Data Interpretation Algorithm for Adaptive Methods of Modeling and Forecasting Time Series

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Abstract: - The paper considers two forms of models: seasonal and non-seasonal analogues of oscillations. The paper analyzes the basic adaptive models: Brown, Holt, and autoregression. The parameters of adaptation and layout are considered by the method of numerical estimation of parameters. The mechanism of reflection of oscillatory (seasonal or cyclic) development of the studied process through a reproduction of the scheme of moving average and the scheme of autoregression is analyzed. The paper determines the optimal value of the smoothing coefficient through adaptive polynomial models of the first and second order. Prediction using the Winters model (exponential smoothing with multiplicative seasonality and linear growth) is proposed. The paper proves that the additive model allows building a model with multiplicative seasonality and exponential tendency. The paper proves statements that allow to choose the right method for better modeling and forecasting of data.

Key-Words: - *A*verage, Holt-Winters model, polynomial time series models, exponential smoothing

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1 Introduction

Effective analysis, modeling, and forecasting of financial and economic processes form the foundation for making informed management decisions across all levels of the economic hierarchy. However, this task is inherently complex and ambiguous, necessitating the use of advanced models and methods to accurately capture the nuances of modern financial and economic processes, [1].

Most often, in the practical construction of forecasts of economic indicators, their seasonality and cyclicality are taken into account. Different mathematical apparatus is used to predict non-seasonal and seasonal processes. The dynamics of many financial and economic indicators have a stable fluctuating component. The study of monthly and quarterly data is often observed within the annual seasonal fluctuations, respectively, in the period of 12 and 4 months. When using daily observations, fluctuations with a weekly (five-day) cycle are often observed. In this case, to obtain more accurate forecast estimates, it is necessary to correctly reflect not only the trend but also the oscillating component. The solution to this problem is possible only with the use of a special class of methods and models, [1], [2], [3], [4], [5].

Seasonal models are based on their non-seasonal counterparts, which are supplemented by means of displaying seasonal fluctuations. Seasonal models can reflect both a relatively constant seasonal wave and a wave that changes dynamically depending on the trend. The first form belongs to the class of additive, and the second - to the class of multiplicative models, [2]. Most models have both of these shapes. The most widely used in practice are Holt-Winters models, [6], and autoregressions, [7].

In short-term forecasting, the dynamics of the development of the studied indicator at the end of the observation period is usually more important than the trend of its development, which has developed on average throughout the prehistory period. The property of dynamic development of financial and economic processes often prevails over the property of inertia, so adaptive methods that take into account information inequality of data are more effective, [8].

Adaptive models and methods have a mechanism of automatic adjustment to change the studied indicator. The forecasting tool is a model, the initial assessment of the parameters of which is carried out on the first few observations. Based on it, a forecast is made, which is compared with actual observations. Next, the model is adjusted according to the magnitude of the forecast error and is used again to predict the next level, until all observations are exhausted. Thus, it constantly "absorbs" new information, adapts to it, and by the end of the observation period reflects the current trend, [9], [10]. The forecast is obtained as an extrapolation of the latest trend. In different forecasting methods, the process of setting up (adapting) the model is carried out in different ways. Basic adaptive models are:

– Brown model, [11];
– Holt-Winters model, [6];
– autoregression model, [7].

The first two models belong to the average mean scheme, the latter to the autoregression scheme, [12]. Numerous adaptive methods based on these models differ in the way of numerical estimation of parameters, determination of adaptation parameters, and layout.

Under the moving average approach, the current level estimation is a weighted average of prior levels, with decreasing weights assigned to observations as they become further removed from the most recent level. In essence, observations closer to the end of the observation period hold greater informational value, [13].

According to the autoregression scheme, the estimate of the current level is the weighted sum of the orders of the models "p" of the previous levels. The information value of observations is determined not by their proximity to the simulated level, but by the closeness of the relationship between them, [14], [15], [16]. Both of these schemes have a mechanism for reflecting the oscillating (seasonal or cyclical) development of the studied process.

Autoregressive Integrated Moving Average (ARIMA) is a popular method for forecasting time series data using a single variable, [17]. The problem with ARIMA is that it doesn't support seasonal data. This is a time series with a repeating cycle. ARIMA expects data that is not seasonal or has a seasonal component removed, for example seasonally adjusted using techniques such as seasonal variance. This method supports direct modeling of the seasonal component of the series called Seasonal Autoregressive Integrated Moving Average SARIMA, [14].

SARIMA, an extension of ARIMA, is specifically designed to handle univariate time series data that exhibit seasonal patterns. The model incorporates seasonal terms that closely resemble the non-seasonal components but account for reversed shifts in the seasonal period.

Prophet is a technique for predicting time series data through an additive model that captures non-linear trends using yearly, weekly, and daily seasonality, along with holiday effects. The method is particularly effective for time series with significant seasonal patterns and a considerable historical data set. Prophet is highly resistant to data gaps and trend shifts, and can typically handle outlier values with ease, [18].

The purpose of the paper is to develop the adaptive methods of modeling and forecasting the time series based on a combination of the adaptive methods of predictive modeling:

– Holt-Winters model, [19],
– moving average model, [20].

Time series analysis is predominantly concerned with predicting real values, which can be characterized as regression problems. As a result, the evaluation metrics described in the paper will concentrate on techniques for assessing the accuracy of predictions for continuous variables.

The main contribution consists of the following:

– the adaptive polynomial models used sequentially allow to increase in the prediction accuracy,
– the data interpretation algorithm for adaptive methods of modeling and forecasting time series is developed,
– the comparison between the Winters model and the Tayle-Wage model shows the good quality of the proposed predictive model.

This paper consists of several sections. In the Methods and Means section, the data interpretation algorithm for adaptive methods of modeling and forecasting time series is given. The next section presents the result of the calculation and data interpretation. The last section concludes this paper by containing the probable decision of appraisal technique.

2 Methods and Means

The time series in adaptive models are presented in the form (Formula 1):

$$u_t = f(a_{1t}, a_{2t}, \dots, a_{pt}, t) + e_t, \quad (1)$$

where t – time indicator; $a_{1t}, a_{2t}, \dots, a_{pt}$ – coefficients of the adaptive model at the moment of time t.

Depending on the shape of the trend and the presence or absence of a periodic component, a certain type of adaptive forecasting should be chosen. To do this, you need to find the optimal value of the smoothing parameters $\beta_1, \beta_2, \beta_3$. They should be used to calculate the coefficients $a_{1t}, a_{2t}, \dots, a_{pt}$.

if the smoothing parameters change, the prediction error increases. However, this approach will not bring the quality of forecasting. The research proposes an algorithm for determining the optimal values of smoothing parameters.

Also, it is important to analyze the effectiveness of the adaptive approach in other methods. Therefore, it is proposed to develop an algorithm that allows you to take into account the accuracy of the forecast, the complexity of the model, and its adequacy and compliance with the object under study.

There are two groups of adaptive models: linear and seasonal.

