Efficient Ranking Function Methods for Fully Fuzzy Linear Fractional Programming problems via Life Problems

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Abstract: - In this paper, we propose two new ranking function algorithms to solve fully fuzzy linear fractional programming (FFLFP) problems, where the coefficients of the objective function and constraints are considered to be triangular fuzzy numbers (TrFN) s. The notion of a ranking function is an efficient approach when you want to work on TrFNs. The fuzzy values are converted to crisp values by using the suggested ranking function procedure. Charnes and Cooper's method transforms linear fractional programming (LFP) problems into linear programming (LP) problems. The suggested ranking functions methods' applicability to actual problems of daily life, which take data from a company as an example, is shown, and it presents decision-making and exact error with the main value problem. The study aims to find an efficient solution to the FFLFP problem, and the simplex method is employed to determine the efficient optimal solution to the original FFLFP problem.

Key-Words:- Linear Fractional Programming, Fully Fuzzy Linear Fractional Programming, Linear Programming Problem, Ranking Function, Triangular Fuzzy Number, Charnes Cooper's Method.

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1 Introduction

A particular class of non-linear programming problems is called linear fractional programming (LFP) problems, in which the constraints are linear equations (inequality) and the objective function is a ratio of linear functions. The significance of this class of problems becomes apparent as there are many situations in corporate, economic, industrial, etc., where it has to be achieved that the ratio of two values, such as input/output, profit/cost, nurse/patient, sales/stock, etc., has to be optimized.

Many academics, including Charnes Cooper [1], Dinkelbach [2], Swarup [3], Guzel [4], Mustafa and Sulaiman [5], [6], and Nawkhass and Sulaiman [7], have developed a variety of ways to solve fractional programming and multi-objective linear fractional programming.

The fuzzy optimization problem is interesting and finds use in many crucial areas [8], [9]. In real-world situations, we frequently face the necessity to optimize an objective function. Due to this, the problem of linear fractional programming is extended to include fully fuzzy programming problems. In the literature, several approaches have been put forward to solve fuzzy programming problems.

Researchers have developed many strategies to find exact, close, rough, or efficient solutions to the fuzzy programming (FP) problem, as introduced by the first technique of Nayak [10] and the approach proposed by Umadevi [11]. Then, the researchers used the ranking function to change the FP problem into an appropriate crisp programming (CP) problem; this is a novel approach to solving FP problems.

A new viewpoint on how to solve fuzzy programming problems is presented by the application of fuzzy set theory [12] to deal with optimization problems. There are other approaches, including linguistic techniques based on Zadeh's work [13] fuzzy goal programming approach [14], Assignment Problem using Genetic Approach [15], etc...

Likewise, after them, the ranking function is used to explain the fuzzy fractional programming (FFP) problems [16], multi-objective LFP problems with fuzzy parameters [17], fully fuzzy multi-objective LP problems [18], and fuzzy LP problems [19], [20].

Many ranking function methods exist, such as the area between the centroid and original point method [21], SD of the PILOT procedure [22], the area method [23]and a revised approach to the PILOT ranking procedure [24], and new ranking procedures [19], new ranking [25], comparing different rankings [26], ranking method [27] efficient algorithm [16], Rouben ranking function [28], etc.

To deal with such kinds of inexact states, LFP problems with fuzzy coefficients were introduced, which are mentioned as FFLFP problems. The FFLFP problem is a fantastic tool for making decisions. With one or more objective functions, such as profit/cost, inventory/sales, output/employees, etc., it is used to model actual problems. The FFLFP problem might be used effectively since the procedure of production planning involves increasing profit by lowering costs with ambiguous values.

Applications of this problem include supply chain management, network banning, traffic, and transportation process of assignment, security, and injury strategic planning, policy decisions, power management, tax problems, and other actual problems.

