Dynamic Behaviors of a Commensalism Model Incorporating Nonselective Harvesting in a Partial Closure

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Abstract: - A commensalism model incorporating nonselective harvesting in a partial closure is proposed and studied in this paper. Local and global stability properties of the equilibria are investigated, respectively. Our study shows that depending on the fraction of the stock available for harvesting, the system may be extinct, partial survival, or two species coexist in a stable state. Numeric simulations are carried out to show the feasibility of the main results.

Key-Words: -Commensalism model; Differential inequality; Global stability

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1 Introduction

Many scholars, [1], [2], [3], [4], [5], [6], [7], [8], have studied the dynamic behaviors of the mutualism model over the last two decades. Many interesting results were obtained, for example, The study, [1], showed that for a cooperative community, stage structure and the death rate of mature species are two of the most important factors that influence the persistence or extinction of the system, and cooperation has no influence on the persistent property of the model.

Commensalism is a a mutually beneficial relationship between two populations, where one population gets the benefit from the other species while the other is neither harmed nor benefited due to the interaction with the previous species, [9]. There are many real-life examples of commensalism, for example, the relationship between squirrel and the oak, the clownfish and the sea anemone, etc. However, only recently have scholars paid attention to this direction, see, [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], and the references cited therein.

The most basic commensalism model is as follows:

$$\begin{aligned} \frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big), \\ \frac{dy}{dt} &= r_2y\Big(1-\frac{y}{K_2}\Big), \end{aligned} \tag{1.1}$$

where the constants r_1, r_2, K_1, K_2, and α are all positive. [15] investigated the local stability property of the system (1.1).

On the other hand, harvesting of the species is required to obtain the resource required for human development. Based on the model (1.1), [19], proposed the following non-selective harvesting Lotka-Volterra amensalism model incorporating partial closure for the populations:

$$\begin{aligned} \frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big)-q_1Emx, \\ \frac{dy}{dt} &= r_2y\Big(1-\frac{y}{K_2}\Big)-q_2Emy, \end{aligned} \tag{1.2}$$

where the constants r_1, r_2, K_1, K_2, and α are all positive. E is the combined fishing effort used to harvest and $m(0 < m < 1)$ is the fraction of the stock available for harvesting. Their study showed that depending on the fraction of the stock available for harvesting, the system may be extinct, partial survival, or two species coexist in a stable state. The dynamic behaviors of the system becomes complicated compared to the dynamic behaviors of the system (1.1).

Vargas-De-León and Gómez-Alcaraz proposed the following two species commensal system in [26], based on the May type cooperation system:

$$\begin{aligned} \frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1+b_1y}\Big), \\ \frac{dy}{dt} &= r_2y\Big(1-\frac{y}{K_2}\Big). \end{aligned} \tag{1.3}$$

By constructing some suitable Lyapunov function and after careful calculation, the authors showed that the unique positive equilibrium of the system (1.3) is globally stable.

Now, inspired by the works of [19], [26], we now incorporate nonselective harvesting in a partial closure to the system (1.3), and this leads to

$$\begin{aligned}\frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1+b_1y}\Big)-q_1Emx,\\ \frac{dy}{dt} &= r_2y\Big(1-\frac{y}{K_2}\Big)-q_2Emy,\end{aligned} \tag{1.4}$$

where r_1, r_2, K_1, K_2 are all positive constants, and $r_i, i=1,2$ are the intrinsic growth rates of the species x and y, respectively. $k_i, i=1,2$ are the carrying capacity of the species x and y, respectively. E is the combined fishing effort used to harvest, and $m(0<m<1)$ is the fraction of the stock available for harvesting, where $q_i, i=1,2$ is the harvesting coefficients. In this system, the first species gets benefit from the second species by means of the second species enlarging the carrying capacity of the first species from K_1 to K_1+b_1y.

We will try to investigate the dynamic behaviors of the systems (1.4), and find out the influence of the harvesting and the fraction of the stock.

The paper is arranged as follows. We will investigate the local and global stability properties of the equilibria of systems (1.4) in sections 2 and 3, respectively. An example together with its numerical simulations is presented in Section 5 to show the feasibility of the main results. We end this paper with a brief discussion.

