Conformable Triple Sumudu Transform with Applications

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Abstract: - One of the important topics in applied mathematics is the topic of integral transformations, due to their importance in electrical engineering applications, including communications in particular, and other sciences. In this work, one of the most important transformations in its three dimensions was presented, which is the triple Sumudu transform, including solving some real-life applications of physics, some of which have not been solved using such an integral transform before. In this work, we extend the Sumudu transform formula to the conformable fractional order, as well as other interesting and significant rules. The general analytical solution of a singular and nonlinear conformable fractional differential equation based on the conformable fractional Sumudu transform is also presented in this paper. The general solutions of several linear and nonhomogeneous conformable fractional differential equations can be obtained using the method we've proposed. As a result, our results reveal that our proposed method is an efficient one that can be used for solving all conformable fractional differential equations. The relationship between the Sumudu integral transform and other important and recently proposed integral transforms are also discussed. Finally, the triple Sumudu transform is used to solve boundary value problems, such as the heat equation with boundary values. The triple Sumudu integral transform is also used to solve linear partial integro-differential equations. The transform capability to handle such equations has been proven via its utilization in three applications.

Key-Words: - Triple Sumudu Transform; Sumudu Transform, Partial differential equation, Integral equation. Transform, Double Transform, partial integro-differential equations, fractional Sumudu transform , fractional equation.

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1 Introduction

It is possible to solve some ordinary differential equations, partial linear ones, and non-linear ones, using different integral transformations in their different dimensions. Solutions to the Burgers equation were presented in, [1] and in [2], the Navier-Stokes equation and similar equations in, [3]. More than one method was used to solve various equations, including the Sumudu transform in, [4] and [5], as well as Storm-Liouville problems in, [6], explaining integration coefficients in, [7], and discussing Laplace applications in, [8]. In, [9] the generalized Lyapunov type appeared, and Hermite-Hadamard variants for fractional integrals were presented in, [10]. In, [11], the transformation properties were presented and deduced. The results were discussed and presented in, [12]. The nonlinear biological model was solved in, [13], and Euler's results were presented in, [14]. All immunodynamic results appeared, [15], [16] and show random behavior at different and random values of the fractional order, [17]. Conversion of symmetry disturbance and residual energy chain path using the analytical method, multidimensional thermal equations are solved, [18]. To solve the new types of equations, the space fractional telegraph equation, the new mixed diffusion equation Yang-Abdel-Aty-Cattani, and the gas dynamics equation using the new fractional symmetry analysis conversion method, [19], [20], [21], [22]. The authors in, [23], By applying the first integration method, prepare the exact solutions to Burger's fractional time equations, [24]. By generalized two-dimensional differential transformation (DTM) to solve the Berger equations paired with fractions of space and time, at present, new concepts and properties of the corresponding derivative have been identified for more information, [25], [26], [27]. Moreover, the authors in, [28], [29] Using the Laplace Transformation matching technique, differential equations are solved. In, [30], the Laplace double congruent transformation method is used to solve partial differential equations, to obtain solutions to a regular and single-dimensional equation for a partially paired burger provided by the authors in, [31]. To determine the exact solutions of the burger fractional equations of time, the first integrated method was used in, [32].

In this work, a new method called congruent triple Sumudu will be proposed, and a solution is done by the decomposition method (CTSDM) to solve a wonderful mixture type for nonlinear equations, but the proposed methods are conformity, analysis method, and triple sumudu conversion method. This article looks at whether they can apply SUMU conformance triple structure decoding methods (CTSDM), [33].

In this article, we will solve regular and single-dimensional identical burger equations and some basic concepts and definitions of compatible derivatives will be published later in this article, [34].

2 The System of Some NLEEs

For the first time, the Korteweg-de Vries equation (KdV) equation is used in this article (ISM), [7]. Later, the authors extended it in, [23]. ISM was initially created in this article for the Schrödinger nonlinear equation (NLSE) and it was then deepened in, [6], such that it now contains a different type of NLEE. These steps are used to create the AKNS method: (i) construct a suitable, 22 linear scattering (eigenvalue) problem using the "space" variable, where the NLEE's resolution serves as the potential; (ii) choose the Eigen functions' "time" dependence so that the eigenvalues hold steady while the potential changes in accordance with the NLEEs; (iii) Select a time for self-functions such that the eigenvalues are preserved. We will quickly find a solution to the direct dispersion issue, and we need to establish how time affects the dispersion data. Rebuild the potential using the dispersion data in (iv). This section focuses on the AKNS method's first phase. As a result, each DE solution yields a scale on M2 with a constant Gaussian curvature of -1. Additionally, the aforementioned definition of DE is equal to saying that DE for you is the problem's integration condition:

$$dv = \Omega v, \qquad v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \tag{1}$$

where d denotes exterior dierentiation, v is a vector, and the 2×2 matrix $\Omega(\Omega_{ij}, i, j = 1,2)$ is traceless:

$$\operatorname{tr} \Omega = 0 \tag{2}$$

The NLPDE is given by Equation (1.2),

$$\Theta \equiv d\Omega - \Omega\Omega = 0, \qquad \Omega = \frac{1}{2}\begin{pmatrix} \omega_2 & \omega_1 - \omega_3 \\ \omega_1 + \omega_3 & -\omega_2 \end{pmatrix}, \tag{3}$$

It is the initial NLPDE that needs to be addressed. The AKNS system provided the following examples, which we demonstrate here.

(a) Equation 1:

$$u_t = \left(u_x^{-1/2}\right)_{xx} + u_x^{3/2}, \tag{4}$$

$$\Omega = \frac{1}{2}\begin{pmatrix} \eta dx - \eta^2 u_x^{-1/2} dt & -\eta e^{-u} dx + B_1 dt \\ \eta e^{u} dx + C_1 dt & -\eta dx + \eta^2 u_x^{-1/2} dt \end{pmatrix}, \tag{5}$$

where $B_1 = \eta((u_x^{-1/2})_x - u_x^{1/2} + \eta u_x^{-1/2})e^{-u}$, $C_1 = \eta((u_x^{-1/2})_x + u_x^{1/2} - \eta u_x^{-1/2})e^{u}$.

(b) Equation 2:

$$2u_t - 2uu_x - u_{xx} = 0, \tag{6}$$

$$\Omega = \frac{1}{2}\begin{pmatrix} \eta dx + \frac{\eta u}{2} dt & (u+\eta)dx + \left(\frac{\eta u}{2} + \frac{u^2}{2} + \frac{u_x}{2}\right)dt \\ (u-\eta)dx + \left(\frac{-\eta u}{2} + \frac{u^2}{2} + \frac{u_x}{2}\right)dt & -\eta dx - \frac{\eta u}{2} dt \end{pmatrix}, \tag{7}$$

(c) Equation 3:

$$u_{xt} = \sinh u, \tag{8}$$

$$\Omega = \frac{1}{2}\begin{pmatrix} \eta dx + \frac{1}{\eta}\cosh u dt & u_x dx - \frac{1}{\eta}\sinh u dt \\ u_x dx + \frac{1}{\eta}\sinh u dt & -\eta dx - \frac{1}{\eta}\cosh u dt \end{pmatrix}. \tag{9}$$

(d) Equation 4:

$$u_{xt} = -2e^{u}, \tag{10}$$

$$\Omega = \frac{1}{2}\begin{pmatrix} \eta dx - \frac{1}{\eta}e^{u}dt & u_x dx + \frac{1}{\eta}e^{u}dt \\ u_x dx - \frac{1}{2\eta}e^{u}dt & -\eta dx + \frac{1}{\eta}e^{u}dt \end{pmatrix}. \tag{11}$$

(e) Equation 5:

$$u_t = u_{xxx} + \left(a + u^2\right)u_x, \tag{12}$$

where a is a constant,

$$\Omega = \frac{1}{2}\begin{pmatrix} \eta dx + \left(\eta^3 + \frac{\eta u^2}{3} + a\eta\right)dt & -\sqrt{\frac{2}{3}}u dx + B_2 dt \\ \sqrt{\frac{2}{3}}u dx + C_2 dt & -\eta dx - \left(\eta^3 + \frac{\eta u^2}{3} + a\eta\right)dt \end{pmatrix}, \tag{13}$$

where $B_2 = \sqrt{2/3}(\eta^2 u + \eta u_x + u^3/3 + u_{xx} + au)$, $C_2 = \sqrt{2/3}(\eta^2 u + u^3/3 - \eta u_x + u_{xx} + au)$.