According to Formula 2, the forecast of linear growth models is shown, [31]:

$$u_{t+\tau} = a_{1t} + a_{2t}\tau, \quad (2)$$

where a - the number of steps of the forecast; a_{1t}, a_{2t} - the coefficients of the adaptive model at a moment of time t

Adaptive models of linear growth include the Holt model, the Braun model, and the Box-Jenkins model. The difference between linear growth models lies in finding the parameters a_{1t}, a_{2t}, [31].

The parameters of the Holt model are found in Formula 3:

$$\begin{cases} a_{1,t} = \beta_1 u_t + (1-\beta_1)(a_{1,t-1} + a_{2,t-1}) \\ a_{2,t} = \beta_2\left(a_{1,t} - a_{1,t-1}\right) + (1-\beta_2)a_{2,t-1} \end{cases} \quad (3)$$

Formula 4 presents the calculation of parameters according to the Tayle-Vage model, [31]:

$$\begin{cases} a_{1,t} = \beta_1 u_{t-1} + (1-\beta_1)\widehat{u_t} \\ a_{2,t} = a_{2,t-1} + \beta_1\beta_2 e_t \quad , \\ e_t = u_t - \widehat{u_t} \end{cases} \quad (4)$$

where $\beta_1, \beta_2, \beta_3$ are the smoothing coefficients that take values from 0 to 1, u_t - the real value of the series level at the t-th step, $\hat{u}_t -$ the predictive value at the t-th step, e_t - the error at the t-th step.

Characterizing the calculation of the parameters of Formulas 3-4, it is possible to highlight a certain feature of adaptive models. It is necessary to calculate a_{1t}, a_{2t} at each step. For the model to give better results, it is necessary to find $\beta_1, \beta_2, \beta_3$, which will most closely correspond to the time series.

The adaptive monoparameter Braun model is used for stationary time series based on simple exponential smoothing:

$$\hat{y}_{t+1} = S_t, S_t = \alpha y_t + (1-\alpha)S_{t-1}, t = 1,2,3,... \quad (5)$$

where y_{t+1} is the prognostic value of time series level in time (t+1), $S_t$is exponential mean, αis adaptation coefficient, y_t is the current time series value.

In this context, the model's value is a weighted average of both the current true value and past model values. The weight, referred to as the smoothing factor or alpha (α), dictates the rate at which the model "forgets" the most recent actual observation. A smaller α places more emphasis on earlier model values, resulting in a smoother series.

Taking the adaptation coefficient α and the warning period τ, it is necessary to approximate the series using an adaptive polynomial model.

The Data Interpretation Algorithm for Adaptive Methods of Modeling and Forecasting Time Series (DIAAMMFTS) is developed in the paper.

DIAAMMFTS consists of the following steps:

Procedure 1:Zero order (p = 0);

Procedure 2:First order (p = 1);

Procedure 3:Second order (p = 2);

Procedure 4:Assess the accuracy and quality of forecasts;

Procedure 5:Make a forecast.

All procedures of DIAAMMFTS are presented below.

Procedure 1.

Procedure 1 developed as a sequence of the following steps:

1. Let $\hat{y}_0 = y_0$.
2. Append array$\hat{y}$using the following formula: $\hat{y}_t = \alpha * y_t + (1-\alpha) * \hat{y}_t - 1$, where y_t is an actual value and $\hat{y}_{t-1}$ is the previous number from the prediction array.
3. Repeat step 2 for all values in a dataset.

Procedure 2.

So far, we have been able to get from our methods at best a forecast only one point ahead (and still nicely smooth the series), this is great, but not enough, so we move to the expansion of exponential smoothing, which will build the forecast two points forward (and also nice to smooth out a number).

This will help us to divide the series into two components - ℓ (level, intercept) and b(trend, slope). The level, or expected value of the series, we predicted using previous methods, and now the same exponential smoothing can be applied to the trend, naively or not very much believing that the future direction of change of the series depends on weighted previous changes.

$$\ell_x = \alpha y_x + (1-\alpha)(\ell_{x-1} + b_{x-1}),$$
$$b_x = \beta(\ell_x - \ell_{x-1}) + (1-\beta)b_{x-1}, \quad (6)$$
$$\hat{y}_{x+1} = \ell_x + b_x.$$

The algorithm is the following:

1. Let $x = 1$, $\hat{y}_0 = y_0$, $\ell_0 = y_0$ and $.b_0 = y_1 - y_0$, where y is our initial dataset.
2. Define new level value using the formula: $\ell_x = \alpha y_x + (1-\alpha)(\ell_{x-1} + b_{x-1})$.
3. Define new trend value using the formula:$b_x = \beta(\ell_x - \ell_{x-1}) + (1-\beta)b_{x-1}$.

4. Defineour prediction $\hat{y}_{x+1} = \ell_x + b_x$.
5. Definex $= x + 1$ and repeat steps 2-5 until $x < n$.

Procedure 3.

This technique involves introducing a third component - seasonality - to the model. Thus, it can only be applied when a specific seasonal pattern is present, which is the case in our scenario. The seasonal component accounts for cyclic fluctuations around the trend and level, and is determined by the length of the seasonal pattern, indicating the duration after which the fluctuations repeat. For each observation in the season, a corresponding component is generated. For instance, if the seasonal pattern is weekly (with a length of 7), seven seasonal components are derived, each representing a specific day of the week.

Therefore, a new system is defined:

$$\begin{gathered} \ell_x = \alpha(y_x - s_{x-L}) + (1-\alpha)(\ell_{x-1} + b_{x-1}), \\ b_x = \beta(\ell_x - \ell_{x-1}) + (1-\beta)b_{x-1}, \\ s_x = \gamma(y_x - \ell_x) + (1-\gamma)s_{x-L}, \\ \hat{y}_{x+m} = \ell_x + mb_x + s_{x-L+1+(m-1)modL.} \end{gathered} \quad (7)$$

The algorithm is the following:

1. Let x = 1, L = 24*7 $\hat{y}_0 = y_0$, $\ell_0 = y_0$ and .$b_0 = \frac{\sum_{i=0}^{L}(y_{i+L} - y_i)/L}{L}$, $s\_num = \frac{y.length}{L}$, where y is the initial dataset, and L is the length of the season in our case we set it to count weeks and s_{num} is the number of seasons.
2. Defineavrg using this formula $\frac{\sum_{i=0}^{L} y_{i*n}}{L}$
3. Count n = n+1. Repeat step 2 until n<s_num.
4. Defines$_0$using formula $\sum_{i=0}^{L}\sum_{j=0}^{s\_num} y_{L*J+i} - avrg_j$.
5. Define new level using the formula $\ell_x = \alpha(y_x - s_{x-L}) + (1-\alpha)(\ell_{x-1} + b_{x-1})$
6. Define new trend using the formula $b_x = \beta(\ell_x - \ell_{x-1}) + (1-\beta)b_{x-1}$
7. Define new s using the formula $s_x = \gamma(y_x - \ell_x) + (1-\gamma)s_{x-L}$
8. Define new result using the formula $\hat{y}_x = (\ell_x + b_x + s_x)$
9. Count x = x+1. Repeat steps 5-8 until x<y.length.
10. Make a prediction using the formula $\hat{y}_{x+m} = \ell_x + mb_x + s_{x-L+1+(m-1)modL}$, where m is the number that indicates how many steps forward we want to predict.

