On the Existence of Positive Periodic Solution of an Amensalism Model with Beddington-DeAngelis Functional Response

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Abstract: - A non-autonomous discrete amensalism model with Beddington-DeAngelis functional response is proposed and studied in this paper. Sufficient conditions are obtained for the existence of positive periodic solution of the system.

Key-Words: -Amensalism model; Positive periodic solution; Beddington-DeAngelis functional response

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1 Introduction

Amensalism is an interaction in which an organism inflicts harm to another organism without any costs or benefits received by the other. In the past decade, numerous works on the mutualism or commensalism model has been published([1]-[25]). However, only recently did scholars paid attention to the amensalism model([26]-[36]). In 2019, Guan and Chen[26] proposed the following two species amensalism model with Beddington-DeAngelis functional response

$$\begin{aligned}\frac{dx_1}{dt} &= x_1\Big(a_1 - b_1x_1 \\ &\quad -\frac{cx_2}{mx_1+nx_1+1}\Big), \qquad (1)\\ \frac{dx_2}{dt} &= x_2\Big(a_2-b_2x_2\Big).\end{aligned}$$

The existence and stability of possible equilibria were investigated. Under some additional assumptions, the authors showed that there are two stable equilibria which implies this system is not asymptotically stable. Based on the stability analysis of equilibria, closed orbits and the saddle connection, they gave some comprehensive bifurcation and global dynamics of the system.

It brings to our attention that the system (1) is an autonomous ones. Model (1) is not well studied yet in the sense that the model is with constant environment. The assumption that the environment is constant is rarely the case in real life. Most natural environments are physically highly variable, and in response, birth rates, death rates, and other vital rates of populations, vary greatly in time. Taking these factors into consideration, then it is naturally to study the nonautonomous case of system (1), i.e,

$$\begin{aligned}\frac{dx_1}{dt} &= x_1\Big(a_1(t) - b_1(t)x_1 \\ &\quad -\frac{c(t)x_2}{m(t)x_1+n(t)x_2+1}\Big), \qquad (2)\\ \frac{dx_2}{dt} &= x_2\Big(a_2(t)-b_2(t)x_2\Big).\end{aligned}$$

It is well known that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have non-overlapping generations, corresponding to system (2), we could propose the following discrete nonautonomous amensalism model with Beddington-DeAngelis functional response

$$\begin{aligned}x_1(k+1) &= x_1(k)\exp\Big\{a_1(k)-b_1(k)x_1(k) \\ &\quad -\frac{c(k)x_2(k)}{m(k)x_1(k)+n(k)x_2(k)+1}\Big\},\\ x_2(k+1) &= x_2(k)\exp\{a_2(k)-b_2(k)x_2(k)\Big\},\end{aligned} \qquad (3)$$

where $\{b_i(k)\}, i = 1, 2, \{c(k)\}\{m(k)\}, \{n(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i = \frac{1}{\omega}\sum_{k=0}^{\omega-1} a_i(k) > 0, i = 1, 2.$ Here we assume that the coefficients of the system (3) are all periodic sequences which having a common integer period. Such an assumption seems reasonable in view of seasonal factors, e.g., mating habits, availability of food, weather conditions, harvesting, and hunting, etc.

The aim of this paper is to obtain a set of sufficient conditions which ensure the existence of positive periodic solution of system (3).

2 Main Results

In the proof of our existence theorem below, we will use the continuation theorem of Gaines and Mawhin([37]).

Lemma 2.1 (Continuation Theorem) *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}$. Suppose*

(a).For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;

(b).$QNx \neq 0$ for each $x \in \partial\Omega \cap KerL$ and

$$deg\{JQN, \Omega \cap KerL, 0\} \neq 0.$$

Then the equation $Lx = Nx$ has at least one solution lying in $DomL \cap \bar{\Omega}$.

