Generating a Set of Consistent Pairwise Comparison Test Matrices in AHP using Particle Swarm Optimization

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Abstract: Pairwise Comparison Matrix (PCM) play a crucial role in multi-criteria decision-making, especially within the framework of the Analytic Hierarchy Process (AHP). It is essential for an expert to systematically assess and compare alternatives, considering various criteria as part of the decision-making process. The AHP relies on consistency to ensure the accuracy of pairwise comparisons made by decision makers, increasing the overall integrity of the decision-making process. The aim of this paper is to present an approach based on Particle Swarm Optimization (PSO) as a metaheuristic method to generate a set of consistent pairwise comparison matrices. Numerical results are presented with different sizes of matrices showing the effectiveness of our algorithm in producing acceptable matrices for the benefit of expets.

Key-Words: - Pairwise Matrix, Analytic Hierarchy Process, Decision Theory, Consistency, Metaheuristics, Particle Swarm Optimizition

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1 Introduction

The Analytic Hierarchy Process (AHP) is a method for multi-attribute decision-making. It was developed in the last cemtry [1], [2]. It is widely used for making decisions including, for instance, social, military, managerial and many other fields, [3], [4], [5].

The main problem of the AHP method in its practical application is the judgment matrix which must be constructed by the decision makers on the basis of their experiences and knowledge. Due to the limitations of experience and knowledge and the complex nature of the decision problem, [6], [7], especially when dealing with a large number of judgments, the pairwise comparison matrix may be inconsistent.

To test whether the matrix is consistent, [8], suggested the use of the consistency ratio CR. Indeed, the pairwise comparison matrix could pass the consistency test when the so-called consistency ratio $CR < 0.1$. However, in many cases, the judgment matrix cannot pass this test to be acceptable and it has to be adjusted.

The consistency of pairwise matrix has been a subject of many studies for several decades, [9], [10], [11], [12]. However, some of the methods developed for revising inconsistent comparison matrices are complicated and difficult to use, while others struggle to preserve the original comparison information as a new matrix must be constructed to replace the original.

Metaheuristic methods such as Genetic Algorithms (GAs), Particle Swarm Optimisation (PSO) and others have been used to study the consistency problem of the pairwise comparison matrix. GAs were developed by [13]. Their inspiration is the theory of evolution of Charles Darwin and natural biological processes, [14]. They entail multiple iterative processes that allow for steady and iterative progress towards optimal solutions. To fully realize the benefits of genetic algorithms as fundamental problem-solving approaches and improve their performance, it is imperative to adapt and incorporate appropriate genetic operators for addressing the specific problem at hand, [15]. In [16], we propose an alternative approach to create consistent pairwise matrices. This approach is based on GAs to define a set of consistent pairwise matrices for different dimensions, to help experts in making appropriate judgment decisions.

Particle swarm optimization (PSO) is a population based optimization algorithm inspired by the group dynamics of flocks of birds or schools of fish. It was proposed in [17] and was first intended for simulating social behaviour. The idea behind PSO is that each particle represents a possible solution to the problem, and the particles move around in the search space according to simple mathematical formulas, [18]. The particles movements are influenced by their own best known positions and the best known positions of the entire swarm.

The aim of this paper is to propose an alternative approach to create consistent pairwise matrices. This approach is based on PSO in defining a set of consistent pairwise matrices for different dimensions to aid experts to define the appropriate judgment decision. Thus, instead of determining a pairwise matrix that can be inconsistent, especially in the case of high order, and trying to modify it, the expert will have pairwise comparison matrices during the algorithm process allowing the identification of the appropriate one.

The remaining of this paper is structured as follows. In the second section, we briefly describe the different steps of the Analytic Hierarchy Process. Section 3 is devoted to the pairwise comparison matrix. In section 4, we present the particle swarm optimization approach to identify consistent judgment matrices. In Section 5, numerical results are performed with different sizes of matrices, showing how effective and accurate is the proposed method to overcome the problem of inconsistency in pairwise comparison matrix in the AHP method.

2 Overview of the AHP Method

The Analytic Hierarchy Process (AHP) serves as a comprehensive theory of measurement, employed in the generation of ratio scales through comparisons, encompassing both discrete and continuous dimensions. These comparative assessments can be drawn from empirical measurements or a foundational scale that captures the inherent hierarchical relationships of preferences and sentiments. The AHP method exhibits a specific focus on evaluating deviations from consistency, measuring such deviations, and assessing dependencies within and between the constituent groups of its structural elements, [19]. The general idea of the AHP method is showing in Figure 1.