The current level is now determined by subtracting the corresponding seasonal component from the current series value, while the trend remains constant. Additionally, the seasonal component is calculated based on the current series value minus the level and the preceding component value. With the inclusion of the seasonal component, we can now make predictions for any desired number of steps (m) into the future.

3 Results

The dataset consists of the dynamics of shares of a company for 25 days, [21].

The time series xt of some economic indicators consisting of n observations will be analyzed.

In Pandas, [22], there is a ready implementation - DataFrame.rolling (window) .mean (). The more we set the width of the interval - the smoother the trend will be. If the data is very noisy, which is especially common, for example, in financial terms, such a procedure can help us see common patterns.

3.1 Adaptive Zero-Order Polynomial Model

The exponential mean has the form, [23]:

$$\begin{gathered} S_t = \alpha x_t + \beta S_{t-1}, \\ \beta = 1 - \alpha. \end{gathered} \quad (8)$$

Taking the adaptation coefficient α = 0.5 and the warning period τ = 1, it is necessary to approximate the series using an adaptive polynomial model, [7], [8], [9], [10].

The initial condition for (5) is given as follows: $S_0 = \hat{a}_{1,0}$, where$\hat{a}_{1,0}$is an average value, for example, the first five observations:

$$\hat{a}_{1,0} = \frac{1}{5}\sum_{t=1}^{5} x_t = 511.$$

The forecast model value with the warning period τ will be determined from the relation

$$\hat{x}_t^* = S_{t-\tau} = 511.$$

The error is determined by the formula 9:

$$E = \frac{(x_t - x_i^*)^2}{x_t} \quad (9)$$

Using the Formula 8 first formula and the accepted value of α = 0.5, calculate (Table 1).

For t = 1

$$S_1 = \alpha x_i + (1-\alpha)S_0 = 0.5 * 520 + 0.5 * 511 = 515.5$$

$$\hat{x}_1^* = S_0 = 511$$

t = 2

$$S_2 = 0.5 * 497 + 0.5 * 515.5 = 506.25$$

$$\hat{x}_2^* = S_1 = 515.5$$

t = 3

$$S_3 = 0.5 * 504 + 0.5 * 506.25 = 505.125$$
$$\hat{x}_3^* = S_2 = 506.25$$

Table 1. Predicting the time series xt one step further (adaptive polynomial model of zero (p = 0) order)

τ	t	x_t	P=0		
			S_t	$\hat{x}_t^*$	Error
1	0		511		
1	1	520	515.5	511	0.16
1	2	497	506.25	515.5	0.68
1	3	504	505.125	506.25	0.01
1	4	525	515.063	505.125	0.75
…					
1	24	545	534.38	523.769	0.83
1	25	529	531.99	534.38	0.05
1	26			531.99	
α	0.5				
β	0.5				

We have made a forecast for one step forward, but it cannot be considered optimal. To obtain an adequate forecast, it is necessary to choose such a value of α that the sum of the squares of the deviations and the error of the forecast was minimal. To determine the optimal value of α, tabulate it from 0.1 to 0.9 in steps of 0.1. Then each time we substitute it in the calculation model to obtain the forecast and the magnitude of the error. Thus, the value of α is selected at which the error will also be minimal.

The distribution of the prediction error with respect to the parameter α is shown in Figure 1.

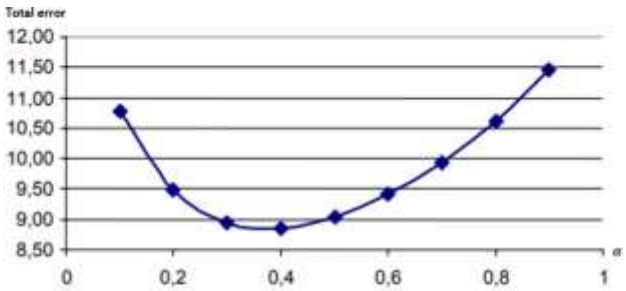

Fig. 1: Dependence of forecasting error on α.

Figure 1 shows that the optimal value for the zero-order model is α = 0.4, which is determined based on the minimum total error E = 8.85. The results of the forecast are shown in Figure 2.

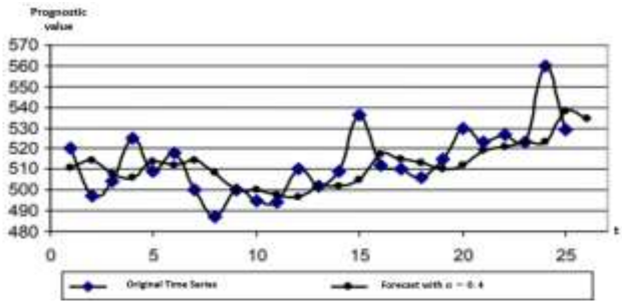

Fig. 2: Forecasting results based on a zero-order polynomial model (p = 0)

Numerical forecasting values are shown in Table 2.

Table 2. The results of the forecast at α = 0.4

τ	t	x_t	P=0		
			S_t	$\hat{x}_t^*$	Error
1	0		511.00		
1	1	520	412.4	511.00	0.16
1	2	497	363.76	412.4	14.4
1	3	504	347.1	363.76	39.02
1	4	525	348.84	347.1	60.28
….					
1	24	560	433.26	523.159	2.46
1	25	529	384.9	433.26	17.33
1	26			384.9	
α	0.4				
β	0.6				

In Table 2 the results of the forecast are given. They are not much different from our original series.

3.2 Adaptive First-Order Polynomial Model

First, according to the time series xt, we find the LSM (Least Squares Method), [24], estimate of the linear trend:

$$\hat{x}_t = \hat{a}_1 + \hat{a}_2 t.$$

Suppose, $\hat{a}_{1,0} = \hat{a}_1$ and $\hat{a}_{2,0} = \hat{a}_2$.

To find the coefficients $\hat{a}_{1,0}$ and $\hat{a}_{2,0}$ on the graph of the time series x_t, the trend line is added (Figure 3). In our case, the trend equation has the form:

$$\hat{x}_t = 498 + 1.2t,$$

where $\hat{a}_{1,0} = \hat{a}_1 = 498$ and $\hat{a}_{2,0} = \hat{a}_2 = 1.2$.