Let Z, Z^+, R and R^+ denote the sets of all integers, nonnegative integers, real numbers, and nonnegative real numbers, respectively. For convenience, in the following discussion, we will use the notation below throughout this paper:

$$I_\omega = \{0, 1, ..., \omega - 1\},$$

$$\overline{g} = \frac{1}{\omega}\sum_{k=0}^{\omega-1} g(k), \quad g^u = \max_{k\in I_\omega} g(k), \quad g^l = \min_{k\in I_\omega} g(k),$$

where $\{g(k)\}$ Ï is an ω-periodic sequence of real numbers defined for $k \in Z$.

Lemma 2.2[38] *Let $g : Z \to R$ be ω-periodic, i. e., $g(k+\omega) = g(k)$. Then for any fixed $k_1, k_2 \in I_\omega$, and any $k \in Z$, one has*

$$g(k) \leq g(k_1) + \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|,$$

$$g(k) \geq g(k_2) - \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|.$$

Lemma 2.3 *Assume that*

$$\bar{a}_1 > \overline{\left(\frac{c}{n}\right)} \tag{4}$$

holds, Then any solution (x_1^*, x_2^*) of the system of algebraic equations

$$\begin{aligned} &\bar{a}_1 - \bar{b}_1 \exp\{u_1\} \\ &-\frac{1}{\omega}\sum_{k=0}^{\omega-1} \frac{c(k)\exp\{u_2\}}{m(k)\exp\{u_1\} + n(k)\exp\{u_2\} + 1} = 0, \\ &\bar{a}_2 - \bar{b}_2 \exp\{u_2\} = 0. \end{aligned} \tag{5}$$

satisfies

$$\ln \frac{\bar{a}_1 - \overline{\left(\frac{c}{n}\right)}}{\bar{b}_1} \leq u_1^* \leq \ln \frac{\bar{a}_1}{\bar{b}_1}, \ u_2^* = \ln \frac{\bar{a}_2}{\bar{b}_2}, \tag{6}$$

Proof. From the second equation of (5), it immediately follows that

$$u_2 = \ln \frac{\bar{a}_2}{\bar{b}_2}. \tag{7}$$

From the first equation of system (5) we have

$$\bar{a}_1 - \bar{b}_1 \exp\{u_1\} \geq 0,$$

thus

$$u_1 \leq \ln \frac{\bar{a}_1}{\bar{b}_1}. \tag{8}$$

From the first equation of system (5), we also have

$$\begin{aligned} 0 &= \bar{a}_1 - \bar{b}_1 \exp\{u_1\} \\ &\quad -\frac{1}{\omega}\sum_{k=0}^{\omega-1} \frac{c(k)\exp\{u_2\}}{m(k)\exp\{u_1\} + n(k)\exp\{u_2\} + 1} \\ &\geq \bar{a}_1 - \bar{b}_1 \exp\{u_1\} - \frac{1}{\omega}\sum_{k=0}^{\omega-1} \frac{c(k)\exp\{u_2\}}{n(k)\exp\{u_2\}} \\ &= \bar{a}_1 - \overline{\left(\frac{c}{n}\right)} - \bar{b}_1 \exp\{u_1\}. \end{aligned}$$

Thus

$$u_1 \geq \ln \frac{\bar{a}_1 - \overline{\left(\frac{c}{n}\right)}}{\bar{b}_1}. \tag{9}$$

This ends the proof of Lemma 2.3.

We now reach the position to establish our main result.

Theorem 2.1 *Assume that (4) holds, then system (3) admits at least one positive ω-periodic solution.*

Proof. Let

$$x_i(k) = \exp\{u_i(k)\}, \quad i = 1, 2,$$

so that system (3) becomes

$$\begin{aligned} &u_1(k+1) - u_1(k) \\ &= a_1(k) - b_1(k)\exp\{u_1(k)\} \\ &\quad -H(u_1(k), u_2(k)), \\ &u_2(k+1) - u_2(k) \\ &= a_2(k) - b_2(k)\exp\{u_2(k)\}. \end{aligned} \tag{10}$$

where

$$H(u_1(k),u_2(k)) = \frac{c(k)\exp\{u_2(k)\}}{m(k)\exp\{u_1(k)\}+n(k)\exp\{u_2(k)\}+1}. \tag{11}$$

