On Some New Uncertain Spaces using Generalized Difference Operator

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Abstract: - It was C. You, who has given the concept of uncertainty theory in 2009. The scenario of this paper is to define a new notion of spaces using generalized Δ- operator and the uncertainty theory. Also, certain basic structures will be given. Moreover, inclusion relations and their counter examples will be given.

Key-Words: - Δ-operator; uncertain variable; convergence
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1 Introduction

Uncertainty theory was introduced in [1] and many researchers have shown wings in this area of study. The random event is given by probability measure. It was Zadeh, [2], who introduced possibility Fuzzy measure. In, [3], the authors have introduced a self-dual measure and the measure of credibility and in, [4], the axiomatic structure for credibility theory have been determined and can be seen in, [5]. In order to analyze the concept that fuzziness and randomness simultaneously in system, a fuzzy random variable have been given in, [6]. In, [7], the results of convergence were given. Further, in, [8], the notion of chance measure was established. The various applications of these structures have been given in, [9], [10].

For a non-void set Ʊ, and a σ-algebra L over Ʊ, we call each entry $v \in$ L as an event. The uncertain measure has been well structured as a set function S which satisfies the following axioms:

a. M{Ʊ} = 1 (normality).

b. M$\{v_1\} \leq$ M$\{v_2\}$ whenever $v_1 \leq v_2$ (monotonicity).

c. $\mathcal{M}\{v_r\} + \mathcal{M}\{v_1^c\} = 1$ for any v_1 (self-duality).

d. For every countable sequence of events $\{v_r\}$ gives

$$\mathcal{M}\{\cup_{r=1}^{\infty} v_r\} \leq \sum_{r=1}^{\infty} \mathcal{M}\{v_r\}$$

(Countable subadditivity).

Using, [11], we have following definitions:

For uncertain variable ζ, the map $\psi(\varrho)$ is said to be uncertainty distribution if

$$\psi(\varrho) = \mathcal{M}\{\tau \in Ʊ |\ \zeta(\tau) \leq \varrho\}$$

An uncertain sequence $\{\varrho_r\}$ is said to converge almost surely (a.s.) to ϱ if we can find an event v having M(v) = 1 in such a way that

$$\lim_{r\to\infty} \|\varrho r(w) - \varrho(w)\| = 0 \text{ for all } w \in v.$$

An uncertain sequence $\{\zeta\}$ is called as convergent in measure to ζ if

$$\lim_{r\to\infty} \mathcal{M}\{\|\zeta_j(\eta) - \zeta(\eta)\| \geq \varepsilon\} = 0, \varepsilon > 0$$

A sequence $\{\zeta_j\}$ is said to be convergence in average to ζ if

$$\lim_{j\to\infty} E[|\zeta_j(\eta) - \zeta(\eta)|] = 0.$$

For uncertain variable $\vartheta, \vartheta_1, \vartheta_2, \cdots$ having respectively the expected values as $\zeta, \zeta_1, \zeta_2, \cdots$. Then, $\{\zeta_j\}$ is said to be convergence in distribution to ζ if corresponding to any continuous point ϑ, we have $\vartheta_n \to \vartheta$.

For a sequence $\vartheta = (\eta_r)$ of whole numbers having $\eta_0 = 0$, $0 < \eta_r < \eta_{r+1}$ and $h_r = \eta_r - \eta_{r-1} \to \infty$ for $r \to \infty$, we call ϑ to be as lacunary sequence as can be seen in, [12], [13], [14], [15], [16], and many others.

Space Ɛ as normal (or solid) if $(\zeta_j) \in$ Ɛ yields $(\beta_j v_j) \in$ Ɛ for each scalar sequence (β_j) having $|\beta_j| \leq 1 \; \forall j \in \mathbb{N}$ as in, [17], [18], [19], and many others.

Lemma 1.1. *Every solid sequence space is monotone.*

In, [20], we have following for T ∈ $\{\ell_\infty, c, C_0\}$:
$T(\Delta) = \{v = (v_i) \in \Lambda : (\Delta v_i) \in T\}$, where and $\Delta v_i = v_i - v_{i-1}$.