2 Local stability of the equilibria

The system always admits the boundary equilibrium $E_1(0,0)$.

If

$$r_1 > Emq_1 \tag{2.1}$$

is true, the system admits the boundary equilibrium $E_2(x_0,0)$, where

$$x_0=\frac{K_1(r_1-Emq_1)}{r_1}. \tag{2.2}$$

If

$$r_2 > Emq_2 \tag{2.3}$$

, the system admits the boundary equilibrium $E_3(0,y_0)$, where

$$y_0=\frac{K_2(r_2-Emq_2)}{r_2}. \tag{2.4}$$

If (2.1) and (2.3) hold, then the system admits a unique positive equilibrium

$$\begin{aligned}x^* &= \frac{(qEm-r_1)\Big(K_2b_1(Emq_2-r_2)-K_1r_2\Big)}{r_1r_2},\\ y^* &= \frac{K_2(r_2-Emq_2)}{r_2}.\end{aligned}$$

We shall now investigate the local stability property of the above equilibria.

Theorem 2.1

(1)Suppose

$$m > \max\Big\{\frac{r_1}{Eq_1},\frac{r_2}{Eq_2}\Big\} \tag{2.5}$$

is true, then $E_1(0,0)$ is locally asymptotically stable, otherwise, it is unstable;

(2) Suppose

$$\frac{r_2}{Eq_2} < m < \frac{r_1}{Eq_1} \tag{2.6}$$

is true, then $E_2(x_0,0)$ is locally asymptotically stable, otherwise, it is unstable;

(3) Suppose

$$\frac{r_1}{Eq_1} < m < \frac{r_2}{Eq_2} \tag{2.7}$$

is true, then $E_3(0,y_0)$ is locally asymptotically stable, otherwise, it is unstable;

(4) Suppose

$$m < \min\Big\{\frac{r_2}{Eq_2},\frac{r_1}{Eq_1}\Big\} \tag{2.8}$$

is true, then $E_4(x^*,y^*)$ is locally asymptotically stable.

Proof. The variational matrix of the system of Eq. (1.4) at (x,y) is

$$J(x,y)=\begin{pmatrix} A_{11} & \dfrac{x^2r_1b_1}{(K_1+b_1y)^2}\\ 0 & A_{22}\end{pmatrix}. \tag{2.9}$$

where

$$\begin{aligned}A_{11} &= r_1\Big(1-\frac{x}{K_1+b_1y}\Big)-\frac{r_1x}{K_1+b_1y}-q_1Em,\\ A_{22} &= r_2\Big(1-\frac{y}{K_2}\Big)-\frac{r_2y}{K_2}-q_2Em.\end{aligned}$$

The characteristic equation of the variational matrix is

$$\lambda^2-tr(J)\lambda+det(J)=0. \tag{2.10}$$

Obviously, both eigenvalues of (2.10) have negative real parts if $tr(J)<0$ and $det(J)>0$, and the corresponding equilibrium solution is asymptotically stable.

(1) For the steady-state solution $E_1(0,0)$,

$$\begin{aligned}tr\Big(J(0,0)\Big) &= r_1+r_2-Emq_1-Emq_2,\\ det\Big(J(0,0)\Big) &= (r_1-Emq_1)(r_2-Emq_2).\end{aligned}$$

Under assumption (2.5), $tr(J(0,0)) < 0, det(J(0,0)) > 0$, and thus $E_1(0,0)$ is locally

asymptotically stable, otherwise, it is unstable;
(2) For the steady-state solution $E_2(x_0, 0)$,

$$\begin{aligned} tr\Big(J(x_0,0)\Big) &= -r_1+r_2+Emq_1-Emq_2, \\ det\Big(J(x_0,0)\Big) &= (Emq_1-r_1)(r_2-Emq_2). \end{aligned}$$