Define

$$l_2=\Big\{u=\{u(k)\},u(k)=(u_1(k),u_2(k))^T\in R^2\Big\}.$$

For $a = (a_1,a_2)^T \in R^2$, define $|a| = \max\{|a_1|,|a_2|\}$. Let $l^\omega \subset l_2$ denote the subspace of all ω sequences equipped with the usual normal form $\|u\| = \max_{k\in I_\omega}|u(k)|$. It is not difficult to show that l^ω is a finite-dimensional Banach space. Let

$$l_0^\omega=\{u=\{u(k)\}\in l^\omega : \sum_{k=0}^{\omega-1}u(k)=0\},$$

$$l_c^\omega=\{u=\{u(k)\}\in l^\omega : u(k)=h\in R^2, k\in Z\},$$

then l_0^ω and l_c^ω are both closed linear subspace of l^ω, and

$$l^\omega=l_0^\omega\oplus l_c^\omega,\quad diml_c^\omega=2.$$

Now let us define $X = Y = l^\omega$, $(Lu)(k) = u(k+1)-u(k)$. It is trivial to see that L is a bounded linear operator and

$$KerL=l_c^\omega,\quad ImL=l_0^\omega,$$

$$dimKerL=2=CodimImL.$$

Then it follows that L is a Fredholm mapping of index zero. Let

$$N(u_1,u_2)^T=(N_1,N_2)^T:=N(u,k),$$

where

$$\begin{cases} N_1 = a_1(k)-b_1(k)\exp\{u_1(k)\} \\ \qquad -H(u_1(k),u_2(k)), \\ N_2 = a_2(k)-b_2(k)\exp\{u_2(k)\}. \end{cases}$$

$$Px=\frac{1}{\omega}\sum_{s=0}^{\omega-1}x(s), x\in X,\quad Qy=\frac{1}{\omega}\sum_{s=0}^{\omega-1}y(s), y\in Y.$$

It is not difficult to show that P and Q are two continuous projectors such that

$$ImP=KerL\quad and\quad ImL=KerQ=Im(I-Q).$$

Furthermore, the generalized inverse (to L) K_p: ImL →Ker$P$∩Dom$L$ exists and is given by

$$K_p(z)=\sum_{s=0}^{k-1}z(s)-\frac{1}{\omega}\sum_{s=0}^{\omega-1}(\omega-s)z(s).$$

Thus

$$QNx=\frac{1}{\omega}\sum_{k=0}^{\omega-1}N(x,k),$$

$$\begin{aligned} Kp(I-Q)Nx &= \sum_{s=0}^{k-1}N(x,s)+\frac{1}{\omega}\sum_{s=0}^{\omega-1}sN(x,s) \\ &\quad -\Big(\frac{k}{\omega}+\frac{\omega-1}{2\omega}\Big)\sum_{s=0}^{\omega-1}N(x,s). \end{aligned}$$

Obviously, QN and $K_p(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, it is not difficult to show that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega\subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Thus, N is L-compact on any open bounded set $\Omega\subset X$. The isomorphism J of ImQ onto Ker$L$ can be the identity mapping, since ImQ=Ker$L$.

Now we are at the point to search for an appropriate open, bounded subset Ω in X for the application of the continuation theorem. Corresponding to the operator equation $Lx=\lambda Nx, \lambda\in(0,1)$, we have

$$\begin{aligned} & u_1(k+1)-u_1(k) \\ = & \lambda\Big[a_1(k)-b_1(k)\exp\{u_1(k)\} \\ & -H(u_1(k),u_2(k))\Big], \end{aligned} \tag{12}$$

$$\begin{aligned} & u_2(k+1)-u_2(k) \\ = & \lambda[a_2(k)-b_2(k)\exp\{u_2(k)\}]. \end{aligned}$$

