Conformable Triple Sumudu Transform with Applications

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Abstract: - One of the important topics in applied mathematics is the topic of integral transformations, due to their importance in electrical engineering applications, including communications in particular, and other sciences. In this work, one of the most important transformations in its three dimensions was presented, which is the triple Sumudu transform, including solving some real-life applications of physics, some of which have not been solved using such an integral transform before. In this work, we extend the Sumudu transform formula to the conformable fractional order, as well as other interesting and significant rules. The general analytical solution of a singular and nonlinear conformable fractional differential equation based on the conformable fractional Sumudu transform is also presented in this paper. The general solutions of several linear and nonhomogeneous conformable fractional differential equations can be obtained using the method we’ve proposed. As a result, our results reveal that our proposed method is an efficient one that can be used for solving all conformable fractional differential equations. The relationship between the Sumudu integral transform and other important and recently proposed integral transforms are also discussed. Finally, the triple Sumudu transform is used to solve boundary value problems, such as the heat equation with boundary values. The triple Sumudu integral transform is also used to solve linear partial integro-differential equations. The transform capability to handle such equations has been proven via its utilization in three applications.

Key-Words: - Triple Sumudu Transform; Sumudu Transform, Partial differential equation, Integral equation. Transform, Double Transform, partial integro-differential equations, fractional Sumudu transform , fractional equation.

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1 Introduction

It is possible to solve some ordinary differential equations, partial linear ones, and non-linear ones, using different integral transformations in their different dimensions. Solutions to the Burgers equation were presented in, [1] and in [2], the Navier-Stokes equation and similar equations in, [3]. More than one method was used to solve various equations, including the Sumudu transform in, [4] and [5], as well as Storm-Liouville problems in, [6], explaining integration coefficients in, [7], and discussing Laplace applications in, [8]. In, [9] the generalized Lyapunov type appeared, and Hermite-Hadamard variants for fractional integrals were presented in, [10]. In, [11], the transformation properties were presented and deduced. The results were discussed and presented in, [12]. The nonlinear biological model was solved in, [13], and Euler's results were presented in, [14]. All immunodynamic results appeared, [15], [16] and show random behavior at different and random values of the fractional order, [17]. Conversion of symmetry disturbance and residual energy chain path using the analytical method, multidimensional thermal
equations are solved, [18]. To solve the new types of equations, the space fractional telegraph equation, the new mixed diffusion equation Yang-Abdel-Aty-Cattani, and the gas dynamics equation using the new fractional symmetry analysis conversion method, [19], [20], [21], [22]. The authors in, [23], By applying the first integration method, prepare the exact solutions to Burger's fractional time equations, [24]. By generalized two-dimensional differential transformation (DTM) to solve the Berger equations paired with fractions of space and time, at present, new concepts and properties of the corresponding derivative have been identified for more information, [25], [26], [27]. Moreover, the authors in, [28], [29] Using the Laplace Transformation matching technique, differential equations are solved. In, [30], the Laplace double congruent transformation method is used to solve partial differential equations, to obtain solutions to a regular and single-dimensional equation for a partially paired burger provided by the authors in, [31]. To determine the exact solutions of the burger fractional equations of time, the first integrated method was used in, [32].

In this work, a new method called congruent triple Sumudu will be proposed, and a solution is done by the decomposition method (CTSDM) to solve a wonderful mixture type for nonlinear equations, but the proposed methods are conformity, analysis method, and triple sumudu conversion methods. This article looks at whether they can apply SUMU conformance triple structure decoding methods (CTSDM), [33].

In this article, we will solve regular and single-dimensional identical burger equations and some basic concepts and definitions of compatible derivatives will be published later in this article, [34].

