Spatial and Non-Spatial Panel Data Estimators: Simulation Study and Application to Personal Income in U.S. States

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Abstract:- The spatial analysis aims to understand and explore the nature of entanglements and interactions between spatial units' locations. The analysis of models involving spatial dependence has received great attention in recent decades. Because ignoring the presence of spatial dependence in the data is very likely to lead to biased or inefficient estimates if we use traditional estimation methods. Therefore, this paper is an attempt to assess the risks involved in ignoring the spatial dependence that characterizes the panel data by using a Monte Carlo simulation (MCS) study for two of the most common spatial panel data (SPD) models; Spatial lag model (SLM) and spatial error model (SEM), by comparing the performance of two estimators; i.e., spatial maximum likelihood estimator (MLE) and non-spatial ordinary least squares (OLS) within-group estimator, across two levels of analysis; Parameter-level in terms of bias and root mean square error (RMSE), and model-level in terms of goodness of fit criteria under different scenarios of spatial units N, time-periods T, and spatial dependence parameters, by using two different structures of spatial weights matrix; inverse distance, and inverse exponential distance. The results show that the non-spatial bias and RMSE of $\hat{\beta}$ are functions of the degree of spatial dependence in the data for both models, i.e., SLM and SEM. If the spatial dependence is small, then the choice of the non-spatial estimator may not lead to serious consequences in terms of bias and RMSE of $\hat{\beta}$. On the contrary, the choice of the non-spatial estimator always leads to has disastrous consequences if the spatial dependence is large. On the other hand, we provide a general framework that shows how to define the appropriate model from among several candidate models through application to a dataset of per capita personal income (PCPI) in U.S. states during the period from 2009 to 2019, concerning three main aspects: educational attainment, economy size, and labour force type. The results confirm that PCPI is spatially dependent lagged correlated.

Key-Words: - Per capita personal income, spatial autoregressive combined model, spatial lag model, spatial error model, spatial weights matrix, simulation.

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1 Introduction

Panel data refer to cross-section units (e.g., individuals, groups, countries, companies) observed over several time periods. In a SPD setting, the cross-section observations are associated with a particular location in space. The data can be observed either at point locations (e.g., housing data) or aggregated over regular or irregular areas (e.g., countries, regions, states, counties). The structure of the interactions between each pair of spatial units is represented by a spatial weights matrix. On a somewhat more formal level, in spatial econometrics, these interactions may relate to the models' dependent variable, to the exogenous variables, to the disturbance term, or various combinations of them.

The analysis of models involving spatial dependence has received great attention in recent decades. Because ignoring the presence of the spatial dependence in the data is very likely to lead to inefficient or biased estimates if we use traditional estimation methods, such as OLS. For instance, when the spatial dependence exists in the data, then this may be an additional source of variation. As we know, ignoring the source of variation can lead to biased estimates, and also the traditional estimators are no longer efficient due to changes in asymptotic variance-covariance matrices (VCMs). Therefore, alternative estimation methods had to be developed that take into account spatial dependence that characterizes the data to obtain more accurate results.

Spatial econometrics was first studied in the 1950s. It was named by Jean Paelinck in the early 70s. Many works have been published since the launch of Cliff and Ord's seminal work, see [1]. More recently, researchers have recognized the importance of introducing this approach in the case of panel data models to take the advantages provided by these models, where SPD models have the same structure of panel data model which capture spatial interactions across spatial units and over time. With panel data available, we can not only improve the efficiency of estimates but also investigate some issues that cannot be addressed by the cross-section data, such as heterogeneity and dependence across time. Besides the additional information regarding the use of the cross-sectional dimension of the data enables accounting for the presence of unobservable heterogeneity among cross-section units. Also, access to information included in both cross-sectional and temporal dimensions enables us to model dynamic relations, see [2, 3, 4, and 5].

Since many applications in spatial econometrics are currently based on panel data. In addition, the attention to the space of geographical units and the interaction between them has become an important feature of the empirical work, see [6, 7, and 8], because this type of data possesses information about the location of the observations that may constitute an additional source of variation, and ignoring this variation may lead to biased estimates, see [9]. Therefore, many of researchers are trying to propose estimation methods that allow for the existence of spatial dependence in panel data models, see [10, 11, and 12].

Although a reasonable amount of literature has been devoted to reviewing the spatial econometrics techniques in the last decades, this paper focus some of the recent theoretical advances in this research area. For this purpose, as follows:

(1) Studying the effect of ignoring spatial dependence in the data by comparing the performance of the spatial and non-spatial estimators for two specifications of SPD models, i.e., SLM and SEM, through a MCS study under different scenarios of N, T, spatial dependence degree, and spatial weights matrix.
(2) Investigating the influence of the structure of spatial weights on the performance of spatial estimators and the goodness of fit model through a simulation study.

Applying the SPD modeling to analyze the determinants of PCPI in U.S. states, and providing a general framework of how to select the appropriate SPD model among several candidates.

This paper is divided into 9 sections as follows: Section 2 introduces the specification of SPD models and some other related terminology, sections 3 illustrates the assumptions of the SPD models, section 4 provides a brief summary for spatial weights matrix, section 5 explains the SLM and its estimation methods, section 6 explains the SEM and its estimation methods, section 7 provides a MCS study, section 8 presents our application to personal income in U.S. States, finally, section 9 includes the concluding remarks.

2 The Specification of SPD Models

As we mentioned previously; spatial econometrics focuses on interaction effects among geographical units, such as counties, regions, etc. in modelling terms; Elhorst [13] defined three types of interaction effects to explain why an observation related to a specific location may be dependent on observations at other locations as stated in

(1) **Endogenous interaction effects among the dependent variable (y)**: measures the dependency of unit (A) in dependent variable y on other units in the same dependent variables. This effect can be denoted by $\mathrm{W_{NT}y}$.
(2) **Exogenous interaction effects among the independent variables (X)**: measures the dependency of unit (A) in dependent variable y on other units in the explanatory variables X. This effect can be denoted by $\mathrm{W_{NT}X}$.
(3) **Interaction effects among the error terms (u):** refers that units may behave similarly because they have the same unobserved characteristics or face similar unobserved environments. This effect can be denoted by $\mathrm{W_{NT}u}$.

A full static model with the above three types of interaction effects can be expressed as:

$$\begin{aligned} \mathrm{y_t} &= \lambda \mathrm{W_N y_t} + \mathrm{X_t}\beta + \mathrm{W_N X_t}\theta + \mathrm{u_t}, \\ \mathrm{u_t} &= \rho \mathrm{W_N u_t} + \varepsilon_t, \mathrm{t} = 1, \dots, \mathrm{T} \end{aligned} \quad (1)$$

where $\mathrm{y_t}$ is a (N×1) vector consisting of one observation of the dependent variable for every spatial unit $(i = 1, \cdots, \mathrm{N})$ in the sample at time $t\ (t = 1, \cdots, \mathrm{T})$, $\mathrm{X_t}$ is a (N×K) matrix of exogenous explanatory variables, u_t reflects the error terms specification of the model, which is assumed to be spatially correlated, and ε_t is a (N×1) vector of $i.i.d.$ disturbance terms, whose elements have zero mean and finite variance σ_ε^2. λ is the spatial autoregressive coefficient, ρ is the spatial autocorrelation coefficient, β and θ are $(K\times 1)$ vectors contain the response parameters of the exogenous explanatory variables. Any vector or a matrix pre-multiplied by W_N denotes its spatially lagged value. W_N is a (N×N) non-negative matrix of known constants describing the spatial arrangement of the units in the sample. Where the element w_{ij} in the matrix represents the prior strength of the interaction between spatial unit i (row) and spatial unit j (column). In other words, the elements of W_N, w_{ij}, are non-zero if i and j are neighbors. By convention, a self-neighbor relation is excluded, so the diagonal elements of W_N are zero.