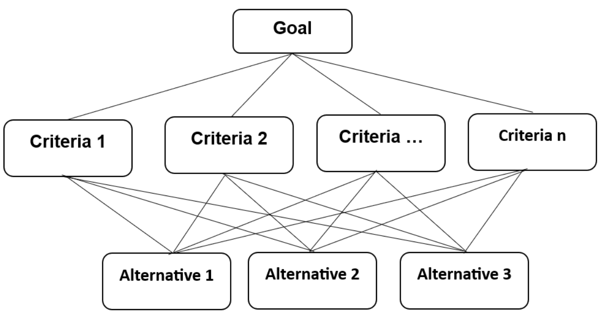

Figure 1: Schema of AHP method

In recent years, AHP method is applied in different fields such as economic, mathematic applications, renewable energy and decision-making problem. A systematic procedure exists for effective application of AHP in decision-making, which can be summarized in the following steps:

- **Step 1 :** Construction of a structural hierarchy that highlights the objectives and identifies the criteria and alternatives. Figure 1. illustrates the hierarchical structure of the information, where, the top level representing the goals, the second level outlining the ranking criteria and the last level consists of the alternatives, [19].

- **Step 2 :** Construction of Pairwise Comparison Matrices (PCM) (Comparative Judgments) for all the criteria and alternatives. The pairwise comparison approach, inspired by the research in [20], is employed. After constructing a hierarchy, the subsequent step involves establishing the priorities of variables at each level through the creation of comparison matrices for all variables in relation to each other. This pairwise comparison reveals the degree to which variable $'A'$ is more favorable or important than variable $'B'$. Table 1 illustrates how an opinion scaling/pairwise comparison evaluation scaling from point one to nine scaling (1-9) is used to quantify these logical preferences.

- **Step 3 :** Weight determination via normalization procedure involves calculating the weights for criteria and the local weight of alternatives based on the pairwise comparison matrices. For each value in a column $'j'$, it is divided by the total of the values in that column. The sum of values in each column is normalized to 1 in the matrix.

$$AW = \begin{bmatrix} \frac{a_{11}}{\sum a_{i1}} & \frac{a_{12}}{\sum a_{i2}} & \cdots & \frac{a_{1n}}{\sum a_{in}} \\ \frac{a_{21}}{\sum a_{i1}} & \frac{a_{22}}{\sum a_{i2}} & \cdots & \frac{a_{2n}}{\sum a_{in}} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \frac{a_{n1}}{\sum a_{i1}} & \frac{a_{n2}}{\sum a_{i2}} & \cdots & \frac{a_{nn}}{\sum a_{in}} \end{bmatrix} \quad (1)$$

- **Step 4 :** Integration of weight and consistency testing: Begin by synthesizing local weights to derive global weights for the alternatives. The eigenvector of matrix will then be computed. The Consistency Index (CI) will be calculated by using (3) below. Subsequently, it is essential to verify the consistency judgment for the appropriate value of $'n'$ using the Consistency Ratio (CR) to ensure the consistency of the pairwise comparison matrix, as outlined in the representation (4).

- **Step 5 :** Assessing the CR : If CR is equal to or less than 0.10 (10%), the consistency level is deemed satisfactory. However, if CR exceeds 0.10, it suggests the presence of significant inconsistencies.

3 Pairwise Comparison Matrix

Pairwise comparison plays a crucial role in the realm of multi-criteria decision analysis (MCDA), [21]. The quantitative utilization of pairwise comparisons in decision analysis was introduced by [22]. Subsequently, we further developed the concept of pairwise comparison, extending it into a widely adopted multi-criteria decision-making approach known as the Analytic Hierarchy Process (AHP) in [23].

The matrix $A = (a_{ij})$ records the pairwise comparisons between alternatives. Each entry a_{ij} represents the numerical answer to the question 'How many times is alternative i better than alternative j?'. Let $\mathbb{R}_+$ to denote the set of positive numbers, $\mathbb{R}_+^n$ be the set of positive vectors of size n, and $\mathbb{R}_+^{n\times n}$ to be the set of positive square matrices of size n with all elements greater than zero. The matrix of pairwise comparison is defined as follows:

$$A = \begin{pmatrix} 1 & a_{12} & \dots & a_{1n} \\ a_{21} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & 1 \end{pmatrix} \quad (2)$$

Definition 1. The matrix $A = (a_{ij}) \in \mathbb{R}_+^{n\times n}$ is said to be pairwise comparison matrix if:

$$a_{ij} = \frac{1}{a_{ji}} \ \forall \ 1 \le i, j \le n$$

The scale used for comparisons indicates the relative importance of one element over another one with respect to a given attribute.

