The Inverse Problem of Determining the Coefficients Elliptic Equation

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Abstract: - The paper considers the inverse problem in determining unknown coefficients in a linear elliptic equation. Theorems of existence, uniqueness and stability of the solution of inverse problems for a linear equation of elliptic type are proved. Using the method of sequential measurements, a regularizing algorithm is constructed to determine several coefficients.

Key-Words: - Inverse problem, elliptic equation, quasilinear elliptic equation, regularizing algorithm, correct problem, unknown coefficient.

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1 Introduction

Inverse problems include the tasks of determining some physical properties of objects, such as density, thermal conductivity, elastic moduli depending on the coordinates or as functions of other parameters.

The solution of inverse problems is carried out, as a rule, within the framework of some mathematical model of the object under study. It consists in determining either the coefficients of differential equations, or the domain in which the operator acts, or the initial conditions.

Inverse problems have a number of features that are unpleasant from a mathematical point of view. First, they are usually non-linear, that is, an unknown function or an unknown parameter enters an operator or functional equation in a non-linear manner. Secondly, the solutions of inverse problems are usually non-unique. Thirdly, inverse problems are not well-posed.

Inverse problems of mathematical physics are currently a rapidly developing part of 1 modern mathematics. An increasing part of mathematical models is becoming harmonious and reliable precisely due to the achievements of the theory of inverse problems.Mathematical models of many established processes of various physical natures lead to elliptic differential equations.It is enough to specify stationary problems of thermal conductivity and diffusion, the problem of determining the current in a conductive medium, and problems of electrostatics.The problems of identification of these models are investigated as inverse problems of mathematical physics.To date, the number of studies of inverse problems for an elliptic equation, ranging from theoretical to specific applied problems, has increased significantly.

The papers [1], [2], [3], [4] present methods for solving various inverse problems with boundary conditions. In [5], the classical solution of a nonlinear inverse boundary value problem is studied.To solve the problem under consideration, a transition is made from the original inverse problem to some auxiliary inverse problem. Monograph [6] is devoted to the theory of inverse problems of mathematical physics and applications of such problems.Modern results on the problem of uniqueness in integral geometry and on inverse problems for partial differential equations [7] are presented very broadly. In [8], the inverse problem for a second-order quasilinear elliptic equation with an unknown coefficient was considered. In the class of continuously differentiable functions, inverse problems of determining the source and coefficient of an elliptic equation in a rectangle are studied [9]. In [10], two inverse problems are considered. A numerical method for solving these inverse problems is proposed.

In this paper, we investigate the correctness of one class of inverse problems of determining the coefficients of an elliptic equation. A regular algorithm for determining the coefficients is constructed.

2 Materials and Methods

Let D, D_1- be bounded regions $(n-1)-$ dimensional and $n-$ dimensional Euclidean spaces, respectively, and an arbitrary point D_1 of the region can be represented in the form $(x,y)=(x_1,x_2,...,x_{n-1},y),$ where $x=(x_1,x_2,...,x_{n-1})\in D.$ By Γ denote the boundary of the area D_1, which is assumed to be sufficiently smooth. Let γ_1,γ_2 –be the parts of the boundary Γ such that their projections on the dimensional $(n-1)$–subspace coincide with the domain $\bar{D}$ and there are sufficiently smooth functions $F_i(x)$ such

$\gamma_i=\{(x,y): y=F_i(x), x\in\bar{D}\},$

$i=1,2.$ that it is obvious $\gamma_i, i=1,2$ that it can be, for example, a piece of the boundary Γ such that their projections on the $(n-1)$–dimensional space cover the domain $\bar{D}.$

