Positive Periodic solution of a discrete Lotka-volterra commensal symbiosis model with Michaelis-Menten type harvesting

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Abstract: -A non-autonomous discrete Lotka-volterra commensal symbiosis model with Michaelis-Menten type harvesting is proposed and studied in this paper. Under some very simple and easily verified condition, we show that the system admits at least one positive periodic solution.

Key-Words: -Commensal symbiosis model, Positive periodic solution, Michaelis-Menten type harvesting.

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1 Introduction

The aim of this paper is to investigate the positive periodic solution of the following discrete commensal symbiosis model with Hassell-Varley type functional response

$$\begin{aligned} N_1(k+1) &= N_1(k)\exp\Big\{a_1(k)-b_1(k)N_1(k) \\ &\quad +c_1(k)N_2(k)\Big\}, \\ N_2(k+1) &= N_2(k)\exp\Big\{a_2(k)-b_2(k)N_2(k) \\ &\quad -\frac{q(k)E(k)}{m_1(k)E(k)+m_2(k)N_2(k)}\Big\}, \end{aligned} \tag{1}$$

where $N_1(k)$ and $N_2(k)$ represent the densities of the first and second species of k-generation, respectively. In view of seasonal factors, e.g., mating habits, availability of food, weather conditions, harvesting, and hunting, etc, we assume that the coefficients of the system (1) are all periodic sequences with a common integer period. More precisely, we assume that the coefficients of the system (1) satisfies

(H_1) $\{b_1(k)\}, \{b_2(k)\}, \{m_1(k)\}, \{m_2(k)\}, \{c_1(k)\}, \{q(k)\}, \{E(k)\}$ are all positive ω-periodic sequences, ω is a fixed positive integer, $\{a_i(k)\}$ are ω-periodic sequences, which satisfies $\overline{a}_i = \frac{1}{\omega}\sum\limits_{k=0}^{\omega-1} a_i(k) > 0, i = 1, 2.$

In the past several years, many scholars paid their attention to study the dynamic behaviors of the commensal symbiosis model, see [1]-[30] and the references cited therein. However, only recently did scholars ([24]-[30]) began to study the influence of harvesting to commensalism model. It is well known that Michaelis-Menten type harvesting ([24]-[26],[29]-[30], [33]-[37]) is more appropriate than the linear harvesting and constant harvesting, and recently, several scholars ([24]-[26],[29]-[31]) began to study the influence of Michaelis-Menten type harvesting to commensalism model, however, most of them were studied the autonomous ones, and only Liu et al [31] and Xue et al[30] began to investigate the nonautonomous case.

In [31], Liu et al proposed the following nonautonomous Lotka-Volterra commensalism model with Michaelis-Menten type harvesting

$$\begin{aligned} \frac{dN_1(t)}{dt} &= N_1(t)\Big(a(t)-b(t)N_1(t)+c(t)N_2(t)\Big), \\ \frac{dN_2(t)}{dt} &= N_2(t)\Big(d(t)-e(t)N_2(t)\Big) \\ &\quad -\frac{q(t)E(t)N_2(t)}{m_1(t)E(t)+m_2(t)N_2(t)}. \end{aligned} \tag{2}$$

Under the assumption that all the coefficients are continuous positive periodic functions with a common period, the authors obtained a set of sufficient conditions which ensure the existence of at least one positive periodic solution of the system.

It is well known that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations. Hence, corresponding to system (2), we propose the discrete type of Lotka-Volterra commensalism model with

Michaelis-Menten type harvesting, i.e., system (1). To the best of our knowledge, this is the first time that the model is proposed. We will focus our attention to the existence of positive periodic solution of system (1).

2 Main Result

In the proof of our existence theorem below, we will use the continuation theorem of Gaines and Mawhin([32]).

Lemma 2.1 (Continuation Theorem) *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}$. Suppose*

(a).For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;

(b).$QNx \neq 0$ for each $x \in \partial\Omega \cap KerL$ and

$$deg\{JQN, \Omega \cap KerL, 0\} \neq 0.$$

Then the equation $Lx = Nx$ has at least one solution lying in $DomL \cap \bar{\Omega}$.

Let Z, Z^+, R and R^+ denote the sets of all integers, nonnegative integers, real unumbers, and nonnegative real numbers, respectively. For convenience, in the following discussion, we will use the notation below throughout this paper:

$$I_\omega = \{0, 1, ..., \omega - 1\},\ \bar{g} = \frac{1}{\omega}\sum_{k=0}^{\omega-1} g(k),$$

$$g^u = \max_{k\in I_\omega} g(k), \quad g^l = \min_{k\in I_\omega} g(k),$$

where $\{g(k)\}$ is an ω-periodic sequence of real numbers defined for $k \in Z$.

