

Multi-Fuzzy Rings

ABDALLAH AL-HUSBAN¹, MOWAFAQ OMAR AL-QADRI², RANIA SAADEH³,
AHMAD QAZZA³, HEBA HAZZA ALMOMANI¹

¹Department of Mathematics, Irbid National University, JORDAN

²Department of Mathematics, Jerash University, JORDAN

³Department of Mathematics, Zarqa University, JORDAN

Abstract: In this article, we generalize the notion of a fuzzy space defined by Dib and Fathi for the multi-membership function by examining and developing the concept for the multi-fuzzy binary operation. This inspired us to study and consider the multi-fuzzy ring theory approach.

Key-Words: - Multi-fuzzy ring, multi-fuzzy subspaces, multi-fuzzy ring and multi-fuzzy subring.

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1 Introduction

In 1965, Zadeh defined the concept of the fuzzy set theory, and it was advanced in many mathematical fields and applications to solve several examples and complicated problems in medical sciences, logic, control engineering, economics, etc [1]. By utilizing the ordered sequences of the membership function, Sebastian and Ramakrishnan found a new type of multi-fuzzy set. The colour of pixels is one of the problems that was explained by the notion of multi-fuzzy sets since it provides new methods to represent the problems. But we should note that is difficult to represent some other problems using another extension of a fuzzy set theory. The multi-fuzzy extensions of functions and multi-fuzzy subgroups were discussed by Sebastian and Ramakrishnan, we considered that the multi-fuzzy sets are an extension of the theories of fuzzy sets and it's called Atanassov intuitionistic fuzzy sets and L-fuzzy sets [2-4]. In 2010 they also studied and introduced some of the elementary properties of multi-fuzzy subgroups [5-7]. We will discuss the development of fuzzy group theory, starting from 1956, when L. A. Zadeh introduced the definition and the concept of a fuzzy set as we mentioned earlier then applied by Chang [8] in fuzzy topological spaces to generalize and define some fundamental definitions in general topology. In 1971, Azriel Rosenfeld defines some applications in groupoids and groups from the elementary theory [9]. Negoita and Ralescu use Rosenfeld's definition to consider a generalization where the unit interval $T = [0,1]$. In 1960, Schweizer and Sklar [10] define the triangular norm then in 1979, Anthony and Sherwood used the Schweizer and Sklar definition

to redefine the fuzzy subgroup of a group[11-13]. Note that the Rosenfeld-Anthony-Sherwood approach was used by many mathematicians to define or investigate fuzzy group theory. Also, a new approach was defined by Youssef and Dib for defining the fuzzy groupoid and the fuzzy subgroupoid [14-17]. Since the concept of a fuzzy universal set is not defined, so we can't define the fuzzy group and fuzzy subgroup. This was the reason for Youssef and Dib to introduce and define the fuzzy universal set and fuzzy binary operation. In 1982, J. Liu introduced in his paper, the concept of fuzzy rings and fuzzy ideals, then in 1985, the notions of fuzzy ideals and fuzzy quotient ring studied by Ren. In 2004, the concept of the intuitionistic fuzzy subgroup was generalized by Zhan and Tan [18], by adding restrictions for the definition of the non-membership function. They also introduced the definitions for the notion of a fuzzy space and an intuitionistic fuzzy function in their paper [18], they made a generalization of Rosenfeld's fuzzy subgroup definition to define the intuitionistic fuzzy subgroup. For any given classical group, we can use the classical binary operation to define the intuitionistic fuzzy subgroup. The definition of the notion of an intuitionistic fuzzy group was introduced by M. Fathi and AR Salleh in 2009 [19], by using the notion of intuitionistic fuzzy space and fuzzy function. To introduce the notion of the intuitionistic fuzzy group we will consider both of their notion of intuitionistic fuzzy space and intuitionistic fuzzy function [18]. In 2021, Jaradat and Al-Husban introduced the concept of multi-fuzzy group spaces on multi-fuzzy space. Notice that the fuzzy set concept will find in many mathematical fields [20-27]. As result, we will use

the multi-fuzzy space to create a new algebraic system, known as the multi-fuzzy ring, by integrating two mathematical fields on fuzzy sets which are fuzzy algebra and multi-fuzzy set theory.

