Sensitivity of the Modified Exponentially Weighted Moving Average Sign-Rank Control Chart to Process Changes in Counted Data

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Abstract: - Control charts are generally used to detect mean process changes in continuous processes. However, counting procedures that characterize product quality are popular in production processes, such as the number of defects or the proportion of defective products. This research aims to investigate using the Modified Exponentially Weighted Moving Average combined with a nonparametric Sign Rank control chart, namely MEWMA-SR chart is a novel tool. Comparative analyses involving different run length measures are conducted to evaluate the proposed scheme against MEWMA, MEWMA-SR, and classical EWMA charts. The performance of the MEWMA-SR chart is assessed using Monte Carlo simulations based on its run-length profiles. It was found that the proposed combination control chart was effective in detecting changes better than other control charts along with presenting applications with real data. A study further validates the proposed chart's practical utility through a case study analyzing COVID-19 mortality data.

Key-Words: - average run length, control chart, discrete distributions, nonparametric statistics, Monte Carlo simulation, number of defective products, nonconformities.

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1 Introduction

Good quality products that meet standards are the main factors that make the industry successful. Therefore, to make the products that meet the needs of consumers, it is necessary to use process control to control production and detect abnormalities or the amount of waste in the production process which in the industrial production process. Without loss of generality, deviations or variations in the production process occur at any time. Individual products produced from the same process may vary. This is what we call variation, such as different weights, different thicknesses, number of defective products, number of nonconformities, etc. There are two types of variations; natural variation which is a normal variation that is hidden in the production process. Another type of variation is variation due to assignable causes, such as human, machine, material, and method causes. Statistical process control (SPC) is crucial and frequently employed across sectors, scientific natural sciences, environmental sciences, medication, and services for process observation. The indispensable tool in SPC is the control chart, which is widely utilized during the process tracking to determine any deviations from the procedure parameters. While variability is an inherent aspect of every resultant process and can compromise quality, typical reasons for variance have no bearing on process conformance, whereas special causes significantly impact process outputs, [1]. Control charts serve three main purposes: to set standards in the production process; to achieve the goal and for use in improving the production process. Initially introduced by [2], control charts have proven highly effective in identifying significant shifts in process outputs. However, these charts lack the sensitivity to detect subtle and sustained deviations in the process location. In contemporary times, sophisticated process monitoring systems encompass exponentially weighted moving averages (EWMA), [3] and cumulative sum (CUSUM) control charts, [4]. Those two control charts, known as memory-type charts, incorporate current and past sample information into their charting structures. While their performance detecting small to moderate shifts is nearly identical, quality practitioners often favor the EWMA chart's simplicity. The increasing adoption of these flowcharts is attributed to their heightened sensitivity to persistent modifications to the process variables. Consequently, they are regularly employed to identify subtle alterations in the location and scale parameter, where even minor changes can lead to significant quality issues. Later, the modified exponentially weighted moving average (MEWMA) control chart, [5], an enhanced version of the EWMA chart, was created to exhibit superior efficacy in detecting subtle changes compared to standard EWMA charts.

Parametric control charts typically function on the presumption that the data come from a normal distribution. However, if the finding stems from an irregular distribution, utilizing the equivalents of such control diagrams for tracking change in the process becomes unsuitable. Consequently, developing an adequate substitute is a nonparametric control (NP) chart. Nonparametric control charts offer several advantages, including ease of use, the absence of a necessity to let the underlying process have a particular parametric distribution, increased durability and resilience to outliers, and removing the necessity of estimating variance while creating location parameter charts.