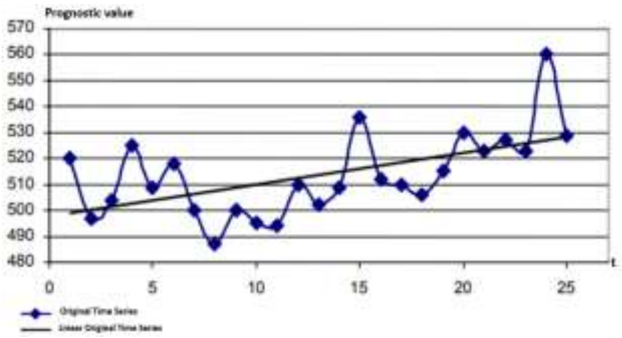

Fig. 3: Estimation of LSM regression line

Exponential averages of the 1st and 2nd order are defined as

$$S_t = \alpha x_t + \beta S_{t-1}, S_t^{[2]} = \alpha S_t + \beta S_{t-1}^{[2]},$$

whereβ=1-α.

Hence the initial conditions are the following:

$$S_0 = \hat{a}_{1,0} - \frac{\beta}{\alpha}\hat{a}_{2,0}, S_0^{[2]} = \hat{a}_{1,0} - \frac{2\beta}{\alpha}\hat{a}_{2,0}.$$

The estimation of the model (predicted) value of the series with the warning period τ is equal to

$$\hat{x}_t^* = \left(2 + \frac{\alpha}{\beta}\tau\right) S_{t-\tau} - \left(1 + \frac{\alpha}{\beta}\tau\right) S_{t-\tau}^{[2]},$$

$$S_0 = \hat{a}_{1,0} - \frac{\beta}{\alpha}\hat{a}_{2,0} = 498 - \frac{0.5}{0.5} * 1.2 = 496.8,$$

$$S_0^{[2]} = \hat{a}_{1,0} - \frac{2\beta}{\alpha}\hat{a}_{2,0} = 498 - 2 * 1.2 = 495.6.$$

Using this formula, the times series is given below:

$$\hat{x}_t^* = \left(2 + \frac{\alpha}{\beta}\tau\right) S_{t-\tau} - \left(1 + \frac{\alpha}{\beta}\tau\right) S_{t-\tau}^{[2]} = \left(2 + \frac{0.5}{0.5} * 1\right) * 496.8 - \left(1 + \frac{0.5}{0.5} * 1\right) * 495.6 = 499.2.$$

The result of the calculation is given in Table 3. The error value is lower than for the parameters presented in Table 2.

Table 3. The results of calculations of the predicted model at α = 0.5

τ	t	x_t	P=1			
			St	**$S_t^{[2]}$**	$\hat{x}_t^*$	**Error**
1	0		496.80	495.60	496.80	495.60
1	1	520	508.40	502.00	508.40	502.00
1	2	497	502.70	502.35	502.70	502.35
1	3	504	503.35	502.85	503.35	502.85
1	4	525	514.18	508.81	514.18	508.81
...						
1	24	560	541.88	532.37	541.88	532.37
1	25	529	535.44	533.90	535.44	533.90
1	26					
α	0.5					
β	0.5					

At t = 1 exponential mean levels are the following:

$$S_1 = \alpha x_1 + \beta S_0 = 0.5 * 520 + 0.5 * 496.8 = 508.4,$$

$$S_1^{[2]} = \alpha S_1 + \beta S_0^{[2]} = 0.5 * 508.4 + 0.5 * 495.6 = 502.0.$$

Based on this, the time series is given as:

$$\hat{x}_t^* = \left(2 + \frac{\alpha}{\beta}\tau\right) S_{t-\tau} - \left(1 + \frac{\alpha}{\beta}\tau\right) S_{t-\tau}^{[2]} = \left(2 + \frac{0.5}{0.5} * 1\right) 508.4 - \left(1 + \frac{0.5}{0.5} * 1\right) 502.0 = 521.2.$$

The results of the calculations are shown in Table 3. For the analyzed dataset, the predicted values are first calculated at α = 0.5 and τ = 1.
Next, it is necessary to determine the optimal value of α, based on the consideration of the minimum total error. To do this, as in the first model, a value of α with the minimum total error is selected.

Figure 4 shows the results of determining the optimal smoothing parameter.

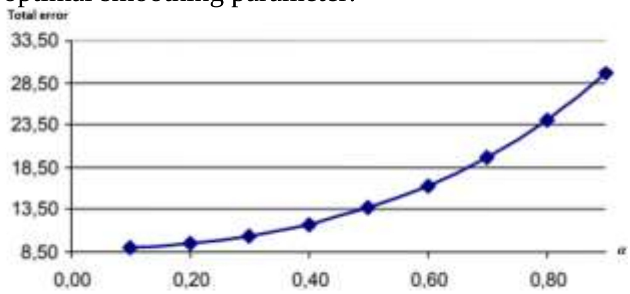

Fig. 4: Determination of the optimal value of α

Figure 4 shows that the minimum error of the predicted model will be at α = 0.1.

The results of forecasting at the selected optimal value of α are shown in Figure 5.

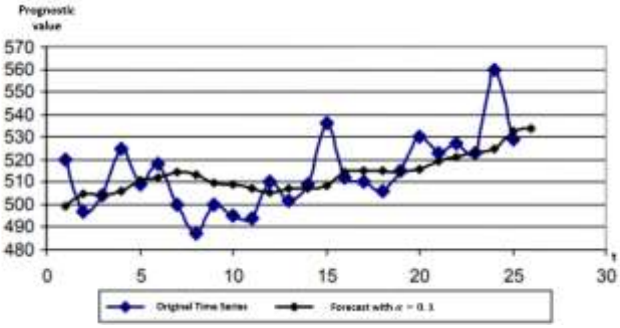

Fig. 5: Forecasting results based on a first-order polynomial model (p = 1)

Thanks to this method, we obtained a smoother series, based on which we were able to calculate predictions for 1 step forward.

3.3 Adaptive Second-Order Polynomial Model

According to the time series xt, we find the LSM estimate of the parabolic trend, [25], [26]:

$$\hat{x}_t = \hat{a}_1 + \hat{a}_2 t + \hat{a}_3 t^2.$$

For the second-order model, the equation of the parabolic trend has the form (see Figure 6):

$$\hat{x}_t = 515.96 - 2.79t + 0.15t^2,$$
$$\hat{a}_{1,0} = \hat{a}_1 = 515.96; \quad \hat{a}_{2,0} = \hat{a}_2 = -2{,}79; \quad \hat{a}_{3,0} = \hat{a}_3 = 0.15.$$

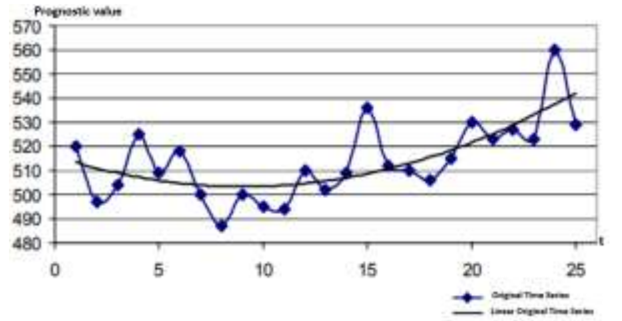

Fig. 6: Finding the LSM estimate of the parabolic trend according to the time series xt