2 Preliminaries

In this section, we will mention many theorems and definitions in the fuzzy set, which we will mainly use in the third section.

Definition 1 [1] Consider the fuzzy set $B \in Q$,

where Q is the universe of discourse, then the set B will be characterized by the membership function

$$\mu_B(m) \text{ where: } \mu_B(m): m \rightarrow [0,1].$$

Definition 2 [2], Consider $B \in Q$ where the set B is

intuitionistic fuzzy set and Q is the universe of discourse (a non-empty set) then we get the form:

$$B = \{ \langle m, \mu_B(m), \gamma_B(m) : m \in Q \rangle \}, \text{ consider the}$$

functions $\mu_B(m): Q \rightarrow [0, 1]$ is the degree of

membership and $\gamma_B(m): Q \rightarrow [0, 1]$ is the degree of

non-membership of each element $m \in Q$ to the set B

respectively, and $0 \leq \mu_B(m) + \gamma_B(m) \leq 1$ for all

$m \in Q$.

Definition 3 [6] Let k be a positive integer number

and the set B is multi-fuzzy set, where $\tilde{B} \in Q$ then \tilde{B} is a set of ordered sequences as follow:

$$\tilde{B} = \{ q (\mu_1(q), \mu_2(q), \dots, \mu_i(q), \dots, \mu_k(q)) : q \in Q \},$$

where $\mu_i \in \tilde{P}(Q), i = 1, 2, 3, \dots, k$. The function

$\mu_{\tilde{B}} = (\mu_1, \mu_2, \dots, \mu_k)$ is called the multi-

membership function of multi-fuzzy set \tilde{B} , where k

is called the dimension of \tilde{B} . The set of all multi-

fuzzy sets of dimension k in Q is denoted by

$$M^k DS(U).$$

Remark 1 [6], Consider the following summation

$$\sum_{i=1}^k \mu_i(q), q \in Q, \text{ where: } \mu_i(m): m \rightarrow [0,1],$$

$$i = 1, 2, 3, \dots, k.$$

1- If $k = 1$ then we get multi-fuzzy set of dimensions 1 , called Zadeh's fuzzy set.

2- If $k = 2$ and $\sum_{i=1}^2 \mu_i(q) \leq 1$, then we get multi-fuzzy set of dimensions 2 , called Atanassov's intuitionistic fuzzy set.

3- If $\sum_{i=1}^k \mu_i(q) \leq 1$, then the multi-fuzzy set of dimensions k is called a normalized multi-fuzzy set.

Definition 4 [6], Let

$$\tilde{B} = \{ q / (\mu_1(q), \mu_2(q), \dots, \mu_k(q)) : q \in Q \} \text{ and}$$

$$\tilde{E} = \{ q / (\gamma_1(q), \gamma_2(q), \dots, \gamma_k(q)) : q \in Q \} \text{ be two}$$

multi-fuzzy sets of dimensions k in Q . Then

\tilde{B} and \tilde{E} satisfying this relations and operations:

$$(1) \tilde{B} \subseteq \tilde{E} \text{ iff } \mu_i(q) \leq \gamma_i(q), \forall q \in Q \text{ and } 1 \leq i \leq k.$$

$$(2) \tilde{B} = \tilde{E} \text{ iff } \mu_i(q) = \gamma_i(q), \forall q \in Q \text{ and } 1 \leq i \leq k.$$

$$(3) \tilde{B} \cup \tilde{E} = \{ q / (\mu_1(q) \vee \gamma_1(q), \mu_2(q) \vee \gamma_2(q), \dots, \mu_k(q) \vee \gamma_k(q)) : q \in Q \}.$$

$$(4) \tilde{B} \cap \tilde{E} = \{ q / (\mu_1(q) \wedge \gamma_1(q), \mu_2(q) \wedge \gamma_2(q), \dots, \mu_k(q) \wedge \gamma_k(q)) : q \in Q \}.$$

$$(5) \tilde{B}^c = \{ q / (\mu_1^c(q), \mu_2^c(q), \dots, \mu_k^c(q)) : q \in Q \}.$$

Definition 5 [17], Let M be a nonempty set then

consider $(M, T) = \{ (m, T) : m \in M \}$ as a fuzzy

space, the ordered pair (m, T) is called a fuzzy

element in (M, T) .