Under assumption (2.6), $tr(J(x_0, 0)) < 0, det(J(x_0, 0)) > 0$, and thus $E_2(x_0, 0)$ is locally asymptotically stable, otherwise, it is unstable;
(3) The system's Jacobian about the equilibrium point $E_3(0, y_0)$ is given by

$$\begin{pmatrix} r_1-q_1Em & 0 \\ 0 & Emq_2-r_2 \end{pmatrix}.$$

Under the assumption (2.7), The matrix's two eigenvalues satisfy $\lambda_1 = r_1 - q_1Em < 0, \lambda_2 = Emq_2 - r_2 < 0$. Consequently, $E_3(0, y_0)$ is locally stable, otherwise, it is unstable;

(4) It should be noted that the positive equilibrium $E_4(x*, y*)$ satisfies

$$\begin{aligned} r_1\Big(1-\frac{x^*}{K_1+b_1y^*}\Big)-q_1Em=0, \\ r_2\Big(1-\frac{y^*}{K_2}\Big)-q_2Em=0. \end{aligned} \tag{2.11}$$

By using (2.11), the Jacobian of the system about the equilibrium point $E_4(x^*, y^*)$ is given by

$$J(x^*,y^*)=\begin{pmatrix} -\dfrac{r_1x^*}{K_1+b_1y^*} & \dfrac{(x^*)^2r_1b_1}{(K_1+b_1y^*)^2} \\ 0 & -\dfrac{r_2y^*}{K_2} \end{pmatrix}. \tag{2.12}$$

$E_4(x^*, y^*)$ is locally asymptotically stable because $tr\Big(J(x^*,y^*)\Big) < 0, det\Big(J(x^*,y^*)\Big) > 0$.

The proof of Theorem 2.1 is finished.

3 Global asymptotical stability

This section tries to obtain some sufficient conditions that could ensure the global asymptotic stability of the equilibria.

Lemma 3.1.[31], System

$$\frac{dy}{dt}=y(a-by) \tag{3.1}$$

has a unique globally attractive positive equilibrium $y^* = \frac{a}{b}$.

Theorem 3.1
(1) Assume that

$$m>\max\Big\{\frac{r_1}{Eq_1},\frac{r_2}{Eq_2}\Big\} \tag{3.2}$$

hold, then $E_1(0, 0)$ is globally asymptotically stable;
(2) Assume that

$$\frac{r_2}{Eq_2}<m<\frac{r_1}{Eq_1} \tag{3.3}$$

hold, then $E_2(x_0, 0)$ is globally asymptotically stable;
(3) Assume that

$$\frac{r_1}{Eq_1}<m<\frac{r_2}{Eq_2} \tag{3.4}$$

hold, then $E_3(0, y_0)$ is globally asymptotically stable;
(4) Assume that

$$m<\min\Big\{\frac{r_2}{Eq_2},\frac{r_1}{Eq_1}\Big\} \tag{3.5}$$

hold, then $E_4(x^*, y^*)$ is globally asymptotically stable.

Proof.
(1) From the second equation of (1.4) and (3.2), we have

$$\frac{dy}{dt}=y\Big(r_2-Eq_2m-\frac{r_2y}{K_2}\Big)<(r_2-Eq_2m)y. \tag{3.6}$$

Hence

$$y(t)<y(0)\exp\{(r_2-Eq_2m)t\}\to 0 \ \text{ as } \ t\to+\infty. \tag{3.7}$$

For above $\varepsilon > 0$, there exists a $T_1 > 0$, such that

$$y(t)<\varepsilon \ \text{ as } \ t>T_1. \tag{3.8}$$

From the first equation of system (1.4), we have, for $t > T_1$,

$$\begin{aligned} \frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1+b_1y}\Big)-q_1Emx \\ &< r_1x\Big(1-\frac{x}{K_1+b_1\varepsilon}\Big)-q_1Emx \\ &< x\Big(r_1-q_1Em\Big). \end{aligned}$$

Hence

$$x(t)<x(T_1)\exp\{(r_1-q_1Em)(t-T_1)\}\to 0 \ \text{ as } \ t\to+\infty. \tag{3.9}$$

