On New Two-Step GMM Estimation of the Panel Vector Autoregressive Models with Missing observations

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Abstract: Few estimation methods were discussed to handle the missing data problem in the panel data models. However, in the panel vector autoregressive (PVAR) model, there is no estimator to handle this problem. The traditional treatment in the case of incomplete data is to use the generalized method of moment (GMM) estimation based on only available data without imputation of the missing data. Therefore, this paper introduces a new GMM estimation for the PVAR model in case of incomplete data based on the mean imputation. Moreover, we make a Monte Carlo simulation study to study the efficiency of the proposed estimator. We compare between two GMM estimators based on the mean squared error (MSE) and relative bias (RB) criteria. The first is the GMM estimation based on the list-wise (LW) and the second is the GMM estimation using the mean imputation (MI) at multi-missing levels. The results showed that the MI estimator provides more efficiency than the LW estimator.

Key-Words: Generalized method of moments; Mean Imputation; Missing Data; Two-Step Estimation; missing observations.

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1 Introduction

The regression of panel data differs from a regular cross section or time series in that it has a double subscript on its variables i and t, where the i subscript represents the cross-section dimension while t denotes the time series dimension, see [1, 2, 3, 4, 5]. Time series vector autoregressive (VAR) models were discovered in the macro-econometrics literature as an alternative to multivariate simultaneous equation models, since all the variables of VAR systems are treated as endogenous variables [6].

The panel vector autoregressive PVAR models include a lagged endogenous variable, and the first difference of the error term will be correlated with all of the explanatory variables. In this situation, the estimators of PVAR models will be biased. Yamamoto and Kunitomo [7] first derived the asymptotic bias for the ordinary least squares estimator of a multivariate autoregressive model with a constant term. They reduced a model without a constant term as a special case, and multivariate autoregressive time series models could be treated as similar to the idea of PVAR models. Missing data patterns have effects on most applied studies in economic fields. As a result, there is a wide literature about how to treat the problem of missing data. However, an efficient method to deal with estimation in an arbitrary generalised method of moment (GMM) setting with a general missing data pattern is not available. Therefore, there are many inefficient methods, such as complete-case analyses, that dominate the empirical literature. In a survey constructed by Abrevaya and Donald [8], they found that few of the empirical research deals with missing data, and in most of these cases, a complete-case estimator is used, i.e., all incomplete observations were discarded. Therefore, the main objective of this paper is to present a Monte Carlo simulation study to study the efficiency of the proposed estimator suggested by Rady et al. [1].

The rest of the paper is organized as follows: section 2 introduces the PVAR model and its assumptions. Section 3 provides the results of the Monte Carlo simulation study. Finally, section 4 offers the conclusions.

2 The PVAR model

In dynamic panel data (DPD), we assume the observations are on many individuals, with many observations on each individual, and the model of interest is a regression model in which the lagged value of the dependent variable is treated as one of the explanatory variables. The error term in the model is assumed to contain a time-invariant individual effect as well as random noise [9, 10, 11, 12, 13, 14]. The basic problem faced in the estimation of DPD models is that a fixed effects treatment tends to the within estimator (least-squares after transformation to deviations from means), which has inconsistent estimators because the within transformation induces a correlation between the lagged dependent variable and the error, see [3, 12, 13, 14]. Holtz-Eakin et al. [15], expanding on the Anderson-Hsiao [16] approach, show how it is implemented to estimate a vector autoregression with time-varying parameters. Arellano and Bond [17] used Monte Carlo studies to evaluate a GMM estimator that is like [15] recommendation, and Kiviet [18] used the simulations to compare these and many other techniques, including the corrected least squares dummy variable estimator, see [3, 12, 13]. Vector autoregressions are now a standard part of the applied econometrician's toolkit. Although their interpretation in terms of causal relationships is controversial,

Holtz-Eakin et al. [15] introduced an estimation and testing for the PVAR model. They used an estimation method as similar to that Anderson and Hisiao [16]. Consider PVAR data with N units observed for $T + P$ consecutive time periods. Each unit i, we observed M outcome variables $y_{it1}, \ldots, y_{itM}$, where $t = 1 - P, \ldots, T$.

The behaviour of $Y_{it} = (Y_{it1}, Y_{it2}, \ldots, Y_{itM})'$ and it is described by the P^{th} order vector autoregression:

$$Y_{it} = \Phi_1 Y_{it-1} + \Phi_2 Y_{it-2} + \ldots + \Phi_p Y_{it-p} + u_{it} \quad (1)$$

Where $u_{it} = \alpha_i + \upsilon_{it}$; $i = 1, \ldots, N$, $t = 1, \ldots, T$, $Y_{it-p} = L^P Y_{it}$; P^{t} lag order of Y_{it}, Φ_p is $M \times M$, u_{it} M-dimensional error term. $\upsilon_{it} = (\upsilon_{it1}, \ldots, \upsilon_{itM})'$, and $\alpha_i = (\alpha_{i1}, \ldots, \alpha_{iM})'$ is a fixed effects in the model.

2.1 Proposed Estimator Assumption

In general, the assumptions of the PVAR model are:

Assumption 1 (Condition of stationarity): The roots of the given determinant:

$$det(I_M - \Phi_{1z} - \cdots - \Phi_{pz}^{p}) = 0 \quad (2)$$

This assumption refers that the process of vector autoregressive is stable.

Assumption 2 (Regularity conditions): The υ_{it} has finite eight-order moments and, as N →∞

$$\frac{1}{N}\sum_{i=1}^{N}\|\alpha_i\|^2 O(1), \frac{1}{N}\sum_{i=1}^{N}\left\|Y_{i1-p}\right\|^2 = O(1),$$

Where $p = 1, \ldots, P$. The conditions about the moment in this assumption make sure that the regular asymptotic behaviour of the least squares estimator is standard [1].

Assumption 3 (The errors): The υ_{it} are independent and identically distributed across i and t:

$$E[\upsilon_{it}] = 0, \quad E[\upsilon_{it}\upsilon'_{it}] = \Omega \quad (3)$$

Where Ω is positive definite matrix and the independence across time can allow for dependence between υ_{it} and υ_{it-P} through their higher-order moments. For simplicity, we can say this assumption as, the error vectors are independent and identically distributed (iid).