Exponential averages of the 1st, 2nd and 3rd order are the following:

$$S_t = \alpha x_t + \beta S_{t-1},$$
$$S_t^{[2]} = \alpha S_t + \beta S_{t-1}^{[2]},$$
$$S_t^{[3]} = \alpha S_t^{[2]} + \beta S_{t-1}^{[3]}.$$

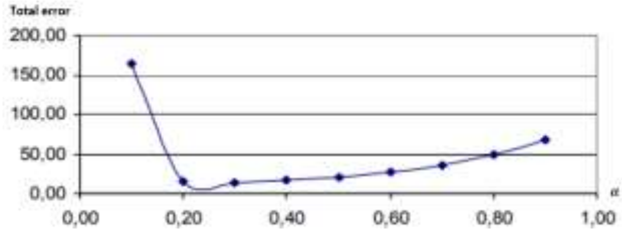

Fig. 7: Determination of the optimal α

From the graph, it is seen that the optimal α is 0.25, as at this value we get the smallest error.

The initial conditions are determined by the following formulas:

$$S_0 = \hat{a}_{1,0} - \frac{\beta}{\alpha}\hat{a}_{2,0} + \frac{\beta(2-\alpha)}{2\alpha^2}\hat{a}_{3,0};$$
$$S_0^{[2]} = \hat{a}_{1,0} - \frac{2\beta}{\alpha}\hat{a}_{2,0} + \frac{\beta(3-2\alpha)}{\alpha^2}\hat{a}_{3,0};$$
$$S_0^{[3]} = \hat{a}_{1,0} - \frac{3\beta}{\alpha}\hat{a}_{2,0} + \frac{3\beta(4-3\alpha)}{2\alpha^2}\hat{a}_{3,0}.$$

The estimate of the model (prediction) with the warning period τ is found in the expression

$$\hat{x}_t^* = [6\beta^2 + (6-5\alpha)\alpha * \tau + \alpha^2\tau^2]\frac{S_{t-\tau}}{2\beta^2} - \left[\begin{matrix}6\beta^2 + (5-4\alpha)\alpha\tau + \\ 2\alpha^2\tau^2\end{matrix}\right]\frac{S_{t-\tau}^{[2]}}{2\beta^2} + +[2\beta^2 + (4-3\alpha)\alpha\tau + \alpha^2\tau^2]\frac{S_{t-\tau}^{[3]}}{2\beta^2}.$$

Next, we determine the optimal value of the smoothing coefficient (see Figure 7). Taking into account the optimally obtained value α = 0.25 (E = 9.06) the forecast is given (see Figure 8).

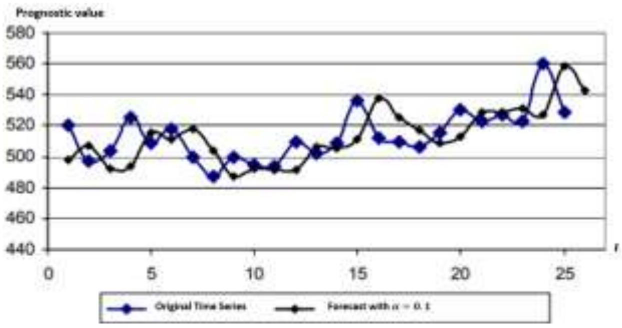

Fig. 8: Forecasting results based on a second-order polynomial model (p = 2)

Next, the proposed model will be compared with existing adaptive models. The Winters model and Tayle-Vage are analyzed.

3.4. Forecasting using the Winters Model (Exponential Smoothing with Multiplicative Seasonality and Linear Growth)

This model is convenient to use with a small amount of initial data. The seasonal model of Winters with linear growth has the form

$$x_t = a_{1,t} f_{v_t k_t} + \varepsilon_t,$$

wherex_t - original time series t = 1, 2, ..., n; $a_{1,t}$ - the parameter characterizes the linear trend of the process, ie the average values of the level of the studied time series x_t at time t;$f_{v_t k_t}$ - seasonality factor for v_t phase of the k_t-th cycle; $v_t = 1,2,\ldots,l$, where$v_t = t - l(k_t - 1)$; l - the number of phases in the full cycle (in monthly time series l = 12, in quarterly l = 4, etc.);εt - random error. It is usually assumed that the vector$\varepsilon = N_n(0, \sigma^2 I_n)$where$\varepsilon = (\varepsilon_1, \ldots, \varepsilon_t, \ldots, \varepsilon_n)^T$; I_n – unit matrix with a size of (n × n).

The adaptive parameters of the model are estimated using a recurrent exponential scheme according to the time series xt, consisting of n observations

$$\begin{cases} \hat{a}_{1,t} = \alpha_1 \dfrac{x_t}{\hat{f}_{v_t,k_{t-1}}} + (1-\alpha)(\widehat{\alpha}_{1,t-1} + \widehat{\alpha}_{2,t-1}) \\ \hat{f}_{v_t k_t} = \alpha_2 \dfrac{x_t}{\hat{a}_{1,t}} + (1-\alpha_2)\hat{f}_{v_t k_{t-1}} \\ \hat{a}_{2,t} = \alpha_3(\hat{a}_{1,t} - \hat{a}_{1,t-1}) + (1-\alpha_3)\hat{a}_{2,t-1} \\ \hat{x}_t^* = (\hat{a}_{1,t-\tau} + \tau \hat{a}_{2,t-\tau})\hat{f}_{v_t k_t - 1}, \end{cases}$$

Where $a_{2,t}$ - the increase of the average level of the series from the moment t - 1 to the moment t; $\hat{x}_t^* = x_\tau(t)$ - the calculated value of the time series, which is determined for the time t with the warning period τ, ie according to the moment (t -τ); α_1, α_2,α_3, - parameters of adaptation of exponential smoothing, and (0 <α_1, α_2, α_3<1).

The increase in α_j(j = 1,2,3) leads to an increase in the weight of later observations, and a decrease in αj leads to an improvement in the smoothing of random deviations. These two requirements are in conflict, and the search for a compromise combination of values is the task of optimizing the model.

Exponential alignment always requires a preliminary estimate of the smoothed value. When the adaptation process is just beginning, there should be initial values prior to the first observation. In our task it is necessary to define the initial conditions:$\hat{a}_{1,0}$;$\hat{a}_{2,0}$;$\hat{f}_{v_t,0}$, where$v_t = 1,2, l$. Thus, the calculated values of$\hat{x}_t^*$are a function of all past values of the original time series xt, parameters α_1, α_2 and α_3 and initial conditions. The influence of the initial conditions on the calculated value depends on the value of the weights αj and the length of the series preceding the moment t. Impacts of$\hat{a}_{1,0}$;$\hat{a}_{2,0}$usually decrease faster than$\hat{f}_{v_t,0}$, $\hat{a}_{1,t}$and$\hat{f}_{2,t}$are reviewed at each step, but$\hat{f}_{v_i,k_i}$only once per cycle.