Studies featuring parametric control charts encompass: In 1991, this research significantly advanced the statistical process control domain by demonstrating the effectiveness of the nonparametric EWMA method for monitoring processes with non-normal or heavy-tailed distributions, [6]. Compared to conventional EWMA schemes, the nonparametric approach performed better in detecting shifts and identifying out-of-control situations, particularly in scenarios with heavy-tailed distributions where traditional methods tend to be less sensitive. The NP structure proposed by [7], recommends the widely recognized EWMA chart to keep track of changes in the process goal or median, utilizing a straightforward sign test statistic. Given the EWMA chart's sensitivity to subtle and enduring shifts, numerous changes have been proposed and examined within the nonparametric exponentially weighted moving average (EWMA) charting structure. In 2011, [8], introduced an EWMA chart based on an SR test (EWMA-SR) designed to monitor small, persistent shifts in the process target or process mean. In 2014, the MEWMA-sign control chart demonstrated superior performance in detecting process shifts, exceeding the benchmarks established by both the EWMA-sign and standard EWMA charts. However, its efficacy in identifying more minor changes and for right-skewed distributions was limited (see detailed in [9]). [10], examined the effectiveness of the EWMA sign chart as a non-parameterized chart for individual measurement. Further research, [11], enhanced the arcsine EWMA for focused on parameter-free determining the average run length (ARL) for detection a change in process mean, especially small change. The EWMA sign and standard EWMA charts can be effectively employed for process monitoring, regardless of whether the quality feature has a normal distribution. However, sign statistics control schemes require transforming process observations into a binomial distribution for optimal performance. Subsequently, in a study by [12], the EWMA-sign chart was recommended as a valuable instrument for finding little and persistent shifts in location parameters. The conclusions indicate that the suggested diagram features a well-designed structure, offering heightened sensitivity for efficient process monitoring. The modified exponentially weighted moving average - -sign control chart (MEWMA-sign) was developed by [13], a novel control chart employing the sign statistic for enhanced change detection. Measured using average run length (ARL) as a performance measure, the MEWMA-Sign chart exhibited superior detection capabilities regarding the EWMA-sign and standard EWMA charts. Nevertheless, its efficacy was diminished in auto corrected data, as documented by [14]. Addressing the limitations of existing nonparametric control charts was developed, employing the Wilcoxon signed-rank statistic for enhanced sensitivity and robustness. Performance evaluations, usually measured using average run length (ARL) as the standard, demonstrate the nonparametric sign rank's superior efficiency compared to the EWMA sign and EWMA-SR control charts, particularly in non-normal data distributions, [15]. Moreover, examining the Extend Exponentially Weighted Moving Average - Sign Rank (EEWMA-SR) control chart to monitor the process mean and continuous distribution revealed superior performance. Recently, a novel MEWMA Wilcoxon sign-rank chart has been devised to identify alterations within the average parameter of a continuous distribution. The evaluation and numerical findings confirm that the MEWMA-SR chart excels, mainly when the magnitude of change and the size of the rational subgroup are small (for further details, refer to [16]).

However, most production processes have a normal distribution and are independent, such as the average amount of mineral water packed, the width of the diameter of the piston ring, etc. But in practice, the quality characteristics of interest may have a non-normal distribution or a discrete distribution, such as proportion. the proportion of defective products or the number of defects, which corresponds to a binomial distribution. Production quality is sometimes measured by the number of nonconformities, which corresponds to a binomial distribution and the Poisson distribution is suitable for counting processes. Consequently, in this research, a new control chart named MEWMA-Sign Rank chart is proposed to be used to detect enumerated data or observed values that have a discrete distribution such as the binomial (10, 0.1) and Poisson(4) distributions. Combining the benefits of both charts, it is a hybrid of the Sign-Rank statistic and the MEWMA chart. The proposed nonparametric control chart can solve problems with processes where the distribution of observed values is unknown or parameters cannot be estimated. Conventional control charts currently in use cannot overcome these limitations. Compared to other control charts and EWMA-Sign control charts, the suggested control chart performs better at identifying small to medium-sized changes. Its effectiveness is rigorously assessed and compared to established control charts through Monte Carlo simulations, employing the out-of-control average run length (ARL_1) metric as the evaluation benchmarking. Additionally, practical applications of the MEWMA-SR chart are demonstrated using real-world data examples.

2 Method

This segment introduces pertinent theory, organized into three parts. Section 2.1 delves into the nonparametric properties of the control chart. Section 2.2 elucidates the conventional control chart's conceptual design as an exponentially weighted moving average (EWMA) control chart and its modification into an exponentially weighted moving average (MEWMA) control chart. Additionally, it discusses the existing nonparametric control chart, transformed into an exponentially weighted moving average sign rank (EWMA-SR) control chart, and proposes the modified exponentially weighted moving average sign rank (MEWMA-SR) control chart. Finally, Section 2.3 outlines the method for evaluating accomplishment.