(3.7) and (3.9) show that

$$\lim_{t\to+\infty}x(t)=0,\ \ \lim_{t\to+\infty}y(t)=0. \tag{3.10}$$

In other words, $E_1(0, 0)$ is globally asymptotically stable;
(2) Similarly to the analysis of (3.6)-(3.8), we can show that

$$y(t)\to 0 \ \text{ as } \ t\to+\infty. \tag{3.11}$$

For arbitrary enough small $\varepsilon > 0$, there exists a $T_2 > 0$, such that

$$y(t) < \varepsilon \text{ as } t > T_2. \tag{3.12}$$

From the first equation of system (1.4), we have, for $t > T_2$,

$$\begin{aligned} \frac{dx}{dt} &< r_1x\Big(1-\frac{x}{K_1+b_1\varepsilon}\Big)-q_1Emx \\ &= x\Big(r_1-q_1Em-\frac{r_1x}{K_1+b_1\varepsilon}\Big). \end{aligned} \tag{3.13}$$

Consider the equation

$$\frac{du}{dt}=u\Big(r_1-q_1Em-\frac{r_1u}{K_1+b_1\varepsilon}\Big).$$

It follows from Lemma 3.1 that

$$\lim_{t\to+\infty}u(t)=\frac{(r_1-q_1Em)(K_1+b_1\varepsilon)}{r_1}.$$

By using the comparison theorem of the differential equation, it follows from (3.13) that

$$\limsup_{t\to+\infty}x(t)\le\frac{(r_1-q_1Em)(K_1+b_1\varepsilon)}{r_1}. \tag{3.14}$$

On the other hand, from the first equation of the system (1.4), we also have

$$\begin{aligned} \frac{dx}{dt} &> r_1x\Big(1-\frac{x}{K_1}\Big)-q_1Emx \\ &= x\Big(r_1-q_1Em-\frac{r_1x}{K_1}\Big). \end{aligned} \tag{3.15}$$

Consider the equation

$$\frac{dv}{dt}=v\Big(r_1-q_1Em-\frac{r_1v}{K_1}\Big).$$

It follows from Lemma 3.1 that

$$\lim_{t\to+\infty}v(t)=\frac{(r_1-q_1Em)K_1}{r_1}.$$

By using the comparison theorem of the differential equation, it follows from (3.15) that

$$\liminf_{t\to+\infty}x(t)\ge\frac{(r_1-q_1Em)K_1}{r_1}. \tag{3.16}$$

It follows from (3.14) and (3.16) that

$$\begin{aligned} &\frac{(r_1-q_1Em)K_1}{r_1} \\ \le\ &\liminf_{t\to+\infty}x(t)\le\limsup_{t\to+\infty}x(t) \\ \le\ &\frac{(r_1-q_1Em)(K_1+b_1\varepsilon)}{r_1}. \end{aligned} \tag{3.17}$$

Since ε is any arbitrary small positive constant, setting $\varepsilon\to0$ in (3.17) results in

$$\lim_{t\to+\infty}x(t)=\frac{(r_1-q_1Em)K_1}{r_1}. \tag{3.18}$$

(3.11) and (3.18) show that $E_2(x_0,0)$ is globally asymptotically stable;

(3) From the second equation of (1.4), we have

$$\frac{dy}{dt}=y\Big(r_2-Eq_2m-\frac{r_2y}{K_2}\Big). \tag{3.19}$$

It follows from Lemma 3.1 that

$$\lim_{t\to+\infty}y(t)=\frac{K_2(r_2-Eq_2m)}{r_2}. \tag{3.20}$$

For above $\varepsilon>0$, there exists an enough large $T_3>0$ such that

$$y(t)<\frac{K_2(r_2-Eq_2m)}{r_2}+\varepsilon \text{ for all } t\ge T_3. \tag{3.21}$$

From the first equation of system (1.4), we have, for $t>T_3$,

$$\begin{aligned} \frac{dx}{dt} &= r_1x\left(1-\frac{x}{K_1+b_1y}\right)-q_1Emx \\ &< r_1x\left(1-\frac{x}{K_1+b_1\Big(\frac{K_2(r_2-Eq_2m)}{r_2}+\varepsilon\Big)}\right) \\ &\quad -q_1Emx \\ &< (r_1-q_1Em)x. \end{aligned} \tag{3.22}$$

Hence

$$x(t)<x(T_3)\exp\{(r_1-q_1Em)(t-T_3)\}\to0 \tag{3.23}$$

as $t\to+\infty$. (3.20) and (3.23) demonstrate that $E_3(0,y_0)$ is globally asymptotically stable;

(4) If condition (3.5) is true, it follows from Theorem 2.1 that $E_1(0,0)$, $E_2(x_0,0)$ and $E_3(0,y_0)$ are all unstable, while $E_4(x^*,y^*)$ is locally asymptotically stable.