To generalize the spatial weights matrix in panel data settings, the weights are assumed to remain constant over time, then the full (NT ×NT) weights matrix becomes:

$$W_{NT} = I_T \otimes W_N = \begin{bmatrix} W_N & 0 & \cdots & 0 \\ 0 & W_N & \cdots & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & 0 & \cdots & W_N \end{bmatrix} \quad (2)$$

Model (1) can be rewritten in a reduced form as follows:

$$y_t = S_N^{-1}(X_t\beta + W_N X_t \theta) + S_N^{-1} B_N^{-1} \varepsilon_t, \quad (3)$$

where:

$$S_N = (I_N - \lambda W_N), \quad (4)$$
$$B_N = (I_N - \rho W_N) \quad (5)$$

Table 1 summarizes the main spatial regression models. These models can be treated as fixed effects (FE) or random effects (RE).

In this paper, we will focus on studying two specifications of SPD models that suffered from some econometric problems; SLM, and SEM which mentioned in Fig. 1. In the first case; it must be dealt with the endogeneity of the spatial lag (SL), and in the second case; the non-spherical nature of the error VCM must be taken into account.

3 Models Assumptions

Even though there are different SPD model specifications, there are some basic common features for all of them. The following common assumptions will be used throughout the text for static SPD models. In addition to these, specific assumptions for some models will be listed when needed.

A1. Assumptions of Spatial Weights Matrices:

(1) The spatial weights matrix (W_N) is non-stochastic matrix with zero diagonals.
(2) The spatial transformation matrices (i.e., $(I_N - \lambda W_N)$) are invertible on the compact parameter spaces of spatial parameters λ and ρ.
(3) The admissible parameter space for the true spatial parameters λ and ρ is [-1, 1].
(4) Row sums of the matrices $W_N, (I_N - \lambda W_N)^{-1}$, and $(I_N - \rho W_N)^{-1}$, before W_N is row-standardized, are uniformly bounded (UB) in absolute values as N goes to infinity.

A2. Assumptions of the Error Components: The relevant disturbances, i.e. $\{\varepsilon_{it}\}, i = 1, \cdots, N$ and $t = 1, \cdots, T$ are $i.i.d$ across i and t with zero mean, and finite variance, and their higher than fourth moments exist, i.e., $E|\varepsilon_{it}|^{4+C} < \infty$ for some c>0.

A3. Assumptions on Covariates: The regressors X_t are non-stochastic and have full rank and their elements are UB in absolute value.

A4. Assumption of N and T: Most studies assume that N is large while T can be finite or large. The case of finite N and large T is of less interest as the incidental parameter problem doesn't occur in this situation.

These assumptions are frequently made in spatial econometrics. For cross-sectional models; see [14], or [15], among others. For panel data models; see [11], or [16], among others.

4 Spatial Weights Matrix

A spatial weights matrix is a representation of the spatial structure in a particular data. It is a key element in spatial models, which represents the spatial dependence structure between locations exogenously, see [17] and [18]. In other words, Anselin [19] mentioned that the weights matrix is the formal expression of spatial dependence between observations.

The problem of choosing the optimal weights matrix is still in the developing phase. In this paper, we focus only on two structures of the spatial weights matrix as follows:

Table 1. SPD Models with Different Combinations of Spatial Interactions

Type of Model		Spatial Interaction Effects	
		Term	Number
SLM	Spatial Lag Model	$W_{NT}y$	1
SEM	Spatial Error Model	$W_{NT}u$	1
SAC	Spatial Autoregressive Combined Model	$W_{NT}y$ & $W_{NT}u$	2
SLX	Spatial Lag of X Model	$W_{NT}X$	K
SDM	Spatial Durbin Model	$W_{NT}y$ & $W_{NT}X$	K+1
SDEM	Spatial Durbin Error Model	$W_{NT}X$ & $W_{NT}u$	K+1
GNS	General Nesting Spatial Model	$W_{NT}y$ & $W_{NT}X$ & $W_{NT}u$	K+2

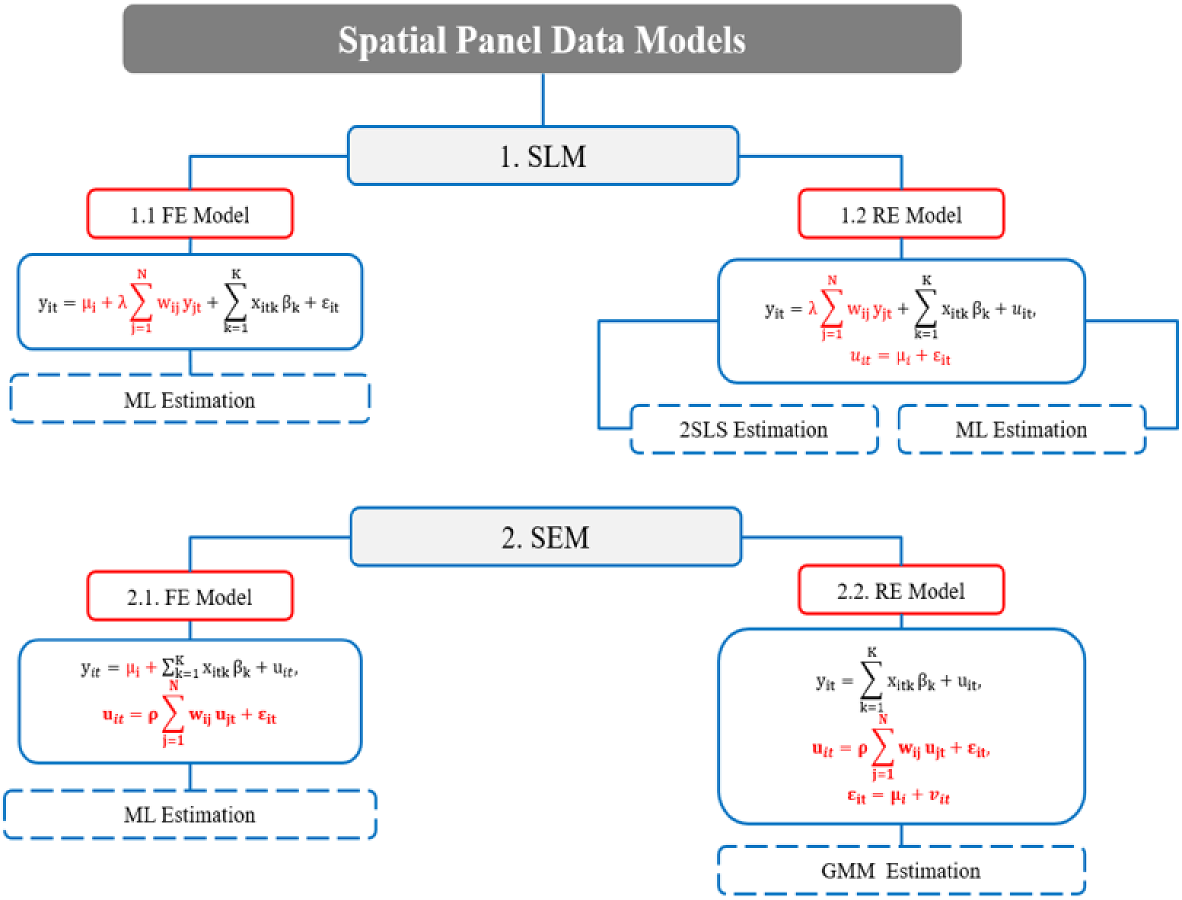


Fig. 1: SPD Models under Consideration.

***Note**: Where μ_i is called the time-invariant individual (spatial) effects or individual-specific effects, and v_{it} is independent and identically distributed $(i.i.d.)$ disturbances.*

(1) **The inverse distance weights:** This method relies on a simple transformation by taking the inverse of the distance separating the spatial units, and respects the Tobler's law: the weights are greater (smaller) as the units are spatially closer (further apart) as in the following equation:

$$w_{ij} = \begin{cases} 1/d_{ij} & \text{if } d_{ij} \leq \ddot{d} \ \forall\, i,j = 1, \dots, N \\ 0 & \forall\, i = j \text{ or } d_{ij} > \ddot{d} \end{cases} \quad (6)$$

where d_{ij} is the distance between region i and region j, and $\dddot{d}$ denotes a threshold distance (or bandwidth).