Let $I_1=\{2,...,n-1\}, I_2=\{1,n\},\ i_0\in I_1.$

Consider the problem of determining $\{a_{i_0}(x), c(x), u(x,y)\}$ from the following conditions:

$$-\sum_{i=1}^{n-1} a_i(x)u_{x_ix_i}-a_n(x)u_{yy}+c(x)u=h(x,y),$$
$$(x,y)\in D_1, \quad (1)$$
$$u(x,y)\big|_{\tilde{A}}=f(\xi,\eta),\ (\xi,\eta)\in\tilde{A}, \quad (2)$$

$$a_{i_0}(x)u_\nu(x,y)\Big|_{\gamma_1}=g_1(\xi,\eta),\ (\xi,\eta)\in\gamma_1 \quad (3)$$

$$\left[a_{i_0}(x)u_\nu(x,y)-c(x)\phi(x,y)\right]_{\gamma_2}=g_2(\xi,\eta),$$
$$(\xi,\eta)\in\gamma_2, \quad (4)$$

here

$0<a_i(x)\in C^\alpha(\bar{D}),$

$i=1,2,...,i_0-1,i_0+1,...,n, h(x,y)\in C^\alpha(\bar{D}_1),$

$f(\xi,\eta)\in C^{2+\alpha}(\Gamma),$

$g_i(\xi,\eta)\in\ \tilde{N}^\alpha(\gamma_i),\ i=1,2$ – set functions,

$0<\alpha<1,\ \nu$ – direction of the internal normal of the surface $\gamma_i,\ u_\nu(x,y)\Big|_{\gamma_i}\equiv\frac{\partial u}{\partial\nu}(x,y)\Big|_{\gamma_i},\ i=1,2.$

Definition.Functions $\{a_{i_0}(x), c(x), u(x,y)\}$ are called the solution of the problem $(1)-(4),$ if there $0<a_{i_0}(x), c(x)\in C(\bar{D}), u(x,y)\in C^2(D_1)\cap C(\bar{D}_1)$ are limits of functions $u_{x_i}(x,y),\ i=1,2,...,n,\ x_n=y$ at $(x,y)\to(\xi,\eta)\in\gamma_i, i=1,2$ and the correlations are satisfied $(1)-(4).$

Below, everywhere constant numbers that do not depend on the estimated values are denoted by $N_i, i=1,2,...,n.$

It is not difficult to verify that if a solution to the problem $(1)-(4)$ exists, then under accepted assumptions about the smoothness of the problem data, $a_{i_0}(x), c(x)\in C^\alpha(\bar{D}),\quad u(x,y)\in C^{2+\alpha}(\bar{D}_1).$ Indeed, under accepted assumptions from the general theory of elliptic equations, it follows, $u(x,y)\in W_p^2(D_1)\subset C^{1+\alpha}(\bar{D}_1)$ that $p>n$ therefore, from the additional (3) and (4) conditions $a_{i_0}(x), c(x)\in C^\alpha(\bar{D}).$ It follows that therefore $u(x,y)\in\ \in C^{2+\alpha}(\bar{D}_1)$

Task (1) can also be written in the following form

$$-\sum_{i=1}^{i_0-1} a_i(x)u_{x_ix_i}-a_{i_0}(x)u_{x_{i_0}x_{i_0}}-\sum_{i=i_0+1}^{n} a_i(x)u_{x_ix_i}+,$$
$$+c(x)u=h(x,y)$$
$$(x,y)\in D_1.$$

Suppose, in addition to the task $(1)-(4)$, there is also a task $(\bar{1})-(\bar{4}),$ where all the functions that $(1)-(4),$ are input to are replaced by the corresponding functions with a dash.

Let 's put

$\upsilon(x,y)=\bar{u}(x,y)-u(x,y),$

$\lambda_{i_0}(x)=\bar{a}_{i_0}(x)-a_{i_0}(x), \delta_{1i}(x)=\bar{a}_i(x)-a_i(x),$

$i=1,2,...,i_0-1,i_0+1,...,n,\ \mu(x)=\bar{c}(x)-c(x),$

$\delta_1(x,y)=\bar{h}(x,y)-h(x,y),$

$\delta_2(\xi,\eta)=\bar{f}(\xi,\eta)-f(\xi,\eta),$

$\delta_3(\xi,\eta)=\bar{\phi}(\xi,\eta)-\phi(\xi,\eta),$

$\delta_{i+3}(\xi,\eta)=\bar{g}_i(\xi,\eta)-g_i(\xi,\eta), i=1,2.$

The uniqueness of the solution of the inverse problem $(1)-(4)$ under the assumption of its existence is proved by Theorem 1.