In the field of implementation in the life of society, scholars have taken many serious steps, including Garrido et al. [26] comparing different ranking functions for solving fuzzy linear programming problems with fuzzy cost coefficients from the field of tourism. Mitlif [20] used an efficient ranking function method to solve LP problems with trapezoidal fuzzy coefficients in food products. Stanojevic et al. [29] and Sapan et al. [30] proposed a new approach for solving FFLFP problems and provided a tool for making good decisions in production planning separately. Also, Sapan [31] presented a new formulation of the FFLFP problem in a real-life case and found an efficient solution. Regarding COVID-19, Sapan and Chakraborty [32] proposed a new method to solve the LP problem under a pentagonal fuzzy environment. Veeramani and Sumathi [33], [34] studied a new approach to solving the LFP problem with triangular fuzzy coefficients about company manufacturing. Many other researchers are constantly trying to do new things for the benefit of society through fuzzy number solutions.

Optimization problems have more formulations of the objective function, such as linear programming, quadratic programming, and fractional programming. The coefficients of problems in the objective function, constraints coefficient, or decision-making variable are sometimes fuzzy numbers. In this work, we focus on the FFLFP problem under TrFNs. Here, we have received an example from companies working in Erbil to enforce our techniques and show the optimal results for making the best decisions as well as to make clear the company's owner in its profits and costs.

Here, we provide two model ranking function techniques for dealing with the FFLFP problem and de-fuzzing any fuzzy number. By taking advantage of Charnes and Cooper’s method, the LFP problem is converted into an LP problem. Also, by using the simplex method, we obtain an optimal solution. The derivations are used to clarify the techniques that are being taught.

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This article is constructed into six sections. In section 2, some important preliminaries on triangular fuzzy numbers and the notions that benefit us from this work are presented. In section 3, we show the mathematical problem formulation. The ranking functions are derived in section 4. An algorithm is illustrated in section 5. A real-life example of an FFLFP problem is included in section 6. In section 7, we describe the discussion. Finally, conclusions are described in section 8.

2 Preliminaries Notions

The fundamental principles of TrFNs with their operations and some concepts are given in this section.

Definition 1 Let X is a universal set. Then the ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$ of A is a subset in X called fuzzy set, Where $\mu_{\tilde{A}}: X \to [0,1]$ is named membership function.

Definition 2 If $\tilde{A}$ is a fuzzy subset convex, normal, and possesses bounded support of the set R is a real number, then $\tilde{A}$ is said to be a fuzzy number.

Definition 3 A fuzzy number $\tilde{A}$ is called TrFN if its representation is in the form $\tilde{A} = (a, b, c)$ with $a \leq b \leq c$, and the membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & otherwise \end{cases}$$

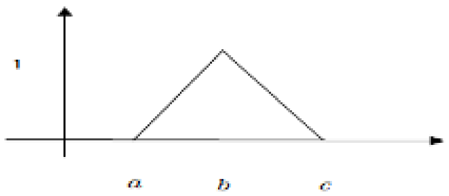

Fig. 1: TrFN Membership Function.

Definition 4 Let form $\tilde{A} = (a_1, b_1, c_1)$ and for $\tilde{B} = (a_2, b_2, c_2)$ be two TrFNs, where $a_1, b_1, c_1, a_2, b_2, c_2 \in R$. Then the arithmetic operations and scalar multiplications are defined by

1- $\tilde{A} + \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$
2- $\tilde{A} - \tilde{B} = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$
3- $L\tilde{A} = (La_1, Lb_1, Lc_1)$ where $L \in R^+$ and $L\tilde{A} = (Lc_1, Lb_1, La_1)$ where $L \in R^-$
4- $\tilde{A} * \tilde{B} = \tilde{A}\tilde{B} =$

$$\begin{cases} (a_1a_2, b_1b_2, c_1c_2), & if \quad a_1 \geq 0 \\ (a_1c_2, b_1b_2, c_1c_2), & if \quad b_1 < 0,\ 0 \leq c_1 \\ (a_1c_2, b_1b_2, c_1c_2), & if \quad b_1 < 0 \end{cases}$$

Definition 5 A TrFN $\tilde{A} = (a_1, b_1, c_1)$ is called non-negative (non-positive) TrFN iff $a_1 \geq 0$ $(c_1 \leq 0)$

Definition 6 D is a crisp set and it is also called a classical set defined as the group of elements present over the universal set U., in this case, a random element is present that may be a part of D or not which means two ways are possible to define the set. These are the first element that would become from set D, or it does not come from D, Crisp set defines the value as either 0 or 1.

