General Formulas in the Table of $\bar{g} - Derivatives$ for $\bar{g} - Functions$

DHURATA VALERA[1,*], AGRON TATO[2]
[1]Department of Mathematics, Faculty of Natural Sciences,
University of Elbasan "Aleksandër Xhuvani",
Elbasan,
ALBANIA

[2]Polytechnic University of Tirana,
Tirana,
ALBANIA

Abstract: - The consistent system of pseudo-arithmetical operations and its generator $\bar{g}$ are applied to $\bar{g} - functions$ that are derived as solutions of some functional equations. Some interesting classes of $\bar{g} - functions$ are built and the $\bar{g} - calculus$ for real functions marks a new development over the years. But, $\bar{g} - derivative$ opens a new perspective highlighting the fundamental properties of these $\bar{g} - transform$ functions. Based on the fundamental properties of these $\bar{g} - functions$, we have further outlined our study in verifying other properties for pseudo-linearity, pseudo-nonlinearity and generalization of the table of $\bar{g} - derivative$ for these transformed functions, with some pseudo-derivative identities as Pseudo-Basic Properties/Pseudo-General Rules. The table of $\bar{g} - derivative$ for the $\bar{g} - functions$ is built as a first attempt and equipped with the Pseudo-Linearity, the Constant Pseudo-Term, the Pseudo-Product, the Pseudo-Quotient and the Pseudo-Chain Rule as $\bar{g} - formulas$ grouped into eight cases. Further, it will be completed for different cases, as more modified functions take part in pseudo-nonlinear combinations, Elementary Transcendental Functions or $\bar{g} - table$ for $\bar{g} - integrals$ of $\bar{g} - functions$ etc., showing once again the importance of generated Pseudo-Analysis with the broad field of its applications as a generalization of Classical Analysis.

Key-Words: - Pseudo-arithmetical operations, generator, $\bar{g} - transform$, $\bar{g} - functions$, $\bar{g} - derivative$, table of $\bar{g} - derivative$, functional equations.

Received: April 15, 2023. Revised: December 2, 2023. Accepted: December 17, 2023. Published: December 31, 2023.

1 Introduction

As a generalization of the Classical Analysis, Pseudo-analysis, [1], is based on a semiring $([a,b], \oplus, \odot))$, where this structure is defined on a real interval $G = [a,b] \subset [-\infty, +\infty]$, denoting the corresponding operation $(\oplus, \odot)$ respectively as pseudo-addition and pseudo-multiplication, [2], [3], [4], [5], [6], [7].

The generator g of the binary operation $\oplus$, was extended into the odd function $\bar{g}$, such that, [7], [8], [9], [10], [11], [12]:

$$\bar{g}(x) = sgn\, x \cdot g(|x|),\ x \in [-\infty, \infty].$$

The pseudo–arithmetical operations (PAO) are extended to the whole extended real line $[-\infty, +\infty] = \bar{R}$, [4], and introduced the operations of pseudo-subtraction and pseudo-division in [13], [14], [15], so the system of PAO, generated by a special generator $\bar{g}$, is the consistent SPAO $\{\overline{\oplus}, \overline{\odot}, \overline{\ominus}, \overline{\oslash}\} = \{\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}}, \overline{\ominus}_{\bar{g}}, \overline{\oslash}_{\bar{g}}\}$, [3], [4].

The role of the consistent SPAO $\{\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}}, \overline{\ominus}_{\bar{g}}, \overline{\oslash}_{\bar{g}}\}$ generated by the generator $\bar{g}$, is shown directly by taking the rational functions, [2], [3], but $\bar{g} - Transform$ is a further development of $\bar{g} - calculus$.

The extended forms of $\bar{g} - calculus$, [3], [10] are:

$x \,\overline{\oplus}_{\bar{g}}\, y = \overline{\oplus}_{\bar{g}}\,(x,y) = \bar{g}^{\,-1}(\bar{g}(x) + \bar{g}(y))$;

$x \,\overline{\odot}_{\bar{g}}\, y = \overline{\odot}_{\bar{g}}\,(x,y) = \bar{g}^{\,-1}(g(x) \cdot \bar{g}(y))$;

$x \,\overline{\ominus}_{\bar{g}}\, y = \overline{\ominus}_{\bar{g}}\,(x,y) = \bar{g}^{\,-1}(\bar{g}(x) - \bar{g}(y))$;

$x \,\overline{\oslash}_{\bar{g}}\, y = \overline{\oslash}_{\bar{g}}\,(x,y) = \bar{g}^{\,-1}(\bar{g}(x)/\bar{g}(y))$.

The function corresponding to a function f introduced by the $g - calculus$, [2] (called $g -$

$function$ and denoted by f_g in general) are derived as solutions of some functionals equations using several results of [16], [17], [18] and four interesting classes of $\bar{g} - functions$ (f_g) are built in [2]. The $\bar{g} - calculus$ for real functions introduced in [3], and investigated in [7], mark a new development in the field of Pseudo-Analysis. Based on the fundamental properties of these $\bar{g} - functions$, [2], [9], [10], [19], [20], [21], for the first time in this paper, we have studied and verified other properties for pseudo-linearity/nonlinearity of $\bar{g} - functions$ and generalization of the table of $\bar{g} - derivative$, [3] of transformed functions, [2]. The eight exceptional $\bar{g} - derivative$ cases are considered for some $\bar{g} - transformed$ functions' pseudo-linear and pseudo-nonlinear combinations with some conditions. Also, the table of $\bar{g} - derivative$ for these $\bar{g} - functions$ is build and equipped with pseudo-derivative identities as Pseudo-Basic Properties/Pseudo-General Rules in the same way as in the Classical Analysis, [22], [23], for the derivative function.

2 Problem Formulation

The definition of $\bar{g} - derivative$ presented in [3], opens a new perspective highlighting the basic properties of all classes of $\bar{g} - functions$ as transformed functions by $\bar{g} - generators$ that are treated in [2], [10], [11]. Based on the basic properties of $\bar{g} - calculus$ for these $\bar{g} - functions$, [2], we have further outlined our study in verifying other properties for pseudo-linearity, pseudo-nonlinearity of $\bar{g} - derivative$ for combinations of some modified functions and generalization of the table of $\bar{g} - derivative$ with general formulas. Some pseudo-derivative identities are found in the form of "*The Pseudo-Rules"* for eight specific cases treated in this paper and are presented as $\bar{g} - formula$ which are listed in a table of $\bar{g} - derivative$.

The Table of Pseudo-Derivative for $\bar{g} - transform$ functions has been built and equipped with several formulas as $\bar{g} - formulas$ for the Pseudo-Linearity Rule, the Constant Pseudo-Term Rule, the Pseudo-Product Rule, the Pseudo-Quotient Rule, the Pseudo-Chain Rule, etc. This opens the line for further studies in $\bar{g} - integrals$ for $\bar{g} - functions$ and even more for the corresponding table of $\bar{g} - integrals$, [17], [18], [19], [22].

2.1 Definition and Some Relations for $\bar{g} - functions$

In this paper, the real functions are continuous from $\overline{R}$ to $\overline{R}$.