Theorem 1.
Let, $g_1(\xi,\eta)\neq 0, \phi(\xi,\eta)\neq 0, NmesD_1<1$, then the solution of the problem $(1)-(4)$ is unique and the following estimate is true:

$$\|\bar{a}_{i_0}(x)-a_{i_0}(x)\|_{C(\bar{D})}+\|\bar{c}(x)-c(x)\|_{C(\bar{D})}+\|\bar{u}-u\|_{W_p^2(D_1)}\leq$$

$$\leq N_1\Big[\sum_{i=1}^{i_0-1}\|\bar{a}_i(x)-a_i(x)\|_{C(\bar{D})}+$$
$$+\sum_{i=i_0+1}^{n}\|\bar{a}_i(x)-a_i(x)\|_{C(\bar{D})}+$$
$$\|\bar{h}(x,y)-h(x,y)\|_{L_p(D_1)}+$$
$$+\|\bar{f}(\xi,\eta)-f(\xi,\eta)\|_{W_p^2(\Gamma)}+$$
$$\|\bar{\phi}(\xi,\eta)-\phi(\xi,\eta)\|_{C(\gamma_2)}+$$
$$+\sum_{i=1}^{2}\|\bar{g}_i(\xi,\eta)-g_i(\xi,\eta)\|_{C(\gamma_i)}\Big],\quad p>n, \qquad (5)$$

N, N_1- positive constants, depending on the data of the solution set.

Proof. From $(\bar{1})-(\bar{4})$, respectively, we subtract $(1)-(4)$. Then we get

$$-\sum_{i=1}^{n}\bar{a}_i(x)\upsilon_{x_ix_i}=\sum_{i=1}^{i_0-1}\delta_{1i}(x)u_{x_ix_i}+\sum_{i=i_0+1}^{n}\delta_{1i}(x)u_{x_ix_i}+$$
$$+\delta_1(x,y)-\bar{c}(x)\upsilon+\lambda_{i_0}(x)u_{x_{i_0}x_{i_0}}-\mu(x)u,$$
$$(x,y)\in D_1, \qquad (6)$$
$$\upsilon(x,y)\big|_{\Gamma}=\delta_2(\xi,\eta),(\xi,\eta)\in\Gamma \qquad (7)$$

$$\lambda_{i_0}(x)=\delta_4(\xi,\eta)u_\nu^{-1}(x,y)\big|_{\gamma_1}-\bar{a}_{i_0}(x)u_\nu^{-1}(x,y)\upsilon_\nu(x,y)\big|_{\gamma_1},$$
$$(\xi,\eta)\in\gamma_1, \qquad (8)$$
$$\mu(x)=\bar{a}_{i_0}(x)\upsilon_\nu(x,y)\big|_{\gamma_2}\phi^{-1}(\xi,\eta)+$$
$$+u_\nu(x,y)\lambda_{i_0}(x)\big|_{\gamma_2}\phi^{-1}(\xi,\eta)-$$
$$-\big[\bar{c}(x)\big|_{\gamma_2}\delta_3(\xi,\eta)+\delta_5(\xi,\eta)\big]\phi^{-1}(\xi,\eta),$$
$$(\xi,\eta)\in\gamma_2 \qquad (9)$$

For a function $\upsilon(x,y)$ satisfying the equation (6) and the condition (7), the estimate is true (10):

$$\|\upsilon(x,y)\|_{W_p^2(D_1)}\leq N_2\Big\{\|\delta_2(\xi,\eta)\|_{W_p^2(\Gamma)}+\Big\|\sum_{i=1}^{i_0-1}\delta_{1i}(x)u_{x_ix_i}+$$
$$+\sum_{i=i_0+1}^{n}\delta_{1i}(x)u_{x_ix_i}+\delta_1(x,y)-\bar{c}(x)\upsilon+\lambda_{i_0}(x)u_{x_{i_0}x_{i_0}}-$$
$$-\mu(x)u\Big\|_{L_p(D_1)}\Big\}, \qquad (10)$$

