential transform proves to be a valuable tool for exploring the fundamental characteristics of a linear system governed via a differential equation under specified initial data. The following examples illustrate the application of the MAHA transform in solving specific initial value problems described by ordinary differential equations.

Consider the linear first order linear Ordinary differential problem:

Example 1:

Consider the differential problem:

$$\frac{dy}{dx} + y = 0, y(0) = 1 \quad (14)$$

Solution: take MAHA transform to this data, we obtain:

$$\begin{aligned} M \left[\frac{dy}{dx} \right] + M[y] &= 0 \\ (uv)^\beta (-uy(0)) + \frac{u}{v} F(u, v) + F(u, v) &= 0 \\ F(u, v) \left[\frac{u}{v} + 1 \right] &= u(uv)^\beta \end{aligned}$$

So

$$F(u, v) = \frac{(uv)^{\beta+1}}{u+v}$$

Required exact solution is:

$$y(x) = e^{-x}$$

Example 2:

Find the exact solution of differential equation:

$$y'' + y = 0, y(0) = y'(0) = 1 \quad (15)$$

Solution: take MAHA technique to above differential data:

$$\begin{aligned} (uv)^\beta (-u) - \frac{u^2}{v} (uv)^\beta + \frac{u^2}{v^2} F(u, v) + F(u, v) &= 0 \\ F(u, v) \left[1 + \frac{u^2}{v^2} \right] &= u(uv)^\beta + \frac{u^2}{v} (uv)^\beta \end{aligned}$$

So

$$F(u, v) = \frac{u^{\beta+1}v^{\beta+2}}{u^2+v^2} + \frac{u^{\beta+2}v^{\beta+1}}{u^2+v^2}$$

The inverse MAHA transform of this equation is:

$$y(x) = \sin x + \cos x \quad (16)$$

Example 3:

Consider the second request linear differential equation:

$$y'' - 3y' + 2y = 0, y(0) = 1, y'(0) = 4 \quad (17)$$

Solution: Take MAHA technique to above differential equation, we have:

$$\begin{aligned} (uv)^\beta (-u) - \frac{u^2}{v} (uv)^\beta + \frac{u^2}{v^2} F(u, v) & \\ - 3[(uv)^\beta (-u) + \frac{u}{v} F(u, v)] & \\ + 2F(u, v) &= 0 \end{aligned}$$

So

$$F(u, v) = \frac{(uv)^\beta (u + \frac{u^2}{v})}{(\frac{u}{v} - 2)(\frac{u}{v} - 1)}$$

Now

$$\frac{u + \frac{u^2}{v}}{(\frac{u}{v} - 2)(\frac{u}{v} - 1)} = \frac{A}{(\frac{u}{v} - 2)} = \frac{B}{(\frac{u}{v} - 1)}$$

After basic computations, we obtain: A=3, B=-2.

$$F(u, v) = \frac{3(uv)^{\beta+1}}{u-2v} - \frac{2(uv)^{\beta+1}}{u-v}$$

The general exact solution is:

$$y(x) = 3e^{2x} - 2e^x$$

Example 4:

Consider the second-order linear nonhomogeneous request differential equation:

$$y'' + 9y = \cos 2x, y(0) = 1, y\left(\frac{\pi}{5}\right) = -1 \quad (18)$$

Solution: since $y'(0)$ is unknown, $y'(0) = a$.

Take MAHA transform of this data and utilizing beginning data, we obtain:

$$\begin{aligned} (uv)^\beta \left[-ua - \frac{u^2}{v} \right] + \frac{u^2}{v^2} F(u, v) + 9F(u, v) & \\ = \frac{u^{\beta+2}v^{\beta+1}}{u^2 + 9v^2} \end{aligned}$$

$$F(u, v) \left[9 + \frac{u^2}{v^2} \right] = \frac{u^{\beta+2} v^{\beta+1}}{u^2 + 9v^2} + (au + \frac{u^2}{v})(uv)^\beta$$