2.1 Sign Rank

Consider $A_t = \{A_{t1}, A_{t2}, ..., A_{tk}\}$ a size-n sample from a procedure characterized by a continuous distribution with process means(α). The distribution between the observation and the desired amount, denoted as $A_{tk} - \alpha$ within groups, can be expressed as Equation (1),

$$\beta_{tk} = A_{tk} - \alpha, t = 1,2,3,...; k = 1,2,3,... \qquad (1)$$

The sign statistic S_t can be formulated as:

$$S_t = \sum_{k=1}^{n} I_{tk} \text{ where } I_{tk} = \begin{cases} 1, & \beta_{tk} > 0 \\ 0, & otherwise \end{cases}$$

The sign statistic is the aggregate count based on observations that adhere to a binomial distribution with the parameter $(n, p = 0.5)$ in the control scenario. The $p = P(\beta > 0)$ is the fraction of the procedure, which is $p = P(\beta \le 0) = P(\beta > 0) = 0.5$ a control process. Conversely, in situations where the process is unmanageable, $q \ne 0.5$.

Determine J_{tk} indicate the absolute difference in rank $|A_{tk} - \alpha|$ inside the t^{th} subset. The definition of the sign rank statistic is as follows:

$$SR_t = \sum_{k=1}^{n} I_{tk} J_{tk} \text{ where } I_{tk} = \begin{cases} 1, & ; \ (A_{tk} - \alpha) > 0 \\ 0 & ; \ (A_{tk} - \alpha) = 0 \\ -1 & ; \ (A_{tk} - \alpha) < 0 \end{cases}$$

2.2 The Features of Control Charts

The examined control chart can be represented as follows:

2.2.1 Exponentially Weighted Moving Average (EWMA) Control Chart

This control chart, introduced in 1959 by [3], is a time-weighted method incorporating historical data. It demonstrates exceptional sensitivity to detecting variation in the procedure, especially when dealing with minor alterations. The EWMA statistic, represented by Equation (2), can be described as:

$$EWMA_t = \theta \bar{Y}_t + (1-\theta) EWMA_{t-1}, \ t = 1,2,... \qquad (2)$$

Here, θ represents the weighting parameter assigned to historical data, ranging between 0 and 1, while $\bar{Y}_t$ it signifies the process mean at time *t*. The starting value Z_0 is typically set to match the μ_0 parameter, with $\bar{Y}_t$ separate and regularly spaced observations. Subsequently, both the mean and variance Z_t can be described as follows:

$E(EWMA) = \mu_0$

and $V(EWMA_t) = \sigma^2 \left(\frac{\theta}{2-\theta}\left(1-(1-\theta)^{2t}\right) \right), t = 1,2,...$ (3)

where μ_0 is the process's mean, and σ^2 is the process's variance. Referring to Equation (3), as t approaches infinity, the variance asymptotically is

$$V(EWMA) = \sigma^2 \left(\frac{\theta}{2-\theta} \right). \quad (4)$$

Consequently, Equation(4) is followed by the control limit of the EWMA chart.

$$UCL_{EWMA} / LCL_{EWMA} = \mu_0 \pm L_1 \sqrt{\sigma^2 \frac{\theta}{2-\theta}}. \quad (5)$$

The coefficient of the control limit for the EWMA chart is represented by L_1, which corresponds to the desired ARL_0. This value can be found to reach the desired ARL_0 using the Monte Carlo simulation method.