Let's put it
$$\chi=\max_{x,y}|\upsilon(x,y)|+\max_x|\lambda_{i_0}(x)|+\max_x|\mu(x)|.$$
Then from the embedding theorems, under the conditions assumed above, we obtain

$$\|\upsilon(x,y)\|_{C^1(\bar{D}_1)}\leq N_3\|\upsilon(x,y)\|_{W_p^2(D_1)}\leq$$
$$\leq N_4\Big[\sum_{i=1}^{i_0-1}\|\delta_{1i}(x)\|_{C(\bar{D})}+\sum_{i=i_0+1}^{n}\|\delta_{1i}(x)\|_{C(\bar{D})}+$$
$$+\|\delta_1(x,y)\|_{L_p(D_1)}+\|\delta_2(\xi,\eta)\|_{W_p^2(\Gamma)}+\chi mes\,D_1\Big] \qquad (11)$$

If we take (11) account the estimate in the right part (8) and (9) then under the assumptions of the theorem we get:
$$\|\lambda_{i_0}(x))\|_{C(\bar{D})}\leq N_5(\delta+\chi mesD_1),$$
$$\|\mu(x)\|_{C(\bar{D})}\leq N_5(\delta+\chi mesD_1) \qquad (12)$$
where δ –is the value contained in curly brackets on the right side (5). It (11) follows from the obtained (12) estimate that $\chi\leq N(\delta+\chi mesD_1)$. Thus, under the condition $NmesD_1<1$ that the evaluation is performed $\chi\leq N\delta$. From the evaluation (11) and (12) the validity of the evaluation follows (5). The uniqueness of the solution of the problem $(1)-(4)$ directly follows from the evaluation (5). Theorem proved.
The existence of a solution to the problem $(1)-(4)$ is proved under additional assumptions.

Lemma. Let $h(x,y)\leq 0, f(\xi,\eta)\geq 0,$ to solve the problem $\sum_{i=1}^{n}a_i(x)\omega_{x_ix_i}=0,\ \omega(x,y)\big|_{\Gamma}=f(\xi,\eta)$ the estimates
$$\omega_\nu(x,y)\big|_{\gamma_1}<0, \omega_\nu(x,y)\big|_{\gamma_2}>0,$$
$a_i(x), i=1,2,...,n\quad c(x)-$ and the strictly positive continuous functions given are correct.Then , to

solve the problem of determining $u(x,y)$ from the conditions $(1)-(2)$, the estimates are correct:

$$\|u\|_{C(\bar{D}_1)} \le \|f\|_{C(\Gamma)} + \|c^{-1}h\|_{C(\bar{D}_1)}, u_\nu(x,y)\Big|_{\gamma_i} \le \le \omega_\nu(x,y)\Big|_{\gamma_i}, i=1,2. \tag{13}$$

Proof. The evaluation of a function $u(x,y)$ in a uniform metric follows from the maximum principle. Now let's prove the validity of the estimate $u_\nu(x,y)\Big|_{\gamma_i} \le \omega_\nu(x,y)\Big|_{\gamma_i}$, $i=1,2.$ Let's $\upsilon = u-\omega.$ Assume that the function $\omega(x,y)$ is a solution to the Dirichlet problem, so it follows from the maximum principle $f(\xi,\eta)\ge 0$, provided that the $\omega(x,y)\ge 0.$ Function $\upsilon(x,y)$ satisfies the equation:

$$-\sum_{i=1}^{n} a_i(x)\upsilon_{x_i x_i} + c(x)\upsilon = h(x,y) - -c(x)\omega(x,y) \tag{14}$$

and a homogeneous boundary condition.The right side of the equation (11) is negative, therefore $\upsilon(x,y)\le 0$. Hence, $\upsilon_\nu(x,y)\Big|_{\gamma_i} \le 0$ and

$u_\nu(x,y)\Big|_{\gamma_i} \le \omega_\nu(x,y)\Big|_{\gamma_i}$, $i=1,2.$ The Lemma is also proved.

Theorem 2. Let where
$h(x,y)\le 0,\ f(\xi,\eta)\ge 0, g_1(\xi,\eta)<0,$
$g_2(\xi,\eta)=0, \phi(\xi,\eta)>0,$
$NmesD_1<1,$ here N – be a positive number determined by the data of the problem $(1)-(4),$
for solving $\sum_{i=1}^{n} a_i(x)\omega_{x_i x_i}=0,\ \omega(x,y)\big|_{\Gamma} = f(\xi,\eta)$
the problem the estimates are correct
$\omega_\nu(x,y)\Big|_{\gamma_1}<0,$ then the problem $(1)-(4)$ has at least one solution.