(f) Equations 6, [33]:

$$\left[u_t - (\alpha g(u) + \beta)u_x\right]_x = -g'(u), \tag{14}$$

where $g(u)$ satisfies $g'' + \mu g = \theta$, such that $\xi^2 = \alpha\eta^2 + \mu$,

$$\Omega = \frac{1}{2}\begin{pmatrix} \eta dx + \left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta\right)dt & -\xi u_x dx + \left(\xi(\alpha g + \beta)u_x + \frac{\xi}{\eta}g'\right)dt \\ \xi u_x dx + \left(\xi(\alpha g + \beta)u_x - \frac{\xi}{\eta}g'\right)dt & -\eta dx - \left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta\right)dt \end{pmatrix}. \tag{15}$$

(g) Equations 7, [25]:

$$\left[u_t - (\alpha g(u) + \beta)u_x\right]_x = g'(u), \tag{16}$$

where $g'' + \mu g = \theta$, and $\xi^2 = \alpha\eta^2 - \mu$,

$$\Omega = \frac{1}{2}\begin{pmatrix} \eta dx + \left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta\right)dt & \xi u_x dx + \left(\xi(\alpha g + \beta)u_x + \frac{\xi}{\eta}g'\right)dt \\ \xi u_x dx + \left(\xi(\alpha g + \beta)u_x - \frac{\xi}{\eta}g'\right)dt & -\eta dx - \left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta\right)dt \end{pmatrix}. \tag{17}$$

Keeping in mind that the problem in (1) is played by the parameter. The equations (1), (2), and (3) are form invariant under the "gauge" transformation, are not unique for a particular NLPDE:

$$v \longrightarrow v' = Av, \quad \Omega \longrightarrow \Omega' = dAA^{-1} + A\Omega A^{-1}, \quad \Theta \longrightarrow \Theta' = A\Theta A^{-1}, \tag{18}$$

where A is 2×2 matrix,

$$\det A = 1. \tag{19}$$

Integrability of (1) is,

$$0 = d^2 v = d\Omega v - \Omega dv = (d\Omega - \Omega\Omega)v, \tag{20}$$

requires the vanishing of the two form:

$$d\Omega - \Omega\Omega = 0. \tag{21}$$

3 The System Describe PSS

The, [8], gave the problem and defined it by:

$$\omega_1 = (r + q)dx + (C + B)dt,$$
$$\omega_2 = \eta dx + 2A dt,$$
$$\omega_3 = (r - q)dx + (C - B)dt, \tag{22}$$

Examples

(a) Equation 1.

$$\omega_1 = \eta \sinh u dx + \left[\eta\left(u_x^{-1/2}\right)_x \cosh u + \eta\left(u_x^{1/2} - \eta u_x^{-1/2}\right)\sinh u\right]dt,$$
$$\omega_2 = \eta dx - \eta^2 u_x^{-1/2} dt,$$
$$\omega_3 = \eta \cosh u dx + \left[\eta\left(u_x^{-1/2}\right)_x \sinh u + \eta\left(u_x^{1/2} - \eta u_x^{-1/2}\right)\cosh u\right]dt. \tag{23}$$

u satisfies equation 10.

(b) Equation 2.

$$\omega_1 = u dx + \left(\frac{u^2}{2} + \frac{u_x}{2}\right)dt,$$
$$\omega_2 = \eta dx + \frac{\eta u}{2}dt,$$
$$\omega_3 = -\eta dx - \frac{\eta u}{2}dt. \tag{24}$$

u satisfies equation 6.

(c) Equation 3.

$$\omega_1 = u_x dx,$$
$$\omega_2 = \eta dx + \frac{\cosh u}{\eta} dt,$$
$$\omega_3 = \frac{\sinh u}{\eta} dt. \tag{25}$$

u satisfies equation 8.
(d) Equation 4.

$$\omega_1 = u_x dx,$$
$$\omega_2 = \eta dx - \frac{e^u}{\eta} dt,$$
$$\omega_3 = -\frac{e^u}{\eta} dt. \tag{26}$$

u satisfies 4.
(e) Equation 5.

$$\omega_1 = -\eta\sqrt{\frac{2}{3}} u_x dt,$$
$$\omega_2 = \eta dx + \left(\eta^3 + \frac{\eta u^2}{3} + a\eta\right) dt,$$
$$\omega_3 = \sqrt{\frac{2}{3}} u dx + \sqrt{\frac{2}{3}}\left(\eta^2 u + \frac{u^3}{3} + u_{xx} + au\right) dt. \tag{27}$$

u satisfies 12.
(f) Equations 6.

$$\omega_1 = -\frac{\xi}{\eta} g' dt,$$
$$\omega_2 = \eta dx + \left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta\right) dt,$$
$$\omega_3 = \xi u_x dx + \xi(\alpha g + \beta) u_x dt. \tag{28}$$

u satisfies the equations I 14.
(g) Equations 7.

$$\omega_1 = \xi u_x dx + \xi(\alpha g + \beta) u_x dt,$$
$$\omega_2 = \eta dx + \left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta\right) dt,$$
$$\omega_3 = \xi\eta g' dt. \tag{29}$$

u satisfies equations II 16.