where $H(u_1(k),u_2(k))$ is defined by (11). Suppose that $u=(u_1(k),u_2(k))^T\in X$ is an arbitrary solution of system (12) for a certain $\lambda\in(0,1)$. Summing on both sides of (12) from 0 to $\omega-1$ with respect to k, we reach

$$\sum_{k=0}^{\omega-1}\Big[a_1(k)-b_1(k)\exp\{u_1(k)\} -H(u_1(k),u_2(k))\Big]=0,$$

$$\sum_{k=0}^{\omega-1}[a_2(k)-b_2(k)\exp\{u_2(k)\}]=0.$$

That is,

$$\sum_{k=0}^{\omega-1}\Bigg(b_1(k)\exp\{u_1(k)\} +H(u_1(k),u_2(k))\Bigg)=\bar{a}_1\omega, \tag{13}$$

$$\sum_{k=0}^{\omega-1}b_2(k)\exp\{u_2(k)\}=\bar{a}_2\omega. \tag{14}$$

From (13) and (14), we have

$$\begin{aligned}
&\sum_{k=0}^{\omega-1}|u_1(k+1)-u_1(k)| \\
=&\ \lambda\sum_{k=0}^{\omega-1}\Big|a_1(k)-b_1(k)\exp\{u_1(k)\} \\
&\qquad -H(u_1(k),u_2(k))\Big| \\
\leq&\ \sum_{k=0}^{\omega-1}|a_1(k)|+\sum_{k=0}^{\omega-1}\Big(b_1(k)\exp\{u_1(k)\} \\
&\qquad +H(u_1(k),u_2(k))\Big) \\
=&\ \sum_{k=0}^{\omega-1}|a_1(k)|+\bar{a}_1\omega \\
=&\ (\bar{A}_1+\bar{a}_1)\omega,
\end{aligned} \tag{15}$$

$$\begin{aligned}
&\sum_{k=0}^{\omega-1}|u_2(k+1)-u_2(k)| \\
=&\ \lambda\sum_{k=0}^{\omega-1}\Big|a_2(k)-b_2(k)\exp\{u_2(k)\}\Big| \\
\leq&\ (\bar{A}_2+\bar{a}_2)\omega.
\end{aligned} \tag{16}$$

where $\bar{A}_1=\frac{1}{\omega}\sum_{k=0}^{\omega-1}|a_1(k)|,\ \ \bar{A}_2=\frac{1}{\omega}\sum_{k=0}^{\omega-1}|a_2(k)|.$

Since $\{u(k)\}=\{(u_1(k),u_2(k))^T\}\in X$, there exist $\eta_i,\delta_i, i=1,2$ such that

$$u_i(\eta_i)=\min_{k\in I_\omega}u_i(k),\ \ u_i(\delta_i)=\max_{k\in I_\omega}u_i(k). \tag{17}$$

By (14), we have

$$\exp\{u_2(\eta_2)\}\sum_{k=0}^{\omega-1}b_2(k)\leq\bar{a}_2\omega.$$

So

$$u_2(\eta_2)\leq\ln\frac{\bar{a}_2}{\bar{b}_2}. \tag{18}$$

It follows from Lemma 2.2, (16) and (18) that

$$\begin{aligned}
u_2(k) &\leq u_2(\eta_2)+\sum_{k=0}^{\omega-1}|u_2(k+1)-u_2(k)| \\
&\leq \ln\frac{\bar{a}_2}{\bar{b}_2}+(\bar{A}_2+\bar{a}_2)\omega \stackrel{def}{=} K_1,
\end{aligned} \tag{19}$$