Table 1 presents a scale ranging from 1 to 9 (that is, from *least important* to *most important*).

It should be noted that, if activity $'i'$ is assigned one of the above numbers and is compared with activity $'j'$, then $'j'$ will have reciprocal values when compared with $'i'$.

Table 1. 1-9 Pairwise comparison scale

Linguistic term	Preference number
Equally important	1
Weakly more important	3
Strongly more important	5
Very strong important	7
Absolutely more important	9
Intermediate values	$2, 4, 6, 8$

Let $\mathcal{A}$ to denote the set of pairwise matrices and $\mathcal{A}^{n\times n}$ denotes the set of pairwise matrices of size n, respectively.

Definition 2. $A = (a_{ij}) \in \mathcal{A}^{n\times n}$ is consistent if :

$$a_{ik} = a_{ij} \times a_{jk} \ \forall 1 \le i, j, k \le n$$

Otherwise, it is said to be inconsistent.

The Theorem of Perron-Frobenius states that each pairwise comparison matrix $A \in \mathcal{A}^{n\times n}$ has a unique positive weight vector $\mathbf{w} = (w_i)$ that satisfies the condition $A\mathbf{w} = \lambda_{\max}\mathbf{w}$ and $\sum_{i=1}^{n} w_i = 1$, with $\lambda_{\max}$ is the maximal or Perron eigenvalue of the given matrix A.

Definition 3. : Let $A \in \mathcal{A}^{n\times n}$ be an pairwise comparison matrix of size n. Its Consistency Index (CI) is given by :

$$CI = \frac{\lambda_{\max} - n}{n-1} \quad (3)$$

So, $CI = 0 \Longleftrightarrow \lambda_{\max} = n$.

The consistency index (CI) is a useful measure for determining the degree to which a pairwise comparison matrix deviates from consistency, [24]. It should be noted that, in [25], the authors proposed a method for establishing an upper bound on the value of CI when the pairwise comparison matrix entries are expressed on a bounded scale.

We has recommended using a discrete scale for the matrix elements, i.e., $\forall\ 1 \le i, j \le n$:

$$a_{ij} \in \ \{\frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \dots, \frac{1}{2}, 2, \dots, 8, 9\}$$

We suggested obtaining a normalized measure of inconsistency.

Definition 4. The random index (RI) of a pairwise comparison matrix A is provided to:

- Generate numerous pairwise comparison matrices, draw each entry above the diagonal independently and uniformly from the Saaty scale mentioned in Table 1.
- Calculate the CI for each pairwise comparison matrix.
- Calculate the average of these values.

Several authors have published random indices. These are based on simulation methods and the number of generated matrices used, see, [26]. The random indices RI_n are reported in Table 2 for $1 \le n \le 10$ as provided by [27], and validated by [28].

Table 2. Random Consistency Index

n	1	2	3	4	5	6	7	8	9	10
$\mathcal{RI}_n$	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Definition 5. Let $A \in \mathcal{A}^{n\times n}$. The Consistency Ratio (CR) of A is defined by :

$$CR = \frac{CI}{RI_n} \tag{4}$$

Definition 6. Let $A \in \mathcal{A}^{n\times n}$. The matrix A is consistent enough to be accepted if $CR < 0,1$.

Theorem 7. *[29], let $A \in \mathcal{A}^{n\times n}$. The matrix A is consistent if and only if $rank(A) = 1$.*

Theorem 8. *[29], let $A = (a_{ij}) \in \mathcal{A}^{n\times n}$. A is consistent, if and only if*

$$\lambda_{max} = n$$

Proof. Following the Theorem 7, if A is consistent, we have $rank(A) = 1$,
Also, all but one of its eignevalues are zero.
However, $Trace(A) = \sum_{i=1}^{n} a_{ii} = n$, and,
$Trace(A) = \sum_k \lambda_k = \lambda_{max}$,
Then, $\lambda_{max} = n$.