Definition 7 The crisp set of elements that belong to $\tilde{A}$ at least to degree $\alpha \in [0,1]$ is named as α- cut set $\tilde{A}_\alpha = \{x \in R / \mu_{\tilde{A}}(x) \geq \alpha\}$

Definition 8 Defuzzification is the process of changing fuzzy parameters into clear (crisp) parameters. The set of all fuzzy sets can be mapped onto the set of all real numbers using a technique.

Theorem 1 A feasible point $x^* \in S$ is said to be an optimal solution to any optimization problem $F(x)$, if there does not exist $x \in S$ such that $F(x^*) \leq F(x)$. Where $S = \{x \in R^n : Ax \leq k, x \geq 0\}$

Remark 1 x^0 is said to be an efficient solution to a linear programming problem if x^0 is feasible and no other solution x^* exists such that $Cx^0 \neq Cx^*$ and $Cx^0 \leq Cx^*$ to maximize the problem and $Cx^0 \geq Cx^*$ to minimize the problem.

3 Mathematical Statement Problems

In this section, we will focus on the formulas we worked on for this paper and used to express our life problems.

3.1 Linear Fractional Programming (LFP) Problem:

Max or (Min). $z = \frac{ax+\alpha}{dx+\beta}$

Subject to:

$Ax \leq k, x \geq 0.$

Where $a, d, x \in R^m, k \in R^n$, A is an $n \times m$ real matrix and $\alpha, \beta \in R$. R is a real number.

3.2 Fully Fuzzy Linear Fractional Programming (FFLFP) problems:

In the FFLFP problem, the variables, and coefficients are completely TrFNs.

The general form of the FFLFP problem is as follows:

Max or (Min). $z = \frac{\tilde{a}\tilde{x}+\tilde{\alpha}}{\tilde{d}\tilde{x}+\tilde{\beta}}$

Subject to:

$\tilde{A}\tilde{x} \leq \tilde{k}, \tilde{x} \geq 0.$

Where $\tilde{a}, \tilde{d}1$ is by n matrices, $\tilde{x}$ is n by 1 matrix, $\tilde{A}$ is m by n matrix, and $\tilde{k}$ is m by 1 matrix. Here all the parameters $\tilde{A} = (A_1, A_2, A_3), \tilde{a} = (a, b, c), \tilde{d} = (d, e, f), \tilde{x} = (x, y, z), \tilde{k} = (k_1, k_2, k_3)$, and $\tilde{\alpha}, \tilde{\beta} \in$TrFNs

4 Ranking Functions Derivation

A ranking function of fuzzy number $R: F(R) \to R$, $F(R)$ is the set of all fuzzy numbers defined on R which maps each fuzzy number into a real number. Here we use two different rankings.

4.1 First Ranking Function

A triangular fuzzy number $\tilde{A} = (a, b, c)$ defines on the x-axis the real points a, b, c we can divide the entire range into six intervals$(a, a), (b, b), (c, c), (a, b), (a, c), (b, c)$.
In the beginning, find the arithmetic mean of each interval, the ranking function is defined by the quadratic mean of an average mean of all such possible intervals. Therefore, the ranking function

$$\mathcal{R}_1(\tilde{A}) = \sqrt{\frac{\left(\frac{a+a}{2}\right)^2 + \left(\frac{b+b}{2}\right)^2 + \left(\frac{c+c}{2}\right)^2 + \left(\frac{a+b}{2}\right)^2 + \left(\frac{a+c}{2}\right)^2 + \left(\frac{b+c}{2}\right)^2}{6}}$$

$$\mathcal{R}_1(\tilde{A}) = \sqrt{\frac{a^2 + b^2 + c^2 + \frac{(a+b)^2+(a+c)^2+(b+c)^2}{4}}{6}}$$

$$\mathcal{R}_1(\tilde{A}) = \sqrt{\frac{a^2 + b^2 + c^2 + \frac{2a^2+2b^2+2c^2+2ab+2ac+2bc}{4}}{6}}$$

$$\mathcal{R}_1(\tilde{A}) = \sqrt{\frac{a^2 + b^2 + c^2 + \frac{a^2+b^2+c^2+ab+ac+bc}{2}}{6}}$$

$$\mathcal{R}_1(\tilde{A}) = \sqrt{\frac{\frac{3a^2+3b^2+3c^2+ab+ac+bc}{2}}{6}}$$

$$\mathcal{R}_1(\tilde{A}) = \sqrt{\frac{3a^2 + 3b^2 + 3c^2 + ab + ac + bc}{12}}$$

Note that, regarding $a, b, and\ c \in \mathbb{R}$ being real numbers in the above ranking function, we have seven cases:

1- If $a, b, and\ c$ are positive real numbers.
2- If a is negative, $b = 0$ and c is positive.
3- If $a = 0$, and $b, and\ c$ are positive.
4- If a is negative and $b, and\ c$ are positive.