First, by n = 8 observations of the time series xt, we find the LSM estimate of the linear trend $\hat{x}_t = a_0 + a_t t$. As a result of the calculation we have

$$\hat{x}_t = 492.46 - 8.5476 * t.$$

Next, the initial conditions are defined:

$$\hat{a}_{1,0} = \hat{a}_0 = 492.46; \quad \hat{a}_{2,0} = \hat{a}_1 = -8.5476.$$

Multiplicative zero-cycle seasonality coefficients, [27]. $\hat{F}_{v_t,0}$ define as the arithmetic mean of seasonality indices$x_t/\hat{x}_t$forv_t-th phase in the original time series (Figure 9):

$$\hat{f}_{1,0} = \frac{1.031 + 1.070}{2} = 1.050; \quad \hat{f}_{2,0} = \frac{0.999 + 1.059}{2} = 1.029;$$

$$\hat{f}_{3,0} = \frac{0.968 + 0.996}{2} = 0.982; \quad \hat{f}_{4,0} = \frac{0.906 + 0.972}{2} = 0.939.$$

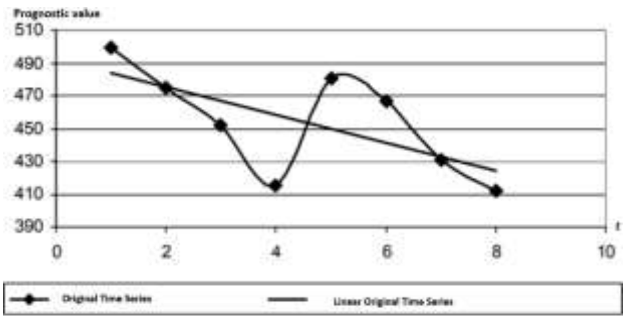

Fig. 9: LSM assessment of a linear trend

We will perform calculations with adaptation parameters $a_1 = 0.2$; $a_2 = 0.3$; $a_3 = 0.4$ and the warning period τ = 1. Estimated values for the 1st cycle ($k_t = 1$, $v_t = t$).

According to the formula for t = 1, we have

$$\hat{x}_1^* = (\hat{a}_{1,0} + \hat{a}_{2,0}) * \hat{f}_{1,0} = (492.46 - 8.5476) * 1.050 = 508.28$$

$$\hat{a}_{1,1} = \alpha_1 * \frac{x_1}{\hat{f}_{1,0}} + (1-\alpha_1)(\hat{a}_{1,0} + \hat{a}_{2,0}) = 0.2\frac{499}{1.050} + (1-0.2)(492.46 - 8.5478) = 482.14$$

$$\hat{f}_{1,1} = \alpha_2 \frac{x_1}{\hat{a}_{1,1}} + (1-\alpha_2) * \hat{f}_{1,0} = 0.3\frac{499}{482.14} + (1-0.3) * 1.050 = 1.046$$

$$\hat{a}_{2,1} = \alpha_3\left(\hat{a}_{1,1} - a_{1,0}\right) + (1-\alpha_3) * \hat{a}_{2,0} = 0.4(482.14 - 492.46) + 0.6(-8.5476) = -9.255$$

t = 2

$$\hat{x}_2^* = \left(\hat{a}_{1,1} + \hat{a}_{2,1}\right) * \hat{f}_{2,0} = (482.14 - 9.255) * 1.029 = 486.55$$

$$\hat{a}_{1,2} = \alpha_1 * \frac{x_2}{\hat{f}_{2,0}} + (1-\alpha_1)(\hat{a}_{1,1} + \hat{a}_{2,1}) = 0.2\frac{475}{1.029} + 0.8(482.14 - 9.255) = 470.64$$

$$\hat{f}_{2,1} = \alpha_2 \frac{x_2}{\hat{a}_{1,2}} + (1-\alpha_2) * \hat{f}_{2,0} = 0.3\frac{475}{470.64} + 0.7 * 1.029 = 1.023$$

$$\hat{a}_{2,2} = \alpha_3\left(\hat{a}_{1,2} - a_{1,1}\right) + (1-\alpha_3) * \hat{a}_{2,1} = 0.4(470.64 - 482.14) + 0.6(-9.255) = -10.153$$

t = 3

$$\hat{x}_3^* = (470.64 - 10.153) * 0.982 = 452.32$$

$$\hat{a}_{1,3} = 0.2\frac{452}{0.982} + 0.8(470.64 - 10.153) = 460.43$$

$$\hat{f}_{3,1} = 0.3\frac{452}{460.43} + 0.7 * 0.982 = 0.982$$

$$\hat{a}_{2,3} = 0.4(460.43 - 470.64) + 0.6 * (-10.153) = -10.179$$

t = 4

$$\hat{x}_4^* = (460.43 - 10.179) * 0.939 = 422.58$$

$$\hat{a}_{1,4} = 0.2\frac{415}{0.939} + 0.8(460.43 - 10.179) = 448.63$$

$$\hat{f}_{4,1} = 0.3\frac{415}{448.63} + 0.7 * 0.939 = 0.934$$

$$\hat{a}_{2,4} = 0.4(448.63 - 460.43) + 0.6 * (-10.179) = -10.825$$

Estimated values for the 2nd cycle (k_t = 2, v_t = t-4). Here we need the seasonality coefficients found for the 1st cycle

$$\hat{f}_{1,1} = 1.046; \quad \hat{f}_{2,1} = 1.023; \quad \hat{f}_{3,1} = 0.982; \quad \hat{f}_{4,1} = 0.934$$

t = 5

$$\hat{x}_5^* = \left(\hat{a}_{1,4} + \hat{a}_{2,4}\right) * \hat{f}_{1,1} = (448.63 - 10.825) * 1.046 = 457.84$$