So

$$F(u, v) = \frac{(uv)^\beta u^2 v^{-1} + (uv)^\beta u^2 v^{-1} (4 + \frac{u^2}{v^2})}{(4 + \frac{u^2}{v^2})(9 + \frac{u^2}{v^2})} + \frac{3(uv)^\beta au}{3(9 + \frac{u^2}{v^2})}$$

After simple computations, we get:

$$A = 0, B = \frac{4}{5}, C = 0, D = \frac{1}{5}$$

So

$$y(x) = \frac{a}{3} \sin(3x) + \frac{1}{5} \cos(2x) + \frac{4}{5} \cos(3x)$$

To evaluate a note that

$$y\left(\frac{\pi}{2}\right) = -1$$

Then we find

$$\frac{12}{5}$$

Then

$$y(x) = \frac{4}{5} \sin(3x) + \frac{1}{5} \cos(2x) + \frac{4}{5} \cos(3x)$$

Example 5:

Tackle the differential equation

$$y'' - 3y' + 2y = 4e^{3x}, y(0) = -3, y'(0) = 5 \quad (19)$$

Solution: Take MAHA technique of this differential problem and applying the initial data:

$$(uv)^\beta \left[-5u + \frac{3u^2}{v^2} \right] + \frac{u^2}{v^2} F(u, v) - 3[(uv)^\beta + 3u] - 3\frac{u}{v} F(u, v) + 2F(u, v) = \frac{4(4v)^{\beta+1}}{4 - 3v}$$

$$F(u, v) \left[\frac{u^2}{v^2} - \frac{3u}{v} + 2 \right] = (uv)^\beta \left(-5u + \frac{3u^2}{v^2} \right) + 9(uv)^\beta u + \frac{4(4v)^{\beta+1}}{4 - 3v}$$

$$F(u, v) = \frac{(uv)^\beta [5u^2 v^{-1} - 38u]}{\left(\frac{u}{v} - 3\right)\left(\frac{u}{v} - 2\right)\left(\frac{u}{v} - 1\right)}$$

So

$$F(u, v) = (uv)^\beta u \left[\frac{A}{\left(\frac{u}{v} - 3\right)} + \frac{B}{\left(\frac{u}{v} - 2\right)} + \frac{C}{\left(\frac{u}{v} - 1\right)} \right]$$

After basic computations, we get: $A = 2, B = 4, C = -9$

$$F(u, v) = \frac{2(uv)^{\beta+1}}{u - 3v} + \frac{4(uv)^{\beta+1}}{u - 2v} - \frac{9(uv)^{\beta+1}}{u - v}$$

Take inverse transform, we get:

$$y(x) = 2e^{3x} + 4e^{2x} - 9e^x$$

3 Applications

3.1 Maha Integral Technique in "Nuclear Physics"

The following application is based on nuclear physics fundamentals.

Consider the first order linear ordinary differential problem:

$$\frac{dN(t)}{dt} = -\alpha N(t) \quad (20)$$

The essential relationship describing radioactive decay is given in above equation, where $N(t)$ during time t represents the number of undecayed atoms left in a sample of radioactive isotope, and α is the decay constant number. We can apply the Maha integral transform M :

$$M\{N'(t)\} + \alpha M\{N(t)\} = 0$$

Therefore

$$(uv)^\beta [-uN(0)] + \frac{u}{v} F(u, v) + \alpha F(u, v) = 0$$

$$\left(\frac{u}{v} + \alpha\right) F(u, v) = (uv)^\beta uN(0)$$

$$F(u, v) \left(\frac{u}{v} + \alpha\right) = u^{\beta+1} v^\beta N_0$$

$$F(u, v) = \frac{u^{\beta+1} v^\beta N_0}{\frac{u}{v} + \alpha}$$

So

$$F(u, v) = N_0 \frac{(uv)^{\beta+1}}{u + \alpha v}$$

The required exact solution is:

$$N(t) = N_0 e^{-\alpha t}$$

This is the proper type of radioactive decay.