For the above situations, the ranking function is

$$\mathcal{R}_1(\tilde{A}) = \sqrt{\frac{3a^2 + 3b^2 + 3c^2 + ab + ac + bc}{12}}$$

5- If $a, b, and\ c$ are negative real numbers then the ranking function is
6- $\mathcal{R}_1(\tilde{A}) = -\sqrt{\frac{3a^2+3b^2+3c^2+ab+ac+bc}{12}}$
7- If $a, and\ b$ are negative real numbers and $c = 0$ then
8- $\mathcal{R}_1(\tilde{A}) = \frac{-1}{2}\sqrt{\frac{3a^2+3b^2+3c^2+ab+ac+bc}{12}}$
9- If $a, and\ b$ are negative and c is positive the ranking function is
10- $\mathcal{R}_1(\tilde{A}) = \frac{1}{2}\sqrt{\frac{3a^2+3b^2+3c^2+ab+ac+bc}{12}}$

4.2 Second Ranking Function

Several approaches for the ranking of fuzzy numbers have been proposed in the past decade. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means.

Now, by using the following triangular membership:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{\lambda(x-a)}{b-a}, & a \le x \le b \\ \lambda & x = b \\ \frac{\lambda(c-x)}{c-b}, & b \le x \le c \end{cases}$$

Then, by using α −cut, where $\alpha \in [0,1]$ and $0 \le \alpha \le \lambda$ and $0 \le \lambda \le 1$,

$\alpha = \frac{\lambda(x-a)}{b-a} \Rightarrow x = a + \frac{\alpha}{\lambda}(b-a) = \tilde{L}(\alpha)$ or $\alpha = \frac{\lambda(c-x)}{c-b} \Rightarrow x = c - \frac{\alpha}{\lambda}(c-b) = \widetilde{U}(\alpha)$

Where, $\tilde{L}(\alpha)$is the lower bound and $\widetilde{U}(\alpha)$ is the upper bound, then presented for arbitrary TrFNs be an ordered pair of function $[\tilde{L}(\alpha), \widetilde{U}(\alpha)]$, where $\tilde{L}(\alpha) \le \widetilde{U}(\alpha)$, suppose that w is a weight for $\tilde{L}(\alpha)$ and $(1-w)$ is a weight for $\widetilde{U}(\alpha)$ where $w \in [0,1]$.

$$\mathcal{R}_2(\tilde{A}) = \frac{\int_0^\lambda \alpha^7 [w\tilde{L}(\alpha) + (1-w)\tilde{U}(\alpha)] d\alpha}{\int_0^\lambda \alpha^7 d\alpha}$$

$$\mathcal{R}_2(\tilde{A}) = \frac{\int_0^\lambda \alpha^7 \left[w(a + \frac{\alpha}{\lambda}(b-a) + (1-w)(c - \frac{\alpha}{\lambda}(c-b)\right] d\alpha}{\int_0^\lambda \alpha^7 d\alpha}$$

$$\mathcal{R}_2(\tilde{A}) = \frac{\int_0^\lambda \left[w(a\alpha^7 + \frac{\alpha^8}{\lambda}(b-a) + (1-w)(c\alpha^7 - \frac{\alpha^8}{\lambda}(c-b)\right] d\alpha}{\int_0^\lambda \alpha^7 d\alpha}$$

$$\mathcal{R}_2(\tilde{A}) = \frac{w(\frac{a\alpha^8}{8} + \frac{\alpha^9}{9\lambda}(b-c) + (1-w)(\frac{c\alpha^8}{8} - \frac{\alpha^9}{9\lambda}(c-b)|_0^\lambda}{\frac{\alpha^8}{8}|_0^\lambda}$$