Conversely, if $\lambda_{max} = n$

$$\begin{aligned} n\lambda_{max} &= \sum_{i,j=1}^{n} a_{ij}\frac{w_j}{w_i} \\ &= n + \sum_{1\le i<j\le n} \left(a_{ij}\frac{w_j}{w_i} + a_{ji}\frac{w_i}{w_j}\right) \\ &= n + \sum_{1\le i<j\le n} \left(y_{ij} + \frac{1}{y_{ji}}\right) \end{aligned}$$

Since $y_{ij} + \frac{1}{y_{ji}} \ge 2$, and $n\lambda_{max} = n^2$, equality is obtained for $y_{ij} = 1$, i.e. $a_{ij} = \frac{w_i}{w_j}$.
Then, $a_{ij}a_{jk} = a_{ik}$ for all i, j and k, which shows that A is consistent.

□

Although it may seem strange to apply a crisp decision rule to the fuzzy concept of ”large inconsistency”, it is necessary to do so in order to ensure clarity and objectivity, [30].

4 Approach based on particle swarm optimization for pairwise matrix

4.1 Particle swarm optimization

Particle swarm optimization (PSO) is a more recent evolutionary computational method compared to the genetic algorithm and the evolutionary programming. While PSO has some common properties of evolutionary computation including randomly searching, iteration time and so on, the classical PSO lacks crossover and mutation operators. PSO mimics the social dynamics of birds: Individual birds share information about their position, velocity and fitness, and then the behavior of the flock is affected in a way to increase the probability of migration to regions with high fitness.

Assuming that:

- The swarm, consists of N particles, in search space with n dimensions, $S \subseteq \mathbf{R}^n$.
- The position of the ith particle is an n-dimensional vector $x_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in S$.
- The velocity of this particle is also an n-dimensional vector $v_i = (v_{i1}, v_{i2}, \ldots, v_{in}) \in S$.
- The optimal previous position visited by the ith particle is a point in S, denoted by $p_i = (p_{i1}, p_{i2}, \ldots, p_{in}) \in S$.
- The index of the particle that achieved the best previous position among the entire swarm is denoted by g, and the iteration counter is denoted by t.

Then, the following update equations are used to manipulate the standard PSO:

$$\begin{aligned} v_{id}(t+1) =& v_{id}(t) + c_1 r_1 \left(p_{id}(t) - x_{id}(t)\right) \\ &+ c_2 r_2 \left(p_{gd}(t) - x_{id}(t)\right) \end{aligned} \tag{5}$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \tag{6}$$

where $i = 1, 2, \ldots, N$ stands for the index of the particle, $d = 1, 2, \ldots, n$ indicates the dth component of the particle, and c_1 and c_2 are some positive numbers representing cognitive and social parameters, respectively.

The variables $(r_1, r_2) \in$ [0,1] are uniformly distributed random numbers.

The standard algorithm for PSO method for an optimization problem is shown below :

Algorithm 1: Particle Swarm Optimization (PSO)

Input : $J(x)$, N, c_1, c_2, ω, $maxiter$, $tolerance$
Output: x^*, min $J(x^*)$

```
Initialize particle velocities v_i and positions x_i for i = 1, ..., N
Initialize the best positions p_i = x_i and global best position of the swarm p_g
Evaluate J(x_i) for each particle
Update p_i and p_g
for k = 1 to maxiter do
    for i = 1 to N do
        Update velocity: v_i = ωv_i + c_1 r_1 (p_i − x_i) + c_2 r_2 (p_g − x_i)
        Update position: x_i = x_i + v_i
        Evaluate J(x_i)
        if J(x_i) < J(p_i) then
            Update p_i
            if J(p_i) < J(p_g) then
                Update p_g
            end
        end
    end
    if |J(p_g) − J(p_old_g)| < tolerance then
        break
    end
end
return p_g, J(p_g)
```

4.2 The proposed approach

4.2.1 Initialization

PSO begins with the generation of a random population of potential solutions to the problem to be solved. This is typically done by randomly generating a set of particles in a way that each particle is a potential solution represented as a set of particles.