4 Method on Bäcklund Transforms

BTs are called classical transformations whose solutions to the same equation are related to the self-transformation of B¨acklund (SBT), that the solutions related to the sg equation are found other transformations related to the solutions of specific equations in, [17], [19], [34], [35].

These conversions are called BTS.

After the classical theory of B¨acklund has arisen in the study of pss.

A NLEE's soliton solutions can be obtained using this method. In the paragraphs that follow, we demonstrate how some NLEEs that describe pss can have their BTs determined systematically using the geometrical features of pss.

Proposition 4.1. On a smooth Riemannian surface M2, given a coframe _1, _2, and related connection one-form _3, there exists a new coframe _1', _2', and new connection one-form _3' that satisfy the conditions listed below:

$$d\overline{\omega}_1' = 0, \qquad d\overline{\omega}_2' = \overline{\omega}_2'\overline{\omega}_1', \qquad \overline{\omega}_3' + \overline{\omega}_2' = 0, \tag{30}$$

we give a proof of, [28].
Proof.

$$\overline{\omega}_1' = \overline{\omega}_1 \cos\phi - \overline{\omega}_2 \sin\phi, \qquad \overline{\omega}_2' = \overline{\omega}_1 \sin\phi + \overline{\omega}_2 \cos\phi, \qquad \overline{\omega}_3' = \overline{\omega}_3 - d\phi. \tag{31}$$

$\overline{\omega}_1', \overline{\omega}_2', \overline{\omega}_3'$ satisfying 30.

$$\overline{\omega}_3 - d\phi + \overline{\omega}_1 \sin\phi + \overline{\omega}_2 \cos\phi = 0, \tag{32}$$

if $u_t = F(u, u_x, \dots, u_{x^k})$ one-forms $\omega_i = f_{i1}dx + f_{i2}dt$, (30) and (32)

$$\omega_3 - d\phi + \omega_1 \sin\phi + \omega_2 \cos\phi = 0, \tag{33}$$

is completely integrable for $\phi(x,t)$ whenever $u(x,t)$ is a local solution of $u_t = F(u, u_x, \dots, u_{x^k})$, [2], [36]. □

Proposition 4.2. Let $u_t = F(u, u_x, \dots, u_{x^k})$ be a NLEE which describe a pss with associated one- forms (1.2). Then, for each solution $u(x,t)$ of $u_t = F(u, u_x, \dots, u_{x^k})$, the system of equations for $\phi(x,t)$,

$$\phi_x - f_{31} + f_{11}\sin\phi + \eta\cos\phi = 0, \qquad \phi_t - f_{32} + f_{12}\sin\phi + f_{22}\cos\phi = 0, \tag{34}$$

$u_t = F(u, u_x, \dots, u_{x^k})$

$$(f_{11}\cos\phi - \eta\sin\phi)dx + (f_{12}\cos\phi - f_{22}\sin\phi)dt \tag{35}$$

using

$$\cos\phi = \frac{2\Gamma}{1+\Gamma^2}, \tag{36}$$

where

$$\Gamma = \frac{\nu_1}{\nu_2}, \tag{37}$$

then (34) is :

$$\frac{\partial\Gamma}{\partial x} = \eta\Gamma + \frac{1}{2}f_{11}\left(1-\Gamma^2\right) - \frac{1}{2}f_{31}\left(1+\Gamma^2\right), \tag{38}$$

$$\frac{\partial\Gamma}{\partial t} = f_{22}\Gamma + \frac{1}{2}f_{12}\left(1-\Gamma^2\right) - \frac{1}{2}f_{32}\left(1+\Gamma^2\right). \tag{39}$$

Construction u' (x) is the next step.

$$u'(x) = u(x) + f(\Gamma,\eta). \tag{40}$$

(a) BT for equation 4.