Also, in, [21], we have following for integer $s \geq 0$:

$$\Delta^s(T) = \{v = (v_k) : (\Delta^s v) \in T\},$$

with $\Delta^s v_r = \Delta^{s-1} v_r - \Delta^{s-1} v_{r+1}$ for every $r \in \mathbb{N}$.

Further, consider sequence $g = (g_j)$ of non-zero complex numbers, then as in, [22], we have

$$\Delta_g^s(\tau) = \{v = (v_j) \in \Lambda : (\Delta_g^s v_j) \in \tau\},$$

where

$$\Delta_g^s v_j = \Delta_g^{s-1} v_j - \Delta_g^{s-1} v_{j+1} = \sum_{\mu=0}^{s} (-1)^\mu \binom{s}{\mu} g_{j+\mu} v_{j+\mu} \forall j \in \mathbb{N}$$

And $\Delta_g^s(\tau)$ is complete having norm

$$\|v\|_\Delta = \sum_{i=1}^{s} |g_i v_i| + \|\Delta_g^s v\|_\infty$$

The various well-structured properties concerning this space can be found in, [17], [23], [24], [25], [26], [27], [28], and many others.

This space was further studied and according to, [9], we have following in hand for fixed integers $s, t \geq 0$:

$$\Delta_t^s(\tau) = \{v = (v_j) \in \Lambda : (\Delta_t^s v_j) \in \tau\}$$

where

$$\Delta_t^s v_j = \Delta_t^{s-1} v_j - \Delta_t^{s-1} v_{j+1} = \sum_{\mu=0}^{s} (-1)^\mu \binom{s}{\mu} v_{j+\mu t} \forall j \in \mathbb{N}$$

Clearly, by taking $s = 1 = t$, we have results obtained in, [20], and choosing $t = 1$, we get results of, [21].

Following these, we involve the Δ-definition with uncertain variables by joining of lacunary sequences and will determine some new results.

2 Main Results

This section deals with the introduction of new sequences of uncertain variables using Δ-operator.

Following the cited references as, [2], [9], [29], [30], [31], [32], [33], [34], [35], we define the following new spaces:

$$[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0 = \left\{\zeta = (\zeta_j) : \lim_{j\to\infty} \frac{1}{h_j} \sum_{i\in I_j} \|g_j \, \Delta_t^s \zeta_j(v)\| = 0\right\}$$

$$[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_c = \left\{\zeta = (\zeta_j) : \lim_{j\to\infty} \frac{1}{h_j} \sum_{i\in I_j} \|g_j \, \Delta_t^s \zeta_j(v) - \sigma(v)\| = 0\right\}$$

and

$$[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_\infty = \left\{\zeta = (\zeta_j) : \sup_j \frac{1}{h_j} \sum_{i\in I_j} \|g_j \, \Delta_t^s \zeta_j(v)\| < \infty\right\}$$

where $\sigma(v) \in$ (Ʊ, L, S).

Theorem 2.1.
The sets $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0, [\mathcal{N}_\theta^\mu, g, \Delta_t^s]_c$ *and* $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_\infty$ *are linear.*

Proof: We prove the result for $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$ only and rest will follow on similar steps. Therefore, suppose that $(x_i), (y_i) \in [\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$, then,

$\lim_{j\to\infty} \frac{1}{h_j} \sum_{i\in I_j} \|g_j \, \Delta_t^s x_j(v)\| = 0$ and $\lim_{j\to\infty} \frac{1}{h_j} \sum_{i\in I_j} \|g_j \, \Delta_t^s y_j(v)\| = 0$

Now for any $a, b \in \mathbb{C}$, we have

$$\lim_{j\to\infty} \frac{1}{h_j} \sum_{i\in I_j} \left\| g_j \Delta_t^s \; a x_j(v) + b y_j(v) \; \right\|$$
$$= \lim_{j\to\infty} \frac{1}{h_j} \sum_{i\in I_j} \left\| a g_j \Delta_t^s x_j(v) + b g_j \Delta_t^s y_j(v) \right\|$$
$$\leq |a| \lim_{j\to\infty} \frac{1}{h_j} \sum_{i\in I_j} \left\| g_j \Delta_t^s x_j(v) \right\| + |b| \lim_{j\to\infty} \frac{1}{h_j} \sum_{i\in I_j} \left\| g_j \Delta_t^s y_j(v) \right\|$$
$$\to 0, \text{ as } j \to \infty.$$