Assumption 4 The time dimension of panel is finite with $T > P$ and the available observations are (Y_{i0}, Y_{i1}, ..., Y_{iT}).

2.2 The Proposed Estimator

This section explains the proposed estimator was introduced by Rady et al. [1]. Consider the standard linear PVAR (1) model as discussed in (1)

$$Y_{it} = \Phi_0 Y_{it-1} + u_{it}. \quad (4)$$

Where Y_i is a (possibly missing) scalar, Y_i is a K-vector of all lagged dependent variable. The first element of Y_i is 1; that is, the model is assumed to contain an intercept. We assume the residuals only satisfies the conditions in (3) to be a linear projection, specifically

$$E(u_{it} | y_{i,t-1}, \ldots, y_{i0}, m_i) = 0 \quad (5)$$

The variable m_i indicates whether or not Y_i is missing for observational unit i:

$$m_i = \begin{cases} 1 & if\ Y_i\ missing \\ 0 & if\ Y_i\ observed \end{cases}$$

The proposed weighted GMM estimator can be introduced as

$$\widehat{\Phi} = (V'Z\widehat{W}Z'V)^{-1}(V'Z\widehat{W}Z'Y) \quad (6)$$

Where the GMM estimator can be implemented using instrumental variables methods. We can define the instrumental-variable matrix Z_i as

$$Z_i = \begin{pmatrix} m_{i1}m_{i1}Y_i' & 0 & \cdots & 0 \\ 0 & m_{i2}m_{i2}Y_i' & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & m_{it}m_{it}Y_i' \end{pmatrix}$$

Which corresponds to (Y_i' 0) using as instruments in each time for which y_{it} is observed. For the statistical properties of this estimator, see [1].

3 Monte Carlo Simulation

A Monte Carlo simulation is a type of simulation that relies on repeated random sampling and statistical analysis to compute the results. This method of simulation is very closely related to random experiments, in which the specific result is unknown in advance.

3.1 Design of the Simulation

The simulated dataset represents the generation of different cross sections $n = 5, 10, 20, 30, 40$ as a complete case, and it is assumed that the number of endogenous variables is three ($M = 3$) so, the Φ matrix is of order 3×3 and it consists of nine coefficients to be estimated. The values of the endogenous variable Y were generated as independent normally distributed random variables. The disturbances were generated as independent normally distributed random variants, with mean zero and standard deviation equal one. The disturbances were allowed to differ for each cross-sectional unit on a given Monte Carlo trial and were allowed to differ between trials. The value of time T was chosen to be fixed is equal nine. Moreover, it was used the R codes employed by Muris [19] in PVAR models and the paper considered by Abonazel [20] that shows a new algorithm that provides researchers with basics and advanced skills about how to create their R-codes and then achieve the simulation study for estimating the missing observations and imputation in all percentages of missingness and then estimate make simulation using sample sizes (cross sections are 5, 10, 20, 30 and 40). The first step of this simulation is examining the optimal lag of the PVAR model for this data and the optimal lag length found to be PVAR (1). After determining the optimal lag of the model, now it said to be sure that the data generated in a complete case is of order 1 so the Φ matrix is of order 3×3. Then it made new four datasets from the complete dataset corresponding to four percentages of missingness (10%, 20%, 30% and 40%). Then it is estimated the PVAR model using (6). This estimation using LW deletion and the proposed two-step GMM estimator based on MI method to make a comparison and these estimations are based on many samples of size: 5, 10, 20, 30 and 40 then it is estimated the MSE at each percentage of missingness for each estimator, also, the MSE and RB were calculated as:

$$MSE = \frac{1}{\mathrm{r}}\sum(\hat{\Phi} - \Phi)^2 \qquad (7)$$

$$RB = \frac{\hat{\Phi} - \Phi}{\Phi} \times 100 \qquad (8)$$

This RB is used to evaluate the percentage of the bias for the missingness level, and MSE and RB are used as criteria to determine the perfect usage of the MI method, with which one of missingness is more efficient at different sample sizes where chosen.

For each of the experimental settings, 500 Monte Carlo trials (r = 500) were used in this simulation because each trial takes the used package in the R program and results were recorded and all simulation results were conducted (see the appendix) and the settings of the model and results of the simulation study are discussed below.

3.2 Simulation Results for Different Sample Size

In this section, the basic objective is to study the relationship between sample size and each of MSE and RB under several percentages of missing observations. These are made to evaluate the LW imputation and MI method.

- **The MSE for missingness at multi percentage levels**

The following section presents the MSE at multi-levels as discussed before. Figure 1 introduces the MSE for the estimator $\hat{\Phi}$with GMM estimation of PVAR with 10%, 20%, 30%, and 40% missing observations, and the estimation provides a comparison between LW imputation and MI among the different sample sizes of 5, 10, 20, 30, and 40. It was found that when the sample size increases, the MSE decreases in both imputation methods, LW and MI. Moreover, overall, the graph shows that MI is more efficient than LW starting from sample size 5. It means when the sample size increases, the MSE for the MI method becomes smaller than LW. In the case of 20% of the missing observations It was found that when the sample size increases, the MSE decreases in both imputation methods, LW and MI. Moreover, overall, the graph shows that MI is more efficient than LW, which means when the sample size increases, the MSE for the MI method becomes