Definition 2.1.1. Let f be a function on $]a, b[\ \subseteq\ ]-\infty, +\infty[$ and the function $\bar{g}$ be a generator of the consistent system of pseudo-arithmetical operations $\{\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}}, \overline{\ominus}_{\bar{g}}, \overline{\oslash}_{\bar{g}}\}$, [2], [9], [10].

The function $f_{\bar{g}}$ given by $f_{\bar{g}}(x) = \bar{g}^{-1}\left(f\left(\bar{g}(x)\right)\right)$ for every $x \in \left(\bar{g}^{-1}(a), \bar{g}^{-1}(b)\right)$ is said to be $\bar{g} - function$ corresponding to the function f.

Based on the definition 2.1.1. of $\bar{g} - function$, can consider the pseudo-arithmetical operations generated by generator $\bar{g}$, as a modified function of arithmetic operations by $\bar{g} - Transform$, [3], [10]:

$$\overline{\oplus}_{\bar{g}}\,(x, y) = \bar{g}^{-1}\left((+)(\bar{g}(x), \bar{g}(y)\right) = f_{\bar{g}-(+)}(x, y);$$

$$\overline{\odot}_{\bar{g}}\,(x, y) = \bar{g}^{-1}\left((\cdot)(\bar{g}(x), \bar{g}(y)\right) = f_{\bar{g}-(\cdot)}(x, y);$$

$$\overline{\ominus}_{\bar{g}}\,(x, y) = \bar{g}^{-1}\left((-)(\bar{g}(x), \bar{g}(y)\right) = f_{\bar{g}-(-)}(x, y);$$

$$\overline{\oslash}_{\bar{g}}\,(x, y) = \bar{g}^{-1}\left((/)(\bar{g}(x), \bar{g}(y)\right) = f_{\bar{g}-(/)}(x, y).$$

Definition 2.1.2. Let f and k be two continuous functions $f: \overline{R} \to \overline{R}$, $h: \overline{R} \to \overline{R}$ and let $\bar{g} - generator$ be extended on $\overline{R}$ (perhaps with some undefined values). The $\bar{g} - composite\ function$ k_g from $\overline{R}$ to $\overline{R}$, for $k(x) = (f \circ h)(x)$ is a function satisfying [9]:

$$k_g(x) = \left((f \circ h)(x)\right)_g = \left(f_g \circ h_g\right)(x).$$

The composition of functions is not commutative, but associative.

Definition 2.1.3. If the function f is differentiable on $]a, b[\ \subseteq\ ]-\infty, +\infty[$ and with the same monotonicity as the function $\bar{g}$, a generator of the consistent SPAO $\{\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}}, \overline{\ominus}_{\bar{g}}, \overline{\oslash}_{\bar{g}}\}$, then we can define the $\bar{g} - derivative$ of $\boldsymbol{f}$ at the point $x \in\ ]a, b[$ as $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} f(x) = \bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f(x)\right)\right)$, when the right part is meaningful [3].

3 Problem Solution

The generalization of the table of $\bar{g} - derivative$ with general formulas for $\bar{g} - functions$ and finding some Pseudo-General Rules are the main problems treated in this session. Eight specific cases are treated for several combinations of $\bar{g} - functions$ and some pseudo-derivative identities are found as "*The Pseudo-Rules"*. These groups of identities are presented as $\bar{g} - formulas$ which are listed in, Table 2, "$\bar{g} - Formulas$ of $\bar{g} - Derivative$ for the $\bar{g} - Functions$" (Appendix 2).

We applied the definitions of $\bar{g}-calculus$, [3], [9], [20], [21], $\bar{g}-functions$, derivative and $\bar{g}-derivative$, [3], for some linear/nonlinear combinations of $\bar{g}-functions$, [2], and also considering some conditions for constants, functions and pseudo-operations that participate in relations for sum, difference, product, quotient or composition of $\bar{g}-functions$ etc.

The domain (*D*) of the sum function $\left(f_{\bar{g}}+h_{\bar{g}}\right)$, difference function $\left(f_{\bar{g}}-h_{\bar{g}}\right)$ or product function $\left(f_{\bar{g}}\cdot h_{\bar{g}}\right)$ is the intersection of the individual domains of the two functions in each combination $\left(D_{f_{\bar{g}}+h_{\bar{g}}}=D_{f_{\bar{g}}-h_{\bar{g}}}=D_{f_{\bar{g}}\cdot h_{\bar{g}}}=D_{f_{\bar{g}}}\cap D_{h_{\bar{g}}}\right)$, [23], [24], [25]. The same request for the domain of the quotient function, except the values that make the function in denominator $\left(h_{\bar{g}}\right)$ equal to zero, so the domain of the quotient function is $D_{f_{\bar{g}}/h_{\bar{g}}}=D_{f_{\bar{g}}}\cap D_{h_{\bar{g}}}$ where $D_{h_{\bar{g}}}=\{x: h_{\bar{g}}(x)\neq 0\}$.

The cases (3.1.÷ 3.8) considered in the paper are described in Leibniz's notation, [23], [24], [25].

3.1 The Constant Pseudo-Factor Rule

($\bar{g}-Derivative\ of\ a\ Constant\ Pseudo\text{-}Multiple\ with\ a\ \bar{g}-Function$)

The pseudo-derivative definition is applied to a pseudo-multiple expression of a constant with a modified function by $\bar{g}-transform$ $(f_{\bar{g}})$, [2], [3], [9], with respect to *x*, as below:

$$\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(\propto\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right)=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(\propto\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(\bar{g}^{-1}\left(\bar{g}(\propto)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)\right)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\left(\bar{g}(\propto)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)\right)\right)=$$
$$=\bar{g}^{-1}\left(\bar{g}(\propto)\cdot\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\right)\right)=$$
$$=\bar{g}^{-1}\left\{\bar{g}(\propto)\cdot\bar{g}\left(\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\right)\right)\right)\right\}=$$
$$=\bar{g}^{-1}\left(\bar{g}(\propto)\cdot\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\right)=$$
$$=\propto\overline{\odot}_{\bar{g}}\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x).$$

This formula is "*schematically the same formula*" as for the derivative function of "*the multiple of a constant with a function/the constant factor*", [23], [24], [25].

- The result 3.1. is $\bar{g}-derivative\ function\ of$ *a Constant Pseudo-Multiple with a* $\bar{g}-Function$.

3.2 The Constant Pseudo-Term Rule

($\bar{g}-Derivative\ of\ a\ Constant$)

As a particular case, the Pseudo-derivative is applied to a constant (any constant), [2], [3], [9], and we find:

$$\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\propto=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}(\propto)\right)=\bar{g}^{-1}\left(\frac{d}{dx}c\right)=$$
$$=\bar{g}^{-1}(0)=0\ \text{ where }\ \bar{g}(\propto)=c\ (constant).$$

This formula is "*schematically the same formula*" as for the derivative function of "*a constant*", [23], [24], [25].

- The result 3.2. is $\bar{g}-derivative\ function$ of "*a constant*".