2 The System of Some NLEEs

For the first time, the Korteweg-de Vries equation (KdV) equation is used in this article (ISM), [7]. Later, the authors extended it in, [23]. ISM was initially created in this article for the Schrödinger nonlinear equation (NLSE) and it was then deepened in, [6], such that it now contains a different type of NLEE. These steps are used to create the AKNS method: (i) construct a suitable, 22 linear scattering (eigenvalue) problem using the "space" variable, where the NLEE's resolution serves as the potential; (ii) choose the Eigen functions "time" dependence so that the eigenvalues hold steady while the potential changes in accordance with the NLEEs; (iii) Select a time for self-functions such that the eigenvalues are preserved. We will quickly find a solution to the direct dispersion issue, and we need to establish how time affects the dispersion data. Rebuild the potential using the dispersion data in (iv). This section focuses on the AKNS method's first phase. As a result, each DE solution yields a scale on M2 with a constant Gaussian curvature of -1. Additionally, the aforementioned definition of DE is equal to saying that DE for you is the problem's integration condition:

\[ d\nu = \Omega \nu, \quad \nu = \left( \begin{array}{c} \nu_1 \\ \nu_2 \end{array} \right) \]

where \( d \) denotes exterior differentiation, \( \nu \) is a vector, and the \( 2 \times 2 \) matrix \( \Omega(\Omega_{ij}, i,j = 1,2) \) is traceless:

\[ \text{tr} \Omega = 0 \]

The NLPDE is given by Equation (1.2),

\[ \Theta \equiv d\Omega - \Omega \Omega = 0, \quad \Omega = \frac{1}{2} \left( \begin{array}{cc} \omega_2 & \omega_1 - \omega_3 \\ \omega_1 + \omega_3 & -\omega_2 \end{array} \right) \]

It is the initial NLPDE that needs to be addressed. The AKNS system provided the following examples, which we demonstrate here.

(a) Equation 1:

\[ u_t = \left( u_x^{-1/2} \right)_{xx} + u_x^{3/2} \]

\[ \Omega = \frac{1}{2} \left( \begin{array}{cc} \eta dx - \eta^2 u_x^{-1/2} dt - \eta e^{-u} dx + B_1 dt \\ \eta e^u dx + C_1 dt - \eta dx + \eta^2 u_x^{-1/2} dt \end{array} \right) \]

(b) Equation 2:

\[ 2u_t - 2uu_x - u_{xx} = 0 \]

\[ \Omega = \frac{1}{2} \left( \begin{array}{cc} \eta dx - \eta u_x dt - \eta u_x^{1/2} dt + \eta u_x^{1/2} dt \\ (u^2 + u^2) dt - u_x dt \end{array} \right) \]

(c) Equation 3:

\[ u_{xt} = \sinh u, \]

\[ \Omega = \frac{1}{2} \left( \begin{array}{cc} \eta dx - \frac{1}{\eta} \cosh u dt - \frac{1}{\eta} \sinh u dt \\ u_x dx - \frac{1}{\eta} \sinh u dt - \eta dx - \frac{1}{\eta} \cosh u dt \end{array} \right) \]
(d) Equation 4:
\[
 u_{xt} = -2e^u, \quad (10)
\]
\[
 \Omega = \frac{1}{2} \begin{pmatrix}
 \eta dx - \frac{1}{\eta} e^u dt & u_x dx + \frac{1}{\eta} e^u dt \\
 u_x dx - \frac{1}{\eta} e^u dt & -\eta dx + \frac{1}{\eta} e^u dt
 \end{pmatrix}.
\]
\[
 \Omega = \frac{1}{2} \left( \begin{array}{cc}
 \eta dx - \frac{1}{\eta} e^u dt & u_x dx + \frac{1}{\eta} e^u dt \\
 u_x dx - \frac{1}{\eta} e^u dt & -\eta dx + \frac{1}{\eta} e^u dt
 \end{array} \right). \quad (11)
\]