Lemma 2.2[40] *Let $g : Z \to R$ be ω-periodic, i. e., $g(k+\omega) = g(k)$. Then for any fixed $k_1, k_2 \in I_\omega$, and any $k \in Z$, one has*

$$g(k) \le g(k_1) + \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|,$$

$$g(k) \ge g(k_2) - \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|.$$

Lemma 2.3 *Assume that $\bar{a}_2 > \overline{(\frac{q}{m_1})}$ hold, any solution (u_1^*, u_2^*) of the system of algebraic equations*

$$\begin{aligned} &\bar{a}_1 - \bar{b}_1 \exp\{u_1\} + \bar{c}_1 \exp\{u_2\} = 0,\\ &\bar{a}_2 - \bar{b}_2 \exp\{u_2\}\\ &-\frac{1}{\omega}\sum_{k=0}^{\omega-1} \frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)\exp\{u_2\}} = 0. \end{aligned} \quad (3)$$

satisfies

$$\begin{aligned} \ln\frac{\bar{a}_1}{\bar{b}_1} \le u_1^* \le \ln\frac{\bar{a}_1 + \bar{c}_1\frac{\bar{a}_2}{\bar{b}_2}}{\bar{b}_1},\\ \ln\frac{\bar{a}_2 - \overline{(\frac{q}{m_1})}}{\bar{b}_2} \le u_2^* \le \ln\frac{\bar{a}_2}{\bar{b}_2}, \end{aligned} \quad (4)$$

Proof. From the second equation of (3), it immediately follows that

$$\bar{a}_2 - \bar{b}_2 \exp\{u_2\} \ge 0. \quad (5)$$

Thus,

$$u_2 \le \ln\frac{\bar{a}_2}{\bar{b}_2}. \quad (6)$$

From the second equation of (3) we also have

$$\bar{a}_2 - \bar{b}_2 \exp\{u_2\} - \frac{1}{\omega}\sum_{k=0}^{\omega-1} \frac{q(k)E(k)}{m_1(k)E(k)} \le 0, \quad (7)$$

So,

$$u_2 \ge \ln\frac{\bar{a}_2 - \overline{(\frac{q}{m_1})}}{\bar{b}_2}. \quad (8)$$

From the first equation of system (3) we have

$$\bar{a}_1 - \bar{b}_1 \exp\{u_1\} \le 0, \quad (9)$$

thus

$$u_1 \ge \ln\frac{\bar{a}_1}{\bar{b}_1}. \quad (10)$$

From the first equation of system (3) and (6), we also have

$$\begin{aligned} 0 &= \bar{a}_1 - \bar{b}_1 \exp\{u_1\} + \bar{c}_1 \exp\{u_2\}\\ &\le \bar{a}_1 - \bar{b}_1 \exp\{u_1\} + \bar{c}_1 \exp\Big\{\ln\frac{\bar{a}_2}{\bar{b}_2}\Big\}\\ &= \bar{a}_1 + \bar{c}_1\frac{\bar{a}_2}{\bar{b}_2} - \bar{b}_1 \exp\{u_1\}. \end{aligned}$$

Thus

$$u_1 \le \ln\frac{\bar{a}_1 + \bar{c}_1\frac{\bar{a}_2}{\bar{b}_2}}{\bar{b}_1}. \quad (11)$$

This ends the proof of Lemma 2.3.

We now reach the position to establish our main result.

Theorem 2.1 *Assume that*

$$\bar{a}_2 > \overline{(\frac{q}{m_1})} \quad (12)$$

hold, system (1) admits at least one positive ω-periodic solution.

Proof. Let

$$N_i(k) = \exp\{u_i(k)\}, \quad i = 1, 2,$$

so that system (1) becomes

$$\begin{aligned} & u_1(k+1) - u_1(k) \\ = \;& a_1(k) - b_1(k)\exp\{u_1(k)\} + c_1(k)\exp\{u_2(k)\}, \\ & u_2(k+1) - u_2(k) \\ = \;& a_2(k) - b_2(k)\exp\{u_2(k)\} \\ & -\frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)\exp\{u_2(k)\}}. \end{aligned} \tag{13}$$