Consequently, $\left(a x_j(v) + b y_j(v)\right) \in [\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$ and the result follows. ◊

Theorem 2.2. *The sets* $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$, $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_c$ *and* $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_\infty$ *of complex certain sequences are normed linear spaces with norm*

$$\|\zeta(v)\|_{\Delta_t^s} = \sum_{k=1}^{n} \|g_k \zeta_k(v)\| + \sup_j \frac{1}{h_j} \sum_{i \in I_j} \|g_j \Delta_t^s \zeta_j(v)\|$$

Proof: It can be proved by using classical techniques.⋄

Theorem 2.3. *If $T = 0, c, \infty$, $t \geq 1$ and $s \geq 1$, then*

$$[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_T \subset [\mathcal{N}_\theta^\mu, g, \Delta_t^s]_T$$

The inclusions are proper.

Proof: Only the case of $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$ will be proved and rest will follow on similar lines. So, let $\zeta_j \in [\mathcal{N}_\theta^\mu, g, \Delta_t^{s-1}]_0$, then we see

$$\lim_{j\to\infty} \frac{1}{h_j} \sum_{i \in I_j} \|g_j \Delta_t^{s-1} \zeta_j(v)\| = 0 \quad (1)$$

We can write

$$\frac{1}{h_j}\sum_{i\in I_j}\left\|g_i\Delta_t^s\zeta_i(v)\right\| = \frac{1}{h_j}\sum_{i\in I_j}\left\|g_i\Delta_t^{s-1}\zeta_i(v) - g_{i+1}\Delta_t^{s-1}\zeta_{i+1}(v)\right\|$$
$$\leq \left(\frac{1}{h_j}\sum_{i\in I_j}\|g_i\Delta_t^{s-1}\zeta_i(v)\| - \frac{1}{h_j}\sum_{i\in I_j}\|g_{i+1}\Delta_t^{s-1}\zeta_{i+1}(v)\|\right)$$

Using (1) and approaching $j \to \infty$ in above inequality yields

$$\frac{1}{h_j} \sum_{i \in I_j} \|g_j \Delta_t^s \zeta_j(v)\| = 0, \{\zeta_j\} \in [\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$$

Consequently, to establish the result is proper, we choose a lacunary sequence $\theta = (2^i)$ and take the sequence of uncertain variables as $(\zeta_i) = (i_{s-1})$ with $g_i = 1$ for each $i \in \mathbb{N}$. Consequently, for all $i \in \mathbb{N}$, we have

$$\Delta_t^s(\zeta_i) = 0 \text{ and } \Delta_t^s\zeta_i = \sum_{r=0}^{s-1}(-1)^r\binom{s-1}{r}\zeta_{i+tr}$$

Hence, $\{\zeta_j\} \in [\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$ but not $[\mathcal{N}_\theta^\mu, g, \Delta_t^{s-1}]_0$ as desired.⋄

Theorem 2.4. *If* T = 0, *c*, ∞, *then, in general, the sets* $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_\tau$ *is not symmetric.*

Proof: The result will be proved for $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$ only and others will be proved similarly by taking $s = 2 = t$ and $g_i = 1$ for each $i \in \mathbb{N}$ and by considering $\theta = (2^i)$ as lacunary sequence.