Proof. The proof is carried out by the method of successive measurements. Let the functions $a_{i_0}^{(s)}(x), c^{(s)}(x), u^{(s)}(x,y)$ have already been found and $0<a_{i_0}^{(s)}(x),\ c^{(s)}(x)\in$
$\in C^\alpha(D),\ u^{(s)}(x,y)\in C^{2+\alpha}(\bar{D}_1)$. Let's consider the problem of determining $u^{(s+1)}(x,y)$ from the conditions:

$$-\sum_{i=1}^{i_0-1} a_i(x)u_{x_i x_i}^{(s+1)} - a_{i_0}^{(s)}(x)u_{x_{i_0} x_{i_0}}^{(s+1)} - \sum_{i=i_0+1}^{n} a_i(x)u_{x_i x_i}^{(s+1)} + +c^{(s)}(x)u^{(s+1)} = h(x,y), \quad (x,y)\in D_1, \tag{15}$$

$$u^{(s+1)}(x,y)\big|_{\tilde{A}} = f(\xi,\eta),\ (\xi,\eta)\in\Gamma_1, \tag{16}$$

This problem has a single solution belonging to $C^{2+\alpha}(\bar{D})$. Function $u^{(s+1)}(x,y)$ in the ratio

$$a_{i_0}^{(s+1)}(x)u_\nu^{(s+1)}(x,y)\big|_{\gamma_1} = g_1(\xi,\eta), \quad (\xi,\eta)\in\gamma_1, \tag{17}$$

$$\left[a_{i_0}^{(s+1)}(x)u_\nu^{(s+1)}(x,y) - c^{(s+1)}(x)\phi(x,y)\right]\Big|_{\gamma_2} = = g_2(\xi,\eta), \quad (\xi,\eta)\in\gamma_2, \tag{18}$$

are used to determine , $a_{i_0}^{(s+1)}(x)$,. $c^{(s+1)}(x)$. It follows from the statement of the lemma that the sequence $\{u^{(s)}(x,y)\}$ is uniformly bounded, $u_\nu^{(s+1)}(x,y)\big|_{\gamma_1} \le \omega_\nu(x,y)\big|_{\gamma_1}$. And therefore, from the condition $(17)-(18)$ under the assumptions assumed above, we obtain that:

$$0 < a_{i_0}^{(s+1)}(x) \le \max g_1(\xi,\eta)\left[\omega_\nu(x,y)\big|_{\gamma_1}\right]^{-1} \equiv \equiv A, s=1,2,..., \tag{19}$$

$$0 < c^{(s+1)}(x) \le \le \max A\left[\phi(\xi,\eta)\right]^{-1}\left[\omega_\nu(x,y)\right]\Big|_{\gamma_2} \equiv \equiv B, s=1,2,.... \tag{20}$$

Now let 's evaluate $\left\|u^{(s)}(x,y)\right\|_{C^1(\bar{D})}$. The function $\upsilon^{(s+1)}(x,y) = u^{(s+1)}(x,y) - \omega(x,y)$ satisfies a homogeneous boundary condition and the equation:

$$-\sum_{i=1}^{i_0-1}\frac{a_i(x)}{a_{i_0}^{(s)}(x)}\upsilon_{x_i x_i}^{(s+1)} - \upsilon_{x_{i_0} x_{i_0}}^{(s+1)} - \sum_{i=i_0+1}^{n}\frac{a_i(x)}{a_{i_0}^{(s)}(x)}\upsilon_{x_i x_i}^{(s+1)} = =\left[a_{i_0}^{(s)}(x)\right]^{-1}\left[h(x,y) - c^{(s)}(x)u^{(s+1)}\right].$$

Therefore, the estimate is correct for the function $\upsilon^{(s+1)}(x,y)$:

$$\left\|\upsilon^{(s+1)}\right\|_{W_p^2(D_1)} \le N\left\|\left[a_{i_0}^{(s)}(x)\right]^{-1}\left[h(x,y)-c(x)u^{(s+1)}\right]\right\|_{L_p(D_1)}$$