Define

$$l_2 = \Big\{ y = \{y(k)\}, y(k) = (y_1(k), y_2(k))^T \in R^2 \Big\}.$$

For $a = (a_1, a_2)^T \in R^2$, define $|a| = \max\{|a_1|, |a_2|\}$. Let $l^\omega \subset l_2$ denote the subspace of all ω sequences equipped with the usual normal form $\|u\| = \max\limits_{k \in I_\omega} |u(k)|$. It is not difficult to show that l^ω is a finite-dimensional Banach space. Let

$$l_0^\omega = \{u = \{u(k)\} \in l^\omega : \sum_{k=0}^{\omega-1} u(k) = 0\},$$

$$l_c^\omega = \{u = \{u(k)\} \in l^\omega : u(k) = h \in R^2, k \in Z\},$$

then l_0^ω and l_c^ω are both closed linear subspace of l^ω, and

$$l^\omega = l_0^\omega \oplus l_c^\omega, \quad diml_c^\omega = 2.$$

Now let us define $X = Y = l^\omega$, $(Lu)(k) = u(k+1) - u(k)$. It is trivial to see that L is a bounded linear operator and

$$KerL = l_c^\omega, \quad ImL = l_0^\omega,$$

$$dimKerL = 2 = CodimImL.$$

Then it follows that L is a Fredholm mapping of index zero. Let

$$N(u_1, u_2)^T = (N_1, N_2)^T := N(u, k),$$

where

$$\begin{cases} N_1 = a_1(k) - b_1(k)\exp\{u_1(k)\} \\ \qquad +c_1(k)\exp\{u_2(k)\}, \\ N_2 = a_2(k) - b_2(k)\exp\{u_2(k)\} \\ \qquad -\dfrac{q(k)E(k)}{m_1(k)E(k) + m_2(k)\exp\{u_2(k)\}}. \end{cases}$$

$$Px = \frac{1}{\omega}\sum_{s=0}^{\omega-1} x(s), x \in X, \quad Qy = \frac{1}{\omega}\sum_{s=0}^{\omega-1} y(s), y \in Y.$$

It is not difficult to show that P and Q are two continuous projectors such that

$$ImP = KerL \quad and \quad ImL = KerQ = Im(I-Q).$$

Furthermore, the generalized inverse (to L) K_p: ImL $\to$Ker$P\cap$Dom$L$ exists and is given by

$$K_p(z) = \sum_{s=0}^{k-1} z(s) - \frac{1}{\omega}\sum_{s=0}^{\omega-1}(\omega - s)z(s).$$

Thus

$$QNx = \frac{1}{\omega}\sum_{k=0}^{\omega-1} N(x, k),$$

$$\begin{aligned} Kp(I-Q)Nx = \;& \textstyle\sum_{s=0}^{k-1} N(x, s) \\ & +\tfrac{1}{\omega}\textstyle\sum_{s=0}^{\omega-1} sN(x, s) \\ & -\left(\tfrac{k}{\omega} + \tfrac{\omega-1}{2\omega}\right)\textstyle\sum_{s=0}^{\omega-1} N(x, s). \end{aligned}$$

Obviously, QN and $K_p(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, it is not difficult to show that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Thus, N is L-compact on any open bounded set $\Omega \subset X$. The isomorphism J of ImQ onto Ker$L$ can be the identity mapping, since ImQ=Ker$L$.

Now we are at the point to search for an appropriate open, bounded subset Ω in X for the application of the continuation theorem. Corresponding to the operator equation $Lx = \lambda Nx, \lambda \in (0, 1)$, we have

$$\begin{aligned} & u_1(k+1) - u_1(k) \\ = \;& \lambda\Big[a_1(k) - b_1(k)\exp\{u_1(k)\} \\ & \quad +c_1(k)\exp\{u_2(k)\}\Big], \\ & u_2(k+1) - u_2(k) \\ = \;& \lambda\Big[a_2(k) - b_2(k)\exp\{u_2(k)\} \\ & -\frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)\exp\{u_2(k)\}}\Big]. \end{aligned} \tag{14}$$

Suppose that $u = (u_1(k), u_2(k))^T \in X$ is an arbitrary solution of system (14) for a certain $\lambda \in (0, 1)$. Summing on both sides of (14) from 0 to $\omega - 1$ with respect to k, we reach

$$\sum_{k=0}^{\omega-1}\Big[a_1(k)-b_1(k)\exp\{u_1(k)\}+c_1(k)\exp\{u_2(k)\}\Big]=0,$$
$$\sum_{k=0}^{\omega-1}\Big[a_2(k)-b_2(k)\exp\{u_2(k)\}-\frac{q(k)E(k)}{m_1(k)E(k)+m_2(k)\exp\{u_2(k)\}}\Big]=0.$$