Setting the uncertainty space (Ʊ, L, M) as $\{\tau_1, \tau_2, \cdots\}$ with the power set and taking any event $v \in$ L such that

$$\mathcal{M}\{v\} = \begin{cases} \sup_{\tau_r \in v} \frac{r}{2r+1}, & if \ \sup_{\tau_r \in v} \frac{r}{2r+1} < 0.5, \\ 1 - \sup_{\tau_r \in v^c} \frac{r}{2r+1}, & if \ \sup_{\tau_r \in v^c} \frac{r}{2r+1} < 0.5, \\ 0.5, & if \ elsewhere. \end{cases}$$

Now consider,

$$\zeta_r(\tau_j) = \begin{cases} r, & when\ j = r, \\ 0, & when\ elsewhere. \end{cases}$$

Clearly, $\{\zeta_r\}$ for $r \in I_m$ and $m = 1, 2, \ldots$ is in $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$. Consider the rearrangement of $\{\zeta_r\}$ as $\{\omega_m\}$ given by $\omega_m(\tau) = \{\zeta_1, \zeta_4, \zeta_9, \zeta_2, \zeta_1 0, \cdots\}$ and is consequently not in $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$. From which, we conclude that $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_0$ is not symmetric.⋄

On similar lines, the following theorem is obvious.

Theorem 2.5. T*he sets* $[\mathcal{N}_\theta^\mu, g, \Delta_t^s]_\tau$ *is not monotone in general for* $\tau = 0, c, \infty$ *and* $s \geq 1$.

3 Structure of Lacunary Convergence using Δ −Operator

This portion of the manuscript deals with the lacunary convergence structure of Δ-operator for uncertain variables. Also, we will establish certain new relation corresponding to them.

Definition 3.1. The uncertain sequence $\{\zeta_j\}$ is called as lacunary strongly convergent almost surely to ζ with respect to difference sequence if for $\varepsilon > 0$, we can find an event v with M(v) = 1 such that

$$\lim_{j\to\infty}\frac{1}{h_j}\sum_{l\in I_j}\|g_l\Delta_t^s\zeta_l(\eta) - \sigma(\eta)\| = 0$$

for all $\eta \in v$.

Definition 3.2. An uncertain sequence $\{\zeta_j\}$ is called as lacunary strongly convergent in measure to

$$\lim_{j\to\infty}\mathcal{M}\left[\left\{\eta \in Ʊ : \frac{1}{h_j}\sum_{i\in I_j}\|g_i\Delta_t^s\zeta_i(\eta) - \sigma(\eta)\| > \varepsilon\right\}\right]$$

ζ if

$$\lim_{j\to\infty}\mathcal{M}\left[\left\{\eta \in Ʊ : \frac{1}{h_j}\sum_{l\in I_j}\|g_l\Delta_t^s\zeta_l(\eta) - \sigma(\eta)\| > \varepsilon\right\}\right] = 0$$

for all $\varepsilon > 0$.

Definition 3.3. Let $\zeta, \zeta_1, \zeta_2, \cdots$ be the uncertainty distributions of uncertain variables. Then the sequence $\{\zeta_j\}$ is called convergence in mean to ζ if

$$\lim_{j\to\infty} E\left[\frac{1}{h_j}\sum\nolimits_{l\in I_j} \|g_l \Delta_t^s \varsigma_l(n) - \sigma(n)\|\right] = 0$$

Definition 3.4. Let ϑ_1, ϑ_2, $\vartheta_3,\cdots$ be uncertain variables with finite expected values ζ, ζ_1, $\zeta_2,\cdots$, respectively. Then the sequence $\{\zeta_j\}$ is said to be lacunary strong convergent in distribution to to ζ w.r.t. difference sequence if

,

$$\lim_{j\to\infty} \frac{1}{h_j}\sum_{l\in I_j} \|g_l \Delta_t^s \vartheta_l(\lambda) - \sigma(\lambda)\| = 0$$

for every complex λ in which $\vartheta(\lambda)$ is continuous.

Definition 3.5. The uncertain sequence $\{\vartheta_i\}$ is said to be convergent uniformly almost surely to ζ if we can find a sequence of events $\{E_j\}$, $\mathrm{M}\{E_j\} \to 0$ such that $\{\vartheta_i\}$ converges uniformly to ζ in $\mho - \varepsilon_j$ for some fixed $j \in \mathbb{N}$.