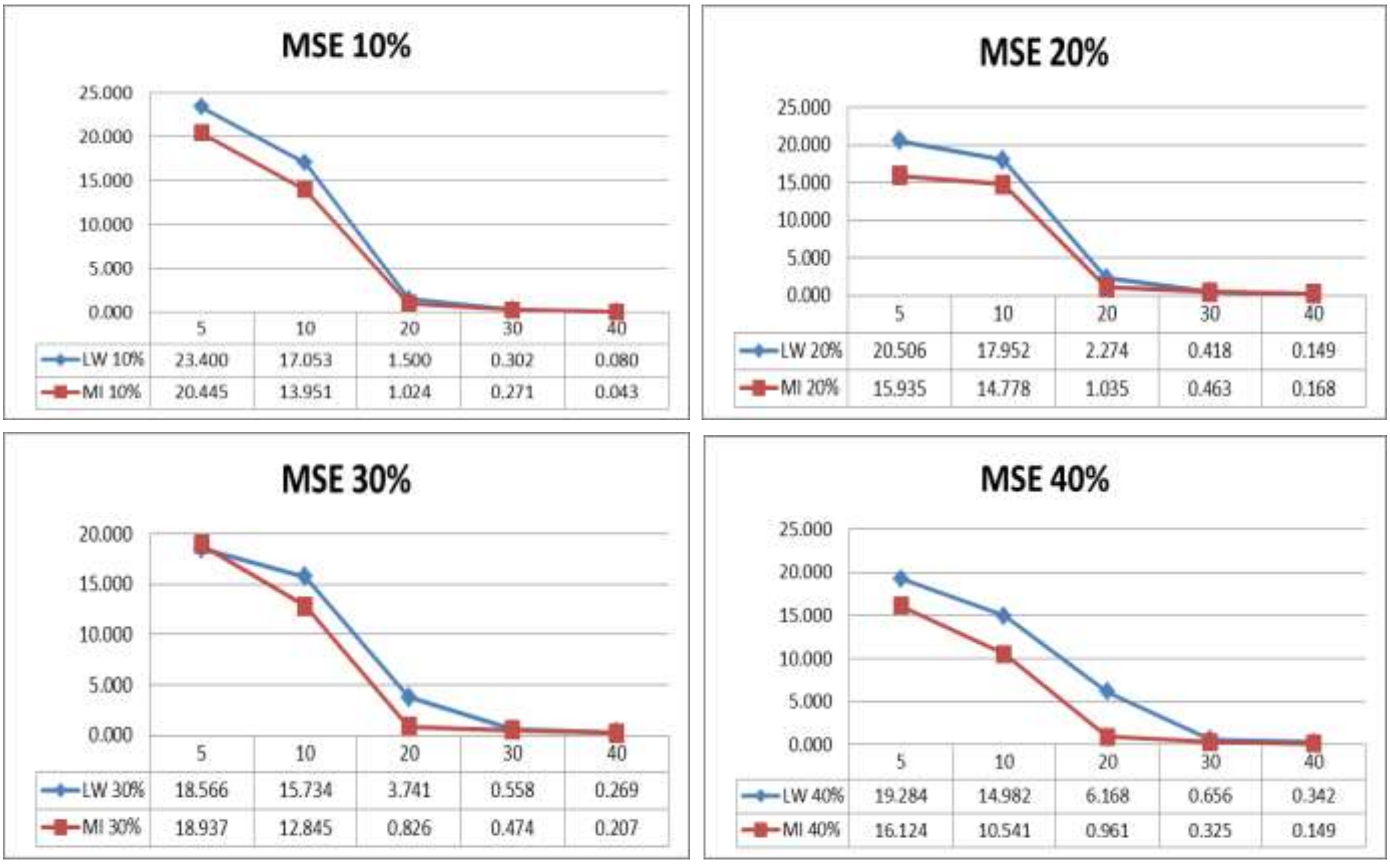

Fig. 1: MSE of $\widehat{\Phi}$ missingness at multi percentage levels

smaller than the LW method and more efficient. With 30% missing, starting from a sample of size 10, when the sample size increases, the MSE decreases in both imputation methods, LW and MI. Furthermore, the graph shows that MI is more efficient than LW at the largest of all sample sizes, which means if the sample size increases, the MSE for the MI method becomes smaller than the LW method, but LW is not efficient at very small samples (5) when the missingness is at level 30%. When the missing observations are 40%, It was seen when the sample size increased, then the MSE decreased in both imputation methods LW and MI except sample size of 30, and the graph shows that MI is more efficient than LW at the most sample sizes which means if the sample size increase, the MSE for MI method becomes more efficient than LW method when the missing observations are 40%.

- **The RB for missingness at multi percentage levels**

The following section it is present the RB at multi-levels of missing observations was discussed as MSE before in the previous section. Figure 2 represents the RB for the estimator $\widehat{\Phi}$ with GMM estimation of PVAR with missing observations at multiple levels, and the estimation provides a comparison between LW imputation and MI among sample sizes of 5, 10, 20, 30, and 40 shown in the graph. It was found that when the sample size increases, the RB decreases in both imputation methods, LW and MI. Moreover, the graph shows that the MI method provides RB less than LW, so it could be said that MI is more efficient than LW at all sample sizes, which means when the sample size increases, the RB for the MI method becomes smaller than the LW method when 10% of observations are missing.

With respect to 20% of the missing observations It was found in the graph that MI is more efficient than LW at sample sizes (5, 10, 20, 30, and 40), which means the RB for the MI method becomes smaller than the LW method at all samples with 20% missing observations, and it means that MI is still more efficient than LW. With the view of missing observations at 30%, it is observed that when the sample size increases, the RB decreases in both imputation methods, LW and MI. The graph shows that MI is more efficient than LW at all samples, which means if the sample size increases, the RB for the MI method becomes smaller than the LW method by a percentage of 30%.