That is,

$$\begin{aligned}&\sum_{k=0}^{\omega-1}b_1(k)\exp\{u_1(k)\}\\ =\;&\bar{a}_1\omega+\sum_{k=0}^{\omega-1}c_1(k)\exp\{u_2(k)\},\end{aligned}\tag{15}$$

$$\begin{aligned}&\sum_{k=0}^{\omega-1}\Big(b_2(k)\exp\{u_2(k)\}+\frac{q(k)E(k)}{m_1(k)E(k)+m_2(k)\exp\{u_2(k)\}}\Big)\\ =\;&\bar{a}_2\omega.\end{aligned}\tag{16}$$

From (14) and (16), we have

$$\begin{aligned}&\sum_{k=0}^{\omega-1}|u_1(k+1)-u_1(k)|\\ =\;&\lambda\sum_{k=0}^{\omega-1}\Big|a_1(k)-b_1(k)\exp\{u_1(k)\}+c_1(k)\exp\{u_2(k)\}\Big|\\ \le\;&\sum_{k=0}^{\omega-1}|a_1(k)|+\sum_{k=0}^{\omega-1}\Big(b_1(k)\exp\{u_1(k)\}+c_1(k)\exp\{u_2(k)\}\Big)\\ =\;&\sum_{k=0}^{\omega-1}|a_1(k)|+\bar{a}_1\omega+2\sum_{k=0}^{\omega-1}c_1(k)\exp\{u_2(k)\}\\ =\;&(\bar{A}_1+\bar{a}_1)\omega+2\sum_{k=0}^{\omega-1}c_1(k)\exp\{u_2(k)\},\end{aligned}\tag{17}$$

$$\begin{aligned}&\sum_{k=0}^{\omega-1}|u_2(k+1)-u_2(k)|\\ =\;&\lambda\sum_{k=0}^{\omega-1}\Big[a_2(k)-b_2(k)\exp\{u_2(k)\}-\frac{q(k)E(k)}{m_1(k)E(k)+m_2(k)\exp\{u_2(k)\}}\Big]\\ \le\;&\sum_{k=0}^{\omega-1}|a_2(k)|+\sum_{k=0}^{\omega-1}b_2(k)\exp\{u_2(k)\}+\frac{q(k)E(k)}{m_1(k)E(k)+m_2(k)\exp\{u_2(k)\}}\Big]\\ \le\;&\sum_{k=0}^{\omega-1}|a_2(k)|+\bar{a}_2\omega\\ \le\;&(\bar{A}_2+\bar{a}_2)\omega.\end{aligned}\tag{18}$$

where $\bar{A}_1=\frac{1}{\omega}\sum_{k=0}^{\omega-1}|a_1(k)|,\ \ \bar{A}_2=\frac{1}{\omega}\sum_{k=0}^{\omega-1}|a_2(k)|$.

Since $\{u(k)\}=\{(u_1(k),u_2(k))^T\}\in X$, there exist $\eta_i,\delta_i, i=1,2$ such that

$$u_i(\eta_i)=\min_{k\in I_\omega}u_i(k),\ \ u_i(\delta_i)=\max_{k\in I_\omega}u_i(k).\tag{19}$$

By (16), we have

$$\exp\{u_2(\eta_2)\}\sum_{k=0}^{\omega-1}b_2(k)\le\bar{a}_2\omega.$$

So

$$u_2(\eta_2)\le\ln\frac{\bar{a}_2}{\bar{b}_2}.\tag{20}$$

It follows from Lemma 2.2, (18) and (20) that

$$\begin{aligned}u_2(k)&\le u_2(\eta_2)+\sum_{k=0}^{\omega-1}|u_2(k+1)-u_2(k)|\\ &\le \ln\tfrac{\bar{a}_2}{\bar{b}_2}+(\bar{A}_2+\bar{a}_2)\omega,\end{aligned}\tag{21}$$

From (16) we also have

$$\exp\{u_2(\delta_2)\}\sum_{k=0}^{\omega-1}b_2(k)\ge\bar{a}_2\omega-\sum_{k=0}^{\omega-1}\Big(\frac{q(k)E(k)}{m_1(k)E(k)}\Big),$$

and so

$$u_2(\delta_2)\ge\ln\frac{\bar{a}_2-\overline{(\frac{q}{m_1})}}{\bar{b}_2}.\tag{22}$$

It follows from Lemma 2.2, (18) and (22) that

$$\begin{aligned} u_2(k) &\geq u_2(\delta_2) - \sum_{k=0}^{\omega-1} |u_2(k+1) - u_2(k)| \\ &\geq \ln \frac{\bar{a}_2 - \overline{(\frac{q}{m_1})}}{\bar{b}_2} - (\bar{A}_2 + \bar{a}_2)\omega, \end{aligned} \tag{23}$$

which together with (21) leads to

$$\begin{aligned} |u_2(k)| &\leq \max\left\{ \left|\ln \tfrac{\bar{a}_2}{\bar{b}_2} + (\bar{A}_2 + \bar{a}_2)\omega\right|, \right. \\ &\left. \left| \ln \frac{\bar{a}_2 - \overline{(\frac{q}{m_1})}}{\bar{b}_2} - (\bar{A}_2 + \bar{a}_2)\omega \right| \right\} \stackrel{\text{def}}{=} H_2. \end{aligned} \tag{24}$$