If we could demonstrate that the solution of system (1.4) is bounded and there is no limit cycle exists, then $E_4(x^*,y^*)$ is globally asymptotically stable.

To begin, we demonstrate that every solution of system (1.4) that starts in R_+^2 is uniformly bounded. Similarly to the analysis of (3.19)-(3.20), we have

$$\lim_{t\to+\infty}y(t)=\frac{K_2(r_2-Eq_2m)}{r_2}\overset{\text{def}}{=}y^*.$$

As a result, for arbitrary small positive constant $\varepsilon>0$, there exists a $T_4>0$ such that

$$y(t)<y^*+\varepsilon \text{ for all } t\ge T_4. \tag{3.24}$$

Similarly to the analysis of (3.18), from the first equation of system (1.4), we have for $t > T_4$,

$$\begin{aligned}\frac{dx}{dt} &< r_1x\Big(1-\frac{x}{K_1+b_1(y^*+\varepsilon)}\Big)-q_1Emx\\ &= x\Big(r_1-q_1Em-\frac{r_1x}{K_1+b_1(y^*+\varepsilon)}\Big)\end{aligned} \tag{3.25}$$

Now consider the equation

$$\frac{du}{dt} = u\Big(r_1-q_1Em-\frac{r_1u}{K_1+b_1(y^*+\varepsilon)}\Big). \tag{3.26}$$

It follows from Lemma 3.1 that

$$\lim_{t\to+\infty} u(t) = \frac{(r_1-q_1Em)\Big(K_1+b_1(y^*+\varepsilon)\Big)}{r_1}. \tag{3.27}$$

From (3.25) and (3.27), using the differential inequality theory, we have

$$\limsup_{t\to+\infty} x(t) \le \frac{(r_1-q_1Em)\Big(K_1+b_1(y^*+\varepsilon)\Big)}{r_1}. \tag{3.28}$$

As a result, there exists a $T_5 > T_4$ such that

$$x(t) < \frac{(r_1-q_1Em)\Big(K_1+b_1(y^*+\varepsilon)\Big)}{r_1}+\varepsilon \overset{\text{def}}{=} \Gamma_1(\varepsilon) \tag{3.29}$$

for all $t \ge T_5$. Let

$$D = \{(x,y)\in R_+^2 : x<\Gamma_1(\varepsilon),\ \ y<y^*+\varepsilon\}.$$

Then every solution of system (1.2) starts in R_+^2 is uniformly bounded on D. Let us now demonstrate that the system admits no limit cycle in the area D. Consider the Dulac function $u(x,y) = x^{-1}y^{-1}$,

$$\begin{aligned}&\frac{\partial(uF_1)}{\partial x}+\frac{\partial(uF_2)}{\partial y}\\ =&\ \frac{r_1\left(1-\frac{x}{b_1y+K_1}\right)-\frac{r_1x}{b_1y+K_1}-q_1Em}{yx}\\ &-\frac{r_1x\left(1-\frac{x}{b_1y+K_1}\right)-q_1Emx}{y\,x^2}\\ &+\frac{r_2\left(1-\frac{y}{K_2}\right)-\frac{r_2y}{K_2}-q_2Em}{xy}\\ &-\frac{r_2y\left(1-\frac{y}{K_2}\right)-q_2Emy}{x\,y^2}\\ =&\ -\frac{b_1r_2\,y^2+K_1r_2y+K_2r_1x}{yx\,(b_1y+K_1)\,K_2}<0,\end{aligned}$$

where

$$F_1(x,y) = r_1x\Big(1-\frac{x}{K_1+b_1y}\Big)-q_1Emx,$$

$$F_2(x,y) = r_2y\Big(1-\frac{y}{K_2}\Big)-q_2Emy.$$

According to [32], there is no closed orbit in area D. As a result, $E_4(x^*,y^*)$ is globally asymptotically stable. This completes the proof of Theorem 3.1.