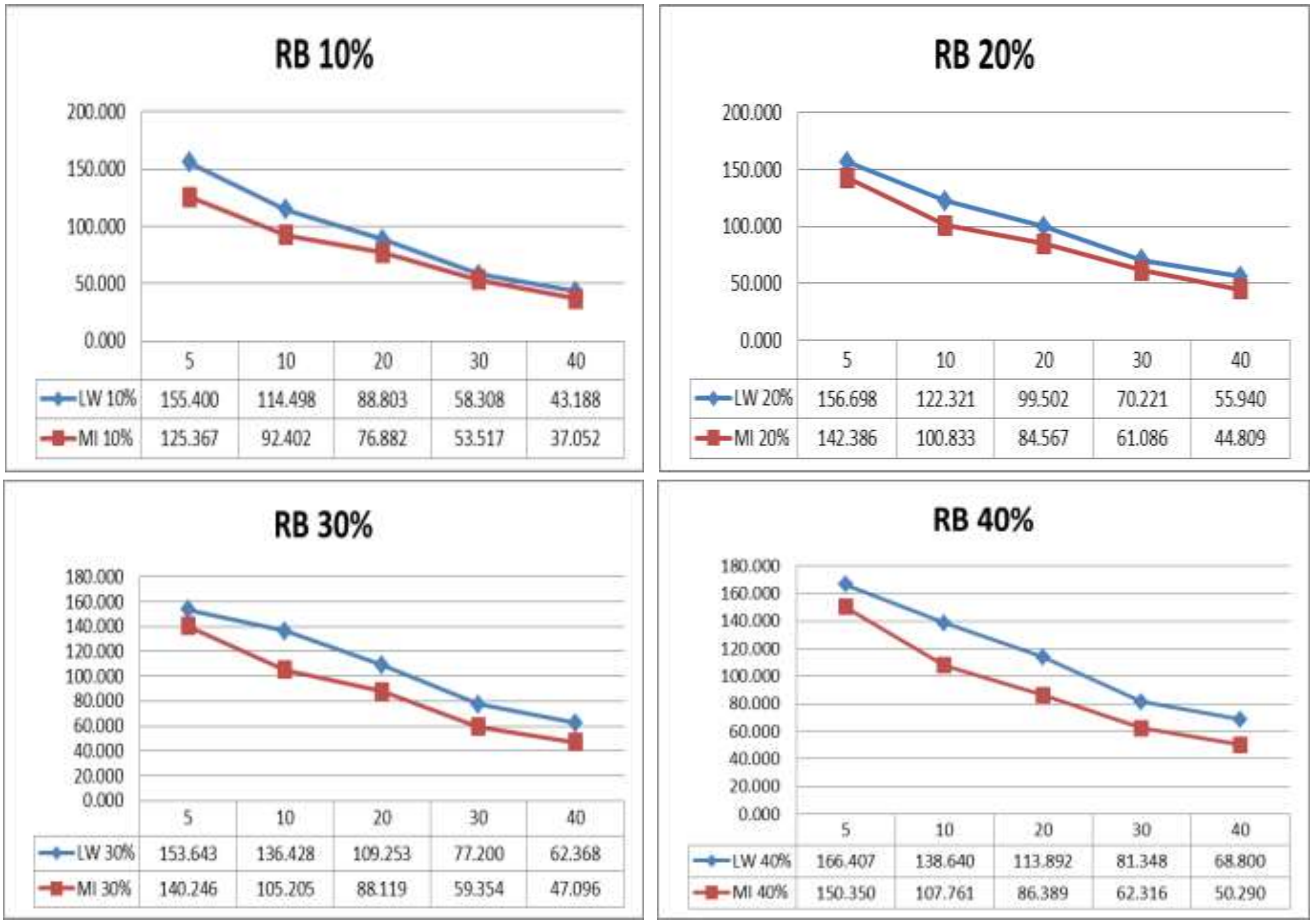

Fig. 2: RB of missingness at multi percentage levels

With respect to 40% missing observations, it was found in the graph that MI is more efficient than LW, which means the RB for the MI method becomes smaller than the LW method at all samples for the estimator with 40% missing observations. This means that MI is more efficient than LW.

3.3 Simulation results for different percentage of missingness fixed sample size

In the next section, the main purpose is to study the relation between the missing observations percentage through calculating MSE and RB in each estimation and compare the LW imputation and MI method for each sample size of sizes (5, 10, 20, 30 and 40) individually.

- **The MSE for each sample size**

The following section is present the MSE at each sample as discussed later.

Regarding Figure 3, it represents MSE for the estimator $\widehat{\Phi}$with GMM estimation of PVAR with a sample size of 5, 10, 20, 30, and 40. The estimation provides a comparison between LW imputation and MI through the missing observation levels of 10%, 20%, 30%, and 40%. For a sample size of 5, it was found that the MSE of the MI method is more efficient than the LW imputation method at missing observation levels of 10%, 20%, and 40%. Overall, the graph shows that MI is more efficient than LW at the most of all missing observation levels except level 30%. It is shown that the difference in MSE between LW and MI is very small, so, at this level, the MSE is roughly similar, which means the MSE for the MI method becomes smaller than the LW method and more efficient. With respect to sample size 10, it was seen that the MSE of the MI method is more efficient than the LW imputation method at all missing observation levels. Overall, the graph shows that the MSE for the MI method becomes smaller than the LW method and more efficient at the start of the missing level. When the missing level increases, the MSE in both LW and MI is decreased, which means they are still better. By viewing a sample of size 20 and the estimation providing comparison between LW imputation and MI through the missing observations of 10%, 20%,

30%, and 40%, it was found that the MSE of the MI method is more efficient than the LW imputation method at all missing observation levels.
The graph shows that the MSE for the MI method is still smaller than the LW method and more efficient, and starting at a missing level of 20%, the MSE of MI is decreased, but inversely, the MSE of LW is increased, which means MI is efficient by increasing the percentage of missing observations at sample size of 20. Regarding a sample of size 30, and the estimation provides a comparison between LW imputation and MI through the missing observations of 10%, 20%, 30%, and 40%, it was found that the MSE of the MI method is more efficient than the LW imputation method at missing observations of 10%, 30%, and 40%. Overall, the graph shows that MI is more efficient than LW at the most of all missing observation levels except level 20%. It is shown that the difference in MSE between LW and MI is small, so the MSE for the MI method is still smaller than the LW method and more efficient at the most of all percentage levels of missing observations.
Moreover, when the missing percentage increases, the MSE of LW increases but the MSE of MI decreases, and the MI is still efficient when a large missing observation level exists (30% and 40%). For sample of size 40, the MI method is more efficient than the LW imputation method at majority of all missing observations levels except level 20%. It is shown that the difference in MSE between LW and MI is small, so the MSE for the MI method is still smaller than the LW method and more efficient at the most of all percentage levels of missing observations. Also, as the percentage of missing observations goes up, so does the MSE of LW. But starting at level 20%, the MSE of MI goes down, and MI still works well when a lot of observations are missing, like 30% or 40%.