From (14) we also have

$$\exp\{u_2(\delta_2)\}\sum_{k=0}^{\omega-1}b_2(k)\geq\bar{a}_2\omega,$$

and so

$$u_2(\delta_2)\geq\ln\frac{\bar{a}_2}{\bar{b}_2}. \tag{20}$$

It follows from Lemma 2.2, (16) and (20) that

$$\begin{aligned}
u_2(k) &\geq u_2(\delta_2)-\sum_{k=0}^{\omega-1}|u_2(k+1)-u_2(k)| \\
&\geq \ln\frac{\bar{a}_2}{\bar{b}_2}-(\bar{A}_2+\bar{a}_2)\omega \stackrel{def}{=} K_2,
\end{aligned} \tag{21}$$

which together with (19) leads to

$$|u_2(k)|\leq\max\Big\{|K_1|,|K_2|\Big\}\stackrel{\text{def}}{=}H_2. \tag{22}$$

It follows from (13) that

$$\begin{aligned}
&\sum_{k=0}^{\omega-1}b_1(k)\exp\{u_1(\eta_1)\} \\
\leq&\ \bar{a}_1\omega-\sum_{k=0}^{\omega-1}H(u_1(k),u_2(k)) \\
\leq&\ \bar{a}_1\omega,
\end{aligned}$$

and so,

$$u_1(\eta_1)\leq\ln\frac{\bar{a}_1}{\bar{b}_1}. \tag{23}$$

It follows from Lemma 2.2, (15) and (23) that

$$\begin{aligned}
u_1(k) &\leq u_1(\eta_1)+\sum_{k=0}^{\omega-1}|u_1(k+1)-u_1(k)| \\
&\leq \ln\frac{\bar{a}_1}{\bar{b}_1}+(\bar{A}_1+\bar{a}_1)\omega \stackrel{\text{def}}{=} M_1.
\end{aligned} \tag{24}$$

It follows from (13) that

$$\begin{aligned}
&\sum_{k=0}^{\omega-1}b_1(k)\exp\{u_1(\delta_1)\} \\
=&\ \bar{a}_1\omega-\sum_{k=0}^{\omega-1}H(u_1(k),u_2(k)) \\
\geq&\ \bar{a}_1\omega-\sum_{k=0}^{\omega-1}\frac{c(k)}{n(k)} \\
\geq&\ \bar{a}_1\omega-\overline{(\frac{c}{n})}\omega,
\end{aligned}$$

where $\overline{(\frac{c}{n})}=\frac{1}{\omega}\sum_{k=0}^{\omega-1}\frac{c(k)}{n(k)}$. And so,

$$u_1(\delta_1)\geq\ln\frac{\bar{a}_1-\overline{(\frac{c}{n})}}{\bar{b}_1}, \tag{25}$$

It follows from Lemma 2.2, (15) and (25) that

$$\begin{aligned}
u_1(k) &\geq u_1(\delta_1)-\sum_{k=0}^{\omega-1}|u_1(k+1)-u_1(k)| \\
&\geq \ln\frac{\bar{a}_1-\overline{(\frac{c}{n})}}{\bar{b}_1}-(\bar{A}_1+\bar{a}_1)\omega\stackrel{\text{def}}{=}M_2.
\end{aligned} \tag{26}$$

It follows from (24) and (26) that

$$|u_1(k)| \le \max\Big\{|M_1|, |M_2|\Big\} \overset{\text{def}}{=} H_1. \qquad (27)$$

Clearly, H_1 and H_2 are independent on the choice of λ.

It follows from (4) and Lemma 2.3 that any solution (x_1^*, x_2^*) of the system of algebraic equations

$$\begin{aligned} &\bar{a}_1 - \bar{b}_1 \exp\{u_1\} \\ &-\frac{1}{\omega}\sum_{k=0}^{\omega-1} \frac{c(k)\exp\{u_2\}}{m(k)\exp\{u_1\} + n(k)\exp\{u_2\} + 1} = 0, \\ &\bar{a}_2 - \bar{b}_2 \exp\{u_2\} = 0 \end{aligned}$$

satisfies

$$\ln \frac{\bar{a}_1 - \overline{\left(\frac{c}{n}\right)}}{b_1} \le u_1^* \le \ln \frac{\bar{a}_1}{\bar{b}_1},\ u_2^* = \ln \frac{\bar{a}_2}{\bar{b}_2}, \qquad (28)$$