It follows from (17) and (21) that

$$\begin{aligned} &\sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \\ \leq\ & (\bar{A}_1 + \bar{a}_1)\omega + 2\sum_{k=0}^{\omega-1} c_1(k)\exp\{u_2(k)\} \\ \leq\ & (\bar{A}_1 + \bar{a}_1)\omega \\ & +2\sum_{k=0}^{\omega-1} c_1(k)\exp\{\ln \tfrac{\bar{a}_2}{\bar{b}_2} + (\bar{A}_2 + \bar{a}_2)\omega\} \\ \leq\ & (\bar{A}_1 + \bar{a}_1)\omega \\ & +2\bar{c}_1\frac{\bar{a}_2}{\bar{b}_2}\omega\exp\{(\bar{A}_2 + \bar{a}_2)\omega\} \stackrel{\text{def}}{=} \Gamma_1, \end{aligned} \tag{25}$$

It follows from (15) and (21) that

$$\begin{aligned} &\sum_{k=0}^{\omega-1} b_1(k)\exp\{u_1(\eta_1)\} \\ \leq\ & \bar{a}_1\omega + \sum_{k=0}^{\omega-1} c_1(k)\exp\{u_2(k)\} \\ \leq\ & \bar{a}_1\omega + \sum_{k=0}^{\omega-1} c_1(k)\exp\{\ln \tfrac{\bar{a}_2}{\bar{b}_2} + (\bar{A}_2 + \bar{a}_2)\omega\} \\ =\ & \bar{a}_1\omega + \bar{c}_1\frac{\bar{a}_2}{\bar{b}_2}\omega\exp\{(\bar{A}_2 + \bar{a}_2)\omega\}, \end{aligned}$$

and so,

$$u_1(\eta_1) \leq \ln \frac{\Delta_1}{\bar{b}_1}, \tag{26}$$

where

$$\Delta_1 = \bar{a}_1 + \bar{c}_1\frac{\bar{a}_2}{\bar{b}_2}\exp\{(\bar{A}_2 + \bar{a}_2)\omega\}.$$

It follows from Lemma 2.2, (25) and (26) that

$$\begin{aligned} u_1(k) &\leq u_1(\eta_1) + \sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \\ &\leq \ln \tfrac{\Delta_1}{\bar{b}_1} + \Gamma_1 \stackrel{\text{def}}{=} M_1. \end{aligned} \tag{27}$$

It follows from (15) that

$$\begin{aligned} &\sum_{k=0}^{\omega-1} b_1(k)\exp\{u_1(\delta_1)\} \\ \geq\ & \bar{a}_1\omega + \sum_{k=0}^{\omega-1} c_1(k)\exp\{u_2(k)\} \\ \geq\ & \bar{a}_1\omega, \end{aligned}$$

and so,

$$u_1(\delta_1) \geq \ln \frac{\bar{a}_1}{\bar{b}_1}, \tag{28}$$

It follows from Lemma 2.2, (25) and (28) that

$$\begin{aligned} u_1(k) &\geq u_1(\delta_1) - \sum_{k=0}^{\omega-1} |u_1(k+1) - u_1(k)| \\ &\geq \ln \tfrac{\bar{a}_1}{\bar{b}_1} - \Gamma_1 \stackrel{\text{def}}{=} M_2. \end{aligned} \tag{29}$$

It follows from (27) and (29) that

$$|u_1(k)| \leq \max\left\{|M_1|, |M_2|\right\} \stackrel{\text{def}}{=} H_1. \tag{30}$$

Clearly, H_1 and H_2 are independent on the choice of λ. Already, in Lemma 2.3, we had showed that under the assumption (12) hold, any solution (u_1^*, u_2^*) of the system of algebraic equations

$$\begin{aligned} &\bar{a}_1 - \bar{b}_1\exp\{u_1\} + \bar{c}_1\exp\{u_2\} = 0, \\ &\bar{a}_2 - \bar{b}_2\exp\{u_2\} \\ &-\frac{1}{\omega}\sum_{k=0}^{\omega-1} \frac{q(k)E(k)}{m_1(k)E(k) + m_2(k)\exp\{u_2\}} = 0. \end{aligned} \tag{31}$$

satisfies

$$\begin{aligned} &\ln \frac{\bar{a}_1}{\bar{b}_1} \leq u_1^* \leq \ln \frac{\bar{a}_1 + \bar{c}_1\frac{\bar{a}_2}{\bar{b}_2}}{\bar{b}_1}, \\ &\ln \frac{\bar{a}_2 - \overline{(\frac{q}{m_1})}}{\bar{b}_2} \leq u_2^* \leq \ln \frac{\bar{a}_2}{\bar{b}_2}, \end{aligned} \tag{32}$$