(2) **The inverse exponential distance weights:** Another possible transformation of the distance can be defined as:

$$w_{ij} = \begin{cases} 1/e^{d_{ij}} & \text{if } d_{ij} \leq \dddot{d}\ \forall\ i,j = 1,\dots,N \\ 0 & \forall\ i = j \text{ or } d_{ij} > \dddot{d} \end{cases} \quad (7)$$

This transformation gives more weights to spatially close units and fewer weights to units that are further apart, for more details, see [9].

5 Spatial Lag Panel Data Model

A SLM or spatial autoregressive (SAR) model includes a spatially lagged dependent variable on the right-hand side of the regression specification, as follows:

$$y_t = \lambda W_N y_t + X_t \beta + \varepsilon_t \quad (8)$$

The SLM can be treated as:

5.1 Fixed Effects

By adding the time-invariant individual FE, μ, to the model (8), the SLM can be rewritten after stacking the observations across individual and time as:

$$y = \lambda (I_T \otimes W_N) y + X\beta + (l_T \otimes I_N)\mu + \varepsilon, \quad (9)$$

where μ is (N×1) vector contains spatial specific effects, $\mu' = [\mu_1 \quad \mu_2 \quad \cdots \quad \mu_N]$, l_T is a (T×1) vector of ones and ⊗ is a Kronecker product. This model suffers from As well known, when N → ∞, there is no consistent estimator of the individual FE, due to the incidental parameter problem, in another words, the No. of parameters goes to ∞ when N goes to ∞. [20] used the transformation in (10) to eliminate the FE from the model (9) and using these transformed variables to estimate the parameters by using ML estimation.

$$Q_0 = I_{NT} - \left(I_N \otimes \frac{l_T l_T'}{T} \right) \quad (10)$$

The transformed model for (9) can be written as:

$$y^* = \lambda (I_T \otimes W_N) y^* + X^* \beta + \varepsilon^*, \quad (11)$$

where:

$$y^* = Q_0 y,\ X^* = Q_0 X,\ \text{and } \varepsilon^* = Q_0 \varepsilon \quad (12)$$

Besides the incidental parameter problem, the endogeneity of $\sum_{j=1}^{N} w_{ij}\, y_{jt}$ violates the assumption of the standard regression model that $E\left[\left(\sum_{j=1}^{N} w_{ij}\, y_{jt}\right) \varepsilon_{it}\right] = 0$. Therefore, the focus in this section will base on ML estimation because the MLE account for the endogeneity of $\sum_{j=1}^{N} w_{ij}\, y_{jt}$, also, the No. of researches considering IV/GMM estimators of SPD models is still sparse. In this context, Elhorst [10] suggested a concentrated likelihood function that can be maximized from the residuals e_0^* of the OLS regression of y^* on X^* and the residuals e_l^* of the OLS regression of $(I_T \otimes W_N) y^*$ on X^*. Then the MLE of λ is obtained by maximizing the following concentrated log-likelihood function:

$$\ln L(\lambda)_{concentrated} = C + T \ln|S_N| - \frac{NT}{2} \ln[(e_0^* - \lambda e_l^*)'(e_0^* - \lambda e_l^*)] \quad (13)$$

where C is a constant not depending on λ. This maximization problem is only solved numerically, since a closed-form solution for λ doesn't exist. Therefore, an iteration procedure must be used, which require λ to be initially fixed to calculate $\hat{\beta}$ and $\hat{\sigma}^2$. Finally, $\hat{\beta}$ and $\hat{\sigma}^2$ are obtained from the first-order conditions of the likelihood function by replacing λ with its numerically estimated value, see [17].

5.2 Random Effects

In contrast to the FE approach, the RE models do not have a problem with a large N. In this context, the SLM can be written in a stacked form across individual and time as:

$$y = \lambda (I_T \otimes W_N) y + X\beta + (l_T \otimes I_N)\mu + u, \quad u = (l_T \otimes I_N)\mu + \varepsilon \quad (14)$$

Assuming that, the unobserved individual effects, μ_i, are uncorrelated with the other explanatory variables in the model, and $\mu_i \sim i.i.d\ (0, \sigma_\mu^2)$.

Additionally, the idiosyncratic error term, $\varepsilon_{it} \sim i.i.d\ (0, \sigma_\varepsilon^2)$, an μ_i and ε_{it} independent from each other. The log-likelihood of the model (14) is:

$$\ln L = -\frac{NT}{2}\ln(2\pi\sigma^2) + T\ln|S_N| + \frac{N}{2}\ln\theta - \frac{1}{2\sigma^2}\sum_{i=1}^{N}\sum_{t=1}^{T}\left[y_{it}^{\bullet} - \lambda\left(\sum_{j=1}^{N} w_{ij}\, y_{jt}\right)^{\bullet} - x_{it}^{\bullet}\beta\right]^2 \quad (15)$$

where the transformed variables are defined as:

$$y_{it}^{\bullet} = y_{it} + \left(\sqrt{\theta} - 1\right)\bar{y}_i,$$
$$x_{it}^{\bullet} = x_{it}' + \left(\sqrt{\theta} - 1\right)\bar{X}_{i.}',$$
$$\left(\sum_{j=1}^{N} w_{ij}\, y_{jt}\right)^{\bullet} = \sum_{j=1}^{N} w_{ij}\, y_{jt} + \left(\sqrt{\theta} - 1\right)\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N} w_{ij} y_{jt} \quad (16)$$

and θ is defined as:

$$0 \le \theta = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + T\sigma_\mu^2} \le 1 \quad (17)$$

By using a similar procedure in SLM with FE, we can estimate β, λ and σ_ε^2, but the subscript * must be replaced by •. Given β, λ and $\sigma_\varepsilon^2, \sqrt{\theta}$ can be estimated by maximizing the concentrated log-likelihood function with respect to $\sqrt{\theta}$.

$$\ln L = -\frac{NT}{2}\ln(e^{\bullet\prime} e^{\bullet}) + \frac{N}{2}\ln\theta \quad (18)$$

where the element of $e^{\bullet}$ is defined as follows:

$$e_{it}^{\bullet} = y_{it}^{\bullet} - \lambda\left(\sum_{j=1}^{N} w_{ij}\, y_{jt}\right)^{\bullet} - X_{it}^{\bullet}\beta \quad (19)$$

6 Spatial Error Panel Data Model

The SAR specification for error vector u_t in time t can be defined as:

$$y_t = X_t\beta + u_t,$$
$$u_t = \rho W_N u_t + \varepsilon_t \quad (20)$$

The SEM can be treated as:

6.1 Fixed Effects

By adding the time-invariant individual FE, μ, to the model (20):

$$Y = X\beta + (l_T \otimes I_N)\mu + u,$$
$$u = \rho(I_T \otimes W_N)u + \varepsilon \quad (21)$$

To eliminate the individual FE, the model (21) is transformed according to the same Q_0-transformation which used for SLM and which defined in (10).

As mentioned in [21], the estimation procedure**s** of SEM with FE can be summarized as follows:

(1) Estimated OLS residuals of the transformed variable**s** can be used to obtain an initial estimate of ρ.

(2) The initial estimate of ρ can be used to compute a (spatial) feasible generalized least squares (FGLS) estimator of β and σ^2 and a new set of estimated GLS residuals.

(3) Then an iterative procedure can be used: the concentrated likelihood and the GLS estimators are alternately computed until convergence.

Lee and Yu [16] proved that the estimation of the SLM or SEM with spatial FE, which is based on the Q_0-transformation, produces biased estimates for σ^2 if N is large and T is fixed, and they called this procedure the direct approach. Starting with the SAC model, and using asymptotic theory, Lee and Yu [16] suggested two methods to obtain consistent results, as follows:

(1) **The first method:** Instead of demeaning, they proposed an alternative procedure to eliminate the spatial FE, reducing the number of observations available for estimation by one observation, i.e., from NT to N(T-1) observations. This procedure is called the transformation approach.