Theorem 3.6. *Consider an uncertain sequence $\{\zeta_i\}$. If it is lacunary strongly convergent in average to ζ w.r.t. difference sequence, then it converges lacunary strongly in measure to ζ, but not conversely.*

Proof: By Markov's inequality, we have for $\varepsilon > 0$ that

$$\leq \lim_{j\to\infty} \frac{E\left[\frac{1}{h}\sum_{i\in I_j} \|g_i \Delta_t^s \varsigma_i(n) - \sigma(n)\|\right]}{\varepsilon} \to 0$$

as $i \in I_j$. Thus $\{\zeta_i\}$ is converges in measure to σ w.r.t. difference sequence.

Now for converse, define uncertainty space ($\mho$, L, M) as $\{\tau_1, \tau_2,\cdots\}$ and consider the event $v \in$ L such that

$$\mathcal{M}\{v\} = \begin{cases} \sup\limits_{\tau_m\in v} \dfrac{1}{m}, & if \ \sup\limits_{\tau_m\in v} \dfrac{1}{m} < 0.5, \\ 1 - \sup\limits_{\tau_m\in v^c} \dfrac{1}{m}, & if \ \sup\limits_{\tau_m\in v^c} \dfrac{1}{m} < 0.5, \\ 0.5, & if \ elsewhere. \end{cases}$$

Also, set the uncertain variables as

$$\varsigma_m(\tau_r) = \begin{cases} m & if \ r = m \\ 0 & if \ elsewhere \end{cases}$$

for every $m \in I_j$ and $\sigma \equiv 0$. Now for $\varepsilon > 0$, we see

$$\lim_{j\to\infty} \mathcal{M}\left[\left\{\eta \in \mho :\ \frac{1}{h_j}\sum_{l\in I_j} \|\Delta_l^s \zeta_l(\eta) - \sigma(\eta)\| > \varepsilon\right\}\right]$$

$$= \lim_{j\to\infty} \mathcal{M}\left[\left\{\eta \in \mho :\ \frac{1}{h_j}\sum_{l\in I_j} \|\Delta_l^s \zeta_l(\eta)\| > \varepsilon\right\}\right]$$

$$= \lim_{j\to\infty} \mathcal{M}\left(\left\{\eta_i\right\}\right)$$

$$= \lim_{j\to\infty} \frac{1}{i} \to 0 \ as \ i \in I_j.$$

This shows that $\{\zeta_i\}$ converges in measure to ζ. But, for all $i \in I_j$, we see the uncertainty distribution of uncertain variable $\|\zeta_i - \zeta\| = \|\zeta_i\|$ is

$$\vartheta_m(\varrho) = \begin{cases} 0, & \text{if } \varrho < 0, \\ 1 - \frac{1}{m} & \text{if } 0 \leq \varrho < m, \\ 1, & \text{if } elsewhere. \end{cases}$$

$$E\left[\frac{1}{h_j}\sum\nolimits_{i\in I_j} \|g_i \Delta_t^s \varsigma_i(n) - \sigma(n)\|\right]$$

$$= \int_0^{+\infty} \mathcal{M}\{\zeta(\varrho)\}d\varrho - \int_{-\infty}^0 \mathcal{M}\{\zeta \leq \varrho\}d\varrho$$

$$= = \int_0^m 1 - \left(1 - \frac{1}{m}\right) d\varrho = 1.$$

Consequently, $\{\zeta_i(\varrho)$ does not converge in mean to $\zeta(\varrho)$ w.r.t. difference sequence.

4 Conclusion

In this work, we have discussed the uncertainty of variables and have defined new type of spaces $\left[\mathcal{N}_\theta^\mu, g, \Delta_t^s\right]_0, \left[\mathcal{N}_\theta^\mu, g, \Delta_t^s\right]_c, and \left[\mathcal{N}_\theta^\mu, g, \Delta_t^s\right]_\infty$ using generalized Δ-operator. We have computed their linear structure and some of their topological structures. The inclusions relations corresponding to these spaces have been given. To support the properness, some example have been given. Symmetry and monotonicity corresponding to these spaces are analyzed. Further, we have constructed some basic ideas of Lacunary convergence using uncertain variables and Δ-operator. In the future, we will continue to study the above proposed structures by applying statistical convergence and the modulus function to obtain the new results.

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Conflict of Interest

I declare that there is no conflict of interest.