$$\mathcal{R}_2(\tilde{A}) = \frac{w(\frac{a\lambda^8}{8} + \frac{\lambda^9}{9\lambda}(b-c) + (1-w)(\frac{c\lambda^8}{8} - \frac{\lambda^9}{9\lambda}(c-b)}{\frac{\lambda^8}{8}}$$

$$\mathcal{R}_2(\tilde{A}) = \frac{w(\frac{a\lambda^8}{8} + \frac{\lambda^8}{9}(b-c) + (1-w)(\frac{c\lambda^8}{8} - \frac{\lambda^8}{9}(c-b)}{\frac{\lambda^8}{8}}$$

$$\mathcal{R}_2(\tilde{A}) = \frac{8}{\lambda^8}[w(\frac{a\lambda^8}{8} + \frac{\lambda^8}{9}(b-c) + (1-w)(\frac{c\lambda^8}{8} - \frac{\lambda^8}{9}(c-b)]$$

$$\mathcal{R}_2(\tilde{A}) = \frac{8\lambda^8}{\lambda^8}[w(\frac{a}{8} + \frac{1}{9}(b-c) + (1-w)(\frac{c}{8} - \frac{1}{9}(c-b)]$$

$$\mathcal{R}_2(\tilde{A}) = 8[w(\frac{9a+8b-8a}{72}) + (1-w)\left(\frac{9c-8c+8b}{72}\right)]$$

$$\mathcal{R}_2(\tilde{A}) = 8[w(\frac{a+8b}{72}) + (1-w)\left(\frac{c+8b}{72}\right)]$$

$$\mathcal{R}_2(\tilde{A}) = 8[w(\frac{a+8b}{72}) + \left(\frac{c+8b}{72}\right) - w\left(\frac{c+8b}{72}\right)]$$

$$\mathcal{R}_2(\tilde{A}) = 8[w((\frac{a+8b}{72} - \frac{c+8b}{72}) + \left(\frac{c+8b}{72}\right)]$$

$$\mathcal{R}_2(\tilde{A}) = 8[w((\frac{a-c}{72}) + \left(\frac{c+8b}{72}\right)]$$

$$\mathcal{R}_2(\tilde{A}) = \frac{8}{72}[w(a-c) + (c+8b)]$$

$$\mathcal{R}_2(\tilde{A}) = \frac{1}{9}[w(a-c) + (c+8b)]$$

4.3 Note
If $\tilde{A}_1$ and $\tilde{A}_2$ are two fuzzy numbers then

1- $\mathcal{R}(\tilde{A}_1 + \tilde{A}_2) = \mathcal{R}(\tilde{A}_1) + \mathcal{R}(\tilde{A}_2)$
2- $\mathcal{R}(\tilde{A}_1 - \tilde{A}_2) = \mathcal{R}(\tilde{A}_1) - \mathcal{R}(\tilde{A}_2)$
3- $\mathcal{R}(\tilde{A}_1) \le \mathcal{R}(\tilde{A}_2) \Rightarrow \tilde{A}_1 \le \tilde{A}_2$
4- $\mathcal{R}(\tilde{A}_1) \ge \mathcal{R}(\tilde{A}_2) \Rightarrow \tilde{A}_1 \ge \tilde{A}_2$
5- $\mathcal{R}(\tilde{A}_1) = \mathcal{R}(\tilde{A}_2) \Rightarrow \tilde{A}_1 \approx \tilde{A}_2$
6- $\mathcal{R}(\sum_{i=1}^n \tilde{A}_i) = \sum_{i=1}^n \mathcal{R}(\tilde{A}_i)$

5 Algorithm
In this algorithm, we describe the steps of the solution to specific problems and similar problems in our problem statement for this work.

1- The LFP problem with fixed coefficients is converted into an LP problem by charnes cooper's method.
2- To obtain the optimal solution, solve the LP problem using the simplex method.
3- Convert the LFP problem to an FFLFP problem (by the values of the problem)
4- Convert the FFLFP problem to an LFP problem, i.e., convert each fuzzy number involved in the FFLFP problem to a crisp number using ($\mathcal{R}_1(\tilde{A})$ and $\mathcal{R}_2(\tilde{A})$).
5- Solve the LFP problem by repeating steps 1-2. This solution is an efficient solution when an optimal solution is required.