(2) **The second method:** It is a bias correction procedure for the parameter's estimates obtained by the direct approach based on ML function that is obtained under the transformation approach. The biases of the SAC model [16] can be conducted on the SLM and SEM models. Where the $\hat{\sigma}^2$ of σ^2 obtained by

the direct approach will be biased. This bias can easily be corrected by:

$$\widehat{\sigma}^2_{\text{bias corrected}} = \frac{T}{T-1}\widehat{\sigma}^2_{\text{direct}} \tag{22}$$

This bias correction will have no any effect if T is large, for more details, see [22].

6.2 Random Effects

In this section, we focus on the approach of Kapoor et al. [11] for specifying the SEM with RE which can be written after stacking across individual and time as follows:

$$\begin{aligned} y &= X\beta + u, \\ u &= \rho(I_T \otimes W_N)u + \varepsilon, \\ \varepsilon &= (l_T \otimes I_N)\mu + v \end{aligned} \tag{23}$$

where μ is a (N×1) vector of cross-sectional random components, $v_{it} \sim i.i.d\,(0, \sigma_v^2)$, and the vectors μ and v are independent of each other and the regressor matrix X. The second line in (23) can be written in a reduced form as follows:

$$\begin{aligned} u &= \left(I_T \otimes B_N^{-1}\right)\varepsilon \\ &= \left(I_T \otimes B_N^{-1}\right)[(l_T \otimes I_N)\mu + v] \end{aligned} \tag{24}$$

The corresponding error VCM is:

$$\begin{aligned} \Omega_u &= E(uu') \\ &= \left(I_T \otimes B_N^{-1}\right)\Omega_\varepsilon\left(I_T \otimes B_N^{-1\prime}\right) \end{aligned} \tag{25}$$

where Ω_ε is VCM of ε. Since μ and v are independent, it implies that:

$$\begin{aligned} \Omega_\varepsilon &= \sigma_\mu^2(l_T \otimes I_N)(l_T \otimes I_N)' + \sigma_v^2 I_{NT} \\ &= \sigma_\mu^2(l_T l_T' \otimes I_N) + \sigma_v^2 I_{NT} \\ &= T\sigma_\mu^2 Q_1 + \sigma_v^2 I_{NT} \end{aligned} \tag{26}$$

where Q_0 is defined in (10) and Q_1 is defined as:

$$Q_1 = I_N \otimes \frac{l_T l_T'}{T} \tag{27}$$

Since $Q_0 + Q_1 = I_{NT}$, then:

$$\Omega_\varepsilon = \sigma_v^2 Q_0 + \sigma_1^2 Q_1 \tag{28}$$

$$\sigma_1^2 = \sigma_v^2 + T\sigma_\mu^2 \tag{29}$$

Kapoor et al. [11] proposed a generalization of generalized method of moments (GMM) estimator provided in [23] for estimating the SAR parameter ρ and the two variance components of the disturbance process σ_1^2 and σ_v^2. Therefore, to estimate the model (23), Kapoor et al. [11] defined three sets of GMM estimators based on the following moment conditions for T ≥ 2:

$$E\begin{bmatrix} \frac{1}{N(T-1)}\varepsilon' Q_0 \varepsilon \\ \frac{1}{N(T-1)}\bar{\varepsilon}' Q_0 \bar{\varepsilon} \\ \frac{1}{N(T-1)}\bar{\varepsilon}' Q_0 \varepsilon \\ \frac{1}{N}\varepsilon' Q_1 \varepsilon \\ \frac{1}{N}\bar{\varepsilon}' Q_1 \bar{\varepsilon} \\ \frac{1}{N}\bar{\varepsilon}' Q_1 \varepsilon \end{bmatrix} = \begin{bmatrix} \sigma_v^2 \\ \sigma_v^2 \frac{1}{N}\mathrm{tr}(W_{NT}' W_{NT}) \\ 0 \\ \sigma_1^2 \\ \sigma_1^2 \frac{1}{N}\mathrm{tr}(W_{NT}' W_{NT}) \\ 0 \end{bmatrix} \tag{30}$$

where:

$$\begin{aligned} \varepsilon = u - \rho\bar{u}, \qquad \bar{\varepsilon} &= \bar{u} - \rho\bar{\bar{u}}, \\ \bar{u} &= (I_T \otimes W_N)u, \\ \bar{\bar{u}} &= (I_T \otimes W_N)\bar{u} \end{aligned} \tag{31}$$

To estimate $\delta^{SEM} = \begin{bmatrix} \beta \\ \rho \\ \sigma_v^2 \\ \sigma_1^2 \end{bmatrix}$, follow the steps:

(1) The First Step:
- The first three moment conditions can be used to obtain the first set of GMM estimators for ρ and σ_v^2.
- The initial estimates obtained ($\hat{\rho}$ and $\widehat{\sigma}_v^2$) are then used to provide an estimate for $\sigma_1^2 = \sigma_v^2 + T\sigma_\mu^2$ based on the fourth-moment condition.

(2) The Second Step: Under the normality assumption of innovation ε_{it}, Kapoor et al. [11] derived the VCM of the sample moments at the true parameter values Ę, whose inverse is to be used as the optimal weighting matrix in a GMM estimator.
The second set of GMM estimators is then defined as the nonlinear least squares estimators

based on all moment conditions weighted by the optimal weighting scheme $\hat{\Xi}^{-1}$.

(3) The Third Step: the third set of GMM estimators is suggested because of computational considerations and is based on a simpler weighting matrix. The third set of GMM estimators uses all moment conditions but a simplified weighting scheme.

(4) The Fourth Step: The FGLS for β can be obtained based on consistent estimates for $\rho, \sigma_{\nu}^2,$ and σ_{μ}^2 that result from previous steps.

7 Monte Carlo Simulation Study

In this section, we focus on trying to achieve two main objectives in two specifications of SPD models, i.e., SLM and SEM with FE, as follows:

(1) Comparing between the finite sample properties of the spatial MLEs (transformation approach), which will be referred to in an abbreviated manner as (the spatial estimator), and the non-spatial OLS within-group estimator, or in a short way (the non-spatial estimator), under different values for temporal and cross-sectional dimensions (N and T), spatial parameters, and spatial weights matrix.

(2) Verifying the impact of the spatial weights structure on the goodness of fit model and performance of estimates in the used SPD models. Where some researches indicate that the choice of the spatial weights matrices are crucial and can affect the findings of the research [24].

7.1 Design of the Simulation

We relied on the general methodology of MCS for Mooney [25], which has been mentioned in most empirical studies, see e.g., [26]. Fig. 2 provides a summary of the used algorithm in our simulation study.

7.2 Simulation Results

This section summarizes the results obtained from our simulation described previously. Each model focuses on the comparison between the non-spatial and spatial estimators in terms of bias and RMSE of $\hat{\beta}$, in addition to, provides the comparison among the two structures of spatial weights matrix in terms of bias and RMSE of $\hat{\lambda}$. Besides, it furnishes information on the goodness of fit criteria for each model and the relationship between biases of ignoring spatial and the degree of spatial dependence in the data.

Table 2. Design of Our MCS

Design Factor	Levels	No. of Levels
Type of SPD Models	SLM and SEM	2
Value of Parameter λ & ρ	$\{\mp 0.2, \mp 0.8\}$	4
Type of Weights Matrix for Data Generation	W_1 or W_2	2
No. of Spatial Units	N={5, 20, 35, 60}	4
No. of Time-periods	T={10, 30, 50}	3
Coordinates of Distance	Coordinates of d_{ij}~ Uniform $(0, 2N)$	1
Spatial FE	$\mu_i \sim N(5, 0.25)$	1
β	5	1
Explanatory Variable	$X_{it} \sim N(0,\ 1)$	1
Error Terms	$\varepsilon_{it} \sim N(0,\ 1)$	1
No. of Unique Simulations	**SLM:** 4 (values of λ) × 2 (types of W) × 4 (values of N) × 3 (values of T) = 96 Simulations. **SEM:** 4 (values of ρ) × 2 (types of W) × 4 (values of N) × 3 (values of T) = 96 Simulations.	