In (10) defined by

$$f_{11} = u_x, \qquad f_{12} = 0,$$

$$f_{21} = \eta, \qquad f_{22} = -\frac{e^u}{\eta},$$

$$f_{31} = 0, \qquad f_{32} = -\frac{e^u}{\eta}, \tag{41}$$

Then (38) becomes

$$\frac{\partial\Gamma}{\partial x} = \eta\Gamma + \frac{u_x}{2}\left(1-\Gamma^2\right). \tag{42}$$

If we choose Γ' and u' as

$$\Gamma' = \frac{1}{\Gamma},$$

$$u' = -u - 2\ln\frac{1-\Gamma}{1+\Gamma}, \tag{43}$$

then the BT:

$$(u'-u)_x = 2\eta\sinh\frac{1}{2}(u'+u), \qquad (u'+u)_t = \frac{2}{\eta}e^{1/2(u-u')}. \tag{44}$$

Equation (41)'s f_11, f_22, and f_32 serve as the BT for Liouville's equation (4) in equation (44).

For equation 2, use (b) BT.

The functions for any Burgers' equation (6) solution u(x,t).

$$f_{11} = u, \qquad f_{12} = \left(\frac{u^2}{2} + \frac{u_x}{2}\right),$$

$$f_{21} = \eta, \qquad f_{22} = \frac{\eta u}{2},$$

$$f_{31} = -\eta, \qquad f_{32} = -\frac{\eta u}{2}. \tag{45}$$

Then (38) becomes, [29]

$$\frac{\partial\Gamma}{\partial x} = \frac{\eta}{2}\left(1+2\Gamma+\Gamma^2\right) + \frac{u}{2}\left(1-\Gamma^2\right). \tag{46}$$

If we choose Γ' and u' as

$$\Gamma' = \frac{1}{\Gamma},$$

$$u' = -u + 4\frac{\partial}{\partial x}\tanh^{-1}\Gamma, \tag{47}$$

the BT:

$$(w'-w)_x = \frac{\eta}{2}\left[1 + \sinh 2(w'+w) + 2\sinh^2(w'+w)\right],$$

$$(w'+w)_t = 4w_x^2 + 4w_{xx} + 2\eta w_x e^{2(w+w')}, \tag{48}$$

(c) Equation 3.

$u(x,t)$ defined by, [30]

$$f_{11} = u_x, \qquad f_{12} = 0,$$

$$f_{21} = \eta, \qquad f_{22} = \frac{\cosh u}{\eta},$$

$$f_{31} = 0, \qquad f_{32} = \frac{\sinh u}{\eta}, \tag{49}$$

Then (38) becomes [34]

$$\frac{\partial\Gamma}{\partial x} = \eta\Gamma + \frac{u_x}{2}\left(1-\Gamma^2\right). \tag{50}$$

Γ' and u' as

$$\Gamma' = \frac{1}{\Gamma},$$

$$u' = -u + 4\tanh^{-1}\Gamma, \tag{51}$$

we get the BT:

$$(u'-u)_x = 2\eta \sinh\frac{(u'+u)}{2},$$
$$(u'+u)_t = \frac{-2}{\eta}\left[\sinh u \cosh\frac{(u'+u)}{2} - \cosh u \sinh\frac{(u'+u)}{2}\right]. \tag{52}$$

c) BT for an equation 1.
(4) defined by

$$f_{11} = \eta \sinh u, \qquad f_{12} = \left[\eta\left(u_x^{-1/2}\right)_x \cosh u + \eta\left(u_x^{1/2} - \eta u_x^{-1/2}\right)\sinh u\right],$$
$$f_{21} = \eta, \qquad f_{22} = -\eta^2 u_x^{-1/2},$$
$$f_{31} = \eta \cosh u, \qquad f_{32} = \left[\eta\left(u_x^{-1/2}\right)_x \sinh u + \eta\left(u_x^{1/2} - \eta u_x^{-1/2}\right)\cosh u\right]. \tag{53}$$

Then (38), [32]

$$\frac{\partial \Gamma}{\partial x} = \eta\Gamma - \frac{-\eta}{2}e^{-u} - \frac{\eta}{2}e^{u}\Gamma^2. \tag{54}$$

choose Γ' and u' as

$$\Gamma' = \Gamma,$$
$$u' = -u - 2\ln\Gamma, \tag{55}$$

then we get the BT:

$$(u'+u)_x = -2\eta + 2\eta\cosh\frac{(u'-u)}{2},$$
$$(u'+u)_t = 2\eta^2 u_x^{-1/2} - 2\eta\left(u_x^{-1/2}\right)_x \sinh\frac{(u'-u)}{2}$$
$$+ 2\eta\left[u_x^{1/2} - \eta u_x^{-1/2}\right]\cosh\frac{(u'-u)}{2}. \tag{56}$$