Remark 3.1. Theorems 2.1 and 3.1 show that if the system (1.2) admits the unique positive equilibrium, then the positive equilibrium is globally asymptotically stable.

Remark 3.2. It follows from Theorems 2.1 and 3.1 that the local stability of the equilibrium also implies the global one.

Remark 3.3. Since

$$\frac{dx^*}{dm} = \frac{\Gamma_1}{r_1r_2} < 0,\ \ \frac{dy^*}{dm} = -\frac{EK_2q_2}{r_2} < 0,$$

where

$$\begin{aligned}\Gamma_1 &= E\Big(K_2b_1q_1(q_2Em-r_2)\\ &\quad +K_2b_1q_2(q_1Em-r_1)-K_1q_1r_1\Big).\end{aligned}$$

Both x^* and y^* are the strictly decreasing functions of m. This means that as more of the stock becomes available for harvesting, both species' final densities decrease. To ensure the coexistence of both species, harvesting should be limited to a small area

$$m < \min\Big\{\frac{r_2}{Eq_2},\frac{r_1}{Eq_1}\Big\}.$$

Otherwise, at least one of the species will be driven to extinction.

4 Numerical simulations

Example 4.1. Let's take $r_1 = 1, E = 4, q_1 = \frac{1}{2}, q_2 = 2, b_1 = 1, r_2 = 2, K_1 = 1, K_2 = 1$. In this case, by simple computation, one could easily see that

$$\frac{r_1}{Eq_1} = \frac{1}{2},\frac{r_2}{Eq_2} = \frac{1}{4},$$

Corresponding to Theorem 3.1, we have
(1) For $m > \frac{1}{2}$, $E_1(0,0)$ is the globally asymptotically stable equilibrium, Fig.1 is the case of $m = 0.7$;
(2) For $\frac{1}{4} < m < \frac{1}{2}$, the boundary equilibrium $E_2(x_0,0)$ is globally asymptotically stable, Fig. 2 is the case of $m = 0.3$;
(3) For $m < \frac{1}{4}$, the positive equilibrium $E_4(x^*,y^*)$

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is globally asymptotically stable, Fig.3 is the case of $m = 0.1$.
Lastly, the dynamic behaviors of the second component x_2 in system (equation. 3.1) with the initial condition $(x(0), y(0)) = (0.5, 0.5)$, $(1, 1)$, $(1.5, 1.5)$ and $(2, 2)$ are presented in Fig.4.

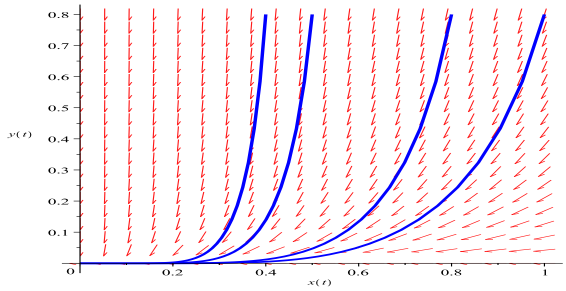

Figure 1: Numeric simulations of system (4.1) with $m = 0.7$, the initial conditions $(x(0), y(0)) = (1, 0.8), (0.4, 0.8), (0.5, 0.8)$, and $(0.8, 0.8)$, respectively.

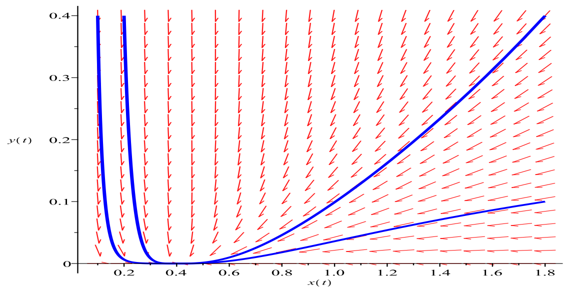

Figure 2: Numeric simulations of system (4.1) with $m = 0.3$, the initial conditions $(x(0), y(0)) = (0.1, 0.4), (1.8, 0.1), (0.2, 0.4)$, and $(1.8, 0.4)$, respectively.