3.2 Blood Glucose Concentration

During continuous intravenous glucose injection, the concentration of glucose in the blood is $G(t)$ exceeding the baseline value at the start of the infusion. The function $G(t)$ satisfies the initial value equation(I, V, P).

$$G'(t) + kG(t) = \frac{\alpha}{\gamma} \quad (21)$$

and $t \in (0, \infty)$, $G(0)=0$.

The variables in this equation are k , α and γ , which respectively represent the constant velocity of elimination, the rate of infusion, and the volume in which glucose is distributed. The Maha technique will be utilized to assess the concentration of glucose present in the blood stream.

Upon bilateral application of the Maha integral to equation (.), we get:

$$M\{G'(t)\} + kM\{G(t)\} = \frac{\alpha}{\gamma} M\{1\} \quad (22)$$

Let $M\{G(t)\} = F(u, v)$. By (I, V, P) and the integral transform outlined in section 2.1. The (22) can be rearranged with the aid of (21) as:

$$\begin{aligned} (uv)^\beta [-uG(0)] + \frac{u}{v} F(u, v) + kF(u, v) \\ - \frac{\alpha}{\gamma} u^\beta v^{\beta+1} = 0 \\ \left(\frac{u}{v} + k\right) F(u, v) = \frac{\alpha}{\gamma} u^\beta v^{\beta+1} \end{aligned}$$

So

$$F(u, v) = \frac{\alpha u^\beta v^{\beta+2}}{\gamma u + kv}$$

After simple computation and using inverse of Maha transform to this expression, we get the concentration of glucose in the blood as:

$$G(t) = \frac{\alpha}{\gamma k} (1 - e^{-kt})$$

3.3 Aorta Pressure

The heart's contraction facilitates the transportation of blood into the aorta. The initial value problem is concerned with the aortic pressure function $P(t)$ as:

$$P'(t) + \frac{c}{k} P(t) = cAsint(wt) \quad (23)$$

where $p(0) = p_0$

c , k , A and P_0 are arbitrary constants. The Maha technique is utilized to derive the pressure in the aorta. With the bilateral application of the Maha transform (23), we get:

$$M\{P'(t)\} + \frac{c}{k} M\{P(t)\} = cAM\{sint(wt)\} \quad (24)$$

By applying the initial value problem and utilizing the transform outlined in section 2.1, the rearrangement of (24) can be expressed as:

$$\begin{aligned} (uv)^\beta [-uP(0)] + \frac{u}{v} F(u, v) + \frac{c}{k} F(u, v) \\ = cA \left[\frac{wu^{\beta+1}v^{\beta+2}}{u^2 + w^2v^2} \right] \\ \left(\frac{u}{v} + \frac{c}{k}\right) F(u, v) \\ = cAw \left[\frac{u^{\beta+1}v^{\beta+2}}{u^2 + w^2v^2} \right] \\ + P_0 \frac{u^{\beta+1}v^\beta}{\frac{u}{v} + \frac{c}{k}} \\ F(u, v) = \frac{cAwu^{\beta+1}v^{\beta+2}}{\left(\frac{u}{v} + \frac{c}{k}\right)(u^2 + w^2v^2)} + P_0 \frac{u^{\beta+1}v^\beta}{\frac{u}{v} + \frac{c}{k}} \end{aligned}$$

After performing basic calculations and applying partition fractions along with the inverse of the Maha transform to the given expression, the resultant value obtained represents the amount of pressure in the aorta:

$$\begin{aligned} P(t) = P_0 e^{-\frac{c}{k}t} + \frac{cAwk^2}{w^2k^2 + c^2} \left(\frac{c}{wk}\right) \sin(wt) \\ - \cos(wt) + e^{-\frac{c}{k}t} \end{aligned}$$

4 Conclusions

A two-parameter integral transform was introduced through this transform. We found that the conversion of linear ordinary differential equations with constant coefficients and higher orders turns into simple algebraic equations that are simpler and easier than the previous two-parameter as mentioned. In this paper, important medical applications were presented, as well as the

application of nuclear physics. The Maha integral transform can be used in future research to solve nonlinear differential problems.