the values given in (53) for f_{11}, f_{22}, and f_{32}
For an equation 5, (e) BT.
We take into consideration the u(x,t) functions defined by for (12);

$$f_{11} = 0, \qquad f_{12} = -\eta\sqrt{\frac{2}{3}}u_x,$$
$$f_{21} = \eta, \qquad f_{22} = \left(\eta^3 + \frac{\eta u^2}{3} + a\eta\right),$$
$$f_{31} = \sqrt{\frac{2}{3}}u, \qquad f_{32} = \sqrt{\frac{2}{3}}\left(\eta^2 u + \frac{u^3}{3} + u_{xx} + au\right), \tag{57}$$

Then (38), [33]

$$\frac{\partial \Gamma}{\partial x} = \eta\Gamma - \sqrt{\frac{1}{6}}u\left(1+\Gamma^2\right). \tag{58}$$

choose Γ' and u' as

$$\Gamma' = \frac{1}{\Gamma},$$
$$u' = u + 2\sqrt{6}\frac{\partial}{\partial x}\tan^{-1}\Gamma, \tag{59}$$

we get the BT:

$$(w'+w)_x = \frac{\eta}{2}\sin 2(w'-w),$$
$$(w'-w)_t = -\left(2w_{xxx} + 16w_x^3 + 2\eta^2 w_x + \frac{a\eta}{\sqrt{6}}\right)$$
$$+ \frac{1}{2}\left(\eta^3 + 8\eta w_x^2 + a\eta\right)\sin 2(w'-w) - 2\eta w_{xx}\cos 2(w'-w). \tag{60}$$

we put $u' = 2\sqrt{6}w_x'$ and $u = 2\sqrt{6}w_x$.
Equation (60) with f_{11}, f_{22}, and f_{32} at (57).
(f) the equations 6.
equations 6 (14), the functions

$$f_{11} = 0, \qquad f_{12} = -\frac{\xi}{\eta}g',$$
$$f_{21} = \eta, \qquad f_{22} = \left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta\right),$$
$$f_{31} = \xi u_x, \qquad f_{32} = \xi(\alpha g + \beta)u_x. \tag{61}$$

Then (38) becomes, [35]

$$\frac{\partial \Gamma}{\partial x} = \eta\Gamma - \frac{\xi}{2}u_x\left(1+\Gamma^2\right). \tag{62}$$

choose Γ' and u′ as

$$\Gamma' = \frac{1}{\Gamma},$$
$$u' = u + \frac{4}{\xi}\tan^{-1}\Gamma, \tag{63}$$

then we get:

$$(u'+u)_x = \frac{2\eta}{\xi}\sin\xi\frac{(u'-u)}{2},$$
$$(u'-u)_t = \frac{2}{\xi}\left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta\right)\sin\xi\frac{(u'-u)}{2} - 2(\alpha g + \beta)u_x$$
$$+ \frac{2}{\eta}g'\cos\xi\frac{(u'-u)}{2}. \tag{64}$$

(g) the equations 7.
$u(x,t)$ defined by

$$f_{11} = \xi u_x, \qquad f_{12} = \xi(\alpha g + \beta) u_x,$$

$$f_{21} = \eta, \qquad f_{22} = \left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta \right),$$

$$f_{31} = 0, \qquad f_{32} = \frac{\xi}{\eta} g', \tag{65}$$

Then (38) becomes

$$\frac{\partial \Gamma}{\partial x} = \eta\Gamma + \frac{\xi}{2} u_x \left(1 - \Gamma^2\right). \tag{66}$$

choose Γ' and u' as

$$\Gamma' = \frac{1}{\Gamma},$$

$$u' = -u + \frac{4}{\xi} \tanh^{-1}\Gamma, \tag{67}$$

we get the BT:

$$(u' - u)_x = \frac{2\eta}{\xi} \sinh \xi \frac{(u' + u)}{2},$$

$$(u' + u)_t = \frac{2}{\xi}\left(\frac{\xi^2 g - \theta}{\eta} + \beta\eta \right) \sinh \xi \frac{(u' + u)}{2} + 2(\alpha g + \beta) u_x - \frac{2}{\eta} g' \cosh \xi \frac{(u' + u)}{2}. \tag{68}$$

Equation (68) is the BT for the equations (16) with f_{11}, f_{22}, and f_{32} at (65).