3.3 The Pseudo-Sum Rule

($\bar{g}-Derivative\ of\ the\ Pseudo\text{-}Addition\ of\ two\ \bar{g}-Functions$)

The pseudo-derivative, applied to a pseudo-sum expression of two modified functions $\left(f_{\bar{g}},\ h_{\bar{g}}\right)$, [2], [3], [9], with respect to *x*, follows these steps as follows, giving us an important conclusion:

$$\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\oplus}_{\bar{g}}\,h_{\bar{g}}(x)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\,\overline{\oplus}_{\bar{g}}\,h_{\bar{g}}(x)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(\bar{g}^{-1}\left(\bar{g}\left(f_{\bar{g}}(x)\right)+\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\left(\bar{g}\left(f_{\bar{g}}(x)\right)+\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\right)+\frac{d}{dx}\bar{g}\left(h_{\bar{g}}(x)\right)\right)=$$
$$=\bar{g}^{-1}\left\{\begin{array}{l}\bar{g}\left(\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\right)\right)\right)+\\+\bar{g}\left(\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)\end{array}\right\}=$$
$$=\bar{g}^{-1}\left\{\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)+\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\right\}=$$
$$=\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\,\overline{\oplus}_{\bar{g}}\,\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x).$$

The formula is "*schematically the same formula*" as for the derivative function of "*the addition of two functions/the sum function*", [23], [24], [25].

- The result 3.3 is $\bar{g}-derivative\ function\ for$ *the Pseudo-Addition of two* $\bar{g}-Functions$.

3.4 The Pseudo-Difference Rule

($\bar{g}-Derivative$ *of the Pseudo-Subtraction of two* $\bar{g}-Functions$)

The pseudo-derivative, applied to a pseudo-difference expression of two modified functions $(f_{\bar{g}},\ h_{\bar{g}})$, [2], [3], [9], with respect to x, follows these steps below:

$$\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\ominus}_{\bar{g}}\,h_{\bar{g}}(x)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\,\overline{\ominus}_{\bar{g}}\,h_{\bar{g}}(x)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(\bar{g}^{-1}\left(\bar{g}\left(f_{\bar{g}}(x)\right)-\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\left(\bar{g}\left(f_{\bar{g}}(x)\right)-\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\right)-\frac{d}{dx}\bar{g}\left(h_{\bar{g}}(x)\right)\right)=$$
$$=\bar{g}^{-1}\left\{\begin{array}{l}\bar{g}\left(\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\right)\right)\right)-\\ -\bar{g}\left(\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)\end{array}\right\}=$$
$$=\bar{g}^{-1}\left\{\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)-\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\right\}=$$
$$=\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\,\overline{\ominus}_{\bar{g}}\,\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x).$$

The formula is "*schematically the same formula*" as for the derivative function of "*the subtraction of two functions/ the difference function*", [23], [24], [25].

- The result 3.4. is $\bar{g}-derivative\ function\ for$ *the Pseudo-Subtraction of two* $\bar{g}-Functions$.

3.5 The General Pseudo-Linearity Rule/ Property for two $\bar{g}-Functions$

($\bar{g}-Derivative$ *of the Pseudo-Linear Combination of one/two* $\bar{g}-Functions$ *and any constant*)

For pseudo-linear combinations of two modified functions $(f_{\bar{g}},\ h_{\bar{g}})$ and any constant $(\propto,\ \lambda)$, [2], [3], [9], the pseudo-derivative is applied so it can be easily verified and shown in the forms below:

- $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left[\left(\propto\overline{\odot}_{\bar{g}}\,f_{\bar{g}}(x)\right)\overline{\oplus}_{\bar{g}}\left(\lambda\,\overline{\odot}_{\bar{g}}\,h_{\bar{g}}(x)\right)\right]=$
$$=\left(\propto\overline{\odot}_{\bar{g}}\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\overline{\oplus}_{\bar{g}}\left(\lambda\,\overline{\odot}_{\bar{g}}\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right).$$
- $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left[\left(\propto\overline{\odot}_{\bar{g}}\,f_{\bar{g}}(x)\right)\overline{\oplus}_{\bar{g}}\,\lambda\right]=\propto\overline{\odot}_{\bar{g}}\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x).$

We are following the evidence shown in the four cases above. We can consider the exceptional cases of modified functions $(f_{\bar{g}},\ h_{\bar{g}})$ and constants $(\propto,\ \lambda)$ in the pseudo-linear combinations, thus reaching the cases treated in points 3.1. to 3.4. as particular cases. Remind here the derivat function of the linear-combination function $(\propto\cdot f+h\cdot\lambda)$ or $(\propto\cdot f+\lambda)$ by the derivate table in Classical Analysis, [23], [24], [25].

The formula is "*schematically the same formula/ property*" as for the derivative function of "*the linear combinations*", [23], [24], [25].

- The result 3.5. is $\bar{g}-derivative\ function\ for$ *pseudo-linear combinations of two modified functions* $(f_{\bar{g}},\ h_{\bar{g}})$ *and for any constant* $(\propto,\ \lambda)$.

3.6 The Pseudo-Product Rule

($\bar{g}-Derivative$ *of the Pseudo-Multiplication of two/ three* $\bar{g}-Functions$)

3.6.1 $\bar{g}-Derivative$ of the Pseudo-Multiplication of two $\bar{g}-Functions$

For the pseudo-multiple expression of two modified functions $(f_{\bar{g}},\ h_{\bar{g}})$ concerning x, [2], [3], [9], we calculate the pseudo-derivative function and find:

$$\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,h_{\bar{g}}(x)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,h_{\bar{g}}(x)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(\bar{g}^{-1}\left(\bar{g}\left(f_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)\right)=$$
$$=\bar{g}^{-1}\left(\frac{d}{dx}\left(\bar{g}\left(f_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)=$$
$$=\bar{g}^{-1}\left\{\begin{array}{l}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\right)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)+\\ +\left(\frac{d}{dx}\bar{g}\left(h_{\bar{g}}(x)\right)\right)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)\end{array}\right\}=$$
$$=\bar{g}^{-1}\left\{\begin{array}{l}\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)+\\ +\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)\end{array}\right\}=$$
$$=\bar{g}^{-1}\left[\begin{array}{l}\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)+\\ +\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)\end{array}\right]=$$

$$= \bar{g}^{-1}\left\{\begin{array}{l}\bar{g}\left[\bar{g}^{-1}\left(\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right]+\\ +\bar{g}\left[\bar{g}^{-1}\left(\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)\right)\right]\end{array}\right\} =$$

$$= \bar{g}^{-1}\left\{\begin{array}{l}\bar{g}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right]+\\ +\bar{g}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right]\end{array}\right\} =$$

$$= \left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right]\overline{\oplus}_{\bar{g}}$$

$$\overline{\oplus}_{\bar{g}}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right].$$

The formula is "*schematically the same formula*" as for the derivative function of "*the product of two functions*", [23], [24], [25].

- The result 3.6.1. is $\bar{g}-derivative\ function$ *of the Pseudo-Multiplication of two* $\bar{g}-Functions$.