- **The RB for each sample size**

The following section is showing the RB at each sample size as previously discussed. With respect to figure 4, it is to introduce the RB for the estimator$\widehat{\Phi}$with GMM. The estimation compares LW imputation and MI using missing observation levels of 10%, 20%, 30%, and 40%. It was found that the RB of the MI method is more efficient than the LW imputation method for most levels of missing observations, except for level 20%, where the RB in both methods is about the same. This means that the RB for the MI method gets smaller and more efficient at level 20%. At a sample size of 10, it was found that the RB of the MI method is more efficient than the LW imputation method at missing observations of 10%, 30%, and 40%. Overall, the graph shows that MI is more efficient than LW at the most of all missing observations levels except level 20%. It is shown that the difference in RB between LW and MI is so small that at this level, the RB in LW and MI is roughly approximately equal, which means the RB for the MI method is still smaller than the LW method and more efficient.
The estimation provides a sample size of 20, and the estimation provides a comparison between LW imputation and MI through the missing observations of 10%, 20%, 30%, and 40%. The RB of the MI method was found to be more efficient than the LW imputation method at all missing observation levels. The graph shows that the RB for the MI method is still smaller than the LW method and more efficient.
Regarding a sample of size 30, it was found that the RB of the MI method is more efficient than the LW imputation method at all missing observation percentages, over all the graph shows that the RB for MI method still smaller than LW method so the MI is more efficient at all conditions about missingness proportions at sample of size 30.
For a sample of size 40, the RB of MI method is more efficient than LW imputation method at all missing observation percentages, over all the graph shows that the RB for MI method smaller than LW method so, it could say that MI is more efficient at all cases of missingness proportions at large sample size as 40.

4 Conclusions

This paper presented a Monte Carlo simulation study to study the two GMM estimators for the PVAR models with missing observations. In our simulation study, we used various sample sizes (small, medium, and large). Furthermore, we ran GMM estimation in the full model using full data and then we eliminated some observations at multi-missing levels. We used LW imputation based on elimination, and we again ran GMM estimation based on the MI at multi-missing levels. We compare the proposed estimator with the LW estimator based on the MSE and RB criteria. We can summarize the final remarks as follows:
1- It was found that the small samples have a large MSE while the large samples has small MSE in each of LW and MI methods so, they are lead to the negative relation between MSE and sample size moreover, over all the results shows that MI method provides more efficiency than LW at all sample sizes which means when the sample size increase,

the MSE for MI method becomes smaller than LW method when the observations are missing so our conclusion is the MI method in our estimator is more efficient than the LW.