No. of Replications	R=1000
Total No. of Simulations	**SLM:** 96×1000= 96000 **SEM:** 96 ×1000= 96000

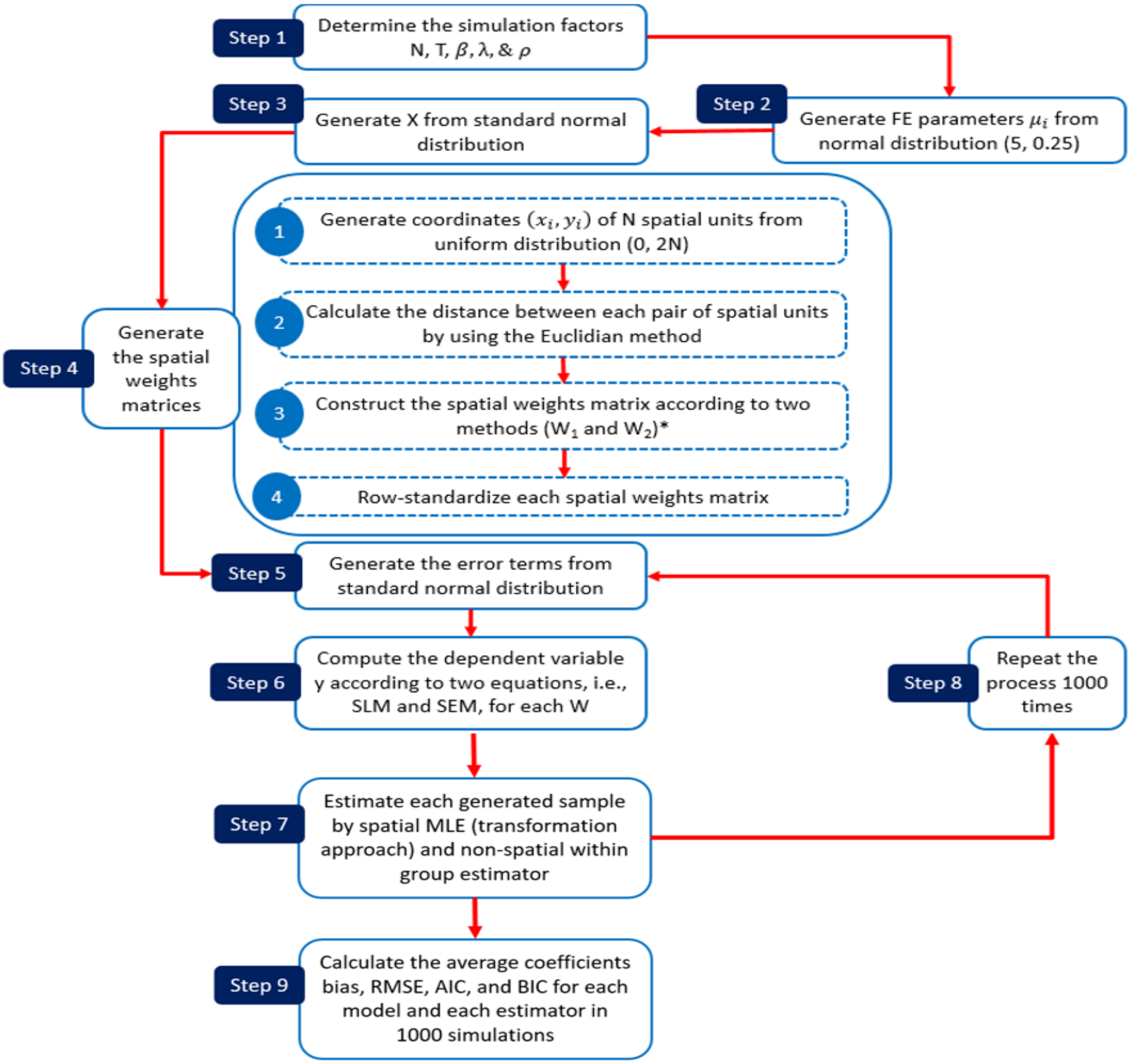


Fig. 2: Algorithm of Our Simulation Study

**Note*: W_1: Inverse distance weights and W_2: inverse exponential distance weights.*

7.2.1 Simulation Results of Spatial Lag Model

The findings show that the bias and RMSE of $\hat{\beta}$ resulting from ignoring the presence of spatial dependence in SLM is a function of the degree or magnitude of spatial dependence in the data. In other words, If the spatial dependence is small, i.e., $\lambda = \{-0.2, 0.2\}$, then the consequences of choosing the non-spatial estimator are not great, where the non-spatial bias and RMSE of $\hat{\beta}$ may be equal to or less than the spatial bias and RMSE of $\hat{\beta}$ in some cases. Quite the contrary, the non-spatial estimator choice definitely brings dire consequences in terms of bias and RMSE of $\hat{\beta}$ when λ is large, see Table 3 - Table 6. On average, the spatial estimator of $\hat{\beta}$ performs mostly satisfactory, where it produces a 19.32% bias less than the non-spatial estimator when λ is small. This percentage reaches 82.02% when λ is large. While, it produces a 19.41% RMSE less than those of the non-spatial estimator when λ is small. This percentage reaches 80.58% when λ is large.

On the other hand, i.e. model-level, the spatial Akaike information criterion (AIC) and Bayesian information criterion (BIC) are always less than their non-spatial counterparts when λ is large. Additionally, the spatial model always performs better than the non-spatial model when N and T are small regardless of the spatial dependence strength and the structure of the spatial weights matrix.

Table 3. Simulation Results of SLM and N = 5

W	T	λ	Spatial Estimator						Non-spatial Estimator			
			Bias $\hat{\lambda}$	RMSE $\hat{\lambda}$	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC
W_1	10	-0.2	0.146	0.153	0.211	0.241	156.1	161.9	0.229	0.261	158.8	172.2
		-0.8	0.163	0.171	0.310	0.339	158.3	164.0	1.295	1.306	215.6	229.0
		0.2	0.110	0.115	0.150	0.182	154.3	160.0	0.175	0.207	159.2	172.6
		0.8	0.031	0.032	0.111	0.139	151.6	157.3	0.473	0.510	342.3	355.7
	30	-0.2	0.074	0.081	0.074	0.092	490.8	499.8	0.119	0.140	466.2	487.2
		-0.8	0.080	0.089	0.099	0.122	493.7	502.7	0.869	0.874	642.3	663.4
		0.2	0.057	0.062	0.065	0.081	487.6	496.6	0.067	0.084	480.5	501.6
		0.8	0.016	0.018	0.079	0.099	484.7	493.7	2.329	2.335	1080.2	1101.2
	50	-0.2	0.083	0.087	0.073	0.088	815.8	826.3	0.147	0.160	771.0	795.7
		-0.8	0.084	0.089	0.113	0.128	821.7	832.2	1.010	1.012	1077.8	1102.5
		0.2	0.065	0.068	0.052	0.064	810.3	820.9	0.078	0.093	795.8	820.4
		0.8	0.019	0.020	0.054	0.068	803.9	814.4	1.497	1.501	1783.2	1807.8
W_2	10	-0.2	0.117	0.123	0.209	0.240	157.4	163.1	0.259	0.289	162.6	176.0
		-0.8	0.104	0.111	0.286	0.317	160.0	165.7	1.656	1.666	247.5	260.9
		0.2	0.094	0.099	0.151	0.183	155.2	161.0	0.188	0.219	163.1	176.5
		0.8	0.027	0.028	0.112	0.140	152.8	158.5	0.495	0.533	354.5	367.9
	30	-0.2	0.049	0.057	0.071	0.089	492.0	501.0	0.131	0.152	476.6	497.6
		-0.8	0.042	0.049	0.083	0.104	494.0	503.0	1.105	1.110	716.0	737.1
		0.2	0.042	0.048	0.065	0.081	489.1	498.1	0.067	0.084	491.4	512.5
		0.8	0.013	0.015	0.076	0.095	486.3	495.4	2.441	2.448	1109.9	1131.0
	50	-0.2	0.060	0.063	0.068	0.082	817.5	828.0	0.171	0.182	792.1	816.8
		-0.8	0.047	0.051	0.090	0.105	822.1	832.7	1.312	1.314	1222.4	1247.1
		0.2	0.051	0.053	0.052	0.064	812.6	823.1	0.088	0.102	817.8	842.4
		0.8	0.015	0.016	0.051	0.064	807.2	817.8	1.460	1.465	1842.5	1867.2