6 Implementation Ranking Functions in Life Problem

One of Erbil's well-known companies produces teak wood household furniture for governmental and non-governmental institutions, such as cupboards, desks, and doors. We only take two types of them: cupboard and desk, if the size of the cupboard is 120*210*70cm and the desk is 75*130*80 cm. The selling price of each cupboard is around 325 dollars, and the desk is around 170 dollars. Of course, to make both kinds of products, there is a need for different kinds of costs, such as the purchase of wood, screws, knobs or pulls, drawer slides, hinges, paint, etc. Thus, the cost of the cupboard is around 125 dollars and the cost of the desk is around 60 dollars, with losses, taxes, and rents (building rent, water, and electricity rent) per week at around 300, 100, and 500 dollars, respectively. In this company, there are two working groups (WG) to create the production of the products: the carpenter group (cutting and tying) and the polishing group (painting and varnishing). Each group consisted of four workers; each worker put in around 5 and 6 hours of carpentry and polishing, except for Friday, which is a holiday, and the fees of 10 and 5 dollars per hour, respectively. Time required for the carpenter's cupboard: 4 h and 2.5 h for the desk, and polish time of 1.25 h and 0.75 h for the cupboard and desk. The company's ability to produce both types is around 36 units per week. Make the above-mentioned problem into a linear fractional programming formulation under fuzzy coefficients and determine how many cupboards and desks should be manufactured to maximize the total profit.

Suppose that x_1 and x_2 are products for cupboards and desks respectively, and arrange carpenters (C), polishing (P), and job time (JT) work per week together the fees per hour in tables (1) as:

Table 1. Total Fees and Hours.

	WG	JT	L=WG×JT	L×6	fees hour	fees per week
C	4	5	20	**120**	120×10	1200
P	4	6	24	**144**	144×5	720
						1920

$\text{income} = 325x_1 + 170x_2$

$\text{cost of material} = 125x_1 + 60x_2$

$\text{costs (losses + tax + rents)} = 300 + 100 + 500 = 900$

$\text{cost function} = 125x_1 + 60x_2 + 900$

Profit function=income –cost function

Profit function$=325x_1 + 170x_2 - (125x_1 + 60x_2 + 900)$

Profit function$=200x_1 + 110x_2 - 900$

To express the above problem in the form of an FFLFP problem, we need to write it as follows:

$$Max.Z = \frac{\text{profit}}{\text{cost}} = \frac{\widetilde{200}x_1 + \widetilde{110}x_2 - \widetilde{900}}{\widetilde{125}x_1 + \widetilde{60}x_2 + \widetilde{900}}$$

Simultaneously, in the above problem there are three constraints:

Carpenter time $\tilde{4}x_1 + \widetilde{2.5}x_2 \leq \widetilde{120}$

Polishing time $\widetilde{1.25}x_1 + \widetilde{0.75}x_2 \leq \widetilde{144}$

Ability product of company $\tilde{1}x_1 + \tilde{1}x_2 \leq \widetilde{36}$

According to the values of the company assume that

$\widetilde{200} = (190,200,210), \widetilde{110} = (100,110,120), \widetilde{900} = (890,900,910),$

$\widetilde{125} = (115,125,135), \widetilde{60} = (50,60,70), \tilde{4} = (3.5,4,4.5), \widetilde{2.5} = (2,2.5,3),$

$\widetilde{1.25} = (1,1.25,1.5), \widetilde{0.75} = (0.5,0.75,1), \tilde{1} = (0,1,2), \widetilde{120} = (115,120,125),$

$\widetilde{144} = (139,144,149), \widetilde{36} = (30,36,42).$

Solution:

Considering our algorithm, we first solve the problem as the usual LFP problem, as in the first step. That is, we first express every coefficient as a real number.