The representation (encoding) of the possible solutions is a crucial step in the process of any metaheuristic. Depending on the problem studied, we distinguish several types of coding, namely, matrix representation, binary coding and real coding. Indeed, the choice of representation influences the performance of particle swarm optimization. In our case, the pairwise matrix, which is the objectif of the problem, can be identified by:

$$n(n-1)/2 \quad \text{elements}$$

from the set:

$$\{2, 3, ..., 9, 1/2, 1/3, ..., 1/9\}$$

Thus, we have to randomly generate m matrices, each of them is encoded by a vector under the form:

$$(a_{1,2}, ..., a_{1,n}, a_{2,3}, ..., a_{2,n}, ..., a_{n-1,n})$$

Example:

3	7	1/3	5	1/6	2

$$\Downarrow$$

$$\begin{pmatrix} 1 & 3 & 7 & 1/3 \\ 1/3 & 1 & 5 & 1/6 \\ 1/7 & 1/5 & 1 & 2 \\ 3 & 6 & 1/2 & 1 \end{pmatrix}$$

4.2.2 Principle of the algorithm

The proposed algorithm, as illustrated in Figure 2, is based on an improved PSO to find an acceptable pairwise matrix with different sizes, depending on the studied application, to help experts to have a panel of choices of acceptable matrices to define by their expertise the desired matrix or to complete an incomplete defined pairwise matrix.

The proposed approach is a combination of an adapted PSO and the AHP method. PSO method aim to exploit the space of possible solutions; however, the AHP method will make it possible to evaluate the acceptability of the matrices by calculating their consistency ratio CR until a stopping test is satisfied. It should be noted that following the desired results, two stopping criteria are considered: The first one consists of get $CR < 0.1$ and the second one is the maximum number of iterations.

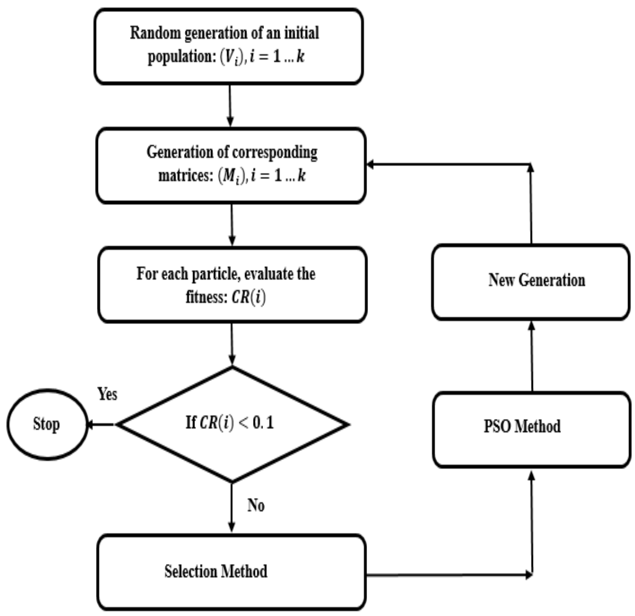

Figure 2: Schema of the proposed approach

4.2.3 The Algorithm

The proposed approach is based on PSO method to propose a set of consistent matrices to experts for choosing the appropriate one for the considered application. The PSO is combined with the AHP method to evaluate the consistency of the matrices by calculating the index CR. The different steps of the proposed approach are described as follows:

Algorithm 2: PSO-based approach to identify consistent pairwise comparison matrix

1: **Step 1** Define the number of criteria n,
2: **Step 2** Initialization: random generation of initial population $V^{(0)}$ of k vectors $(V_i^{(0)}), i = 1, ..., k$; of length $n(n-1)/2$ from the set $\{2, 3, ..., 9, 1/2, 1/3, ..., 1/9\}$,
3: **Step 3** Encoding: transform each (particle) vector of $V^{(0)} = (V_i^{(0)})$ to a population of pairwise matrices $M^{(0)} = (M_i^{(0)})$ with $i = 1, ..., k$,
4: **Step 4** Evaluation: calculate $CR(i)$ for each matrix $M_i^{(0)}$ for $i = 1, ..., k$,
5: **Step 5** Update particles position and particles velocity using (5) and (6)
6: **Step 6** Repeat the step 4 with $M^{(1)}$ replace $M^{(0)}$,
7: **Step 7** The process continue until a stopping test is satisfied.

5 Numerical results and discussion

Several simulations were conducted to demonstrate PSO's ability and efficiency in identifying consistent matrices with varying criteria, resulting in matrices of different sizes. Therefore, the size of the population as well as the maximum number of iterations were adjusted based on the matrix size to be identified.

The considered PSO parameters are described bellow:

- $c_1 = 1.5$ Cognitive Coefficient,
- $c_2 = 1.5$ Social coefficient,
- $w = 0.7$ Inertia weight.