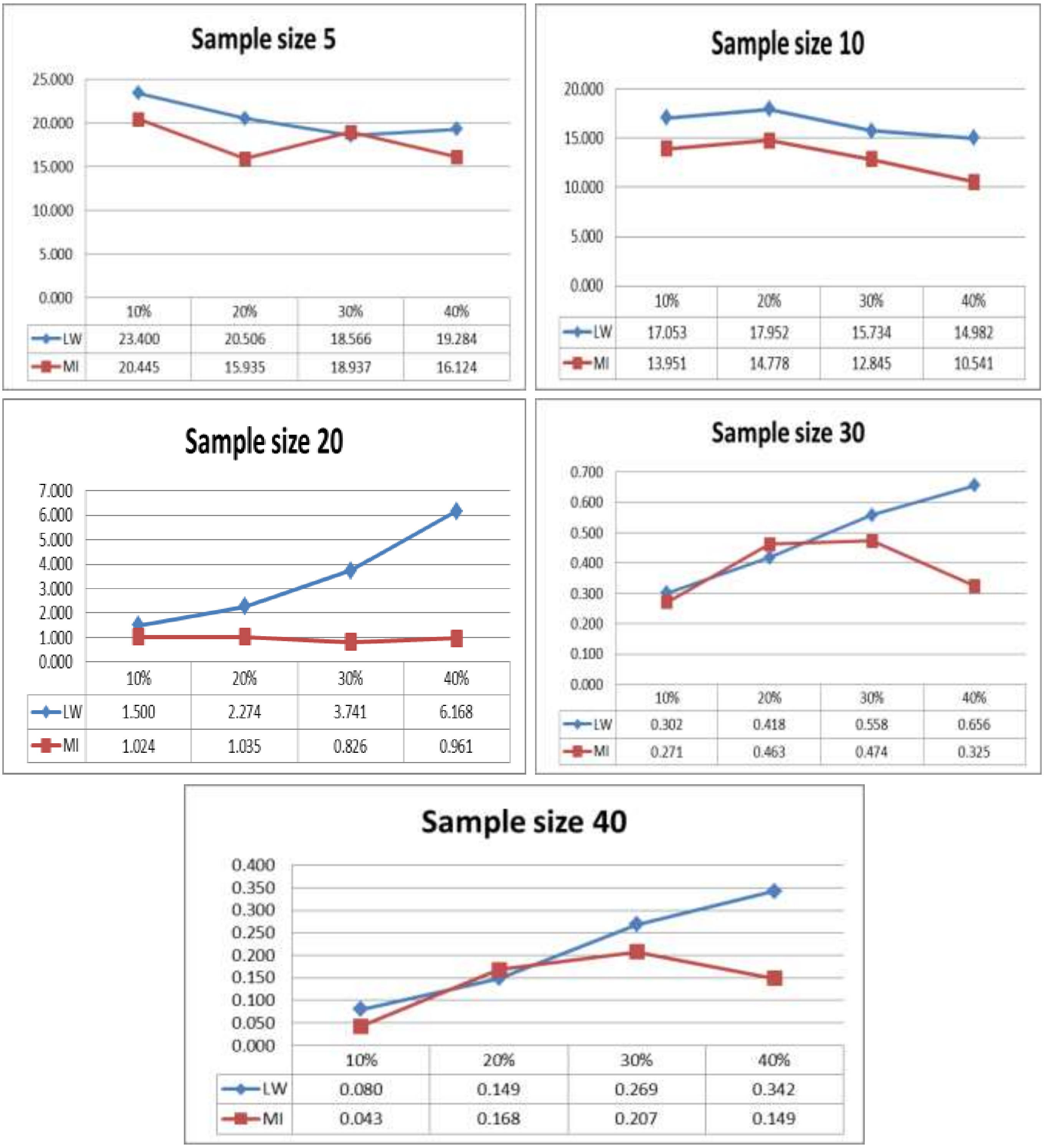

Fig. 3: MSE of $\hat{\Phi}$ at various sample size

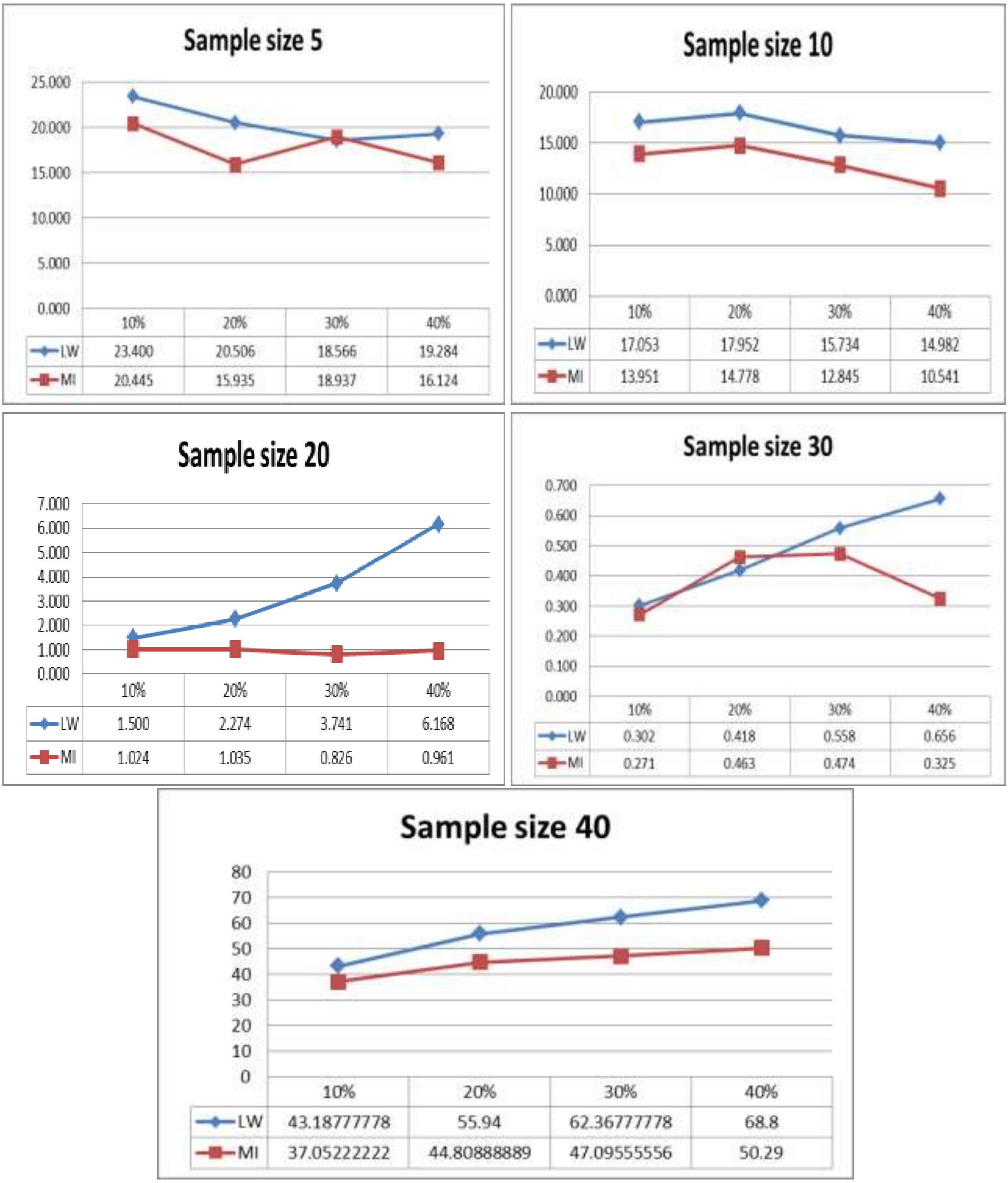

Fig. 4: RB of $\widehat{\Phi}$ at various sample size

2- When we evaluate the RB among the various sample sizes, It was found that when the sample size increase, the value of RB decrease in both imputation methods LW and MI moreover, the results are show that MI method progress RB less than LW method so, we can says that the MI is more optimal than LW at all sample sizes which means when the sample size increase, the RB for MI method becomes smaller than LW method even though the missing observations at very small samples.

3- From studying the results of MSE for the estimator $\widehat{\Phi}$ with GMM estimation of PVAR among the various missing percentages as a comparison between LW imputation and MI through the missing observations 10%, 20%, 30%, and 40% with different sample sizes, it was found that the MSE of MI method is more efficient than LW imputation method at all missing observation levels except very few cases whose studied , over all the results supports that the MSE for MI method

4- Becomes still smaller than LW method and so we can say that the MI more efficient than LW particularly starting of missing level 20%, the MSE in MI is more efficient.

5- By evaluating the results of the RB in our study using GMM estimation of PVAR model with several sample of sizes and the estimation provides comparison between LW imputation and MI through the missing observations 10%, 20%, 30%, and 40%, it was found that the RB of MI method is less than LW imputation method at most of all missing observation percentages, over all those results are introduce the MI as optimal method for handling the missing observations in PVAR models.

6- Finally, we can conclude that the MI method is more efficient than LW method in PVAR models if the data set contains missing pattern and these results are compatible with the estimator was presented in of Rady et al. [1].