2.2.2 Modified Exponentially Weighted Moving Average (MEWMA) Control Chart

The commencement of the MEWMA control chart aimed to improve detection performance by incorporating the additional term $k(S_i - S_{t-1})$ into the EWMA statistic. This modification resulted in MEWMA statistics surpassing the EWMA chart for the identical dataset, [5]. The statistics of the MEWMA chart are as follows:

$$MEWMA_t = \theta \overline{Y}_t + (1-\theta)MEWMA_{t-1} + k(S_t - S_{t-1}),\ t = 1,2,... \quad (6)$$

In the Equation, k represents a constant. When k is set to 1, the MEWMA chart statistic takes a similar form with $k=1$ (see details [5]). The mean and deviation of $MEWMA_t$ for an in-control process are as follows:

$$E(MEWMA) = \mu_0 \quad (7)$$

and

$$V(MEWMA) = \sigma^2 \left(\frac{\theta + 2\theta k + 2k^2}{2-\theta} \right). \quad (8)$$

The upper and lower control limits of the MEWMA chart are explained as follows:

$$UCL_{MEWMA} / LCL_{MEWMA} = \mu_0 \pm L_2 \sqrt{\sigma^2 \frac{\theta + 2\theta k + 2k^2}{2-\theta}}. \quad (9)$$

The MEWMA chart's control limit width is determined by the control limit coefficient L_2, and its value is chosen to achieve a specific average run length to a false alarm (ARL_0).

2.2.3 Exponentially Weighted Moving Average-Sign Rank (EWMA-SR) Control Chart

According to [7], this widely adopted control chart for production processes assumes a normal distribution. Nonetheless, it becomes evident that production processes may exhibit non-normal distributions. A nonparametric approach, known as the EWMA-SR control chart, is introduced to address this, intended to monitor changes in the process mean. The statistic known as EWMA-SR is mathematically defined as Equation (10).

$$EWMA-SR_t = \theta SR_t + (1-\theta)EWMA-SR_{t-1},\ t = 1,2,... \quad (10)$$

where θ is the constant with a range of $0 < \theta < 1$. The mean and asymptotic variance of EWMA-SR_t for the controlled process are as outlined below:

$$E(EWMA-SR) = 0 \quad (11)$$

and

$$V(EWMA-SR) = \frac{\theta}{2-\theta}\left(\frac{n(n-1)(2n+1)}{6} \right). \quad (12)$$

As a result, the upper and lower control limits for the EWMA-SR chart correspond to the following definitions:

$$UCL_{EWMA-SR} / LCL_{EWMA-SR} = \pm L_3 \sqrt{\frac{\theta}{2-\theta}\left(\frac{n(n+1)(2n+1)}{6} \right)} \quad (13)$$

where L_3 is a coefficient used to calculate the EWMA-SR chart's upper and lower control limits, and the desired ARL_0 chooses it, the mean quantity of samples is expected before a false alarm occurs.

2.2.4 Modified Exponentially Weighted Moving Average-Sign Rank (MEWMA-SR) Control Chart

The MEWMA sign gave rise to the MEWMA-SR chart by incorporating a supplementary rank phase, which is particularly effective whenever the procedure undergoes slight changes. SR statistics are beneficial for monitoring the process median. The MEWMA-SR chart has the following statistical value:

$$MEWMA-SR_t = \theta SR_t + (1-\theta)MEWMA-SR_{t-1} + k(SR_t - SR_{t-1}), \quad t = 1,2,... \tag{14}$$

The control process's MEWMA-SR mean and asymptotic variance are as follows:

$$E(MEWMA-SR) = 0 \tag{15}$$

and

$$V(EWMA-SR) = \frac{\theta + 2\theta k + 2k^2}{2-\theta}\left(\frac{n(n-1)(2n+1)}{6}\right). \tag{16}$$

Consequently, the MEWMA-SR control chart's asymptotic control limit is as follows:

$$UCL_{MEWMA-SR} / LCL_{MEWMA-SR} = \pm L_4\sqrt{\frac{\theta + 2\theta k + 2k^2}{2-\theta}\left(\frac{n(n+1)(2n+1)}{6}\right)} \tag{17}$$

where L_4 represents the coefficient of control limit for the MEWMA-SR chart, aligned with the specified ARL_0, the MEWMA-SR will detect if samples are out of control for $MEWMA\text{-}SR_t > UCL_{MEWMA\text{-}SR}$ or $MEWMA\text{-}SR_t < LCL_{MEWMA\text{-}SR}$.