Since $\hat{x}_5^*$refers to the 2nd cycle (k_t = 2) when choosing$\hat{f}_{v_t,k_t-1}$based on the fact that v_t = 5-4 = 1

$$\hat{a}_{1,5} = 0.2\frac{481}{1.046} + 0.8(448.63 - 10.825) = 442.24$$

$$\hat{f}_{1,2} = 0.3\frac{481}{442.24} + 0.7 * 1.046 = 1.058$$

$$\hat{a}_{2,5} = 0.4(442.24 - 448.63) + 0.6 * (-10.825) = -9.053$$

t = 6

$$\hat{x}_6^* = (442.24 - 9.053) * 1.023 = 443.15$$

$$\hat{a}_{1,6} = 0.2\frac{467}{1.023} + 0.8(442.24 - 9.053) = 437.85$$

$$\hat{f}_{2,2} = 0.3\frac{467}{437.85} + 0.7 * 1.023 = 1.036$$

$$\hat{a}_{2,6} = 0.4(437.85 - 442.24) + 0.6 * (-9.053) = -7.187$$

t = 7

$$\hat{x}_7^* = (437.85 - 7.187) * 0.982 = 422.95$$

$$\hat{a}_{1,7} = 0.2\frac{431}{0.982} + 0.8(437.85 - 7.187) = 432.30$$

$$\hat{f}_{3,2} = 0.3\frac{431}{432.30} + 0.7 * 0.982 = 0.987$$

$$\hat{a}_{2,7} = 0.4(432.30 - 437.85) + 0.6 * (-7.187) = -6.531.$$

t = 8

$$\hat{x}_8^* = (432.30 - 6.531) * 0.934 = 397.88$$

$$\hat{a}_{1,8} = 0.2\frac{412}{0.934} + 0.8(432.30 - 6.531) = 428.79$$

$$\hat{f}_{4,2} = 0.3\frac{412}{428.79} + 0.7 * 0.934 = 0.942$$

$$\hat{a}_{2,8} = 0.4(428.79 - 432.30) + 0.6 * (-6.531) = -5.323$$

t = 9 (forecast)

$$\hat{x}_9^* = \left(\hat{a}_{1,8} + \hat{a}_{2,8}\right) * \hat{f}_{1,8} = (428.79 - 5.323) * 1.058 = 448.16$$

The calculated values and the forecast t$\hat{x}_t^*$, obtained from the time series x_t are presented in Figure 10.

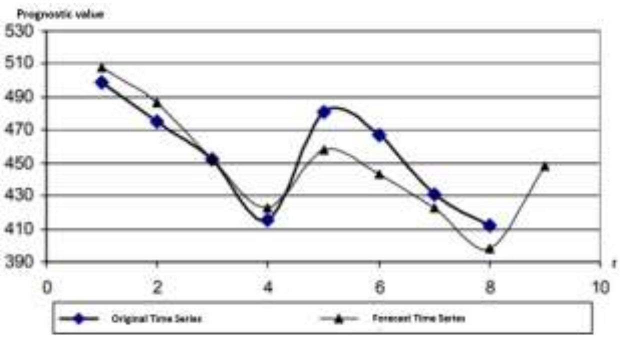

Fig. 10: Forecasting results based on a third-order polynomial model (p = 3)

From the presented graph we can conclude that the model of exponential smoothing with multiplicative seasonality of Winters is better than the regression model but worse than the proposed adaptive model. The forecast results of Winters can be improved by selecting the optimal values of α.

3.5 Production Forecast based on the Tayle-Wage Model

Additive modeling is an approach of interest in economic research, as it enables the construction of a model featuring exponential trends and multiplicative seasonality. This can be achieved by converting the initial time series values into their logarithmic equivalents, transforming the exponential trend into a linear one, and the multiplicative seasonality into an additive one.

Suppose the observation x_t refer to the v_t-th phase of the k_t-th cycle, where $v_t = t - l\,(k_t - l)$, l is the number of phases in the cycle (for the quarterly time series l = 4, and the monthly l = 12).

The model with additive seasonality and linear growth can be represented as

$$x_t = a_{1,t} + g_{v_t k_t} + \varepsilon_t$$
$$a_{1,t} = a_{1,t-1} + a_{2,t},$$

Where x_t- the average value of the level of the time series at time t after excluding seasonal fluctuations; $a_{2,t}$- additive growth rate from time t-1 to time t; $g_{v_t k_t}$– additive seasonality factor for the v_t-th phase of the k_t-th cycle; ε_t – white noise.

Estimates of model parameters will be sought at smoothing coefficients α_1, α_2, α_3, where ($0 < \alpha1, \alpha2, \alpha3 < 1$) on the following adaptation procedures:

$$\hat{a}_{1,t} = \alpha_1\left(x_t - \hat{g}_{v_t,k_{t-1}}\right) + (1-\alpha_1)(\hat{a}_{1,t-1} + \hat{a}_{2,t-1})$$
$$\hat{g}_{v_t k_t} = \alpha_2\left(x_t - \hat{a}_{1,t}\right) + (1-\alpha_2)\,\hat{g}_{v_t,k_{t-1}}$$
$$\hat{a}_{2,t} = \alpha_3\left(\hat{a}_{1,t} - \hat{a}_{1,t-1}\right) + (1-\alpha_3)\hat{a}_{2,t-1}$$
$$\hat{x}_t^* = \hat{a}_{1,t-\tau} + \tau * a_{2,t-\tau} + \hat{g}_{v_t,k_{t-1}}.$$

The initial conditions of exponential smoothing are determined by the original time series x_t (t = 1,2, …, n).

First, on the time series x_t, which contains n = 8 observations, we find the LSM - an estimate of the linear regression equation

$$\hat{x}_t = \hat{\theta} + \hat{\theta}_1 t = 7.0071 - 0.1905t$$
$$\hat{a}_{1,0} = \hat{\theta}_0 = 7.0071; \quad \hat{a}_{2,0} = \hat{\theta}_1 = -0.1905.$$

The calculated values of x_t and deviations $\Delta_t = x_t - \hat{x}_t$ are given below. Then the initial values of additive seasonality coefficients are equal

$$\hat{g}_{1,0} = \frac{0.38 - 0.15}{2} = 0.1144$$
$$\hat{g}_{2,0} = \frac{-0.13 - 0.16}{2} = -0.1451$$
$$\hat{g}_{3,0} = \frac{-0.34 + 0.33}{2} = -0.0046$$
$$\hat{g}_{4,0} = \frac{0.05 + 0.02}{2} = 0.0359.$$

We will perform calculations for adaptation parameters $\alpha_1 = 0.1$; $\alpha_2 = 0.4$; $\alpha_3 = 0.3$ and the warning period $\tau = 1$.

First loop: $v_t = t$; $k_t = 1$; $\tau = 1$,initial data for calculation:

$\hat{g}_{1,0} = 0.1144 \quad \hat{g}_{2,0} = -0.1451 \quad \hat{g}_{3,0} = -0.0046 \hat{g}_{4,0} = 0.0359.$