3.6.2 $\bar{g}-Derivative$ of the Pseudo-Multiplication of three $\bar{g}-Functions$

First, we must note that the domain of the product function $\left(f_{\bar{g}}\cdot h_{\bar{g}}\cdot k_{\bar{g}}\right)$ is the intersection of the individual domain $(D_{f_{\bar{g}}}, D_{h_{\bar{g}}}, D_{k_{\bar{g}}})$ of the three functions $\left(D_{f_{\bar{g}}\cdot h_{\bar{g}}\cdot k_{\bar{g}}} = D_{f_{\bar{g}}}\cap D_{h_{\bar{g}}}\cap D_{k_{\bar{g}}}\right)$, [23], [24], [25]. The pseudo-derivative, applied to a pseudo-multiplication expression of three modified functions $\left(f_{\bar{g}}, h_{\bar{g}}, k_{\bar{g}}\right)$ with respect to *x*, [2], [3], [9], follows the steps below, giving us an important conclusion:

$$\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, k_{\bar{g}}(x)\right) =$$

$$= \bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, k_{\bar{g}}(x)\right)\right) =$$

$$= \bar{g}^{-1}\left\{\frac{d}{dx}\bar{g}\left[\left(\bar{g}^{-1}\left(\begin{array}{c}\bar{g}\left(f_{\bar{g}}(x)\right)\cdot\\ \cdot\,\bar{g}\left(h_{\bar{g}}(x)\right)\end{array}\right)\right)\overline{\odot}_{\bar{g}}\, k_{\bar{g}}(x)\right]\right\} =$$

$$= \bar{g}^{-1}\left\{\frac{d}{dx}\bar{g}\left[\left(\bar{g}^{-1}\left[\begin{array}{c}\bar{g}\left(\bar{g}^{-1}\left(\begin{array}{c}\bar{g}\left(f_{\bar{g}}(x)\right)\cdot\\ \cdot\,\bar{g}\left(h_{\bar{g}}(x)\right)\end{array}\right)\right)\cdot\\ \cdot\,\bar{g}\left(k_{\bar{g}}(x)\right)\end{array}\right]\right)\right]\right\} =$$

$$= \bar{g}^{-1}\left\{\frac{d}{dx}\left(\bar{g}\left(f_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)\cdot\bar{g}\left(k_{\bar{g}}(x)\right)\right)\right\} =$$

= in Appendix 1.1, see the full proof of case 3.6.2 =

$$= \left[\left(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}k_{\bar{g}}(x)\right)\right]\overline{\oplus}_{\bar{g}}$$

$$\overline{\oplus}_{\bar{g}}\left[\left(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, k_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\right]\overline{\oplus}_{\bar{g}}$$

$$\overline{\oplus}_{\bar{g}}\left[\left(k_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\right].$$

The formula is "*schematically the same formula*" as for the derivative function of "*the multiplication of three functions/the product function*", [23], [24], [25].

- The result 3.6.2. is $\bar{g}-derivative\ function$ *of the Pseudo-Multiplication of three* $\bar{g}-Functions$.

3.7 The Pseudo-Quotient Rule

($\bar{g}-Derivative\ of\ the\ Pseudo\text{-}Division\ of\ two\ \bar{g}-Functions$)

We calculate the pseudo-derivative function for the pseudo-division expression of two modified functions $\left(f_{\bar{g}},\ h_{\bar{g}}\right)$ with respect to *x*, [2], [3], [9], with the conditions for values of function h: for each values of $x\in\,]\bar{g}^{-1}(a),\bar{g}^{-1}(b)[$, $h_{\bar{g}}(x)\neq 0$, and get:

$$\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\oslash}_{\bar{g}}\, h_{\bar{g}}(x)\right) =$$

$$= \bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\,\overline{\oslash}_{\bar{g}}\, h_{\bar{g}}(x)\right)\right) =$$

$$= \bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(\bar{g}^{-1}\left(\frac{\bar{g}\left(f_{\bar{g}}(x)\right)}{\bar{g}\left(h_{\bar{g}}(x)\right)}\right)\right)\right) =$$

$$= \bar{g}^{-1}\left(\frac{d}{dx}\left(\frac{\bar{g}\left(f_{\bar{g}}(x)\right)}{\bar{g}\left(h_{\bar{g}}(x)\right)}\right)\right) =$$

= in Appendix 1.2, see the full proof of case 3.7=

$$= \left\{\begin{array}{l}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right]\\ \overline{\ominus}_{\bar{g}}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right]\end{array}\right\}\overline{\oslash}_{\bar{g}}$$

$$\overline{\oslash}_{\bar{g}}\left\{h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right\}.$$

Again we have found an interesting conclusion for 3.7. The formula is "*schematically the same formula*" as for the derivative function of "*the division of two functions/ the quotient function*", [23], [24], [25].

- The result 3.7 is $\bar{g}-derivative\ function$ *of the Pseudo-Division of two* $\bar{g}-Functions$.

3.8 The Pseudo-Chain Rule

$(\bar{g} - derivative\ for\ the\ composition\ of\ two\ \bar{g} - functions)$

If the function $\boldsymbol{y} = f(u)$ is differentiable on $\boldsymbol{u} = h(x)$ and also, the function $\boldsymbol{u} = h(x)$ is differentiable with respect to $\boldsymbol{x}$, then the composite function $\mathbf{y} = f(h(x))$ is differentiable, and we recall, [9], the relationship as a $\bar{g} - composite\ function$:

$$[(f \circ h)(x)]_{\bar{g}} = [f(h(x))]_{\bar{g}} =$$
$$= \bar{g}^{-1}\left[f\left(\bar{g}\left(\bar{g}^{-1}(h(\bar{g}(x)))\right)\right)\right] =$$
$$= \bar{g}^{-1}\left[f\left(\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right] =$$
$$= f_{\bar{g}}\left(h_{\bar{g}}(x)\right) = \left(f_{\bar{g}} \circ h_{\bar{g}}\right)(x).$$

We note with $\boldsymbol{y}$ the composition of two $\bar{g} - functions$, so our function is presented as $\bar{g} - composite\ function$, $\boldsymbol{y} = [(f \circ h)(x)]_{\bar{g}}$ or $\boldsymbol{y} = \left(f_{\bar{g}} \circ h_{\bar{g}}\right)(x)$ and in this case, $\boldsymbol{u} = h_{\bar{g}}(x)$, [9].

The *Pseudo-Chain Rule* is:

$$\frac{d^{\overline{\oplus}_{\bar{g}}} y}{dx} = \frac{d^{\overline{\oplus}_{\bar{g}}}}{du} f_{\bar{g}}(u)\ \overline{\odot}_{\bar{g}}\ \frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} u(x).$$

The pseudo-derivative definition is applied to a pseudo-composition expression of two functions or composition of two modified functions by $\bar{g} - transform$ $(f_{\bar{g}}, h_{\bar{g}})$, [2], [3], [9], as below:

$$\frac{d^{\overline{\oplus}_{\bar{g}}} \boldsymbol{y}}{dx} = \frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}[(f \circ h)(x)]_{\bar{g}} = \frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}} \circ h_{\bar{g}}\right)(x) =$$
$$= \bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(\left(f_{\bar{g}} \circ h_{\bar{g}}\right)(x)\right)\right) =$$
$$= \bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}\left(h_{\bar{g}}(x)\right)\right)\right) =$$
$$= \bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(\bar{g}^{-1}\left[f\left(\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right]\right)\right) =$$
$$= \bar{g}^{-1}\left(\frac{d}{dx}f\left(\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right) =$$
$$= \bar{g}^{-1}\left(\frac{d}{dx}f(\bar{g}(u))\right) =$$
$$= \frac{d^{\overline{\oplus}_{\bar{g}}}}{du} f_{\bar{g}}(u)\ \overline{\odot}_{\bar{g}}\ \frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} u(x) =$$
$$= \frac{d^{\overline{\oplus}_{\bar{g}}}}{dh_{\bar{g}}(x)} f_{\bar{g}}\left(h_{\bar{g}}(x)\right)\ \overline{\odot}_{\bar{g}}\ \frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} h_{\bar{g}}(x).$$

The formula is is "*schematically the same formula*" as for the derivative function of "*the composition of two functions*", [23], [24], [25].