Definition 6 [16], Let M be a nonempty set. Then the intuitionistic fuzzy space is the set of all ordered triples that denoted by $(m, T, T) = \{(m, r, s) : r, s \in T\}$ with the conditions $r + s \leq 1$ and $m \in M$. The intuitionistic fuzzy element of (M, T, T) defined as (m, T, T) , and the condition $r, s \in T$ with $r + s \leq 1$ is hold.

Definition 7 [18], We can define the intuitionistic fuzzy subgroup of the group G , by consider an intuitionistic fuzzy set $B = \{(m, \mu_B(m), \nu_B(m)) : m \in G\}$ in a group G with the binary operation (\circ) and satisfying the following conditions:

$$(1) \mu_B(m \circ n) \geq \min\{\mu_B(m), \mu_B(n)\} \text{ and } \nu_B(m \circ n) \leq \max\{\nu_B(m), \nu_B(n)\}, \forall m, n \in B,$$

$$(2) \mu(m^{-1}) \geq \mu(m) \text{ and } \nu(m^{-1}) \leq \nu(m) \text{ for all } m \in B, T_i^M)$$

Definition 8 [25], Let M be a non-empty set. A multi-fuzzy space $(M, T_i^M = [0, 1])$ is the set of all ordered sequences $(m, T_i^M), m \in M; w \square ere i \in \square$ that is, $(M, T_i^M) = \{(m, T_i^X) : m \in M\}$, where $(m, T_i^M) = \{(m, r_i) : r_i \in T_i^M\}$. The ordered sequences (m, T_i^M) is called a multi-fuzzy element in the multi-fuzzy space (M, T_i^M) . Also, the ordinary set of ordered sequences is a multi-fuzzy space. The first component in the ordered sequence indicates the ordinary element, while a list of potential multi-membership values is indicated by the second component. The Dib's fuzzy group is defined as A multi-fuzzy space of dimension 1, and The Fathi's intuitionistic fuzzy group is defined as a multi-fuzzy space of dimension 2.

Definition 9 [6], Let B be a multi-fuzzy subset of the ordinary group G with the ordinary binary operation (\circ) , then B will be a multi-fuzzy subgroup of G if and only if satisfying the following conditions:

$$(1) B(m \circ n) \geq \min\{B(n), B(n)\},$$

$$(2) B(m^{-1}) \geq B(m), \text{ for all } m, n \in G.$$

Also, we can define Rosenfeld's fuzzy subgroup if the multi-fuzzy subgroup of dimension 1, and define Zhan's intuitionistic fuzzy subgroup if the multi-fuzzy subgroup of dimension 2.

Definition 10 [25] Let Q_0 denote the support of Q ,

that is $Q_0 = \{m \in M : B_i(m) > 0, \text{ where } i \in \mathbb{N}\}$. A

multi-fuzzy subspace Q of the multi-fuzzy space

(M, T_i^M) is the collection of all ordered sequences

$(m, q(m)_i)$, where $m \in Q_0$ for some $Q_0 \subset M$ and

$q(m)_i \subset T_i^M$ that contains besides the zero element at least another one element.