(c) Equation 5:
\[
 u_t = u_{xxx} + \left( a + u^2 \right) u_x, \quad (12)
\]
where \( a \) is a constant,
\[
 \Omega = \frac{1}{2} \left( \begin{array}{cc}
 \eta dx + \left( \eta^2 + \frac{\eta u}{3} + \alpha \eta \right) dt & -\sqrt{\frac{\eta}{3}} u dx + B dx dt \\
 \sqrt{\frac{\eta}{3}} u dx + C dx dt & -\eta dx - \left( \eta^2 + \frac{\eta u}{3} + \alpha \eta \right) dt
 \end{array} \right). \quad (13)
\]

where \( B = \sqrt{\frac{2}{3}} \left( \frac{\eta}{2} u + \eta u + u^3/3 + u_{xx} + au \right), C = \sqrt{\frac{2}{3}} \left( \eta^2 u + u^3/3 - \eta u + u_{xx} + au \right) \).

(f) Equations 6, [33]:
\[
 [u_t - (a g(u) + \beta) u_x]_x = -g'(u), \quad (14)
\]
where \( g(u) \) satisfies \( g'' + \mu g = \theta \), such that \( \xi^2 = a \eta^2 + \mu \),
\[
 \Omega = \frac{1}{2} \left( \begin{array}{cc}
 \eta dx + \left( \frac{\xi x - \theta}{\eta} + \beta \eta \right) dt & -\xi u dx + \left( \xi (\alpha g + \beta) u - \frac{\xi}{\eta} \right) dt \\
 \xi u dx + \left( \xi (\alpha g + \beta) u_x - \frac{\xi}{\eta} \right) dt & -\eta dx - \left( \frac{\xi x - \theta}{\eta} + \beta \eta \right) dt
 \end{array} \right). \quad (15)
\]

(g) Equations 7, [25]:
\[
 [u_t - (a g(u) + \beta) u_x]_x = g'(u), \quad (16)
\]
where \( g'' + \mu g = \theta \), and \( \xi^2 = a \eta^2 - \mu \),
\[
 \Omega = \frac{1}{2} \left( \begin{array}{cc}
 \eta dx + \left( \frac{\xi x - \theta}{\eta} + \beta \eta \right) dt & -\xi u dx + \left( \xi (\alpha g + \beta) u - \frac{\xi}{\eta} \right) dt \\
 \xi u dx + \left( \xi (\alpha g + \beta) u_x - \frac{\xi}{\eta} \right) dt & -\eta dx - \left( \frac{\xi x - \theta}{\eta} + \beta \eta \right) dt
 \end{array} \right). \quad (17)
\]

Keeping in mind that the problem in (1) is played by the parameter. The equations (1), (2), and (3) are form invariant under the "gauge" transformation, are not unique for a particular NLPDE:
\[
 v \rightarrow \psi = Av, \quad \Omega \rightarrow \Omega' = dA A^{-1} + A \Omega A^{-1}, \quad \Theta \rightarrow \Theta' = A \Theta A^{-1}, \quad (18)
\]
where \( A \) is \( 2 \times 2 \) matrix,
\[
 \det A = 1. \quad (19)
\]
Integrability of (1) is,
\[
 0 = d^2 v = d\Omega \psi - \Omega d\psi = (d\Omega - \Omega \Omega') v, \quad (20)
\]
requires the vanishing of the two form:
\[
 d\Omega - \Omega \Omega = 0. \quad (21)
\]

3 The System Describe PSS

The, [8], gave the problem and defined it by:
\[
 \omega_1 = (r + q) u dx + (C + B) dt, \\
 \omega_2 = \eta dx + 2A dt, \\
 \omega_3 = (r - q) u dx + (C - B) dt, \quad (22)
\]

Examples

(a) Equation 1.
\[
 \omega_1 = \eta \sinh u dx + \left( \eta \left( u_x^{1/2} \right) \cosh u + \eta \left( u_x^{1/2} - u_x^{1/2} \right) \sinh u \right) dt, \\
 \omega_2 = \eta dx - \eta u_x^{1/2} dt, \\
 \omega_3 = \eta \cosh u dx + \left( \eta \left( u_x^{1/2} \right) \sinh u + \eta \left( u_x^{1/2} - u_x^{1/2} \right) \cosh u \right) dt. \quad (23)
\]
\( u \) satisfies equation 10.