To show the efficiency of the PSO and its ability to provide experts with a number of acceptable matrices $(CR < 0.1)$ in a reasonable time, numerical experiments are developed using an Intel (R) Core(TM) i3-6006U CPU @ 2.00 GHz RAM 4.00 GB.

Figure 3 and Figure 4 show the evolution of CR during the iterative process for different numbers of criteria, namely 4, 5, 6, 8, 10, 12 and 15, using particle swarm optimisation. These show that the proposed algorithm achieves a better evolution of the CR in a shorter period of time.

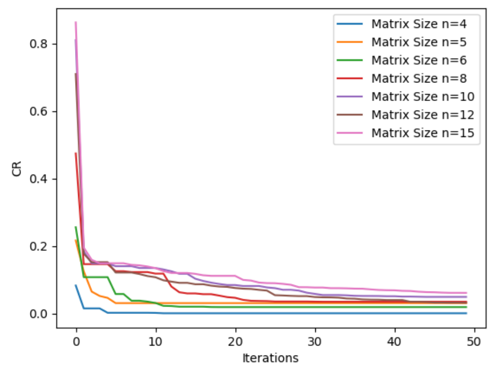

Figure 3: Evolution of the CR during the iterative process for different size of matrices

Table 3 and Table 4 present a comparison of PSO's performance in producing satisfactory pairwise matrices. Specifically, we provide the necessary iteration count to achieve a consistent pairwise matrix with $CR < 0.1$. The results indicate that the PSO algorithm suggested enables the generation of a suitable pairwise matrix after a minimum number of iterations, which can increase depending on the size of the matrix. In particular, it necessitates fewer iterations to fulfill the requested criteria. Furthermore, throughout the iterative process, the CR progressively decreases, thereby empowering experts to base their decisions on data from previous iterations.

Table 5 shows the number of required iterations to obtain the initial pairwise matrix with a $CR < 0.1$. In all cases, the algorithm is initiated by inconsistent pairwise matrices, which corresponds to randomly generated matrices that exhibit higher individual consistency with a CR greater than 0.1, except in the case of matrix of size 4 where we manage to generate consistent matrices at the first iteration. As a result, the consistency rate of matrices starts at 0% for all examples examined and gradually increases after a few iterations until a high consistency rate is achieved. In other words, the experts will have a set containing an increasing number of consistent matrices as the PSO method proceeds, according to their expertise and discussions, to find an acceptable pairwise consistent matrix, thereby avoiding inconsistent matrices.

The aim of the research is to demonstrate the significance and effectiveness of PSO in the generation of reliable pairwise matrices in accordance with Saaty's criteria. Thus, PSO offers a fascinating approach to assist specialists in determining the appropriate matrix for their study's application.