- The result 3.8. is $\bar{g} - derivative\ function\ of\ the\ Composition\ of\ two\ \bar{g} - Functions,\ or\ of\ the\ \bar{g} - Composite\ of\ two\ Function.$

4 Results and Discussion

The Pseudo-Derivative function for some pseudo-linear or pseudo-nonlinear combinations of $\bar{g} - transformed\ functions$, [2], with some conditions for constants, functions and PAO that participate in relations for sum, difference, product, quotient or composition of $\bar{g} - functions$ directed us to Pseudo-Rules for Pseudo-Linearity, the Constant Pseudo-Term, the Pseudo-Product, the Pseudo-Quotient, the Pseudo-Chain cases.

Based on the results we found from the implementation of the $\bar{g} - derivative$ definition 2.1.2, [3], for each case 3.1.÷ 3.8 in this study, we record that all the pseudo-identities formulas are "*schematically the same formula*" as in Classical Analysis, [23], [24] for the derivative function of:

- "*the multiple of a constant with a function"*/ "*The Constant Factor or Multiple Rule";*
- "*a constant"*/"*The Constant Term Rule*";
- "*the addition of two functions"/"The Sum Function*" (case for two functions);
- "*the subtract of two functions"/"The Difference Function*" (case for two functions);
- *"the linear- combination" (case for one or two functions)/"Linearity Property";*
- "*the multiplication of two functions"/"The Product Function*" (case for two and three functions);
- "*the division of two functions"/"The Quotient Function*";
- "*the composition of two functions*"/"*The Chain Rule".*

We applied $\bar{g} - derivative$ for a pseudo-linear combination of two $\bar{g} - functions$ (case 3.5) and some pseudo-derivative identities as $\bar{g} - formula$ are founded for *The Pseudo-Linearity Rule*, after four cases treated before (3.1.÷ 3.4) because we tried to follow the same line with the table of derivative functions in Classical Analysis, [23], [24], as well as the sequence in the consistent SPAO $\{\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}}, \overline{\ominus}_{\bar{g}}, \overline{\oslash}_{\bar{g}}\}$.

But, we emphasize that we can take into consideration the rules below:

- *The Constant Pseudo-Factor Rule (case 3.1);*
- *The Constant Pseudo-Term Rule (case 3.2)*
- *The Pseudo-Sum Rule (case 3.3);*
- *The Pseudo-Difference Rule (case 3.4)*;

as exceptional cases for "*The Pseudo-Linearity Rule",* [24]. All the results for each treated case

3.1.÷ 3.8 are arranged in a $\bar{g}-derivative\ table$ as $\bar{g}-formulas$ in appendix. We can present this $\bar{g}-table$ as a generalization form of the derivatives table for Classical Analysis, [23], [24]. An interesting problem will be applying the $\bar{g}-derivative$ definition for more than three modified functions in pseudo-linear/nonlinear combinations or their $\bar{g}-composite$, as for Elementary Transcendental Functions etc. These cases will be the perspectives of our study.

5 Conclusion

The main problems treated in this paper are the generalization of the table of $\bar{g}-derivative$ for $\bar{g}-functions$ with general formulas and finding some Pseudo-General Rules as $\bar{g}-formulas$ to equip the $\bar{g}-table$. Eight specific pseudo-derivatives cases are treated for several combinations of $\bar{g}-functions$ and some pseudo-derivative identities are found in the form of "*The Pseudo-Rules*". These pseudo-derivative identities are arranged in five groups and presented as $\bar{g}-formula$ listed in a table of $\bar{g}-derivatives$ for the $\bar{g}-functions$ as a first attempt in, Table 2, "$\bar{g}-Formulas$ of $\bar{g}-Derivative$ for the $\bar{g}-Functions$" (Appendix 2). The table of $\bar{g}-derivative$ for the $\bar{g}-functions$, [2], [9], is equipped explicitly with several pseudo-derivative identities as *Pseudo-Basic Properties/ Pseudo-General Rules*:

- *The Pseudo-Linearity Rule* (The Constant Pseudo-Factor Rule, The Pseudo-Sum Rule, The Pseudo-Difference Rule);
- *The Constant Pseudo-Term Rule*;
- *The Pseudo-Product Rule* (case for two and three functions as pseudo-nonlinearity formulas);
- *The Pseudo-Quotient Rule*;
- *The Pseudo-Chain Rule* (pseudo-combination of two pseudo-functions).

In the following, the Table 2, of $\bar{g}-Derivative$ for the $\bar{g}-Functions$ (Appendix 2), will be completed with more $\bar{g}-Formulas$, showing once again the importance of generated Pseudo-Analysis with the broad field of its applications, [19], [20], [21], further using *mathematic induction* for more modified functions to take part in pseudo-nonlinear combinations, for Elementary Transcendental Functions, etc. A perspective line is open in $\bar{g}-integrals$ for $\bar{g}-functions$ and their $\bar{g}-table$, [17], [18], [19], [20], [21], [22], as a generalization form.

References:

[1] E. Pap, Pseudo-analysis and its applications, *Tatra Mountains Math. Publ.* 12 (1997), pp. 1-12.

[2] J. Rybárik, g-FUNCTIONS, *Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak. Ser. Mat.* 25,1 (1995), 29-38.

[3] E. Pap, (1993): g-calculus, *Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak. Ser. Mat.* 23,1(1993), f. 145-156.

[4] R. Mesiar, J. Rybárik, Pseudo-arithmetical operations, *Tatra Mauntains Math., Publ.*, 2 (1993), pp. 185-192.

[5] E. Pap, "Application of decomposable measures on nonlinear difference equations", *Novi Sad J. Math.* 31, 2 (2001), pp. 89-98.

[6] E. Pap, *Pseudo-additive measures and their applications*, in: E. Pap (Ed.), *Handbook of Measure Theory*, Vol. II. Elsevier, Amsterdam, 2002, pp. 1405-1465.

[7] N. Ralević, Some new properties of g-calculus, *Univ. u Novom Sadu Zb. Rad. Prirod.-Mat. Fak. Ser. Mat.* 24,1 (1994), pp. 139-157.

[8] M. Sugeno, T. Murofushi, Pseudo-additive measures and integrals. *J. Math. Anal. Appl.* 122 (1987), 197-222.