(b) Equation 2.
\[
 \omega_1 = u dx + \left( \frac{u_x^2}{2} + \frac{u_x}{2} \right) dt, \\
 \omega_2 = \eta dx + \frac{\eta u_x}{2} dt, \\
 \omega_3 = -\eta dx - \frac{\eta u_x}{2} dt. \quad (24)
\]
\( u \) satisfies equation 6.

(c) Equation 3.
\( u \) satisfies equation 8.

(d) Equation 4.

\[
\begin{align*}
\omega_1 &= u_x \, dx, \\
\omega_2 &= \eta \, dx + \frac{\cosh u}{\eta} \, dt, \\
\omega_3 &= \frac{\sinh u}{\eta} \, dt.
\end{align*}
\]  

(25)

\( u \) satisfies 4.

(e) Equation 5.

\[
\begin{align*}
\omega_1 &= -\eta \sqrt{\frac{2}{3}} u_x \, dt, \\
\omega_2 &= \eta \, dx + \left( \eta^2 + \frac{\eta u_x^2}{3} + a \eta \right) \, dt, \\
\omega_3 &= \sqrt{\frac{2}{3}} u_x \, dx + \sqrt{\frac{2}{3}} \left( \eta^2 u + \frac{u_x^2}{3} + u_{xx} + au \right) \, dt.
\end{align*}
\]  

(26) u satisfies 12.

(f) Equations 6.

\[
\begin{align*}
\omega_1 &= -\frac{\xi}{\eta} \, g' \, dt, \\
\omega_2 &= \eta \, dx + \left( \frac{\xi^2 \eta - \theta}{\eta} + \beta \eta \right) \, dt, \\
\omega_3 &= \xi u_x \, dx + \xi (\alpha g + \beta) u_x \, dt.
\end{align*}
\]  

(28)

(29) \( u \) satisfies equations II 16.

4 Method on Bäcklund Transforms

BTs are called classical transformations whose solutions to the same equation are related to the self-transformation of Bäcklund (SBT), that the solutions related to the sg equation are found other transformations related to the solutions of specific equations in, [17], [19], [34], [35].

These conversions are called BTS.

After the classical theory of Bäcklund has arisen in the study of pss.

A NLEE's soliton solutions can be obtained using this method. In the paragraphs that follow, we demonstrate how some NLEEs that describe pss can have their BTs determined systematically using the geometrical features of pss.

Proposition 4.1. On a smooth Riemannian surface M2, given a coframe \( _1, _2, \) and related connection one-form \( _3, \) there exists a new coframe \( _1', _2', \) and new connection one-form \( _3' \) that satisfy the conditions listed below:

\[
\begin{align*}
d\omega_1 &= 0, \\
d\omega_2 &= \omega_3 \omega_1', \\
\omega_3 + \omega_2' &= 0,
\end{align*}
\]  

(30)

we give a proof of, [28].

Proof:

\[
\begin{align*}
\omega_1 &= \omega_1 \cos \phi - \omega_2 \sin \phi, \\
\omega_2 &= \omega_2 \sin \phi + \omega_1 \cos \phi, \\
\omega_3 &= \omega_3 - d\phi.
\end{align*}
\]  

(31)

\( \omega_1', \omega_2', \omega_3' \) satisfying 30.

\[
\begin{align*}
\omega_3' - d\phi + \omega_1' \sin \phi + \omega_2' \cos \phi &= 0,
\end{align*}
\]  