2.3 Average Run Length and Calculation Step

The statistical process control literature's run length properties provide a way to assess a control chart's effectiveness. When evaluating the control chart's performance and change detection capabilities, the average run length (ARL) is an essential measurement. ARL_0 should be big enough to reduce false alarms. On the other hand, the out-of-control ARL (ARL_1) needs to be small enough to quickly identify changes when the process goes out of control. The ARL is mathematically expressed in Equation (18), where RL represents the number of samples required before the system becomes uncomfortable for the initial instance.

$$ARL = \frac{\sum_{i=1}^{t} RL_i}{N}. \tag{18}$$

By using 370 and 500 as the in-control case parameters, a 100,000 iteration (N) Monte Carlo simulation can be used to investigate the properties of the control chart's run length. Consequently, choosing the control limit coefficient is critical to align the resulting value with approximately the ARL_0 equivalent.

The steps detailing the process to identify a solution are outlined as follows:

Step 1: Generate N random samples from a specific distribution, including the binomial and Poisson distributions.
Step 2: Compute the proposed tracking numerical data and assess L at ARL_0 values of 370 and 500.
Step 3: Identify the control chart's weighting values (θ) and the data variation level (η) when the process is out of control.
Step 4: Compute the statistics and control limits for the control chart using a subsample size of 5.
Step 5: Until the data exceeds the control limit, record the control chart's run length (RL).
Step 6: Count the ARL1 and assess the control chart's efficacy by repeating the process 100,000 times (N).

Ultimately, the average run length value was computed for each control chart. The suggested chart's performance was then contrasted with the EWMA chart, MEWMA chart, EWMA-SR chart, and MEWMA-SR chart. A control chart with a lower average run length to signal (ARL_1) under a specific shift is considered more effective because it detects the change more quickly.

3 Results

The MEWMA-SR chart's performance is examined in this study. The findings of a simulation study contrasting the suggested chart with the third chart currently in use are shown in Section 3.1. A section based on the findings is presented in Section 3.2.

3.1 Comparison Performance of the Control Chart

For discrete distributions such as the Poisson (5) and binomial (10,0.1) distributions, a simulation study is conducted to assess the accuracy of the MEWMA based on sign rank. The numerical outcomes for ARL_0 and ARL_1 were computed using Equation (18) through Monte Carlo simulation. The simulation was implemented using R programming under ARL_0 = 370 and 500. The parameters under in-control conditions are specified: a sample size *(n)* of 5 and weighting (θ) parameters for a control chart set at 0.05. The out-of-control parameters for the process distribution are delineated, with variations in the magnitude (η) of change ranging from 0.01, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.50, to 1.00. Furthermore, we assessed the efficacy of the proposed chart (MEWMA-SR) by contrasting it with the results obtained from existing charts. Our analysis revealed that the chart exhibiting the lowest Average Run Length for detecting out-of-control conditions (ARL_1) is considered the most potent.

In this experiment, Table 1 presents the numerical outcomes for a binomial (10, 0.1) distribution with a subgroup size of 5, resulting in an average run length under in-control conditions (ARL_0) equal to 370. When the process is changed from 0.01 to 0.20, the EWMA chart outperforms all other charts, according to the Monte Carlo method analysis. Beyond that, when the process changes more than 0.2, the MEWMA chart works well regarding change detection.

Next, Table 2 displays the numerical results for a Poisson (4) distribution with a subgroup size of 5, yielding an ARL_0 equal to 370. The calculation found that when the magnitude size (η) was 0.01, the MEWMA-SR chart performed better than the others. Conversely, the MEWMA chart outperformed when the magnitude size (η) exceeded 0.0 5 because the MEWMA chart can provide the lowest ARL_1 value.

Furthermore, the performance study of the MEWMA-SR chart analyzed the ARL value under in-control conditions set at 500 for binomial and Poisson arithmetic distributions. Table 3 and Table 4 display the numerical results for binomial (10,0.1) and Poisson (4) distributions, respectively. Establish the subsample size as 5. The study outcomes revealed that an investigation assessing the efficiency of the current charts and the suggested control chart under an ARL_0 of 500 produced analogous analytical outcomes to those observed under an ARL_0 of 370.