Table 4. Simulation Results of SLM and N = 20

W	T	λ	Spatial Estimator						Non-spatial Estimator			
			Bias $\hat{\lambda}$	RMSE $\hat{\lambda}$	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC
W_1	10	-0.2	0.180	0.185	0.068	0.085	644.9	654.8	0.072	0.089	608.7	681.3
		-0.8	0.142	0.150	0.089	0.106	653.2	663.1	0.402	0.409	791.3	863.9
		0.2	0.169	0.172	0.059	0.073	635.0	644.9	0.061	0.076	599.9	672.4
		0.8	0.064	0.064	0.059	0.074	619.4	629.3	0.062	0.077	798.0	870.5
	30	-0.2	0.195	0.197	0.074	0.082	1941.7	1954.9	0.065	0.074	1788.2	1884.9
		-0.8	0.156	0.158	0.111	0.118	1973.8	1987.0	0.459	0.461	2355.9	2452.6
		0.2	0.176	0.177	0.039	0.048	1912.8	1925.9	0.038	0.046	1791.6	1888.3
		0.8	0.062	0.062	0.042	0.052	1868.7	1881.9	0.375	0.378	3358.8	3455.5
	50	-0.2	0.142	0.143	0.024	0.031	3291.3	3306.0	0.048	0.055	2970.2	3078.2
		-0.8	0.110	0.112	0.034	0.041	3320.8	3335.5	0.392	0.393	3915.9	4023.8
		0.2	0.131	0.132	0.033	0.040	3262.2	3276.9	0.025	0.031	2997.9	3105.9
		0.8	0.047	0.047	0.076	0.082	3219.1	3233.8	0.750	0.751	6010.7	6118.7
W_2	10	-0.2	0.032	0.035	0.066	0.083	656.9	666.8	0.162	0.176	728.0	800.6
		-0.8	0.010	0.012	0.084	0.102	659.9	669.8	4.222	4.226	1509.7	1582.3
		0.2	0.033	0.035	0.058	0.073	652.3	662.1	0.067	0.083	727.8	800.4

		0.8	0.010	0.011	0.061	0.077	648.5	658.4	3.677	3.681	1568.7	1641.3
	30	-0.2	0.034	0.035	0.062	0.071	1987.6	2000.8	0.185	0.190	2126.0	2222.7
		-0.8	0.009	0.009	0.084	0.094	1998.6	2011.8	4.663	4.664	4490.6	4587.3
		0.2	0.038	0.039	0.034	0.042	1972.2	1985.4	0.045	0.054	2132.2	2228.9
		0.8	0.015	0.016	0.054	0.064	1948.0	1961.2	3.800	3.802	4631.2	4727.9
	50	-0.2	0.026	0.027	0.024	0.030	3325.7	3340.4	0.162	0.165	3505.0	3613.0
		-0.8	0.006	0.007	0.027	0.033	3337.9	3352.7	4.199	4.199	7378.2	7486.2
		0.2	0.030	0.031	0.037	0.044	3306.1	3320.9	0.056	0.063	3530.2	3638.1
		0.8	0.013	0.013	0.087	0.093	3273.6	3288.3	3.944	3.945	7728.5	7836.5

Table 5. Simulation Results of SLM and N = 35

W	T	λ	Spatial Estimator						Non-spatial Estimator			
			Bias $\hat{\lambda}$	RMSE $\hat{\lambda}$	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC
W_1	10	-0.2	0.281	0.284	0.094	0.107	1111.8	1123.4	0.072	0.084	1050.4	1193.1
		-0.8	0.235	0.239	0.124	0.136	1133.1	1144.7	0.388	0.392	1271.7	1414.4
		0.2	0.258	0.259	0.064	0.077	1089.1	1100.7	0.058	0.071	1040.7	1183.4
		0.8	0.094	0.094	0.047	0.060	1055.8	1067.4	0.069	0.083	1337.2	1479.9
	30	-0.2	0.223	0.224	0.028	0.035	3430.8	3445.6	0.046	0.054	3080.3	3263.7
		-0.8	0.182	0.184	0.043	0.051	3471.0	3485.8	0.293	0.295	3731.5	3914.9
		0.2	0.213	0.214	0.024	0.031	3389.1	3403.9	0.031	0.039	3072.6	3256.0
		0.8	0.086	0.086	0.056	0.063	3312.7	3327.6	0.068	0.075	4666.9	4850.3
	50	-0.2	0.176	0.177	0.032	0.038	5770.8	5787.2	0.034	0.040	5109.9	5312.2
		-0.8	0.142	0.143	0.022	0.027	5815.5	5831.9	0.242	0.243	6186.3	6388.6
		0.2	0.175	0.175	0.051	0.055	5727.4	5743.8	0.020	0.025	5110.3	5312.6
		0.8	0.077	0.077	0.084	0.087	5624.9	5641.3	0.214	0.215	8192.9	8395.2
W_2	10	-0.2	0.035	0.037	0.066	0.079	1146.9	1158.5	0.224	0.231	1280.1	1422.9
		-0.8	0.008	0.009	0.077	0.092	1153.5	1165.1	4.998	5.001	2741.9	2884.7
		0.2	0.044	0.045	0.048	0.059	1135.2	1146.7	0.048	0.060	1255.7	1398.4
		0.8	0.019	0.019	0.060	0.073	1120.5	1132.1	2.646	2.650	2683.2	2826.0
	30	-0.2	0.015	0.016	0.025	0.032	3490.6	3505.5	0.177	0.180	3768.2	3951.6
		-0.8	0.004	0.004	0.027	0.034	3497.8	3512.6	4.776	4.777	8097.2	8280.5
		0.2	0.017	0.018	0.030	0.037	3480.3	3495.2	0.037	0.044	3762.2	3945.6
		0.8	0.005	0.006	0.046	0.054	3475.4	3490.3	3.423	3.424	8223.3	8406.7
	50	-0.2	0.014	0.014	0.045	0.050	5826.4	5842.8	0.138	0.140	6200.7	6403.0
		-0.8	0.003	0.004	0.039	0.046	5838.3	5854.7	4.355	4.355	13290.4	13492.7
		0.2	0.016	0.016	0.060	0.064	5815.7	5832.1	0.076	0.079	6223.6	6425.9
		0.8	0.006	0.006	0.082	0.086	5797.5	5813.9	3.967	3.968	13711.6	13913.9

Table 6. Simulation Results of SLM and N = 60

W	T	λ	Spatial Estimator						Non-spatial Estimator			
			Bias $\hat{\lambda}$	RMSE $\hat{\lambda}$	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC
W_1	10	-0.2	0.407	0.409	0.061	0.071	1875.2	1888.4	0.037	0.045	1785.3	2057.9
		-0.8	0.387	0.389	0.091	0.100	1921.1	1934.3	0.149	0.155	2028.1	2300.7
		0.2	0.360	0.361	0.038	0.048	1830.6	1843.8	0.032	0.041	1779.0	2051.6
		0.8	0.122	0.122	0.034	0.043	1764.7	1777.9	0.042	0.052	2060.6	2333.2
	30	-0.2	0.311	0.312	0.027	0.032	5859.6	5876.1	0.023	0.029	5236.3	5577.0
		-0.8	0.268	0.270	0.018	0.023	5938.3	5954.8	0.141	0.143	5957.6	6298.3
		0.2	0.302	0.303	0.042	0.047	5775.9	5792.4	0.018	0.023	5217.4	5558.1
		0.8	0.120	0.120	0.041	0.046	5607.2	5623.7	0.068	0.071	6065.9	6406.7
	50	-0.2	0.355	0.355	0.015	0.018	9641.3	9659.4	0.018	0.022	8695.1	9067.5
		-0.8	0.328	0.329	0.037	0.041	9830.3	9848.3	0.125	0.126	9958.6	10331.0
		0.2	0.319	0.319	0.024	0.028	9471.0	9489.0	0.014	0.018	8688.5	9060.9
		0.8	0.113	0.113	0.050	0.053	9173.9	9191.9	0.128	0.129	12227.8	12600.2
W_2	10	-0.2	0.030	0.031	0.054	0.064	1969.5	1982.7	0.159	0.164	2250.7	2523.3
		-0.8	0.007	0.008	0.083	0.094	1982.3	1995.5	6.479	6.481	4784.6	5057.2
		0.2	0.034	0.035	0.032	0.040	1952.3	1965.5	0.159	0.164	2256.7	2529.3
		0.8	0.013	0.013	0.082	0.093	1928.3	1941.5	6.619	6.620	4855.7	5128.3
	30	-0.2	0.018	0.018	0.037	0.043	5992.1	6008.6	0.109	0.111	6533.1	6873.9