(32)

if \( u_t = F(u, u_x, \ldots, u_x^k) \) one-forms \( \omega_1 = f_1 \, dx + f_12 \, dt, \) (30) and (32)

\[
\begin{align*}
\omega_3 - d\phi + \omega_1' \sin \phi + \omega_2' \cos \phi &= 0,
\end{align*}
\]  

(33)

is completely integrable for \( \phi(x, t) \) whenever \( u(x, t) \) is a local solution of \( u_t = F(u, u_x, \ldots, u_x^k), \) [2], [36]. □
Proposition 4.2. Let $u_t = F(u, u_x, \ldots, u_{x^k})$ be a NLEE which describe a pss with associated one-forms (1.2). Then, for each solution $u(x, t)$ of $u_t = F(u, u_x, \ldots, u_{x^k})$, the system of equations for $\phi(x, t)$,

$$\phi_t - f_{31} + f_{31} \sin \phi + q \cos \phi = 0, \quad \phi_t - f_{32} + f_{32} \sin \phi + f_{32} \cos \phi = 0,$$

(34) for each solution $u(x, t)$ of $u_t = F(u, u_x, \ldots, u_{x^k})$.

Using

$$\cos \phi = \frac{2\Gamma}{1 + \Gamma^2},$$

(36) where

$$\Gamma = \frac{\nu_1}{\nu_2},$$

(37) then (34) is:

$$\frac{\partial \Gamma}{\partial x} = \eta \Gamma + \frac{1}{2} f_{11} \left(1 - \Gamma^2\right) - \frac{1}{2} f_{31} \left(1 + \Gamma^2\right),$$

(38)

$$\frac{\partial \Gamma}{\partial t} = f_{22} \Gamma + \frac{1}{2} f_{12} \left(1 - \Gamma^2\right) - \frac{1}{2} f_{32} \left(1 + \Gamma^2\right).$$

(39)

Construction $u'(x)$ is the next step.

$$u'(x) = u(x) + f(\Gamma, \eta).$$

(40)

(a) BT for equation 4.

In (10) defined by

$$f_{11} = u_x, \quad f_{12} = 0,$$

$$f_{21} = \eta, \quad f_{22} = -\frac{e^u}{\eta},$$

$$f_{31} = 0, \quad f_{32} = -\frac{\eta}{\eta},$$

(41)

Then (38) becomes

$$\frac{\partial \Gamma}{\partial x} = \eta \Gamma + \frac{u_x}{2} \left(1 - \Gamma^2\right).$$

(42)

If we choose $\Gamma'$ and $u'$ as

$$\Gamma' = \frac{1}{\Gamma},$$

$$u' = -u - 2 \ln \frac{1 - \Gamma}{1 + \Gamma},$$

(43)

then the BT:

$$(u' - u)_x = 2\eta \sinh \frac{1}{2}(u' + u), \quad (u' + u)_t = \frac{2}{\eta} e^{1/2(u' - u)},$$

(44)

Equation (41)'s $f_{11}, f_{22},$ and $f_{32}$ serve as the BT for Liouville's equation (4) in equation (44).

For equation 2, use (b) BT.

The functions for any Burgers' equation (6) solution $u(x,t)$.

$$f_{11} = u, \quad f_{12} = \left(\frac{u_x^2 + \frac{u_x}{2}}{2}\right),$$

$$f_{21} = \eta, \quad f_{22} = \frac{\eta u}{2},$$

$$f_{31} = -\eta, \quad f_{32} = -\frac{\eta u}{2}.$$  

(45)

Then (38) becomes, [29]

$$\frac{\partial \Gamma}{\partial x} = \frac{\eta}{2} \left(1 + 2\Gamma + \Gamma^2\right) + \frac{u}{2} \left(1 - \Gamma^2\right).$$

(46)

If we choose $\Gamma'$ and $u'$ as

$$\Gamma' = \frac{1}{\Gamma'},$$

$$u' = -u + 4 \frac{\partial}{\partial x} \tanh^{-1} \Gamma,$$

(47)

the BT:

$$(w' - w)_x = \frac{\eta}{2} \left[1 + \sinh 2(w' + w) + 2 \sinh^2(w' + w)\right],$$

$$(w' + w)_t = 4w_x^2 + 4w_{xx} + 2\eta w_x e^{2(w' + w)}.$$  

(48)

(c) Equation 3.