Let $H = H_1 + H_2 + H_3$, where $H_3 > 0$ is taken sufficiently enough large such that

$$H_3 > \left|\ln\frac{\bar{a}_1}{\bar{b}_1}\right| + \left|\ln\frac{\bar{a}_1+\bar{c}_1\frac{\bar{a}_2}{\bar{b}_2}}{\bar{b}_1}\right| + \left|\ln\frac{\bar{a}_2-\overline{(\frac{q}{m_1})}}{\bar{b}_2}\right| + \left|\ln\frac{\bar{a}_2}{\bar{b}_2}\right|,$$

and define

$$\Omega = \Big\{u(t) = (u_1(k), u_2(k))^T \in X : \|u\| < H\Big\}.$$

It is clear that Ω verifies requirement (a) in Lemma 2.1. When $u \in \partial\Omega \cap KerL = \partial\Omega \cap R^2$, u is constant vector in R^2 with $||u|| = B$. Then

$$QNu = \begin{pmatrix} \bar{a}_1 - \bar{b}_1\exp\{u_1\} + \bar{c}_1\exp\{u_2\} \\ \Delta \end{pmatrix} \neq 0.$$

where

$$\Delta = \bar{a}_2 - \bar{b}_2\exp\{u_2\} - \frac{1}{\omega}\sum_{k=0}^{\omega-1}\frac{q(k)E(k)}{m_1(k)E(k)+m_2(k)\exp\{u_2\}}.$$

In order to compute the Brouwer degree, let us consider the homotopy

$$H_\mu u = \mu QNu + (1-\mu)Gu, \qquad (2.31)$$

where

$$Gu = \begin{pmatrix} \bar{a}_1 - \bar{b}_1\exp\{u_1\} + \bar{c}_1\exp\{u_2\} \\ \bar{a}_2 - \bar{b}_2\exp\{u_2\} \end{pmatrix}.$$

From the definition of H, it follows that $0 \notin H_\mu(\partial\Omega \cap KerL)$ for $\mu \in [0,1]$. In addition, one can easily show that the algebraic equation $Gu = 0$ has a unique solution in R^2. Note that $J = I$ since $ImQ = KerL$, by the invariance property of homotopy, direct calculation produces

$$\begin{aligned} & deg(JQN, \Omega\cap KerL, 0) \\ = \ & deg(QN, \Omega\cap KerL, 0) \\ = \ & deg(G, \Omega\cap KerL, 0) \\ = \ & \mathrm{sgn}\Big(\Gamma\Big) = 1 \neq 0, \end{aligned}$$

where

$$\Gamma = \bar{b}_1\bar{b}_2\exp\{u_1^*\}\exp\{u_2^*\}$$

and $deg(\cdot,\cdot,\cdot)$ is the Brouwer degree. By now we have proved that Ω verifies all requirements in Lemma 2.1. Hence (13) has at least one solution $(u_1^*(k), u_2^*(k))^T$ in $DomL \cap \bar{\Omega}$. And so, system (1) admits a positive periodic solution $(N_1^*(k), N_2^*(k))^T$, where $N_i^*(k) = \exp\{u_i^*(k)\}, i = 1, 2$, This completes the proof of Theorem 2.1.

3. Example

Now let us consider the following example.

Example 3.1.

$$\begin{aligned} & N_1(k+1) \\ = \ & N_1(k)\exp\Big\{0.5 - 0.25\cos(\pi k) \\ & -(1+0.5\sin(\pi n+\tfrac{\pi}{4}))N_1(k) \\ & +(0.5+0.3\sin(\pi k+\tfrac{\pi}{3}))N_2(k)\Big\}; \\ & N_2(k+1) \\ = \ & N_2(k)\exp\Big\{1.5 + 0.5\sin(\pi k+\tfrac{\pi}{4}) \\ & -(1+0.3\cos(\pi k+\tfrac{\pi}{6}))N_2(k) \\ & \frac{0.5+0.2\sin(\pi k+\frac{\pi}{3})}{2+N_2(k)}\Big\}. \end{aligned} \qquad (33)$$