5 Discussion

Based on the traditional Lotka-Volterra commensalism model (1.1), [19], proposed a Lotka-Volterra commensalism model with non-selective harvesting in a partial closure, their study showed that partial closure plays an important role in the persistence and stability property of the system.

Inspired by the works of [19], [26], in this paper, based on a commensalism model proposed by Vargas-De-León, Gómez-Alcaraz (we called it May type commensalism model), we also propose a May type commensalism model incorporating nonselective harvesting in the partial closure.

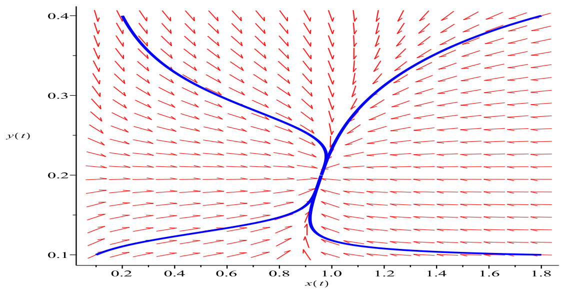

Figure 3: Numeric simulations of system (4.1) with $m = 0.1$, the initial conditions $(x(0), y(0)) = (0.1, 0.1), (1.8, 0.1), (0.2, 0.4)$, and $(1.8, 0.4)$, respectively.

The dynamic behaviors of our model (1.4) is similar to that of the model (1.2). However, it seems that our conditions are very simple. We believe this is because in the model (1.2), the commensalism species y influence the intrinsic rate of the first species x, indeed, the intrinsic growth rate of the first species changed from r_1 to $r_1 + r_1\alpha\frac{y}{K_1}$, while in our model, the commensalism species y only increased the carrying capacity of the first species (carrying capacity changed from K_1 to $K_1 + b_1y$), and has no influence on the intrinsic growth rate of the first species.

Already, [26], previously demonstrated that May type commensalism model admits a unique positive equilibrium, which is globally asymptotically stable, implying that the species could coexist in the long run. When we incorporate harvesting to the system, the dynamic behaviors of the system are changed dramatically. Theorem 3.1 shows that all four possible equilibria may be globally attractive, depending on the harvesting effort and the area that could be harvested. That is, the dynamic behaviors of the system with harvesting are more complicated than the system without harvesting. Harvesting is one of the most important factors that leads to the extinction of the species.

It seems that different assumptions may lead to different phenomena, maybe it is interesting to study the influence of nonlinear harvesting on the system (1.3), we leave this for future study.

Many scholars, [22], [23], [24], [25], [26], [39], have recently studied the influence of the Allee effect's impact on commensalism system. For Merdan type Allee effect, [22], [23], [24], [25], [26] show that the dynamic behaviors of the system with the Allee effect are similar to those of the system without the Allee effect, however, for the system with Additive Allee effect, [39], recently showed that the dynamic behaviors could be very complicated. It brings to our attention that, to this day, no scholar has proposed and studied the influence of the Allee effect on the May-type commensalism model. We will try to do some work in this direction.

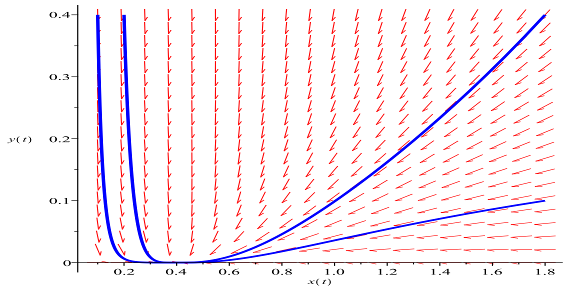

Figure 4: Dynamic behaviors of the second component x_2 in system (equation. 3.1) with the initial condition $(x(0), y(0)) = (0.5, 0.5)$, $(1, 1)$, $(1.5, 1.5)$ and $(2, 2)$, respectively.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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