[9] Dh. Valera, E. Valera, Some Applications for Nonlinear and Pseudo-Nonlinear Functional Equations. *Comput. Sci. Appl. Ethan Publishing Company, ETHAN*, America, (2015), Vol. 2, Number 3: 94-105.

[10] Dh. Valera, B. Shyti, S. Paralloj, Communications to the Pseudo-Additive Probability Measure and the Induced Probability Measure Realized by $\bar{g}-Transform$. *Mathematics and Statistics*, (2024), 12(1), 24 - 30. DOI: 10.13189/ms.2024.120104.

[11] A. Kolesárová, A note on the ⊕-measure based integrals, *Tatra Mauntains Math.,Publ.*,3 (1993), pp.173-182.

[12] A. Kolesárová, Integration of real functions with respect to a ⊕-measure, *Math., Slovaca*, *46* (1996), No. 1, pp. 41-52.

[13] I. Marinová, Integrations with respect to a ⊕-measure, *Math. Slovaca 36*, (1986), pp.15-22

[14] A. Markova, Some remark on pseudo-linear algebra, *Tatra Mauntains Math.*, Publ., 6 (1995), pp. 123-130.

[15] J. Kalinowski, On equivalence and rank preserving operators, *novi sad J. Math.* (2002), Vol 32, No. 1, pp. 133-139.

[16] J. Aczél, Lectures on Functional Equations and their Applications, *Academic Press*, New

York, 1966.
[17] D Zhang, R Mesiar, E Pap, Choquet type integrals for single-valued functions with respect to set-functions and set-multifunctions, *Information Sciences* 630, (2023), pp. 252-270.
[18] D Zhang, R Mesiar, E Pap, Pseudo-integral and generalized Choquet integral, *Fuzzy Sets and Systems* 446, (2022), pp.193-221.
[19] E. Pap, Pseudo-Analysis as a Tool of Information Processing, (2022), 81(1), 116; https://doi.org/10.3390/proceedings2022081116.
[20] D Zhang, R Mesiar, E Pap, Jensen's inequality for Choquet integral revisited and a note on Jensen's inequality for generalized Choquet integral, *Fuzzy Sets and Systems*, (2022), 430, pp.79-87.
[21] D Zhang, R Mesiar, E Pap, Jensen's inequalities for standard and generalized asymmetric Choquet integrals, *Fuzzy Sets and Systems* (2023), 457, pp.119-124.
[22] Darweesh, A.H., Maghrabi, A.M.D., On Fractional φ- and biφ-calculi, *WSEAS Transactions on Systems*, vol. 22, (2023), pp. 87-100. DOI:10.37394/23202.2023.22.10
[23] Derivative Table. Queen's College, [Online]. https://qcweb.qc.edu.hk/math/Resource/AL/Derivative%20Table.pdf (Accessed Date: 28 January 2024).
[24] Table of Derivatives, https://personal.math.ubc.ca/~feldman/m263/formulae.pdf (Accessed Date: 28 January 2024).
[25] Operations on Functions and Composition of Functions. https://www.webassign.net/ebooks/youngat3demo/young9780470648018/c03/young9780470648018c03_5_0_body.htm (Accessed Date: 28 January 2024).

Contribution of individual authors to the creation of a scientific article (Ghostwriting policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of funding for research presented in a scientific article or scientific article itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare.

Appendix 1: The full proof for two cases 3.6.2 and 3.7 of session 3.

1.1 The full proof of case 3.6.2, for $\bar{g}-Derivativ$ e of the Pseudo-Multiplication of three $\bar{g}-Functions$

$$\frac{d^{\overline{\oplus}_{\bar g}}}{dx}\left(f_{\bar g}(x)\,\overline{\odot}_{\bar g}\,h_{\bar g}(x)\,\overline{\odot}_{\bar g}\,k_{\bar g}(x)\right)=\bar g^{-1}\left(\frac{d}{dx}\bar g\left(f_{\bar g}(x)\,\overline{\odot}_{\bar g}\,h_{\bar g}(x)\,\overline{\odot}_{\bar g}\,k_{\bar g}(x)\right)\right)=$$

$$=\bar g^{-1}\left(\frac{d}{dx}\bar g\left(\bar g^{-1}\left(\bar g\left(f_{\bar g}(x)\right)\cdot\bar g\left(h_{\bar g}(x)\right)\right)\overline{\odot}_{\bar g}\,k_{\bar g}(x)\right)\right)=$$

$$=\bar g^{-1}\left\{\frac{d}{dx}\bar g\left[\left(\bar g^{-1}\left(\bar g\left(f_{\bar g}(x)\right)\cdot\bar g\left(h_{\bar g}(x)\right)\right)\right)\overline{\odot}_{\bar g}\,k_{\bar g}(x)\right]\right\}=$$

$$=\bar g^{-1}\left\{\frac{d}{dx}\bar g\left[\left(\bar g^{-1}\left[\bar g\left(\bar g^{-1}\left(\bar g\left(f_{\bar g}(x)\right)\cdot\bar g\left(h_{\bar g}(x)\right)\right)\right)\cdot\bar g\left(k_{\bar g}(x)\right)\right]\right)\right]\right\}=$$

$$=\bar g^{-1}\left\{\frac{d}{dx}\left(\bar g\left(f_{\bar g}(x)\right)\cdot\bar g\left(h_{\bar g}(x)\right)\cdot\bar g\left(k_{\bar g}(x)\right)\right)\right\}=$$

$$=\bar g^{-1}\left\{\frac{d}{dx}\left(\bar g\left(f_{\bar g}(x)\right)\cdot\bar g\left(h_{\bar g}(x)\right)\cdot\bar g\left(k_{\bar g}(x)\right)\right)\right\}=$$

$$=\bar g^{-1}\left\{\left[\left(\bar g\left(f_{\bar g}(x)\right)\cdot\bar g\left(h_{\bar g}(x)\right)\right)\frac{d}{dx}\bar g\left(k_{\bar g}(x)\right)\right]+\left[\left(\bar g\left(h_{\bar g}(x)\right)\cdot\bar g\left(k_{\bar g}(x)\right)\right)\frac{d}{dx}\bar g\left(f_{\bar g}(x)\right)\right]\right.$$
$$\left.+\left[\left(\bar g\left(k_{\bar g}(x)\right)\cdot\bar g\left(f_{\bar g}(x)\right)\right)\frac{d}{dx}\bar g\left(h_{\bar g}(x)\right)\right]\right\}=$$

$$=\bar g^{-1}\left\{\left[\left(\bar g\left(f_{\bar g}(x)\right)\cdot\bar g\left(h_{\bar g}(x)\right)\right)\cdot\bar g\left(\bar g^{-1}\left(\frac{d}{dx}\bar g\left(k_{\bar g}(x)\right)\right)\right)\right]+\left[\left(\bar g\left(h_{\bar g}(x)\right)\cdot\bar g\left(k_{\bar g}(x)\right)\right)\cdot\bar g\left(\bar g^{-1}\left(\frac{d}{dx}\bar g\left(f_{\bar g}(x)\right)\right)\right)\right]\right.$$
$$\left.+\left[\left(\bar g\left(k_{\bar g}(x)\right)\cdot\bar g\left(f_{\bar g}(x)\right)\right)\cdot\bar g\left(\bar g^{-1}\left(\frac{d}{dx}\bar g\left(h_{\bar g}(x)\right)\right)\right)\right]\right\}=$$