Here, corresponding to system (1), we take

$$\begin{aligned} a_1(k) &= 0.5 - 0.25\cos(\pi k), \\ b_1(k) &= 1 + 0.5\sin(\pi n + \tfrac{\pi}{4}), \\ c_1(k) &= 0.5 + 0.3\sin(\pi k + \tfrac{\pi}{3}), \\ a_2(k) &= 1.5 + 0.5\sin(\pi k + \tfrac{\pi}{4}), \\ b_2(k) &= 1 + 0.3\cos(\pi k + \tfrac{\pi}{6}), \\ q(k) &= 0.5 + 0.2\sin(\pi k + \tfrac{\pi}{3}), \\ E(k) &= 1, \ \ m_1(k) = 2, m_2(k) = 1. \end{aligned}$$

Obviously, in system (33)

$$\bar{a}_2 = 1.5 > 0.25 = \overline{(\frac{q}{m_1})}$$

It follows from Theorem 2.1 that system (33) admits at least one positive 2-period solution.

3 Conclusion

In this paper, we propose a discrete Lotka-volterra commensal symbiosis model with Michaelis-Menten type harvesting, it seems that this is the first time such kind of modelling was proposed. We show that under some suitable condition, the system could admits at least one positive periodic solution, which means that two species could coexistent in a fluctuation state.

We will investigate the persistent property and stability property of the system in the future.

References:

[1] Chen, F. D., Xie X. D., et al, Dynamic behaviors of a stage-structured cooperation model, *Commun. Math. Biol. Neurosci.*, Vol. 2015, 2015, pp.1-19.

[2] Yang K., Miao Z., et al, Influence of single feedback control variable on an autonomous Holling-II type cooperative system, Journal of Mathematical Analysis and Applications, Vol. 435, No. 1, 2016, pp. 874-888.

[3] Xie X. D., Chen F. D. , et al, Note on the stability property of a cooperative system incorporating harvesting, *Discrete Dyn. Nat. Soc.*, Vol. 2014, 2014, pp.1-5.

[4] Xue Y. L., Chen F. D., et al. Dynamic behaviors of a discrete commensalism system, *Annals of Applied Mathematics*, Vol. 31, No. 4, 2014, pp.452-461.

[5] Xue Y. L., Xie X. D., et al. Almost periodic solution of a discrete commensalism system, *Discrete Dynamics in Nature and Society*, Volume 2015, Article ID 295483, 11 pages.

[6] Miao Z. S., Xie X. D., et al, Dynamic behaviors of a periodic Lotka-Volterra commensal symbiosis model with impulsive, *Commun. Math. Biol. Neurosci.*, Vol. 2015, 2015, 15 pages.

[7] Wu R. X., Lin L., et al, A commensal symbiosis model with Holling type functional response, *J. Math. Computer Sci.*, 16 (2016) 364-371.

[8] Xie X. D. , Miao Z. S., et al, Positive periodic solution of a discrete Lotka-Volterra commensal symbiosis model, *Commun. Math. Biol. Neurosci.*, Vol. 2015 , 2015, 10 pages.

[9] Lei C., Dynamic behaviors of a stage-structured commensalism system, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 301.

[10] Lin Q., Allee effect increasing the final density of the species subject to the Allee effect in a Lotka-Volterra commensal symbiosis model, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 196.

[11] Chen B., Dynamic behaviors of a commensal symbiosis model involving Allee effect and one party can not survive independently, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 212.

[12] Wu R., Li L. and Lin Q., A Holling type commensal symbiosis model involving Allee effect, *Commun. Math. Biol. Neurosci.*, Vol. 2018, 2018, Article ID 6.

[13] Lei C., Dynamic behaviors of a Holling type commensal symbiosis model with the first species subject to Allee effect, *Commun. Math. Biol. Neurosci.*, Vol. 2019, 2019, Article ID 3.

[14] Vargas-De-León C. and Gómez-Alcaraz G., Global stability in some ecological models of commensalism between two species, *Biomatemática*, Vol.23, 2013, pp. 139-146.

[15] Chen F., Xue Y., Lin Q., et al, Dynamic behaviors of a Lotka-Volterra commensal symbiosis model with density dependent birth rate, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 296.

[16] Han R. and Chen F., Global stability of a commensal symbiosis model with feedback controls, *Commun. Math. Biol. Neurosci.*, Vol. 2015, 2015, Article ID 15.

[17] Chen F., Pu L. and Yang L., Positive periodic solution of a discrete obligate Lotka-Volterra model, *Commun. Math. Biol. Neurosci.*, Vol. 2015, 2015, Article ID 14.

[18] Guan X. and Chen F., Dynamical analysis of a two species amensalism model with Beddington-DeAngelis functional response and Allee effect on the second species, *Nonlinear Analysis: Real World Applications*, Vol. 48, 2019, pp.71-93.