Table 1. ARL_1 of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for binomial distribution when $n = 5$ and $ARL_0 = 370$

Shift (η)	EWMA		MEWMA		EWMA-SR		MEWMA-SR	
	$L_1 = 6.246$		$L_2 = 6.540$		$L_3 = 8.401$		$L_4 = 8.142$	
	ARL_1	SD	ARL_1	SD	ARL_1	SD	ARL_1	SD
0.01	**189.142**	**0.806**	190.510	0.857	192.790	0.830	195.714	0.819
0.05	**155.752**	**0.572**	156.258	0.528	165.290	0.537	171.266	0.519
0.10	**95.277**	**0.207**	95.314	0.296	103.529	0.278	105.631	0.295
0.15	**65.029**	**0.081**	70.401	0.079	75.162	0.083	79.689	0.085
0.20	**57.286**	**0.056**	59.428	0.057	65.116	0.063	67.241	0.058
0.25	48.599	0.041	**45.928**	**0.043**	52.678	0.040	54.962	0.042
0.30	45.284	0.025	**43.627**	**0.028**	48.675	0.026	50.636	0.29
0.35	39.488	0.018	**37.553**	**0.019**	45.179	0.020	45.682	0.022
0.40	34.989	0.015	**34.643**	**0.016**	36.329	0.017	36.337	0.017
0.50	29.682	0.014	**28.575**	**0.015**	28.627	0.013	30.174	0.013
1.00	26.365	0.014	**26.203**	**0.016**	26.980	0.005	26.749	0.005

Bold is the minimum of ARL_1, and SD is the standard deviation of RL.

Table 2. ARL_1 of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for Poisson distribution when $n = 5$ and $ARL_0 = 370$

Shift (η)	EWMA		MEWMA		EWMA-SR		MEWMA-SR	
	$L_1=2.357$		$L_2=2.419$		$L_3=5.924$		$L_4=6.122$	
	ARL_1	SD	ARL_1	SD	ARL_1	SD	ARL_1	SD
0.01	246.156	0.824	245.617	0.833	242.629	0.864	**239.349**	**0.843**
0.05	180.623	0.762	**178.208**	**0.735**	179.915	0.702	180.337	0.741
0.10	96.312	0.402	**95.718**	**0.412**	97.524	0.436	101.647	0.537
0.15	50.371	0.286	**48.104**	**0.281**	50.323	0.308	52.162	0.255
0.20	32.572	0.149	**29.753**	**0.152**	31.552	0.158	32.584	0.137
0.25	19.962	0.085	**17.312**	**0.081**	23.749	0.093	24.520	0.082
0.30	17.203	0.052	**15.955**	**0.055**	16.527	0.059	16.249	0.063
0.35	16.495	0.035	**14.188**	**0.032**	15.679	0.037	15.973	0.038
0.40	11.433	0.075	**10.410**	**0.079**	12.910	0.076	11.993	0.070
0.50	6.857	0.023	**5.820**	**0.027**	7.503	0.024	6.012	0.026
1.00	4.431	0.008	**4.318**	**0.005**	5.571	0.006	4.504	0.001

Bold is the minimum of ARL_1, and SD is the standard deviation of RL.

Table 3. ARL_1 of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for binomial distribution when $n = 5$ and $ARL_0 = 500$

Shift (η)	EWMA		MEWMA		EWMA-SR		MEWMA-SR	
	$L_1 = 7.256$		$L_2 = 7.895$		$L_3 = 9.267$		$L_4 = 9.531$	
	ARL_1	SD	ARL_1	SD	ARL_1	SD	ARL_1	SD
0.01	**284.390**	**0.824**	287.643	0.855	294.620	0.873	298.679	0.837
0.05	**202.679**	**0.569**	206.460	0.585	212.962	0.527	215.830	0.520
0.10	**138.152**	**0.257**	140.433	0.299	142.918	0.294	146.372	0.281
0.15	**85.629**	**0.148**	87.207	0.172	89.410	0.134	91.252	0.122
0.20	**62.598**	**0.054**	64.185	0.059	66.901	0.051	68.008	0.050
0.25	59.320	0.045	**58.369**	**0.046**	61.557	0.040	63.948	0.043
0.30	45.965	0.029	**44.873**	**0.025**	46.809	0.022	47.338	0.027
0.35	39.165	0.021	**38.785**	**0.019**	40.467	0.017	46.867	0.018
0.40	34.808	0.018	**32.038**	**0.015**	35.817	0.014	37.056	0 .014
0.50	31.229	0.012	**30.258**	**0.012**	30.456	0.011	31.191	0.011
1.00	26.347	0.010	**25.213**	**0.011**	26.648	0.009	26.153	0.008

Bold is the minimum of ARL_1, and SD is the standard deviation of RL.