According to the formula for t = 1, we have

$$\hat{x}_1^* = \hat{a}_{1,0} + \hat{a}_{2,0} + \hat{g}_{1,0} = 7.0071 - 0.1905 + 0.1144 = 6.93$$
$$\hat{a}_{1,1} = 0.1 * (7.2 + 0.1144) + (1 - 0.1) * (7.0071 - 0.1905) = 6.844$$
$$\hat{g}_{1,1} = 0.4 * (7.2 - 6.844) + 0.6 * 0.1144 = -0.211$$
$$\hat{a}_{2,1} = 0.3 * (6.844 - 7.0071) + 0.7 * (-0.1905) = -0.182$$

t = 2

$$\hat{x}_2^* = 6.844 - 0.182 - 0.1451 = 6.52$$
$$\hat{a}_{1,2} = 0.1 * (6.5 + 0.1451) + 0.9 * (6.844 - 0.182) = 6.6595$$
$$\hat{g}_{2,1} = 0.4 * (6.5 - 6.6595) + 0.6 * (-0.1451) = -0.1508$$
$$\hat{a}_{2,2} = 0.3 * (6.6595 - 6.844) + 0.7 * (-0.182) = -0.183$$

t = 3

$$\hat{x}_3^* = 6.6595 - 0.183 - 0.0046 = 6.472$$
$$\hat{a}_{1,3} = 0.1 * (6.1 + 0.0046) + 0.9 * (6.6595 - 0.183) = 6.4394$$
$$\hat{g}_{3,1} = 0.4 * (6.1 - 6.4394) + 0.6 * (-0.0046) = -0.1385$$
$$\hat{a}_{2,3} = 0.3 * (6.4394 - 6.6595) + 0.7 * (-0.183) = -0.194$$

t = 4

$$\hat{x}_4^* = 6.4394 - 0.194 + 0.0359 = 6.281$$
$$\hat{a}_{1,4} = 0.1 * (6.3 - 0.0359) + 0.9 * (6.4394 - 0.194) = 6.2472$$
$$\hat{g}_{4,1} = 0.4 * (6.3 - 6.2472) + 0.6 * 0.0359 = 0.0427$$
$$\hat{a}_{2,4} = 0.3 * (6.2472 - 6.4394) + 0.7 * (-0.194) = -0.194$$

Second loop: vt = t-4; kt = 2). Initial data for calculation:

$\hat{g}_{1,1} = 0.211 \quad \hat{g}_{2,1} = -0.1508 \quad \hat{g}_{3,1} = -0.1385 \hat{g}_{4,1} = 0.0427$

t = 5

$\hat{x}_5^* = 6.2472 - 0.194 + 0.211 = 6.265$

$\hat{a}_{1,5} = 0.1 * (5.9 - 0.211) + 0.9 * (6.2472 - 0.194) = 6.0172$

$\hat{g}_{1,2} = 0.4 * (5.9 - 6.0172) + 0.6 * 0.211 = 0.0799$

$\hat{a}_{2,5} = 0.3 * (6.0172 - 6.2472) + 0.7 * (-0.194) = -0.204$

t = 6

$\hat{x}_6^* = 6.0172 - 0.204 - 0.1508 = 5.662$

$\hat{a}_{1,6} = 0.1 * (5.8 + 0.1508) + 0.9 * (6.0172 - 0.204) = 5.8165$

$\hat{g}_{2,2} = 0.4 * (5.7 - 5.8165) + 0.6 * (-0.1508) = -0.1371$

$\hat{a}_{2,6} = 0.3 * (5.8165 - 6.0172) + 0.7 * (-0.204) = -0.203$

t = 7

$\hat{x}_7^* = 5.8165 - 0.203 - 0.1385 = 5.475$

$\hat{a}_{1,7} = 0.1 * (6 + 0.1385) + 0.9 * (5.8165 - 0.203) = 5.6658$

$\hat{g}_{3,2} = 0.4 * (6 - 5.6658) + 0.6 * (-0.1385) = 0.0352$

$\hat{a}_{2,7} = 0.3 * (5.6658 - 5.8165) + 0.7 * (-0.203) = -0.188$

t = 8

$\hat{x}_8^* = 5.6658 - 0.188 + 0.0427 = 5.521$

$\hat{a}_{1,8} = 0.1 * (5.5 + 0.0427) + 0.9 * (5.6658 - 0.188) = 5.4761$

$\hat{g}_{4,2} = 0.4 * (5.5 - 5.4761) + 0.6 * 0.0427 = 0.0352$

$\hat{a}_{2,8} = 0.3 * (5.4761 - 5.6658) + 0.7 * (-0.188) = -0.188$

t = 9 (forecast)

$\hat{x}_9^* = \hat{a}_{1,8} + a_{2,8} + g_{1,2} = 5{,}4761 - 0{,}188 + 0{,}799 = 5{,}368$

As estimates $\hat{g}_{v_t,0}$ takes the average values of the deviations $\Delta_t = x_t - \hat{x}_t$, corresponding to the vt-th phase of the original time series, where $v_t = 1, \dots \dots, l$.

Calculated according to the Tayle-Wage model, the values of the time series $\hat{x}_t^*$ are presented in Figure 11, where they are presented with the original time series x_1.

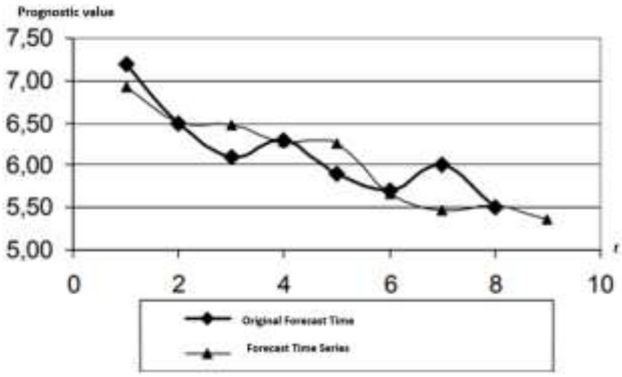

Fig. 11: Forecasting results using the Tayle-Wage model

The graph shows that our forecast is not that far from the original series and that it maintains the trends.

The comparison of the proposed forecasting model DIAAMMFTS with existing models is given in Table 4. Mean Squared Error (MSE), [28], [29], [30], is used for all the models.

Table 4. The comparison of forecasting models.

Model	MSE
DIAAMMFTS	0.23
Tayle-Wage model	0.31
Winters model	0.39

The error values are expressed in squared units of the predicted values. A mean squared error of zero denotes flawless accuracy, or the absence of errors.

4 Conclusion

In this work, the different adaptive methods are analyzed. The Data Interpretation Algorithm for Adaptive Methods of Modeling and Forecasting Time Series (DIAAMMFTS) is developed in the paper. This method is based on a 5-step procedure and shows promising forecast skills.

Also, we implemented a program that builds models using these methods. Based on the obtained results and the characteristics of the models calculated by the program, the results were analyzed and a comparison of the methods used in the work was carried out, based on which a conclusion was made about the most efficient models for each specific situation.

The results of this work are the following:

- time-series research and identification of characteristics that affect the adequacy and accuracy of models;
- characteristics of time series dynamics that influence the choice of forecasting model were determined.
- the new data interpretation algorithm for adaptive methods of modeling and forecasting time series is developed,
- the comparison with Winters model and the Tayle-Wage model shows the good quality of the proposed predictive model;
- there is implemented a program that builds models and calculates forecasts by adaptive methods;
- the adaptive polynomial models used sequentially allow to increase the prediction accuracy.

The implemented program showed good results, which allows us to conclude that these adaptive models are effective in predicting economic or conventional computational processes.

The model of exponential smoothing with multiplicative seasonality of Winters is better than the regression model but worse than the proposed adaptive model. The forecast results of Winters can be improved by selecting the optimal values of α.

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