Let $H = H_1 + H_2 + H_3$, where $H_3 > 0$ is taken sufficiently enough large such that

$$H_3 > \left|\ln \frac{\bar{a}_2}{\bar{b}_2}\right| + \left|\ln \frac{\bar{a}_1}{\bar{b}_1}\right| + \left|\ln \frac{\bar{a}_1 - \overline{\left(\frac{c}{n}\right)}}{b_1}\right|.$$

Let $H = H_1 + H_2 + H_3$, and define

$$\Omega = \Big\{u(k) = (u_1(k), u_2(k))^T \in X : \|u\| < H\Big\}.$$

It is clear that Ω verifies requirement (a) in Lemma 2.1. When $u \in \partial\Omega \cap KerL = \partial\Omega \cap R^2$, u is constant vector in R^2 with $||u|| = B$. Then

$$\begin{aligned} &QNu \\ =& \begin{pmatrix} \bar{a}_1 - \bar{b}_1 \exp\{u_1\} - \Delta_1 \\ \bar{a}_2 - \bar{b}_2 \exp\{u_2\} \end{pmatrix} \\ \ne& \ 0. \end{aligned}$$

where

$$\Delta_1 = \frac{1}{\omega}\sum_{k=0}^{\omega-1} \frac{c(k)\exp\{u_2\}}{m(k)\exp\{u_1\} + n(k)\exp\{u_2\} + 1}.$$

In order to compute the Brouwer degree, let us consider the homotopy

$$H_\mu u = \mu QNu + (1-\mu)Gu, \qquad (29)$$

where

$$Gu = \begin{pmatrix} \bar{a}_1 - \bar{b}_1 \exp\{u_1\} \\ \bar{a}_2 - \bar{b}_2 \exp\{u_2\} \end{pmatrix}.$$

From the definition of H, it follows that $0 \notin H_\mu(\partial\Omega \cap KerL)$ for $\mu \in [0, 1]$. In addition, one can easily show that the algebraic equation $Gu = 0$ has a unique solution in R^2. Note that $J = I$ since $ImQ = KerL$, by the invariance property of homotopy, direct calculation produces

$$\begin{aligned} &deg(JQN, \Omega \cap KerL, 0) \\ =& \ deg(QN, \Omega \cap KerL, 0) \\ =& \ deg(G, \Omega \cap KerL, 0) = \operatorname{sgn}\Big(\Gamma\Big) = 1 \ne 0, \end{aligned}$$

where

$$\Gamma = \bar{b}_1\bar{b}_2 \exp\{u_1^*\}\exp\{u_2^*\}$$

and $deg(., ., .)$ is the Brouwer degree. By now we have proved that Ω verifies all requirements in Lemma 2.1. Hence (4) has at least one solution $(u_1^*(k), u_2^*(k))^T$ in $DomL \cap \bar{\Omega}$. And so, system (3) admits a positive periodic solution $(x_1^*(k), x_2^*(k))^T$, where $x_i^*(k) = \exp\{u_i^*(k)\}, i = 1, 2$, This completes the proof of the claim.

3 Numeric simulations

Now let us consider the following two examples.
Example 3.1.

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\Big\{1.5 - x_1(k) \\ &\quad - \frac{(2+\sin(\pi k))x_2(k)}{1 + x_2(k) + 0.1x_1(k)}\Big\}, \\ x_2(k+1) &= x_2(k)\exp\Big\{1.5 \\ &\quad -(3 + \cos(\pi k + \tfrac{\pi}{3}))x_2(k)\Big\}. \end{aligned} \qquad (30)$$

Corresponding to system (3), here we choose $a_1(k) = 1.5, b_1(k) = 1, c(k) = 2 + \sin(\pi k), m(k) = 0.1, n(k) = 1, a_2(k) = 1.5, b_2(k) = 3+\cos(\pi k+\frac{\pi}{3})$. One could easily check that the condition of Theorem 2.1 holds, and consequently, system (30) admits at least one positive 2-period solution. Numeric simulations (Fig.1, Fig. 2) also support this assertion.