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Appendix

Table A.1 The MSE for the estimator $\hat{\phi}$ with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=5

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	8.4189	6.3545	8.5974	7.3949	7.0723	7.0002	6.8345	6.1951
$\hat{\phi}_{12}$	14.6600	15.1800	13.3700	10.8200	9.7000	9.0800	6.1100	5.2800
$\hat{\phi}_{13}$	12.1800	12.0200	9.2700	7.1800	6.0630	5.7200	5.0340	3.9100
$\hat{\phi}_{21}$	20.4200	19.6000	12.9300	12.8400	10.5800	11.0400	10.1000	9.8100
$\hat{\phi}_{22}$	36.0100	32.0500	26.5200	26.8200	27.9500	28.5600	37.3700	27.5300
$\hat{\phi}_{23}$	32.0800	27.3500	34.4800	30.3800	32.7900	30.4900	35.6900	25.8100
$\hat{\phi}_{31}$	39.7700	37.4300	28.4900	26.3900	30.3700	28.7600	25.1500	25.0200
$\hat{\phi}_{32}$	27.8300	22.6400	19.7500	15.1400	15.9100	14.5800	29.3900	28.1200
$\hat{\phi}_{33}$	19.2300	11.3800	31.1500	6.4500	26.6600	35.2000	17.8800	13.4400

Table A.2 The RB for the estimator $\hat{\phi}$ with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=5

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	214.28	189.06	204.80	179.43	204.65	201.42	181.43	176.26
$\hat{\phi}_{12}$	77.61	81.14	88.62	82.75	93.44	91.80	96.23	85.31
$\hat{\phi}_{13}$	122.29	107.16	129.58	114.34	124.10	111.90	119.67	116.71
$\hat{\phi}_{21}$	166.19	180.66	179.81	196.75	175.50	173.73	137.11	187.54
$\hat{\phi}_{22}$	84.43	83.12	88.53	87.03	98.61	97.33	96.51	90.27
$\hat{\phi}_{23}$	123.47	103.73	134.51	123.84	144.14	143.53	127.26	100.44
$\hat{\phi}_{31}$	114.02	95.87	109.99	109.06	107.76	102.21	125.82	126.07
$\hat{\phi}_{32}$	257.79	165.39	251.49	189.14	211.17	133.66	195.40	139.55
$\hat{\phi}_{33}$	238.52	122.17	222.95	199.13	223.42	206.63	418.23	331.00

Table A.3 The MSE for the estimator $\hat{\phi}$ with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=10

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	2.5870	2.3250	8.2870	8.0700	7.2450	6.5421	5.9550	2.0210
$\hat{\phi}_{12}$	11.1900	9.7400	8.9790	7.0000	6.0621	3.9600	3.4800	2.9800
$\hat{\phi}_{13}$	10.5300	10.1700	8.9700	8.6500	5.2400	5.0710	4.8010	3.2300
$\hat{\phi}_{21}$	15.7300	13.4600	12.0100	11.9000	9.9230	8.9000	8.0460	7.6600
$\hat{\phi}_{22}$	30.0000	21.1200	25.8400	23.1000	22.2100	15.5400	25.0600	19.7400
$\hat{\phi}_{23}$	17.8000	8.7400	28.5400	20.2700	29.8000	22.2600	21.3800	16.2100
$\hat{\phi}_{31}$	27.5000	27.8500	22.4700	21.7900	22.8800	22.2200	24.2200	19.7100
$\hat{\phi}_{32}$	26.0400	21.4700	15.8100	11.0700	15.7700	10.0400	26.6400	11.3100
$\hat{\phi}_{33}$	12.1000	10.6800	30.6600	21.1500	22.4800	21.0700	15.2600	12.0100

Table A.4 The RB for the estimator $\hat{\phi}$ with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=10

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	137.11	96.37	124.62	118.08	192.70	145.66	166.44	108.49
$\hat{\phi}_{12}$	66.06	60.04	77.89	69.56	80.85	77.48	94.38	81.25
$\hat{\phi}_{13}$	130.56	98.54	113.73	102.53	114.09	104.35	120.73	98.45
$\hat{\phi}_{21}$	127.51	124.30	177.02	98.78	153.39	110.82	134.51	82.56
$\hat{\phi}_{22}$	73.48	60.20	78.38	74.40	93.84	83.99	77.62	63.37
$\hat{\phi}_{23}$	99.93	94.41	132.95	90.24	125.25	103.07	122.39	92.52
$\hat{\phi}_{31}$	87.45	86.51	98.86	79.01	97.34	84.30	105.96	102.68
$\hat{\phi}_{32}$	105.20	96.86	118.36	109.68	150.49	99.05	127.40	124.48
$\hat{\phi}_{33}$	203.18	114.38	179.09	165.22	219.91	138.13	298.33	216.06

Table A.5 The MSE for the estimator $\hat{\phi}$ with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=20

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	0.2374	0.1421	0.3274	0.1183	0.4327	0.0809	0.8377	0.0752
$\hat{\phi}_{12}$	0.1354	0.1170	0.1833	0.1304	0.2552	0.1150	0.6729	0.1125
$\hat{\phi}_{13}$	0.0362	0.0307	0.1503	0.0316	0.9296	0.0326	4.2738	0.0331
$\hat{\phi}_{21}$	0.2474	0.1532	0.3429	0.1433	0.4521	0.1044	0.8321	0.1092
$\hat{\phi}_{22}$	0.1749	0.1669	0.1967	0.1608	0.2804	0.1549	0.9990	0.1550
$\hat{\phi}_{23}$	0.0398	0.0367	0.1623	0.0384	1.0208	0.0396	4.5601	0.0402
$\hat{\phi}_{31}$	8.1621	5.1458	9.5889	5.3978	11.2147	4.3870	15.0301	5.1165
$\hat{\phi}_{32}$	3.4784	2.9226	4.3058	2.8593	6.2234	2.0904	13.5185	2.6569
$\hat{\phi}_{33}$	0.9914	0.4992	5.2104	0.4346	12.8616	0.4309	14.7841	0.3520