$$=\bar g^{-1}\left\{\left[\left(\bar g\left(f_{\bar g}(x)\right)\cdot\bar g\left(h_{\bar g}(x)\right)\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}k_{\bar g}(x)\right)\right]+\left[\left(\bar g\left(h_{\bar g}(x)\right)\cdot\bar g\left(k_{\bar g}(x)\right)\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}f_{\bar g}(x)\right)\right]\right.$$
$$\left.+\left[\left(\bar g\left(k_{\bar g}(x)\right)\cdot\bar g\left(f_{\bar g}(x)\right)\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}h_{\bar g}(x)\right)\right]\right\}=$$

$$=\bar g^{-1}\left\{\left[\left(\bar g\left[\bar g^{-1}\left(\bar g\left(f_{\bar g}(x)\right)\cdot\bar g\left(h_{\bar g}(x)\right)\right)\right]\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}k_{\bar g}(x)\right)\right]\right.$$
$$+\left[\left(\left(\bar g\left[\bar g^{-1}\left(\bar g\left(h_{\bar g}(x)\right)\cdot\bar g\left(k_{\bar g}(x)\right)\right)\right]\right)\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}f_{\bar g}(x)\right)\right]$$
$$\left.+\left[\left(\left(\bar g\left[\bar g^{-1}\left(\bar g\left(k_{\bar g}(x)\right)\cdot\bar g\left(f_{\bar g}(x)\right)\right)\right]\right)\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}h_{\bar g}(x)\right)\right]\right\}=$$

$$=\bar g^{-1}\left\{\left[\left(\bar g\left[f_{\bar g}(x)\,\overline{\odot}_{\bar g}\,h_{\bar g}(x)\right]\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}k_{\bar g}(x)\right)\right]+\left[\left(\left(\bar g\left[h_{\bar g}(x)\,\overline{\odot}_{\bar g}\,k_{\bar g}(x)\right]\right)\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}f_{\bar g}(x)\right)\right]\right.$$
$$\left.+\left[\left(\left(\bar g\left[k_{\bar g}(x)\,\overline{\odot}_{\bar g}\,f_{\bar g}(x)\right]\right)\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}h_{\bar g}(x)\right)\right]\right\}=$$

$$=\bar g^{-1}\left\{\left[\bar g\left[\bar g^{-1}\left(\left(\bar g\left[f_{\bar g}(x)\,\overline{\odot}_{\bar g}\,h_{\bar g}(x)\right]\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}k_{\bar g}(x)\right)\right)\right]\right]+\left[\bar g\left[\bar g^{-1}\left(\left(\bar g\left[h_{\bar g}(x)\,\overline{\odot}_{\bar g}\,k_{\bar g}(x)\right]\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}f_{\bar g}(x)\right)\right)\right]\right]\right.$$
$$\left.+\,\bar g\left[\bar g^{-1}\left(\left(\bar g\left[k_{\bar g}(x)\,\overline{\odot}_{\bar g}\,f_{\bar g}(x)\right]\right)\cdot\bar g\left(\frac{d^{\overline{\oplus}_{\bar g}}}{dx}h_{\bar g}(x)\right)\right)\right]\right\}=$$

$$= \bar{g}^{-1}\Bigg\{\bar{g}\Bigg\{\bar{g}^{-1}\Bigg\{\Bigg[\bar{g}\Bigg[\Big(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,h_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}k_{\bar{g}}(x)\Bigg)\Bigg]\Bigg] + \Bigg[\bar{g}\Bigg[\Big(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,k_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\Bigg)\Bigg]\Bigg]\Bigg\}\Bigg\}$$

$$+ \Bigg[\bar{g}\Bigg[\Big(k_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,fh_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\Bigg)\Bigg]\Bigg]\Bigg\} =$$

$$= \bar{g}^{-1}\Bigg\{\bar{g}\Bigg\{\Bigg[\Big(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,h_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}k_{\bar{g}}(x)\Bigg)\Bigg]\,\overline{\oplus}_{\bar{g}}\,\Bigg[\Big(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,k_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\Bigg)\Bigg]\Bigg\}$$

$$+ \Bigg[\bar{g}\Bigg[\Big(k_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,f_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\Bigg)\Bigg]\Bigg]\Bigg\} =$$

$$= \bar{g}^{-1}\Bigg\{\bar{g}\Bigg\{\Bigg[\Big(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,h_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}k_{\bar{g}}(x)\Bigg)\Bigg]\,\overline{\oplus}_{\bar{g}}\,\Bigg[\Big(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,k_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\Bigg)\Bigg]\Bigg\}$$

$$+ \Bigg[\bar{g}\Bigg[\Big(k_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,f_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\Bigg)\Bigg]\Bigg]\Bigg\} =$$

$$= \bar{g}^{-1}\Bigg\{\bar{g}\Bigg\{\bar{g}^{-1}\Bigg\{\bar{g}\Bigg\{\Bigg[\Big(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,h_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}k_{\bar{g}}(x)\Bigg)\Bigg]\,\overline{\oplus}_{\bar{g}}\,\Bigg[\Big(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,k_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\Bigg)\Bigg]\Bigg\}$$

$$+ \bar{g}\Bigg\{\Big(k_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,f_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\Bigg)\Bigg\}\Bigg\}\Bigg\}\Bigg\} =$$

$$= \bar{g}^{-1}\Bigg\{\bar{g}\Bigg\{\Bigg[\Big(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,h_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}k_{\bar{g}}(x)\Bigg)\Bigg]\,\overline{\oplus}_{\bar{g}}\,\Bigg[\Big(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,k_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\Bigg)\Bigg]\Bigg\}$$

$$+ \bar{g}\Bigg\{\Big(k_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,f_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\Bigg)\Bigg\}\Bigg\} =$$

$$= \Bigg[\Big(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,h_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}k_{\bar{g}}(x)\Bigg)\Bigg]\,\overline{\oplus}_{\bar{g}}\,\Bigg[\Big(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,k_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\Bigg)\Bigg]\,\overline{\oplus}_{\bar{g}}$$

$$\overline{\oplus}_{\bar{g}}\,\Bigg[\Big(k_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\,f_{\bar{g}}(x)\Big)\,\overline{\odot}_{\bar{g}}\Bigg(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\Bigg)\Bigg].$$