$u(x, t)$ defined by, [30]

$$f_{11} = u_x, \quad f_{12} = 0,$$

$$f_{21} = \eta, \quad f_{22} = \frac{\cosh u}{\eta},$$

$$f_{31} = 0, \quad f_{32} = \frac{\sinh u}{\eta}.$$  

(49)

Then (38) becomes [34]

$$\frac{\partial \Gamma}{\partial x} = \eta \Gamma + \frac{u_x}{2} \left(1 - \Gamma^2\right).$$

(50)

$\Gamma'$ and $u'$ as

$$\Gamma' = \frac{1}{\Gamma},$$

$$u' = -u + 4 \tanh^{-1} \Gamma,$$

(51)

we get the BT:
c) BT for an equation 1.

(4) defined by

\[ f_{11} = \eta \sinh u, \quad f_{22} = \left[ \eta \left( u^{1/2} \right) \cosh u + \eta \left( u^{1/2} - \eta u^{-1/2} \right) \sinh u \right], \]
\[ f_{31} = \eta u_x, \quad f_{32} = \left[ \eta \left( u^{1/2} \right) \sinh u + \eta \left( u^{1/2} - \eta u^{-1/2} \right) \cosh u \right]. \]

(53)

Then (38), [32]

\[ \frac{\partial \Gamma'}{\partial x} = \frac{\eta}{2} u - \frac{\eta}{2} e^{-u} + \frac{\eta}{2} e^{u} \Gamma', \]

(54)

choose \( \Gamma' \) and \( u' \) as

\[ \Gamma' = \Gamma, \quad u' = \frac{-\pi}{2} - 2 \ln \Gamma, \]

(55)

then we get the BT:

\[ (u' + u)_x = -2 \eta + 2 \eta \cosh \frac{u' - u}{2}, \]
\[ (u' + u)_y = 2 \eta^2 u_x^{1/2} - 2 \eta \left( u_x^{-1/2} \right) \sinh \frac{u' - u}{2} + 2 \eta \left[ u_x^{1/2} - \eta u_x^{-1/2} \right] \cosh \frac{u' - u}{2}. \]

(56)

the values given in (53) for \( f_{11}, \ f_{22}, \) and \( f_{32} \)

For an equation 5, (e) BT.

We take into consideration the \( u(x,t) \) functions defined by for (12);

\[ f_{11} = 0, \quad f_{12} = -\eta \sqrt{\frac{2}{3}} u_x, \]
\[ f_{21} = \eta u, \quad f_{22} = \left( \eta^2 + \frac{\eta u_x^2}{3} + a \eta \right), \]
\[ f_{31} = \sqrt{\frac{2}{3}} u, \quad f_{32} = \sqrt{\frac{2}{3}} \left( \eta^2 u + \frac{u_x^2}{3} + u_x + a u \right). \]

(57)

Then (38), [33]

\[ \frac{\partial \Gamma}{\partial x} = \frac{\eta}{2} u - \frac{1}{2} u \left( 1 + \Gamma^2 \right), \]

(58)

choose \( \Gamma' \) and \( u' \) as

\[ \Gamma' = \frac{1}{\Gamma}, \quad u' = u + 2 \sqrt{\frac{3}{6}} \frac{\partial}{\partial x} \tan^{-1} \Gamma, \]

(59)

we get the BT:

\[ (u' + u)_x = \frac{\eta}{2} \sinh 2(u' - u), \]
\[ (u' - u)_x = -\left( 2 u_x + 16 u_x + 2 \eta u_x + \frac{a u_x}{\sqrt{6}} \right) + \frac{1}{2} \left( \eta^2 + 8 \eta u_x + a \eta \right) \sinh 2(u' - u) - 2 \eta u_x \cos 2(u' - u). \]

(60)

we put \( u' = 2 \sqrt{6} w_x \) and \( u = 2 \sqrt{6} w_x \).