Step1:

Problem 1

$$Max.Z = \frac{200x_1 + 110x_2 - 900}{125x_1 + 60x_2 + 900}$$

Subject to:

$4x_1 + 2.5x_2 \leq 120$

$1.25x_1 + 0.75x_2 \leq 144$

$x_1 + x_2 \leq 36$

$x_1, x_2 \geq 0$

By Charnes and cooper's method we have

$Max.z = 200y_1 + 110y_2 - 900t$

Subject to:

$125y_1 + 60y_2 + 900t = 1$

$4y_1 + 2.5y_2 - 120t \leq 0$

$1.25t_1 + 0.75t_2 - 144t \leq 0$

$y_1 + y_2 - 36t \leq 0$

$y_1, y_2, t \geq 0$

Step2: By using the simplex method we obtain an optimal solution

$y_1 = 0.004585, y_2 = 0.00367, t = 0.000229, z = 1.114678899$

$x_1 = \frac{y_1}{t} = \frac{0.004585}{0.000229} \approx 20, x_2 = \frac{y_2}{t} = \frac{0.00367}{0.000229} \approx 16, Z = \frac{z}{t} = \frac{1.114678899}{0.000229} \approx 4867.593$

Step3: Convert the LFP problem into the FFLFP problem

$$Max.Z = \frac{(190,200,210)x_1 + (100,110,120)x_2 - (890,900,910)}{(115,125,135)x_1 + (50,60,70)x_2 + (890,900,910)}$$

Subject to:

$(3.5,4,4.5)x_1 + (2,2.5,3)x_2 \leq (115,120,125)$

$(1,1.25,1.5)x_1 + (0.5,0.75,1)x_2 \leq (139,144,149)$

$(0,1,2)x_1 + (0,1,2)x_2 \leq (30,36,42)$

$x_1, x_2 \geq 0$

Step4: Convert the FFLFP problem into an LFP problem using ($\mathcal{R}_1(\tilde{A})$ and $\mathcal{R}_2(\tilde{A})$).

Initially, we will use the first-ranking function method $\mathcal{R}_1(\tilde{A})$, then

$$\mathcal{R}_1(\tilde{A}) = \sqrt{\frac{3a^2 + 3b^2 + 3c^2 + ab + ac + bc}{12}}$$

$$\mathcal{R}_1(\widetilde{200}) = \sqrt{\frac{3(190)^2 + 3(200)^2 + 3(210)^2 + 190 * 200 + 190 * 210 + 200 * 210}{12}} = 200.104.$$

$\therefore \mathcal{R}_1(\widetilde{110}) = 110.189, \mathcal{R}_1(\widetilde{125}) = 125.166, \mathcal{R}_1(\widetilde{900}) = 900.023,$

$\mathcal{R}_1(\widetilde{60}) = 60.346, \mathcal{R}_1(\widetilde{120}) = 120.043, \mathcal{R}_1(\widetilde{144}) = 144.036, \mathcal{R}_1(\widetilde{36}) = 36.207,$

$\mathcal{R}_1(\tilde{4}) = 4.0129, \mathcal{R}_1(\widetilde{2.5}) = 2.5207, \mathcal{R}_1(\widetilde{1.25}) = 1.2603, \mathcal{R}_1(\widetilde{0.75}) = 0.7671,$

$\mathcal{R}_1(\tilde{1}) = 1.1902.$

Therefore, the LFP problem is as follows:

$$Max.Z = \frac{200.104x_1 + 110.189x_2 - 900.023}{125.166x_1 + 60.346x_2 + 900.023}$$

Subject to:

$4.0129x_1 + 2.5207x_2 \leq 120.043$

$1.2603x_1 + 0.7671x_2 \leq 144.036$

$1.1902x_1 + 1.1902x_2 \leq 36.207$

$x_1, x_2 \geq 0$

Step5: Solve the LFP problem by repeating steps 1-2, and then we get an optimal solution.

$$x_1 = \frac{y_1}{t} = \frac{0.006091}{0.000216} \approx 30, x_2 = \frac{y_2}{t} = \frac{0.000294}{0.000216} \approx 2, Z = \frac{z}{t} = \frac{1.09651}{0.000216} \approx 5076.5$$

The second time, we will use the second-ranking function method $\mathcal{R}_2(\tilde{A})$),

$\mathcal{R}_2(\tilde{A}) = \frac{1}{9}[w(a-c) + (c+8b)]$,

Let$w = 0$, then

$$\mathcal{R}_2(\tilde{A}) = \frac{1}{9}[(c+8b)], \mathcal{R}_2(\widetilde{200}) = \frac{1}{9}[(210 + 8 * 200)] = 201.111.$$