Table 4. ARL_1 of EWMA, MEWMA, EWMA-SR, and MEWMA-SR chart for Poisson distribution when $n = 5$ and $ARL_0 = 500$

Shift (η)	EWMA		MEWMA		EWMA-SR		MEWMA-SR	
	$L_1=4.715$		$L_2=4.285$		$L_3=7.206$		$L_4=7.965$	
	ARL_1	SD	ARL_1	SD	ARL_1	SD	ARL_1	SD
0.01	325.827	0.841	323.548	0.827	322.516	0.855	**320.410**	**0.867**
0.05	239.218	0.628	**237.824**	**0.675**	243.172	0.612	245.283	0.698
0.10	178.319	0.375	**175.257**	**0.381**	181.691	0.407	182.341	0.341
0.15	86.257	0.119	**84.597**	**0.194**	86.519	0.217	89.716	0.521
0.20	23.457	0.061	**21.964**	**0.059**	25.464	0.053	25.725	0.056
0.25	15.084	0.048	**13.821**	**0.047**	18.631	0.049	20.611	0.050
0.30	8.286	0.026	**7.213**	**0.025**	12.081	0.027	15.328	0.027
0.35	7.374	0.014	**6.528**	**0.014**	10.856	0.016	11.529	0.016
0.40	5.105	0.009	**4.898**	**0.008**	6.248	0.006	6.855	0.006
0.50	4.258	0.003	**3.689**	**0.003**	5.998	0.003	6.283	0.003
1.00	2.599	0.001	**1.382**	**0.001**	4.859	0.001	5.806	0.001

Bold is the minimum of ARL_1, and SD is the standard deviation of RL.

3.2 Real Applications

This section involves studying the comparative efficiency of EWMA, MEWMA, EWMA-SR, and MEWMA-SR in detecting changes in the average values of the number of deaths from COVID-19. The data is collected weekly from April 1, 2021, to May 31, 2021, totalling 61 points. The data exhibits a Poisson distribution.

The study results show that all four control charts efficiently identify the data's mean variations. Figure 1 demonstrates that the statistics of the EWMA chart fall within the upper and lower bounds, leading to the conclusion that the EWMA chart is unable to detect changes in the data. The MEWMA chart fails to detect changes in data like the EWMA chart because its statistics do not exceed the upper and lower limits, as illustrated in Figure 2. Next, Figure 3 shows that the EWMA-SR chart signals a change in the data as early as the 22^{nd} observation. Finally, Figure 4 demonstrates that the MEWMA-SR chart can identify a shift in the data as early as the 4^{th} observation. Upon comparing the performance of the charts as mentioned above, it can be inferred that the MEWMA-SR chart is the most responsive to mean shifts due to its reliance on weighted moving averages that prioritize recent observations.