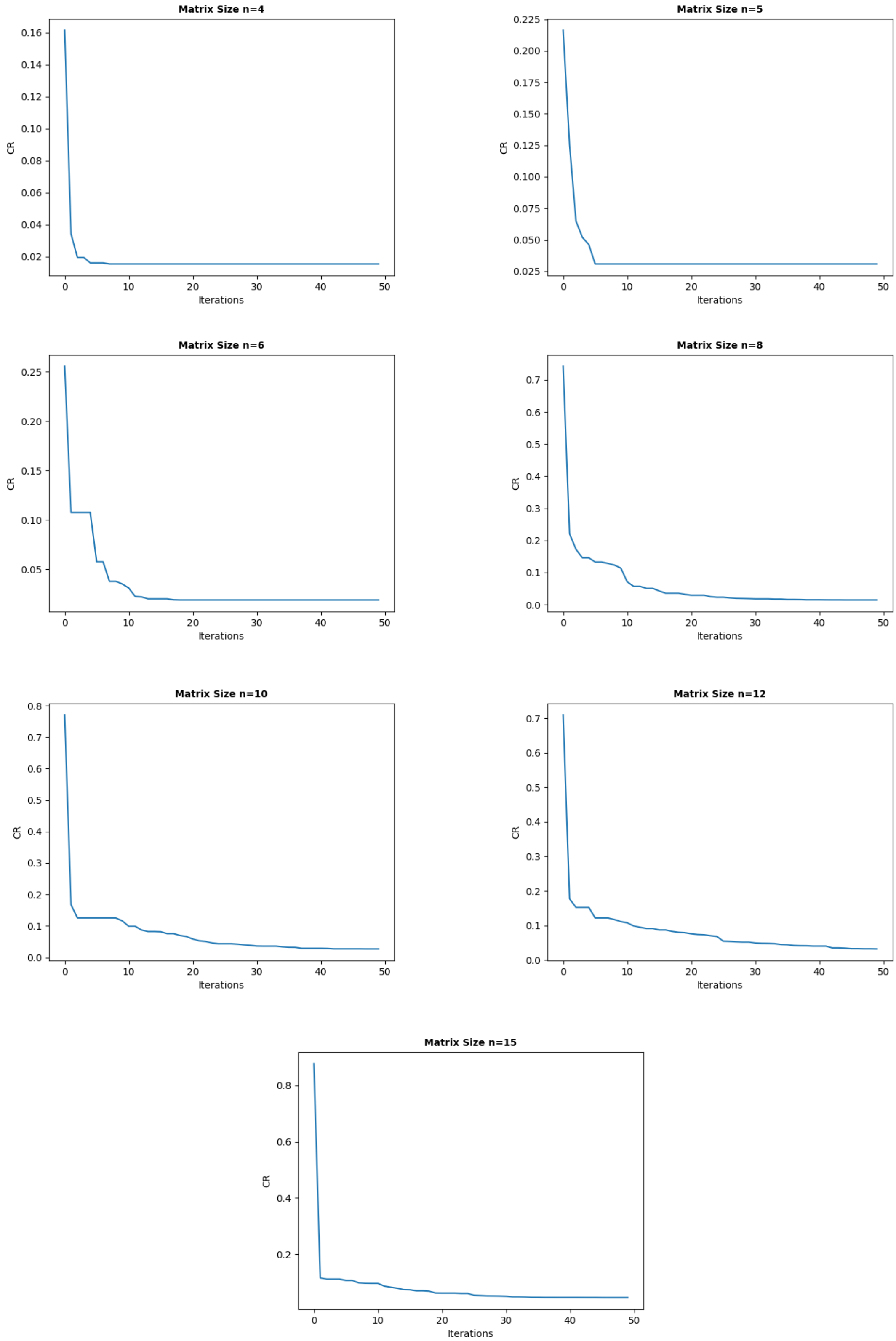

Figure 4: The performance of different criteria in the PSO algorithm for the production of a consistent pairwise comparison matrix

Table 3. Comparing the consistency of PSO process performance using different criteria.

Number of criteria(size of PCM)	4	5	6	8	10	12	15
Consistency Ration (CR)	0.083	0.065	0.58	0.080	0.096	0.098	0.097
Number of Iteration	1	3	6	13	17	12	23

Table 4. Numerical experiment with a variety of matrices

Matrix size	Population	Initial Min_CR	Initial Max_CR	first $CR < 0.1$	Iterations
4	20	0.161	2.584	0.015	1
5	20	0.358	1.666	0.008	4
6	20	0.311	1.840	0.051	10
8	30	0.742	1.469	0.015	14
10	40	0.771	1.544	0.027	20
12	50	0.914	1.261	0.039	35
15	60	0.878	1.277	0.046	50

Table 5. Evolution of the percentage of consistent matrices according to different criteria

Criteria	Min_CR	Max_CR	% of matrices with $CR < 0.1$	Iterations
4	0.083	2.597	2.5%	1
	0.015	1.901	27.5%	3
	0.002	0.596	90%	10
	0.001	0.093	100%	50
5	0.216	2.013	0%	1
	0.065	1.198	7.5%	3
	0.031	0.711	30%	10
	0.031	0.031	100%	50
8	0.474	1.320	0%	1
	0.158	1.238	0%	3
	0.059	0.494	15%	15
	0.035	0.241	95%	50
12	0.710	1.280	0%	1
	0.111	0.933	0%	10
	0.040	0.259	82.5%	40
	0.032	0.421	87.5%	50
15	0.863	1.208	0%	1
	0.120	0.641	0%	15
	0.078	0.731	40%	30
	0.061	0.154	82.5%	50

6 Conclusion

In this study, particle swarm optimization (PSO) was utilized as a metaheuristic approach to establish consistent pairwise matrices when employing the AHP method. The diverse outcomes achieved for matrices of varying sizes illustrate that PSO is a viable option to assist experts in selecting from a range of consistent matrices or in defining incomplete pairwise matrices. Additionally, the performance of PSO can be enhanced to produce pairwise consistent matrices, particularly for large matrices, within a reasonable time frame.

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