1.2 The full proof of case 3.7, for $\bar{g}-Derivative$ of the Pseudo-Division for two $\bar{g}-Functions$

$$\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\Big(f_{\bar{g}}(x)\,\overline{\oslash}_{\bar{g}}\,h_{\bar{g}}(x)\Big) = \bar{g}^{-1}\Big(\frac{d}{dx}\bar{g}\Big(f_{\bar{g}}(x)\,\overline{\oslash}_{\bar{g}}\,h_{\bar{g}}(x)\Big)\Big) =$$

$$= \bar{g}^{-1}\Bigg(\frac{d}{dx}\bar{g}\Bigg(\bar{g}^{-1}\Bigg(\frac{\bar{g}\big(f_{\bar{g}}(x)\big)}{\bar{g}\big(h_{\bar{g}}(x)\big)}\Bigg)\Bigg)\Bigg) = \bar{g}^{-1}\Bigg(\frac{d}{dx}\Bigg(\frac{\bar{g}\big(f_{\bar{g}}(x)\big)}{\bar{g}\big(h_{\bar{g}}(x)\big)}\Bigg)\Bigg) =$$

$$= \bar{g}^{-1}\left\{\frac{\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\right)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)-\left(\frac{d}{dx}\bar{g}\left(h_{\bar{g}}(x)\right)\right)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)}{\left(\bar{g}\left(h_{\bar{g}}(x)\right)\right)^2}\right\} =$$

$$= \bar{g}^{-1}\left\{\frac{\bar{g}\left(\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(f_{\bar{g}}(x)\right)\right)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)-\bar{g}\left(\bar{g}^{-1}\left(\frac{d}{dx}\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)}{\left(\bar{g}\left(h_{\bar{g}}(x)\right)\right)^2}\right\} =$$

$$= \bar{g}^{-1}\left\{\frac{\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)-\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)}{\bar{g}\left(h_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)}\right\} =$$

$$= \bar{g}^{-1}\left\{\frac{\bar{g}\left[\bar{g}^{-1}\left(\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right]-\bar{g}\left[\bar{g}^{-1}\left(\bar{g}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\cdot\bar{g}\left(f_{\bar{g}}(x)\right)\right)\right]}{\bar{g}\left(\bar{g}^{-1}\left(\bar{g}\left(h_{\bar{g}}(x)\right)\cdot\bar{g}\left(h_{\bar{g}}(x)\right)\right)\right)}\right\} =$$

$$= \bar{g}^{-1}\left\{\frac{\bar{g}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right]-\bar{g}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right]}{\bar{g}\left(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right)}\right\} =$$

$$= \bar{g}^{-1}\left\{\frac{\bar{g}\left\{\bar{g}^{-1}\left[\bar{g}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right]-\bar{g}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right]\right]\right\}}{\bar{g}\left(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right)}\right\} =$$

$$= \bar{g}^{-1}\left\{\frac{\bar{g}\left\{\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right]\overline{\ominus}_{\bar{g}}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right]\right\}}{\bar{g}\left(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right)}\right\} =$$

$$= \left\{\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right]\overline{\ominus}_{\bar{g}}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right]\right\}\overline{\oslash}_{\bar{g}}\left\{h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right\}.$$

Appendix 2: The Table of $\bar{g}-Derivative$ for the $\bar{g}-Functions$

Table 2		$\bar{g}-Formulas$ of $\bar{g}-Derivative$ for the $\bar{g}-Functions$
No.	**$\bar{g}-Derivative$ of:**	**$\bar{g}-Formula$**
1	*a Constant Pseudo-Multiple with a $\bar{g}-Function$* ($\propto\neq 0$)	▪ *The Constant Pseudo-Factor Rule* $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(\propto\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right) =\propto\overline{\odot}_{\bar{g}}\, \frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} f_{\bar{g}}(x)$
2	a Constant $\propto$	▪ *The Constant Pseudo-Term Rule* $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\propto= 0$
3	*the Pseudo-Addition of two $\bar{g}-Functions$*	▪ *The Pseudo-Sum Rule* $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\oplus}_{\bar{g}}\, h_{\bar{g}}(x)\right) = \frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} f_{\bar{g}}(x)\,\overline{\oplus}_{\bar{g}}\,\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} h_{\bar{g}}(x)$
4	*the Pseudo-Subtraction of two $\bar{g}-Functions$*	▪ *The Pseudo-Difference Rule* $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\ominus}_{\bar{g}}\, h_{\bar{g}}(x)\right) = \frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} f_{\bar{g}}(x)\,\overline{\ominus}_{\bar{g}}\,\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} h_{\bar{g}}(x)$
5	*the Pseudo-Linear Combination of one/two $\bar{g}-Functions$ and any constant*	▪ *The General Pseudo-Linearity Rule/Property for two $\bar{g}-Functions$* $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left[\left(\propto\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right)\overline{\oplus}_{\bar{g}}\left(\lambda\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right)\right] = \; = \left(\propto\overline{\odot}_{\bar{g}}\,\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} f_{\bar{g}}(x)\right)\overline{\oplus}_{\bar{g}}\left(\lambda\,\overline{\odot}_{\bar{g}}\,\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} h_{\bar{g}}(x)\right)$ ▪ *The General Pseudo-Linearity Rule/ Property for one $\bar{g}-Function$* $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left[\left(\propto\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right)\overline{\oplus}_{\bar{g}}\,\lambda\right] = \propto\overline{\odot}_{\bar{g}}\,\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} f_{\bar{g}}(x)$
6	*the Pseudo-Multiplication of two/three $\bar{g}-Functions$*	▪ *The Pseudo-Product Rule for two $\bar{g}-Functions$* $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right) = \; = \left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right]\overline{\oplus}_{\bar{g}}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right]$ ▪ *The Pseudo-Product Rule for three $\bar{g}-Functions$* $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, k_{\bar{g}}(x)\right) = \; = \left[\left(f_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} k_{\bar{g}}(x)\right)\right]\overline{\oplus}_{\bar{g}} \; \overline{\oplus}_{\bar{g}}\left[\left(h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, k_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} f_{\bar{g}}(x)\right)\right]\overline{\oplus}_{\bar{g}} \; \overline{\oplus}_{\bar{g}}\left[\left(k_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} h_{\bar{g}}(x)\right)\right]$
7	*the Pseudo-Division of two $\bar{g}-Functions$* Condition: $h_{\bar{g}}(x)\neq 0$	▪ *The Pseudo-Quotient Rule* $\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx}\left(f_{\bar{g}}(x)\,\overline{\oslash}_{\bar{g}}\, h_{\bar{g}}(x)\right) = \; = \left\{\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} f_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right]\overline{\ominus}_{\bar{g}}\left[\left(\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} h_{\bar{g}}(x)\right)\overline{\odot}_{\bar{g}}\, f_{\bar{g}}(x)\right]\right\}\overline{\oslash}_{\bar{g}}\left\{h_{\bar{g}}(x)\,\overline{\odot}_{\bar{g}}\, h_{\bar{g}}(x)\right\}$
8	*the Composition of two $\bar{g}-Functions$* Conditions: $\boldsymbol{y}=\left(f_{\bar{g}}\circ h_{\bar{g}}\right)(x),\ \boldsymbol{u}= h_{\bar{g}}(x)$	▪ *The Pseudo-Chain Rule* $\frac{d^{\overline{\oplus}_{\bar{g}}}y}{dx} = \frac{d^{\overline{\oplus}_{\bar{g}}}}{du} f_{\bar{g}}(u)\,\overline{\odot}_{\bar{g}}\,\frac{d^{\overline{\oplus}_{\bar{g}}}}{dx} u(x)$