The presence of the two parameters in this integral transformation facilitates and simplifies difficult mathematical calculations, and thus we obtain an accurate solution to ordinary differential equations without difficulty. This integral transformation will also be of highly efficient use in the future in the most important applications in solving differential and integral equations of all types and classes.

References:

- [1] A. Belafhal, R. EL Aitouni, T. Usman, Unification of Integral Transform and Their Applications, *Partial Differential Equations in Applied Mathematics*, vol. 12, 100695, 2024. DOI: 10.1016/j.padiff.2024.100695.
- [2] C. Constanda, Solution techniques for elementary partial differential equations. *Chap- man and Hall/CRC*, 1st Edition, New York, pp, 272, 2002. <https://doi.org/10.1201/9781420057515>.
- [3] D. G. Duffy, Transform methods for solving partial differential equations. CRC press, 2nd Edition, New York, pp 728, 2004. <https://doi.org/10.1201/9781420035148>.
- [4] T. M. Elzaki, The new integral transform 'Elzaki transform', *Glob. J. Pure Appl. Math.*, vol. 7, no. 1, pp. 57-64, 2011.
- [5] G. Gharib, M. Alsoudi and I. Benkemache, Solving Non Linear Fractional Newell Whitehead Equation By Atomic Solution, *WSEAS Transactions on Mathematics*, 22 pp. 114-119, 2023. <https://doi.org/10.37394/23206.2023.22.14>.
- [6] J. A. Jasim, E.A. Kuffi and S. A. Mehdi, A Review on the Integral Transforms, *Journal of University of Anbar for Pure Science*, vol. 17, no. 2, pp. 273-310, 2023. DOI: 10.37652/juaps.2023.141302.1090.
- [7] A. Kamal and H. Sedeeg, The New Integral Transform Kamal Transform, *Adv. Theor. Appl. Math.*, vol. 11, no. 4, pp. 451-458, 2016.
- [8] S. Khalid, KS. Aboodh, The New Integral Transform Aboodh Transform, *Glob. J. Pure Appl. Math.*, vol. 9, no. 1, pp. 35-43, 2013.
- [9] M. A. M. Mahgoub, The new integral transform 'Mahgoub Transform', *Adv. Theor. Appl. Math.*, vol. 11, no. 4, pp. 391-398, 2016.
- [10] M. A. M. Mahgoub and A. A. Alshikh, An application of new transform Mahgoub Transform to partial differential equations, *Math. theory Model.*, vol. 7, no. 1, pp. 7-9, 2017.
- [11] M. Mohand, A. Mahgoub, The New Integral Transform Sawi Transform. *Adv. Theory Appl. Math.*, vol. 14, no. 1, pp. 81-87, 2019.
- [12] F. Mubarak, MZ. Iqbal, A. Moazzam, U. Amjed, MU. Naeem, Substitution Method Using The Laplace Transformation for Solvin Partial Differential Equation Involving More Than Two Independent Variables, *Bulletin of Mathematics and Statistics Research*, vol. 9, no. 3, pp. 104-116, 2021.
- [13] RZ. Saadeh, BF Ghazal, A New Approach on Transforms: Formable Integral Transform and its Applications, *Axioms*, vol. 10, no. 4, pp. 332, 2021.
- [14] R. Saadeh, A. Qazza, K. Amawi, A New Approach Using Integral Transform to Solve Cancer Models, *Fractal and Fractional*, vol. 6, no. 9, pp. 490, 2022. 10.3390/fractalfract6090490.
- [15] R.Saadeh, A. Qazza, A. Burqan, A New Integral Transform: ARA Transform and its Properties and Applications, *Symmetry*, vol. 10, no. 6, pp. 925, 2020. 10.3390/sym12060925.
- [16] D. Thakur, Utilizing the Upadhyaya Transform to Solve the Linear Second Kind Volterra Intergral Equation, *The Review of Contemporay Scientific and Academic Studies*, vol. 3, no. 4, pp. 6, 2023. 10.55454/rcsas.3.04.2023.007.

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