5 Conclusions

The congruent triple sumudu conversion of fractional partial derivatives was also proven and in this study, the method of placing congruent triple SUMUDU was presented

The congruent triple Sumudu transformation of fractional partial derivatives was also proved, and in this study, the method of placing the congruent triple SUMUDU was corrected and solved based on the congruent triple SUMUDU to solve the congruent two-dimensional BURGERS equations that are regular and irregular. Moreover, Examples were given to explain the proposed methods of solution by applying MATLAB to represent the solutions

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References:

[1] M. J. Ablowitz and JB Been, Integrable Fractional Modified Korteweg-de Vries, Sine-Gordon, and Sinh-Gordon Equations, LD Carr, *J. Phys. Math A*, arXiv:2203.13755 (2022).

[2] S. S. Chern and K. Tenenblat, "Pseudospherical surfaces and evolution equations," *Studies in Applied Mathematics*, vol. 74, no. 1, pp. 55–83, 1986.

[3] Youzhi Tu, "Multi-Cuspon Solutions of the Wadati-Konno-Ichikawa Equation by Riemann-Hilbert Problem Method, *Open Journal of Applied Sciences*, 2020, 10, 100-109.

[4] Z. M. Sipus and B. Divjak, "Surfaces of Constant Curvature in the Pseudo Galilean Space," *International Journal of Mathematics and Mathematical*, Vol. 2012, Article ID 375264, 2012.

[5] D. W. Yoon, "Classification of rotational surfaces in pseudo-galilean space," *Glasnik Matemati ˇ CKI*, Vol. 50(70), 453 – 465, 2015.

[6] M. J. Ablowitz and J.T. Cole, Nonlinear optical waveguide lattices: Asymptotic analysis, solitons, and topological insulators, *Physica D.*, 40, (2022) 133440.

[7] R. Sasaki, "Soliton equations and pseudospherical surfaces," *Nuclear Physics B*, vol. 154, no. 2, pp. 343–357, 1979.

[8] A. H. Khater, D. K. Callebaut, and S. M. Sayed, "Conservation laws for some nonlinear evolution equations which describe pseudo-spherical surfaces," *Journal of Geometry and Physics*, vol. 51, no. 3, pp. 332–352, 2004.

[9] A. H. Khater, A. A. Abdalla, A. M. Shehatah, D. K. Callebaut, and S. M. Sayed, "Bäcklund transformations and exact solutions for self-dual *SU*(3) Yang-Mills fields," *Il Nuovo Cimento della Società Italiana di Fisica B*, vol. 114, no. 1, pp. 1–10, 1999.

[10] A. H. Khater, D. K. Callebaut, and O. H. El-Kalaawy, "Bäcklund transformations and exact soliton solutions for nonlinear Schrödinger-type equations," *Il Nuovo Cimento della Società Italiana di Fisica B*, vol. 113, no. 9, pp. 1121–1136, 1998.

[11] A. H. Khater, D. K. Callebaut, and R. S. Ibrahim, "Bäcklund transformations and Painlevé analysis: exact soliton solutions for the unstable nonlinear Schrödinger equation modeling electron beam plasma," *Physics of Plasmas*, vol. 5, no. 2, pp. 395–400, 1998.

[12] A. H. Khater, D. K. Callebaut, and A. R. Seadawy, "Nonlinear dispersive instabilities in Kelvin-Helmholtz magnetohydrodynamic flow," *Physica Scripta*, vol. 67, pp. 340–349, 2003.

[13] A. H. Khater, O. H. El-Kalaawy, and D. K. Callebaut, "B¨acklund transformations for Alfvén solitons in a relativistic electron-positron plasma," *Physica Scripta*, vol. 58, pp. 545–548, 1998.

[14] K. Chadan and P. C. Sabatier, *Inverse Problems in Quantum Scattering Theory*, Springer, New York, NY, USA, 1977.

[15] A. H. Khater, W. Malfliet, D. K. Callebaut, and E. S. Kamel, "Travelling wave solutions of some classes of nonlinear evolution equations in (1+1) and (2+1) dimensions," *Journal of Computational and Applied Mathematics*, vol. 140, no. 1-2, pp. 469–477, 2002.

[16] K. Konno and M.Wadati, "Simple derivation of B¨acklund transformation from Riccati form of inverse method," *Progress of Theoretical Physics*, vol. 53, no. 6, pp. 1652–1656, 1975.