Definition 11 [25] A multi-fuzzy binary operation

$\underline{D} = (D(m, n), d_{(mn)_i})$ on the multi-fuzzy space

(M, T_i^M) is a multi-fuzzy function from

$(M, T_i^M) \times (M, T_i^M) \rightarrow (M, T_i^M)$ with multi-co-

membership functions $d_{(mn)_i}$ with the following conditions:

$$(i) \quad d_{(mn)_i}(r_i, s_i) \neq 0 \text{ if}$$

$$r_i \neq 0, s_i \neq 0, w \square ere i \in \mathbb{N}$$

(ii) $d_{(mn)_i}(r_i, s_i)$ are onto, that is,

$$d_{(mn)_i}(T_i^M \times T_i^M) = T_i^M, m, n \in M.$$

Definition 12 [25], The multi-fuzzy group is defined as a multi-fuzzy algebraic system

$((M, T_i^M), \underline{D})$ if and only if for every

$(m, T_i^M), (y, T_i^M), (z, T_i^M) \in (X, T_i^M)$ the following conditions are satisfied:

1-Associative:

$$((m, T_i^M) \underline{D} (n, T_i^M)) \underline{D} (z, T_i^M) = (m, T_i^M) \underline{D} ((n, T_i^M) \underline{D} (z, T_i^M)),$$

$$\text{that is } ((mDn)Dz, T_i^M) = (mD(nDz), T_i^M).$$

2-Existence of a multi-fuzzy identity element

(e, T_i^M) , for which

$$(m, T_i^M) \underline{D} (e, T_i^M) = (e, T_i^M) \underline{D} (m, T_i^M) = (m, T_i^M),$$

that is

$$(mDe, T_i^M) = (eDm, T_i^M) = (m, T_i^M).$$

3-Every multi-fuzzy element (m, T_i^M) has an

inverse (m^{-1}, T_i^M) such that:

$$(m, T_i^M) \underline{D} (m^{-1}, T_i^M) = (m^{-1}, T_i^M) \underline{D} (m, T_i^M) = (e, T_i^M).$$

From (1), (2) and (3), it follows that

$((M, T_i^M), \underline{D})$ is a multi-fuzzy group over the multi-

fuzzy space (M, T_i^M) .

3 Multi-Fuzzy Ring

The main result of this paper is to define the concept of the multi-fuzzy ring by using two multi-fuzzy binary operations and adding them, to generate a multi-fuzzy space with similar conditions to the ordinary cases and multi-fuzzy.

Definition 13 A multi-fuzzy ring

$((M, T_i^M); \underline{D}^+, \underline{D}^*)$ is considered as a multi-fuzzy

space (M, T_i^M) with two multi fuzzy binary operations.

$$\underline{D}^+ = (D^+, d_{(mn)_i}^+) \quad \underline{D}^* = (D^*, d_{(mn)_i}^*)$$

+	0	1	2		0	1	2
0	0	1	2		0	0	0
1	1	2	0		1	0	1
2	2	0	1		2	0	2

And satisfying the following conditions:

1. $((M, T_i^M); \underline{D}^+)$ is a commutative multi-fuzzy group,
2. $((M, T_i^M); \underline{D}^*)$ is a multi-fuzzy semigroup,
3. The distributive laws

$$(m, T_i^M) \underline{D}^* ((n, T_i^M) \underline{D}^+ (z, T_i^M)) = (m, T_i^M) \underline{D}^* (n, T_i^M) \underline{D}^+ (m, T_i^M) \underline{D}^* (z, T_i^M)$$

$$((m, T_i^M) \underline{D}^+ (n, T_i^M)) \underline{D}^* (z, T_i^M) = (m, T_i^M) \underline{D}^* (z, T_i^M) \underline{D}^+ (n, T_i^M) \underline{D}^* (z, T_i^M)$$

holds for all

$$(m, T_i^M), (n, T_i^M), (z, T_i^M) \in (M, T_i^M).$$

Example 1 Let the set $\square_3 = \{0,1,2\}$ and defined a multi-fuzzy binary operation

- 1- $\underline{D}^+ = (D^+, d_{(mn)_i}^+)$
- 2- $\underline{D}^* = (D, d_{(mn)_i}^*)$

over the multi fuzzy space (\square_3, T_i^M) by taking the set as follows:

$$1 - D^+(m, n) = m +_3 n,$$

$$2 - d_{aa}^+(r_i(m), s_i(m)) = r_i \cdot s_i.$$

$$3 - D^*(m, n) = m *_3 n.$$

$$4 - d_{aa}^*(r_i(m), s_i(m)) = r_i \cdot s_i.$$

Where $+_3$ refers addition modulo 3, $(*)$ refers to multiplication modulo 3, where (\times_3) refers to multiplication modulo 3.