	-0.8	0.004	0.005	0.022	0.028	6014.0	6030.4	5.810	5.811	14017.7	14358.4
	0.2	0.020	0.021	0.063	0.067	5970.1	5986.5	0.194	0.195	6567.5	6908.2
	0.8	0.008	0.008	0.109	0.112	5930.0	5946.5	6.770	6.770	14356.6	14697.3
50	-0.2	0.023	0.023	0.016	0.020	10005.4	10023.4	0.130	0.131	10856.8	11229.2
	-0.8	0.006	0.007	0.044	0.049	10059.1	10077.2	6.040	6.040	23327.3	23699.7
	0.2	0.023	0.024	0.026	0.031	9962.1	9980.1	0.177	0.178	10922.2	11294.6
	0.8	0.008	0.008	0.073	0.076	9896.2	9914.2	6.662	6.663	23906.3	24278.7

As for the level of comparison between the spatial weight structures, Fig. 3 displays the pattern of spatial bias and RMSE of $\hat{\beta}$ under the influence of N and T for each structure of the spatial weights ignoring the values of λ. The results indicate that each structure of the weights matrix has a specific pattern of the spatial bias and RMSE of $\hat{\beta}$. In all structures, the pattern of spatial bias and RMSE of $\hat{\beta}$ are irregular with increasing N or T, however, it can be argued that the spatial estimator produces a large bias and small RMSE when T is small.

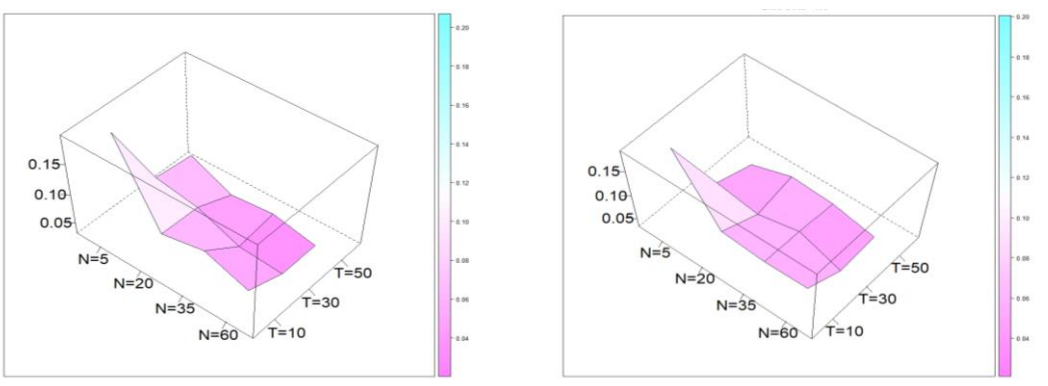
On the other hand; Fig. 4 displays the spatial $\hat{\lambda}$ bias under the influence of N and λ for each structure of the spatial weights ignoring the values of T. our conclusion from this Figure can be pointed that each structure of spatial weights has a specific pattern of the spatial $\hat{\lambda}$ bias according to the used values of N and λ. Interestingly, the result of the spatial $\hat{\lambda}$ bias is in favor of W_2.

Fig. 5 shows the behavior of the spatial $\hat{\lambda}$ RMSE across different combinations of N, T, and the structure of the weights matrix. The results confirm that the results of $\hat{\lambda}$ RMSE are always in favor of W_2.

For a comparison between the two structures of weights matrix in terms of the spatial AIC and BIC. The results show that the two structures of weights matrix seem to provide very similar levels of spatial AIC and BIC, However, the SLM based on W_2 returns a slightly higher value of spatial AIC and BIC in all cases, see Table 3 - Table 6.

W1 W$_2$

Bias



RMSE

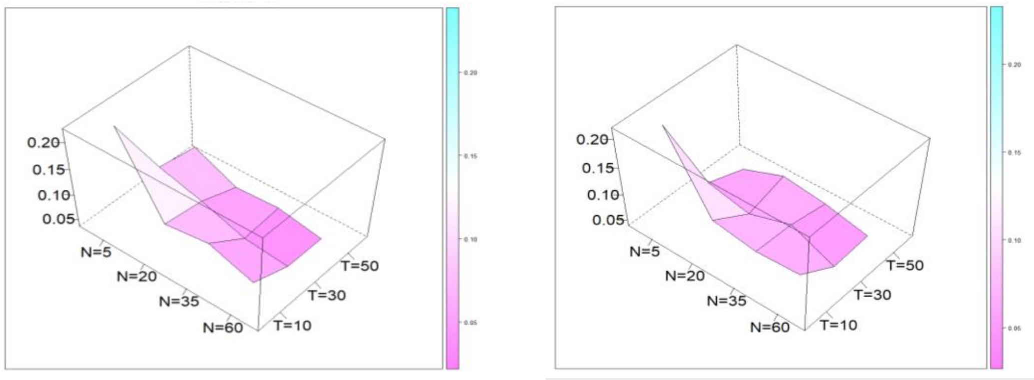


Fig. 3: The Spatial bias and RMSE of $\hat{\beta}$ in SLM at Different Values of N, T, and W for All λ