Hence , taking into account the uniform limitation of sequences $\left\{a_{i_0}^{(s)}(x)\right\}, \left\{u^{(s)}(x,y)\right\}$, we obtain:

$$\left\|u^{(s+1)}\right\|_{W_p^2(D_1)} \le \left\|\omega\right\|_{W_p^2(D_1)} + \left\|\upsilon^{(s+1)}\right\|_{W_p^2(D_1)} \le$$

$$\le \left\|\omega\right\|_{W_p^2(D_1)} + N_1 mesD_1\left[\alpha^{(s)}\right]^{-1},$$

where N_1 is determined by the data of the problem and $a_{i_0}^{(s)}(x) \ge \alpha^{(s)} > 0$. Then from the embedding theorems we have the following estimate:

$$\left\|u^{(s+1)}(x,y)\right\|_{C^1(\bar{D}_1)} \le$$

$$\le N_2\left\|u^{(s+1)}\right\|_{W_p^2(D_1)} \le N_3\left[1+mesD_1\left[\alpha^{(s)}\right]^{-1}\right],$$

where $N_3 > 0$ is determined by the data of the problem $(1)-(4)$.

Let $\alpha^{(0)} = N_3^{-1}\left[g_0 - N_3 mesD_1\right] \equiv a_0 > 0$, where $g_0 = \min\limits_{(\xi,\eta)\in\Gamma}\left|g_1(\xi,\eta)\right|$. Prove that under the conditions $\alpha^{(s)} = a_0, s = 1,2,...,$. of the theorem, the proof is carried out by induction.Let us $\alpha^{(k)} = a_0,\ \ k = 1,2,...,s.$, check $\alpha^{(k+1)} = a_0.$, that it follows from the relation $(17),(19)$ that:

$$\alpha^{(s+1)} =$$

$$\square\ g_0\left[N_3 + N_3 mesD_1\left(\alpha^{(s)}\right)^{-1}\right]^{-1} =$$

$$= g_0\left[N_3 + N_3^2 mesD_1\left(g_0 - N_3 mesD_1\right)^{-1}\right]^{-1} = a_0,$$

Therefore,

$$\left\|u^{(s)}(x,y)\right\|_{C^1(\bar{D}_1)} \le N_3(1+mesD_1 a_0^{-1}).$$

Given this estimate in (17), we get

$$0 < a_0 \le a_{i_0}^{(s)}(x) \le A.$$

Then it follows from the general theory of elliptic equations that, under the conditions of the theorem, the sequence $\left\{u^{(s)}(x,y)\right\}$ is uniformly bounded by the norm $W_p^2(D_1),\ \ p > n.$, therefore $u^{(s)}(x,y)$ compact in $C^1(\bar{D}_1).$, while from the condition (17), (18), it follows that the sequence $\left\{a_{i_0}^{(s)}(x)\right\}, \left\{c^{(s)}(x)\right\}$ will be compact in $C(\bar{D})$. Hence and from follows $(13)-(14)$ compactness in the system $(15)-(18)$ moving to the limit $s \to +\infty$, we obtain that there is a function satisfying the conditions $\left\{a_{i_0}(x), c(x), u(x,y)\right\}$ of the problem . The theorem is proved.

Let $i_0 \in I_2$. The Problem (1) also be written in the following form

$$-a_{\frac{n}{i_0}}(x)u_{x_{\frac{n}{i_0}}x_{\frac{n}{i_0}}} - a_{i_0}(x)u_{x_{i_0}x_{i_0}} - \sum_{i=2}^{n-1} a_i(x)u_{x_i x_i} + ,$$

$$+c(x)u = h(x,y)\ \ (x,y) \in D_1$$

Theorems 1,2 are proved anologically.

3 Results

Thus, the inverse problem in determining unknown coefficients in a linear elliptic equation was considered. Existence, uniqueness, and stability theorems for the solution of inverse problems for a linear equation of elliptic type are proved. Using the method of successive approximations, a regularizing algorithm is constructed to determine the coefficients.

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