Thus $((\square_3, T_i^M); \underline{D}^+, \underline{D}^*)$ is a multi-fuzzy ring using multi-fuzzy spaces.

Example 2 Let the set $M = \{a\}$ and defined the multi-fuzzy binary operation as follow:

- 1- $\underline{D}^+ = (D^+, d_{(mn)_i}^+)$,
- 2- $\underline{D}^* = (D^*, d_{(mn)_i}^*)$.

Over the multi-fuzzy spaces (M, T_i^M) such that:

$$D^+(a, a) = a \text{ and } d_{aa}^+(r_i, s_i) = r_i \wedge s_i.$$

$$D^*(a, a) = a \text{ and } d_{aa}^*(r_i, s_i) = r_i \wedge s_i.$$

Where, the multi-fuzzy space (M, T_i^M) with $\underline{D}^+, \underline{D}^*$

defined a multi-fuzzy ring $((M, T_i^M); \underline{D}^+, \underline{D}^*)$.

Definition 14 The multi-fuzzy ring will be commutative if (m, T_i^M) and $(n, T_i^M) \in (M, T_i^M)$, we have

$$(m, T_i^M) \underline{D}^*(n, T_i^M) = (n, T_i^M) \underline{D}^*(m, T_i^M).$$

Definition 15 Considered the following

$((M, T_i^M); \underline{D}^+, \underline{D}^*)$ be a multi-fuzzy ring satisfying the following:

- (1) A multi fuzzy element in $((M, T_i^M); \underline{D}^+, \underline{D}^*)$ is a multi-fuzzy unity denoted by $(1, T_i^M)$, if $(1, T_i^M) \underline{D}^*(a, T_i^M) = (a, T_i^M) \underline{D}^*(1, T_i^M) = (a, T_i^M)$

for all $(a, T_i^M) \in ((M, T_i^M); \underline{D}^+, \underline{D}^*)$. A multi fuzzy ring having a unity is called a multi fuzzy ring with unity.

- (2) A multi fuzzy element in $(a, T_i^M) \in ((M, T_i^M); \underline{D}^+, \underline{D}^*)$ is called a unit if there exist a multi fuzzy element $(b, T_i^M) \in ((M, T_i^M); \underline{D}^+, \underline{D}^*)$ such that $(a, T_i^M) \underline{D}^*(b, T_i^M) = (b, T_i^M) \underline{D}^*(a, T_i^M) = (1, T_i^M)$

In the following theorem, we will demonstrate the correspondence relationship between the multi-fuzzy ring and the intuitionistic fuzzy ring.

Theorem 2 Associated to each multi-fuzzy ring $((M, T_i^M), \underline{D}^+, \underline{D}^*)$ is an intuitionistic fuzzy ring $((M, T, T), \underline{D}^+, \underline{D}^*)$ which is isomorphic to the complex fuzzy ring $((M, T_i^M); \underline{D}^+, \underline{D}^*)$ by the correspondence $(m, T_i^M) \leftrightarrow (m, T, T): m \in M$.

Proof. Let $((M, T_i^M), \underline{D})$ be a multi fuzzy ring. Restrict the multi fuzzy binary operations $\underline{D}^+ = (D^+, d_{mn}^+)$, $\underline{D}^* = (d^*, d_{mn}^*)$ to the intuitionistic fuzzy case for $m \in M$ using the isomorphic $(m, T_i^M) \leftrightarrow (m, T, T)$ such that $D^+ = (D^+, d_{mn}^+)$, $D^* = (D^*, d_{mn}^+)$ respectively. Thus, the resultant structure $((M, T, T), \underline{D})$ is an intuitionistic fuzzy ring in the sense of Fithi.

4 Conclusion

In this paper, we create a new algebraic system by generalizing and studying the Dib definitions of fuzzy rings that are based on multi-fuzzy spaces. In the future we attend to modify new definitions and inequalities and introduce new theorems related to them with more applications [28-36].

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