Equation (60) with \( f_{11}, \ f_{22}, \) and \( f_{32} \) at (57).

(f) the equations 6.

equations (6) (14), the functions

\[ f_{11} = 0, \quad f_{12} = -\frac{s}{\eta} g', \]
\[ f_{21} = \eta, \quad f_{22} = \left( \frac{\eta^2 - \frac{s}{\eta} \theta}{\eta} + \beta \eta \right), \]
\[ f_{31} = \theta u_x, \quad f_{32} = \theta \left( \alpha u + \beta \right) u_x. \]

(61)

Then (38) becomes, [35]

\[ \frac{\partial \Gamma}{\partial x} = \frac{\eta}{2} u_x \left( 1 + \Gamma^2 \right), \]

(62)

choose \( \Gamma' \) and \( u' \) as

\[ \Gamma' = \frac{1}{\Gamma}, \quad u' = u + \frac{4}{\xi} \tan^{-1} \Gamma, \]

then we get:

\[ (u' + u)_x = \frac{2 \eta}{\xi} \sinh \frac{1}{2} \left( u' - u \right), \]
\[ (u' - u)_x = \frac{2}{\xi} \left( \frac{\xi^2 - \theta}{\eta} + \beta \eta \right) \sinh \frac{1}{2} \left( u' - u \right) - 2 \left( \alpha u + \beta \right) u_x \]
\[ + \frac{2}{\eta} g' \cos \xi \frac{1}{2} \left( u' - u \right). \]

(64)

(g) the equations 7.

\( u(x, t) \) defined by
Then (38) becomes
\[ f_{11} = \xi u_x, \quad f_{12} = \xi(\alpha g + \beta) u_x, \]
\[ f_{21} = \eta, \quad f_{22} = \left( \frac{\xi^2 \eta - \theta}{\eta} + \beta \eta \right), \]
\[ f_{31} = 0, \quad f_{32} = \frac{\xi}{\eta} g'. \]

Then (38) becomes
\[ \frac{\partial \Gamma}{\partial x} = \eta \Gamma + \frac{\xi}{2} u_x (1 - \Gamma^2), \]
choose \( \Gamma' \) and \( u' \) as
\[ \Gamma' = \frac{1}{\Gamma}, \]
\[ u' = -u + \frac{4}{\xi} \tanh^{-1} \Gamma, \]
we get the BT:
\[
\begin{align*}
(u' - u)_x &= \frac{2 \eta}{\xi} \sinh \frac{\xi}{2} (u' + u), \\
(u' + u)_x &= \frac{2}{\xi} \left( \frac{\xi^2 \eta - \theta}{\eta} + \beta \eta \right) \sinh \frac{\xi}{2} (u' + u) + 2(\alpha g + \beta) u_x, \\
& \quad - \frac{2}{\eta} g' \cosh \frac{\xi}{2} (u' + u).
\end{align*}
\]
Equation (68) is the BT for the equations (16) with \( f_{11}, f_{22}, \) and \( f_{32} \) at (65).

5 Conclusions
The congruent triple sumudu conversion of fractional partial derivatives was also proven and in this study, the method of placing congruent triple SUMUDUDU was presented.

The congruent triple Sumudu transformation of fractional partial derivatives was also proved, and in this study, the method of placing the congruent triple SUMUDUDU was corrected and solved based on the congruent triple SUMUDUDU to solve the congruent two-dimensional BURGERS equations that are regular and irregular. Moreover, Examples were given to explain the proposed methods of solution by applying MATLAB to represent the solutions.

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Conflict of Interest
The authors declare that they have no Conflicts of Interest.

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