[17] M. Marvan, "Scalar second-order evolution equations possessing an irreducible *sl*2-valued zero curvature representation," *Journal of Physics A*, vol. 35, no. 44, pp. 9431–9439, 2002.

[18] R. M. Miura, *B¨acklund Transformations, the Inverse Scattering Method, Solitons, and Their Applications*, vol. 515 of *Lecture Notes in Mathematics*, Springer, New York, NY, USA, 1976.

[19] E. G. Reyes, "Pseudo-spherical surfaces and integrability of evolution equations," *Journal of Differential Equations*, vol. 147, no. 1, pp. 195–230, 1998.

[20] E. G. Reyes, "Conservation laws and Calapso-Guichard deformations of equations describing pseudo-spherical surfaces," *Journal of Mathematical Physics*, vol. 41, no. 5, pp. 2968–2989, 2000.

[21] A. V. Shchepetilov, "The geometric sense of the Sasaki connection," *Journal of Physics A*, vol. 36, no. 13, pp. 3893–3898, 2003.

[22] V. E. Zakharov and A. B. Shabat, "Exact theory of two-dimensional self-focusing and one-dimensional self–modulation of waves in nonlinear Media," *Soviet Physics*, vol. 34, pp. 62–69, 1972.

[23] J. A. Cavalcante and K. Tenenblat, "Conservation laws for nonlinear evolution equations," *Journal of Mathematical Physics*, vol. 29, no. 4, pp. 1044–1049, 1988.

[24] R. Beals, M. Rabelo, and K. Tenenblat, "Bäcklund transformations and inverse scattering solutions for some pseudospherical surface equations," *Studies in Applied Mathematics*, vol. 81, no. 2, pp. 125–151, 1989.

[25] M. Crampin, "Solitons and SL(2, R)," *Physics Letters A*, vol. 66, no. 3, pp. 170–172, 1978.

[26] A. C. Scott, F. Y. F. Chu, and D. W. McLaughlin, "The soliton: a new concept in applied science," *Proceedings of the IEEE*, vol. 61, pp. 1443–1483, 1973.

[27] E. G. Reyes, "On Geometrically integrable equations and Hierarchies of pseudospherical type," *Contemporary Mathematics*, vol. 285, pp. 145–155, 2001.

[28] M. L. Rabelo and K. Tenenblat, "A classification of pseudospherical surface equations of type $u_t = u_{xxx} + G(u, u_x, u_{xx})$," *Journal of Mathematical Physics*, vol. 33, no. 2, pp. 537–549, 1992.

[29] A. Sakovich and S. Sakovich, "On transformations of the Rabelo equations," *SIGMA. Symmetry, Integrability and Geometry*, vol. 3, pp. 1–8, 2007.

[30] M. J. Ablowitz, R. Beals, and K. Tenenblat, "On the solution of the generalized wave and generalized sine-Gordon equations," *Studies in Applied Mathematics*, vol. 74, no. 3, pp. 177–203, 1986.

[31] S. M. Sayed, O. O. Elhamahmy, and G. M. Gharib, "Travelling wave solutions for the KdV-Burgers-Kuramoto and nonlinear Schr¨odinger equations which describe pseudospherical surfaces," *Journal of Applied Mathematics*, vol. 2008, Article ID 576783, 10 pages, 2008.

[32] S. M. Sayed, A. M. Elkholy, and G. M. Gharib, "Exact solutions and conservation laws for Ibragimov-Shabat equation which describe a pseudo-spherical surface," *Computational & Applied Mathematics*, vol. 27, no. 3, pp. 305–318, 2008.

[33] K. Tenenblat, *Transformations of Manifolds and Applications to Differential Equations*, vol. 93 of *Pitman Monographs and Surveys in Pure and Applied Mathematics*, Addison Wesley Longman, Harlow, UK, 1998.

[34] M. C. Nucci, "Pseudopotentials, Lax equations and Bäcklund transformations for nonlinear evolution equations," *Journal of Physics A*, vol. 21, no. 1, pp. 73–79, 1988.

[35] S. M. Sayed and G. M. Gharib, “Canonical reduction of self-dual Yang-Mills equations to Fitzhugh-Nagumo equation and exact solutions,” *Chaos, Solitons and Fractals*, vol. 39, no. 2, pp. 492–498, 2009.

[36] Radwan Abu-Gdairi and Mohammed H. Al-smadi, An Efficient Computational Method for 4th-order Boundary Value Problems of Fredholm IDEs, Applied Mathematical Sciences, Vol. 7, 2013, no. 96, pp.4791-4774.

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