Similarly, for all fuzzy numbers of this example applied that$\mathcal{R}_2(\tilde{A})$, we obtain the LFP problem as follows:

$$Max. Z = \frac{201.111x_1 + 111.111x_2 - 901.111}{126.111x_1 + 61.111x_2 + 901.111}$$

Subject to:

$4.055x_1 + 2.555x_2 \leq 120.555$

$1.277x_1 + 0.777x_2 \leq 144.555$

$1.111x_1 + 1.111x_2 \leq 36.666$

$x_1, x_2 \geq 0$

By repeating step 1-2 in algorithm (5), we get optimal solution

$$x_1 = \frac{y_1}{t} = \frac{0.005382}{0.000223} \approx 25, x_2 = \frac{y_2}{t} = \frac{0.001971}{0.000223} \approx 9, Z = \frac{z}{t} = \frac{1.10066711}{0.000223} \approx 4935.726$$

This time let $w = 1$. Then, the $\mathcal{R}_2(\tilde{A}) = \frac{1}{9}[1(a-c) + (c+8b)]$, and the form of the problem is as follows:

$$Max. Z = \frac{198.888x_1 + 108.888x_2 - 898.888}{123.888x_1 + 58.888x_2 + 898.888}$$

Subject to:

$3.944x_1 + 2.444x_2 \leq 119.444$

$1.222x_1 + 0.722x_2 \leq 143.444$

$0.888x_1 + 0.888x_2 \leq 35.333$

$x_1, x_2 \geq 0$

By repeating steps 1-2 in algorithm (5), we get an optimal solution

$x_1 \approx 15, x_2 \approx 25, Z \approx 4763.05$

To avoid lengthening the solution, we will indicate the other values of w where $w \in [0,1]$ in the table (2), where P1 is Problem 1 and FR is the first-ranking function method.

Table 2. Shows the optimal solution to Problem 1 as well as both ranking function methods.

No.	w	x_1	x_2	$Max. Z$	Total profit	Exact error
1	P 1	20	16	4867.59	2947.59	0
2	FR	30	2	5076.5	3156.5	-208.9
3	0	25	9	4935.72	3015.72	-68.13
4	0.125	24	11	4928.02	3008.02	-60.42
5	0.25	23	13	4899.55	2979.55	-31.96
6	0.375	22	15	4872.63	2952.63	-5.038
7	0.5	20	16	4867.59	2947.59	0
8	0.625	19	19	4843.65	2923.65	23.942
9	0.75	18	21	4821.43	2901.43	46.156
10	0.875	17	23	4780.93	2860.93	86.661
11	1	15	25	4763.05	2843.05	104.54

7 Discussion

In table 2, it will be clear to us that the high and low numbers of both types of products are different, meaning that if the number of one of the products made is larger, the other is smaller to reach the optimal solution. At the same time, in the values of w, we note that both products will make some of them so that the company can make the most profits. If we look at when the value of w=0.5 the solution is the same as in Problem 1. The value of w in the second-ranking function method is the most profitable when it starts at zero and the least profitable when it reaches one. And it's clear that through the first ranking function method, the problem is going to get the most profits and count the best profit per week. To get the total profit, we must subtract the (carpentry and polishing) fees from the maximum value of the objective function. And to see the effect and evaluation of both of our ranking function methods, we needed to find the exact rate of error compared to the main value of Problem 1, and it made us realize that both ranking function methods are very effective in analyzing and reaching an optimal solution for each example of TrFN of LFP problem. In the conclusion section, we have explained how we can use our methods in future work.

8 Conclusion

In this work, we have discussed the FFLFP problem and shown a solution method that is very effective and suitable for problems with fuzzy number coefficients. Researchers have tried and solved it in many ways. One of the ways is called the ranking function method. In this study, we have proposed two novel ranking function methods that are effortless to calculate. At the same time, it is very effective and useful because we have created a living example in one of the companies in Erbil to show the effectiveness of both types of ranking methods. These ranking methods can be expanded and used for other problems, such as linear programming problems, quadratic programming problems, and transportation problems in fuzzy environments. In particular, the first ranking method may be expanded to solve the FFLFP problem with fuzzy numbers that are trapezoidal, pentagonal, and hexagonal.

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