[19] Li T., Lin Q., et al, Positive periodic solution of a discrete commensal symbiosis model with Holling II functional response, *Commun. Math. Biol. Neurosci.*, Vol. 2016, 2016, Article ID 22.

[20] Ji M. and Liu M., Optimal harvesting of a stochastic commensalism model with time delay, *Physica A: Statistical Mechanics and its Applications*, Vol. 527, 2019, pp. 121284.

[21] Li T. and Wang Q., Stability and Hopf bifurcation analysis for a two-species commensalism system with delay,*Qualitative Theory of Dynamical Systems*, Vol. 20, No. 3, 2021, pp. 1-20.

[22] Chen L., Liu T., et al, Stability and bifurcation in a two-patch model with additive Allee effect, *AIMS Mathematics*, Vol. 7, No. 1, 2022, pp. 536-551.

[23] Zhu Z., Chen Y., et al. Stability and bifurcation in a Leslie-Gower predator-prey model with Allee effect, *International Journal of Bifurcation and Chaos*, Vol. 32, No. 03, 2022, pp. 2250040.

[24] Puspitasari N., Kusumawinahyu W. M. and Trisilowati T., Dynamical analysis of the symbiotic model of commensalism in four populations with Michaelis-Menten type harvesting in the first commensal population, *JTAM (Jurnal Teori dan Aplikasi Matematika)*, Vol. 5, No. 2, 2021, pp. 392-404.

[25] Chen B., The influence of commensalism on a Lotka-Volterra commensal symbiosis model with Michaelis-Menten type harvesting, *Advances in Difference Equations*, Vol. 2019, 2019, Article ID 43.

[26] Liu Y., Xie X., et al, Permanence, partial survival, extinction, and global attractivity of a nonautonomous harvesting Lotka-Volterra commensalism model incorporating partial closure for the populations, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 211.

[27] Deng H. and Huang X., The influence of partial closure for the populations to a harvesting Lotka-Volterra commensalism model, *Commun. Math. Biol. Neurosci.*, Vol. 2018, 2018, Article ID 10.

[28] Puspitasari N. and Kusumawinahyu W. M. , Trisilowati T., Dynamic analysis of the symbiotic model of commensalism and parasitism with harvesting in commensal populations, *JTAM (Jurnal Teori dan Aplikasi Matematika)*, Vol. 5, No. 1, 2021, pp. 193-204.

[29] Jawad S., Study the dynamics of commensalism interaction with Michaels-Menten type prey harvesting, *Al-Nahrain Journal of Science*, 25(1)(2022) 45-50.

[30] Xue Y., Xie X., et al, Almost periodic solutions of a commensalism system with Michaelis-Menten type harvesting on time scales, *Open Mathematics*, 17(1)(2019) 1503-1514.

[31] Liu Y., Guan X., et al. On the existence and stability of positive periodic solution of a nonautonomous commensal symbiosis model with Michaelis-Menten type harvesting, *Commun. Math. Biol. Neurosci.*, Vol. 2019, 2019, Article ID 2.

[32] Gaines R. E. and Mawhin J. L., ``Coincidence Degree and Nonlinear Differential Equations", *Springer-Verlag*, Berlin, 1977.

[33] Fan M. and Wang K., Periodic solutions of a discrete time nonautonomous ratio-dependent predator-prey system, *Math. Comput. Modell.* Vol. 35, No. 9-10, 2020, pp.951-961.

[34] Lin Q. and Xie X., et al, Dynamical analysis of a logistic model with impulsive Holling type-II harvesting, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID: 112.

[35] Yu X. and Zhu Z., et al, Stability and bifurcation analysis in a single-species stage structure system with Michaelis-Menten-type harvesting, *Advances in Difference Equations*, Vol. 2020, 2020, Article ID: 238.

[36] Zhu Z., Chen F., et al, Dynamic behaviors of a discrete May type cooperative system incorporating Michaelis-Menten type harvesting, *IAENG International Journal of Applied Mathematics*, Vol.50, No.3, 2020, pp.1-10.

[37] Yu X., Zhu Z., et al, Dynamic behaviors of a single species stage structure model with Michaelis-Menten-type juvenile population harvesting, *Mathematics*, 2020, Vol.8, No.8, 2020, pp.1-12.

[38] Zhu Z., Wu R., et al, Dynamic behaviors of a Lotka-Volterra commensal symbiosis model with non-selective Michaelis-Menten type harvesting, *IAENG International Journal of Applied Mathematics*, Vol.50, No.2, 2020, pp.1-10.

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