Table A.6 The RB for the estimator $\hat{\phi}$with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=20

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	127.47	87.65	111.39	102.17	135.52	112.80	96.56	89.07
$\hat{\phi}_{12}$	43.37	38.38	68.56	61.24	66.70	66.17	70.65	62.66
$\hat{\phi}_{13}$	75.95	87.24	108.78	90.89	109.42	92.13	95.35	93.64
$\hat{\phi}_{21}$	101.59	77.20	115.33	75.52	126.99	66.33	116.82	67.26
$\hat{\phi}_{22}$	67.10	54.63	78.55	74.72	88.63	77.76	76.29	59.54
$\hat{\phi}_{23}$	77.90	90.24	116.58	84.71	101.99	96.63	98.06	97.42
$\hat{\phi}_{31}$	86.44	85.06	79.86	76.46	82.49	79.31	104.37	100.67
$\hat{\phi}_{32}$	79.58	62.79	94.03	80.97	94.20	90.35	107.34	104.43
$\hat{\phi}_{33}$	139.83	108.75	122.44	114.42	177.34	111.59	259.59	102.81

Table A.7 The MSE for the estimator $\hat{\phi}$with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=30

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	0.0367	0.0181	0.0672	0.2124	0.3860	0.2480	0.1045	0.2550
$\hat{\phi}_{12}$	0.0483	0.0312	0.0868	0.0925	0.1231	0.0956	0.1494	0.1005
$\hat{\phi}_{13}$	0.0007	0.0004	0.0028	0.0005	0.0084	0.0006	0.0185	0.0006
$\hat{\phi}_{21}$	0.0356	0.1701	0.0639	0.0583	0.0908	0.0803	0.1101	0.2046
$\hat{\phi}_{22}$	0.0395	0.0661	0.0701	0.0626	0.1031	0.0566	0.1263	0.0550
$\hat{\phi}_{23}$	0.0008	0.0008	0.0028	0.0010	0.0080	0.0012	0.0207	0.0012
$\hat{\phi}_{31}$	0.9028	1.0659	1.5729	2.1551	2.2108	2.0048	2.9942	2.0549
$\hat{\phi}_{32}$	1.6281	1.0806	1.8222	1.5683	1.9443	1.7593	2.0253	0.2317
$\hat{\phi}_{33}$	0.0262	0.0096	0.0722	0.0131	0.1518	0.0188	0.3511	0.0225

Table A.8 The RB for the estimator $\hat{\phi}$with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=30

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	25.44	22.31	34.51	23.05	37.69	26.66	41.01	38.60
$\hat{\phi}_{12}$	35.98	34.95	48.35	42.42	57.08	53.79	60.93	55.33
$\hat{\phi}_{13}$	68.02	81.90	102.39	83.99	92.21	61.94	66.58	65.93
$\hat{\phi}_{21}$	53.85	50.36	73.11	74.80	84.86	48.83	92.58	50.67
$\hat{\phi}_{22}$	67.01	54.30	77.30	72.37	64.43	63.48	66.15	58.22
$\hat{\phi}_{23}$	66.11	66.08	105.73	74.48	98.12	82.31	94.39	81.23
$\hat{\phi}_{31}$	53.80	44.49	44.88	42.53	53.11	57.62	60.70	62.41
$\hat{\phi}_{32}$	30.67	28.65	39.03	40.49	45.64	43.34	65.22	60.97
$\hat{\phi}_{33}$	123.89	98.61	106.69	95.64	161.66	96.22	184.57	87.48

Table A.9 The MSE for the estimator $\hat{\phi}$with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=40

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	0.0920	0.0900	0.0300	0.1300	0.1500	0.1200	0.1700	0.1300
$\hat{\phi}_{12}$	0.0300	0.0211	0.0410	0.0400	0.0700	0.0333	0.1000	0.0400
$\hat{\phi}_{13}$	0.0058	0.0001	0.0025	0.0018	0.0020	0.0011	0.0012	0.0008
$\hat{\phi}_{21}$	0.0200	0.0126	0.1000	0.0900	0.0813	0.0800	0.0900	0.0813
$\hat{\phi}_{22}$	0.0200	0.0300	0.0300	0.0200	0.0400	0.0211	0.0800	0.0200
$\hat{\phi}_{23}$	0.0008	0.0007	0.0008	0.0007	0.0006	0.0005	0.0006	0.0002
$\hat{\phi}_{31}$	0.3800	0.1900	0.7700	0.7090	1.0900	0.8500	1.7100	1.0000
$\hat{\phi}_{32}$	0.1600	0.0330	0.3500	0.5100	0.9540	0.7400	0.8600	0.0382
$\hat{\phi}_{33}$	0.0100	0.0091	0.0200	0.0100	0.0300	0.0200	0.0700	0.0300

Table A.10 The RB for the estimator $\hat{\phi}$with GMM estimation of PVAR with levels of missing observations and comparison between LW and MI at n=400

Missing%	10%		20%		30%		40%	
Estimate	LW	MI	LW	MI	LW	MI	LW	MI
$\hat{\phi}_{11}$	25.41	19.33	28.92	22.12	37.01	22.81	47.20	35.24
$\hat{\phi}_{12}$	34.97	22.98	43.02	41.08	57.57	45.96	68.16	45.74
$\hat{\phi}_{13}$	59.63	54.69	51.40	24.62	81.08	57.09	59.15	58.51
$\hat{\phi}_{21}$	52.47	48.87	72.72	60.98	71.32	45.18	90.67	47.01
$\hat{\phi}_{22}$	48.96	45.48	77.32	67.07	56.65	46.66	59.26	40.93
$\hat{\phi}_{23}$	65.79	62.68	99.34	71.31	96.76	73.71	89.17	73.92
$\hat{\phi}_{31}$	24.94	22.94	37.44	36.30	44.69	37.34	56.14	40.81
$\hat{\phi}_{32}$	36.01	22.83	34.63	29.73	39.71	34.99	39.61	37.64
$\hat{\phi}_{33}$	40.51	33.67	58.67	50.07	76.52	60.12	109.84	72.81