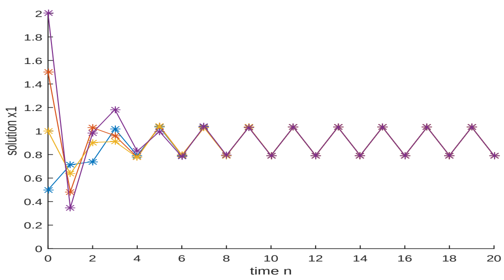

Figure 1: Dynamic behaviors of the first component x_1 in system (30) with the initial condition $(x(0), y(0)) = (0.5, 0.5)$, $(1, 1)$, $(1.5, 1.5)$ and $(2, 2)$, respectively.

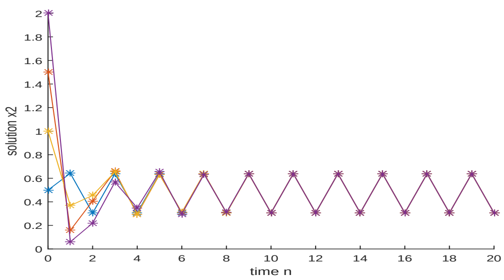

Figure 2: Dynamic behaviors of the second component x_2 in system (30) with the initial condition $(x(0), y(0)) = (0.5, 0.5)$, $(1, 1)$, $(1.5, 1.5)$ and $(2, 2)$, respectively.

Example 3.2.

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\Big\{3 - x_1(k) \\ &\quad -\frac{(2+\sin(\pi k))x_2(k)}{1 + x_2(k) + 0.1x_1(k)}\Big\}, \\ x_2(k+1) &= x_2(k)\exp\Big\{3 \\ &\quad -(3+\cos(\pi k + \tfrac{\pi}{3}))x_2(k)\Big\}, \end{aligned} \tag{31}$$

Corresponding to system (3), here we change $a_1(k), a_2(k)$ to 3, other coefficients are the same as system (30). Numeric simulations (Fig.3, Fig. 4) show that system (31) admits one positive periodic solution. However, the other solutions need more time to approach to the periodic solution.

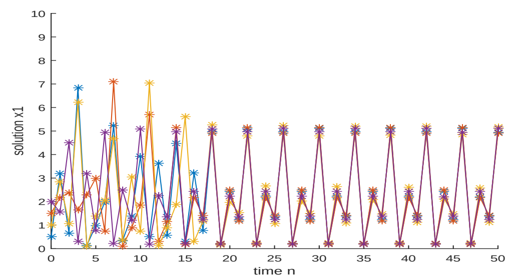

Figure 3: Dynamic behaviors of the first component x_1 in system (31) with the initial condition $(x(0), y(0)) = (0.5, 0.5)$, $(1, 1)$, $(1.5, 1.5)$ and $(2, 2)$, respectively.

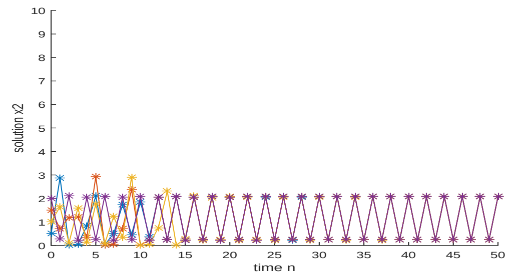

Figure 4: Dynamic behaviors of the second component x_2 in system (31) with the initial condition $(x(0), y(0)) = (0.5, 0.5)$, $(1, 1)$, $(1.5, 1.5)$ and $(2, 2)$, respectively.

4 Discussion

In this paper, we proposed a discrete amensilism model with with Beddington-DeAngelis functional response, by using the coincidence degree theory, sufficient conditions which ensure the existence of positive periodic sequences solution are established. Numeric simulations are carried out to show the feasibility of the main result.

We mention here that we did not investigate the stability property of the system, however, numeric simulations (Fig.1, 2, 3 and 4) showed that the periodic solution is unique and globally asymptotically stable in system (30) and (31). We leave this for future investigation.

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