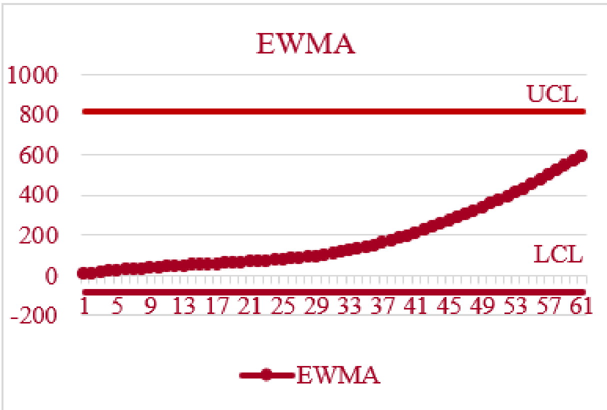

Fig. 1: The performance of the EWMA chart

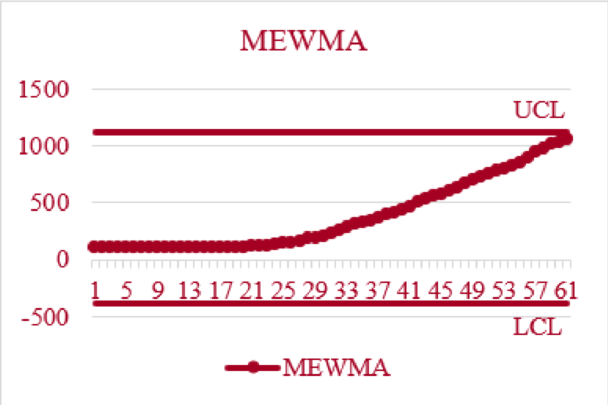

Fig. 2: The performance of the MEWMA chart

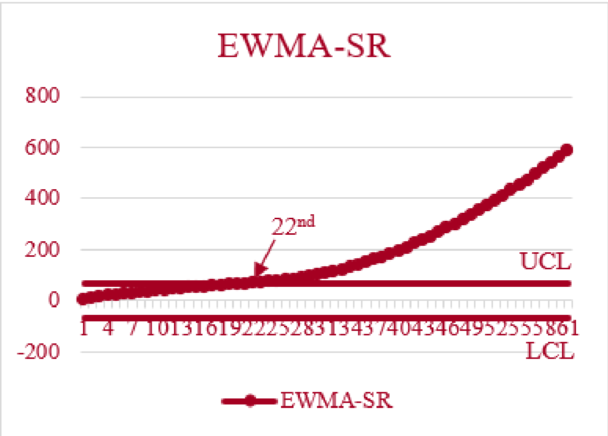

Fig. 3: The performance of the EWMA-SR chart

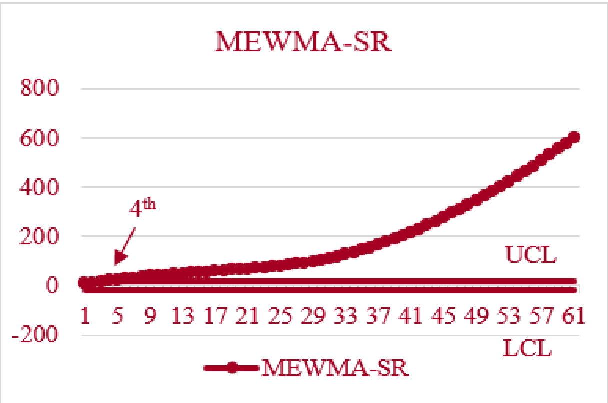

Fig. 4: The performance of the MEWMA-SR chart

4 Conclusions

This research proposes a control chart that combines the MEWMA chart with nonparametric sign rank statistics to improve mean monitoring for discrete distributions such as binomial and Poisson distributions. The control chart utilizes the ARL statistic to evaluate its effectiveness. The results demonstrate that the suggested chart is the best choice for identifying small changes in all distributions, achieving the least ARL_1 compared to the EWMA, MEWMA, EWMA-SR, and MEWMA-SR charts, except for the binomial distribution. However, the EWMA chart exhibits superior performance in detecting significant shifts. Additionally, the results indicated that the proposed chart successfully detected shifts when applied to accurate data. Therefore, the suggested chart provides quality practitioners with an alternate method for implementing a suitable and effective control chart for discrete distribution, consistent with previous research for continuous distributions. The benefit of this research is that nonparametric control charts, such as MEWMA-SR charts, exhibit superior performance compared to parametric control charts. They can overcome the limitation of assuming known parameters or the need to estimate distribution parameters from process observations. On the foundation of this work, future studies can monitor the variation process and the process means in discrete distributions. Finally, it holds for actual data with different distributions.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- P. S.: writing an original draft, software, data analysis, data curation, proof, reviewing, and editing.
- Y. A.: investigation, methodology, validation, reviewing, and editing.
- S. S.: conceptualization, investigation, writing-review and editing, funding acquisition, project administration, reviewing and editing.

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Conflict of Interest

The authors have no conflict of interest to declare.