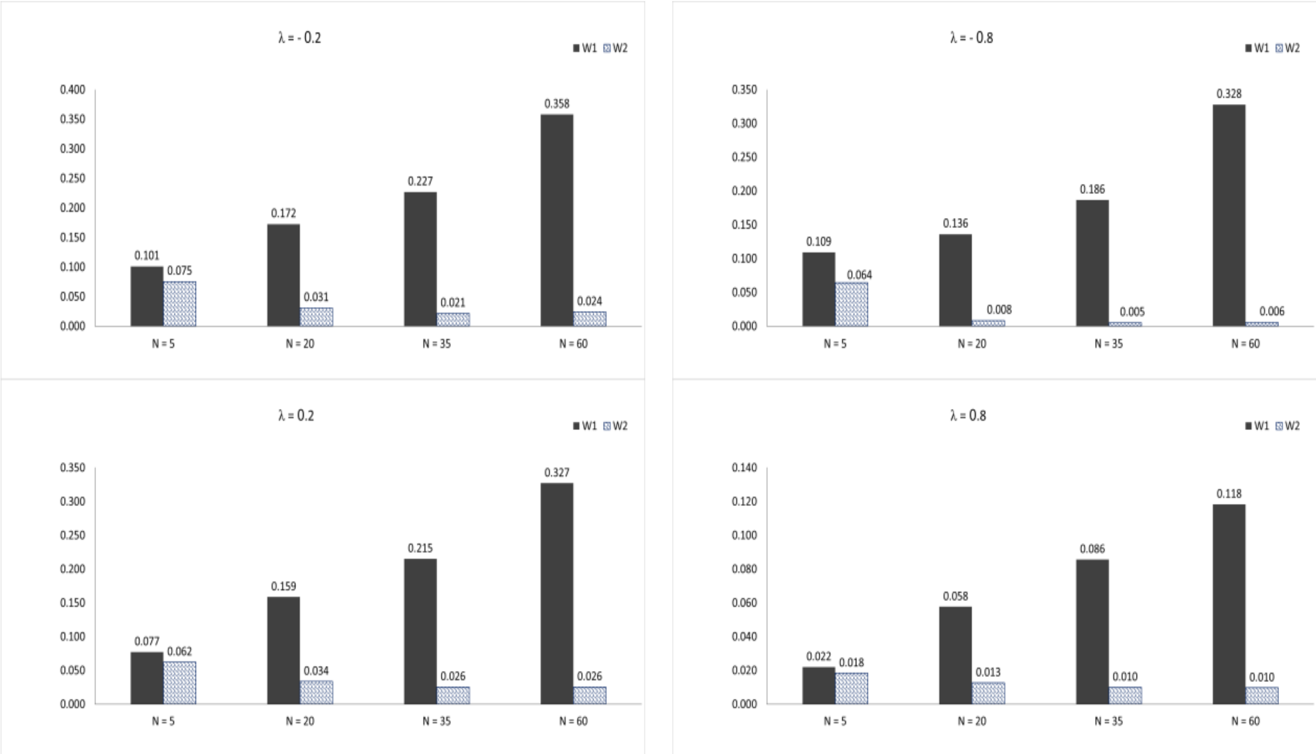


Fig. 4: $\hat{\lambda}$ Bias in SLM at Different Values of N, λ, and W for All T

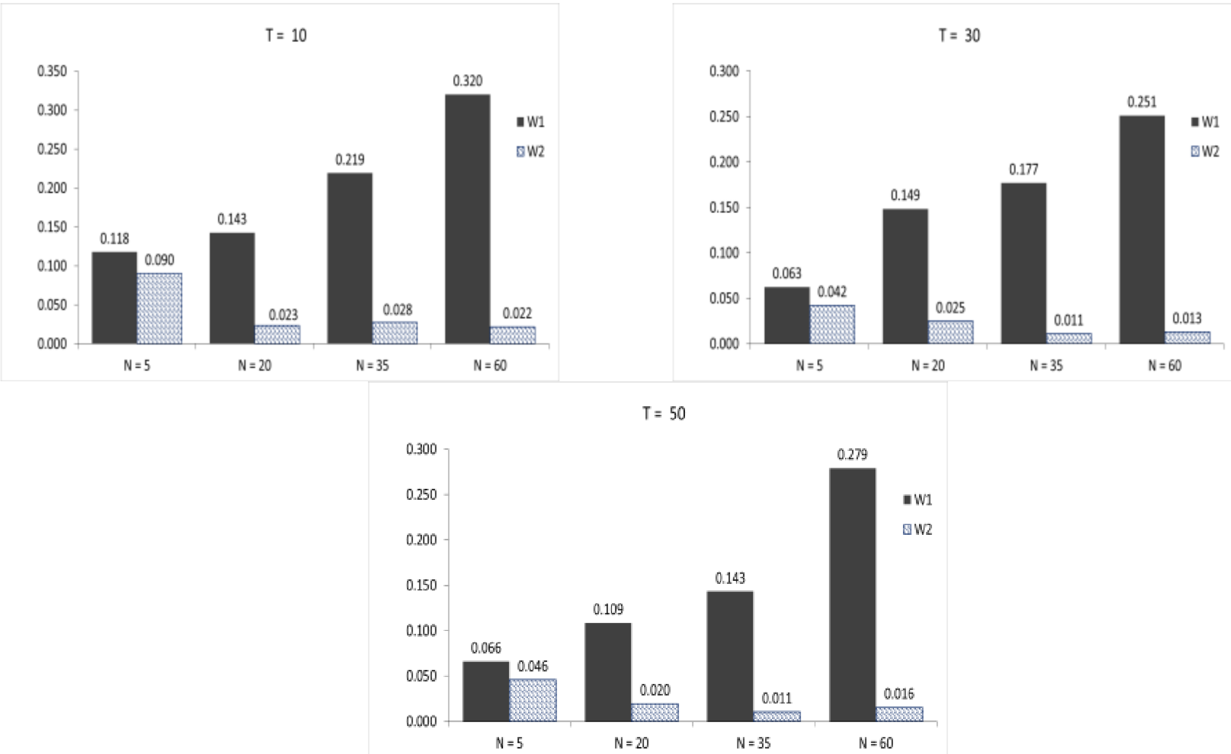


Fig. 5: $\hat{\lambda}$ RMSE in SLM at Different Values of N, T, and W for All λ

Table 7. Simulation Results of SEM and N = 5

W	T	ρ	Spatial Estimator						Non-spatial Estimator			
			Bias $\hat{\rho}$	RMSE $\hat{\rho}$	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC
W_1	10	-0.2	0.202	0.259	0.101	0.127	144.4	150.1	0.107	0.134	152.3	165.7
		-0.8	0.208	0.261	0.098	0.122	143.0	148.7	0.137	0.172	167.8	181.2
		0.2	0.152	0.197	0.102	0.126	144.7	150.4	0.101	0.127	148.1	161.5
		0.8	0.043	0.058	0.091	0.113	144.6	150.4	0.161	0.203	207.7	221.1
	30	-0.2	0.104	0.130	0.068	0.087	428.1	437.2	0.071	0.089	438.6	459.7
		-0.8	0.112	0.140	0.063	0.079	427.8	436.8	0.082	0.101	486.0	507.1
		0.2	0.076	0.096	0.063	0.079	428.0	437.0	0.064	0.082	435.7	456.7
		0.8	0.023	0.030	0.056	0.072	428.3	437.3	0.136	0.171	675.6	696.7
	50	-0.2	0.081	0.104	0.050	0.062	712.6	723.2	0.051	0.063	726.2	750.8
		-0.8	0.089	0.111	0.048	0.060	709.7	720.2	0.057	0.071	801.8	826.4
		0.2	0.061	0.077	0.044	0.054	711.5	722.1	0.045	0.056	723.3	747.9
		0.8	0.016	0.021	0.041	0.051	712.0	722.5	0.086	0.111	1141.9	1166.5
W_2	10	-0.2	0.179	0.223	0.101	0.127	144.5	150.3	0.108	0.135	152.6	165.9
		-0.8	0.160	0.200	0.095	0.118	143.5	149.3	0.148	0.186	173.2	186.6
		0.2	0.140	0.180	0.102	0.126	144.7	150.4	0.101	0.126	148.3	161.7
		0.8	0.042	0.057	0.089	0.110	144.6	150.3	0.167	0.211	211.2	224.6
	30	-0.2	0.094	0.117	0.068	0.087	428.2	437.3	0.071	0.090	439.3	460.4
		-0.8	0.087	0.108	0.062	0.077	428.2	437.3	0.088	0.108	503.1	524.1
		0.2	0.072	0.090	0.062	0.079	428.0	437.0	0.064	0.082	436.3	457.4
		0.8	0.022	0.029	0.055	0.070	428.2	437.3	0.143	0.180	687.0	708.1
	50	-0.2	0.073	0.092	0.050	0.062	712.8	723.3	0.051	0.064	727.3	752.0
		-0.8	0.067	0.084	0.047	0.058	710.2	720.8	0.061	0.076	830.4	855.0
		0.2	0.057	0.072	0.044	0.054	711.6	722.1	0.045	0.056	724.3	748.9
		0.8	0.016	0.020	0.040	0.050	711.9	722.5	0.089	0.114	1161.0	1185.7

Table 8. Simulation Results of SEM and N = 20

W	T	ρ	Spatial Estimator						Non-spatial Estimator			
			Bias $\hat{\rho}$	RMSE $\hat{\rho}$	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC
W_1	10	-0.2	0.135	0.169	0.055	0.070	570.1	580.0	0.054	0.068	594.6	667.1
		-0.8	0.131	0.162	0.053	0.066	569.5	579.4	0.058	0.073	629.3	701.9
		0.2	0.116	0.146	0.054	0.067	568.7	578.6	0.052	0.065	584.8	657.4
		0.8	0.042	0.056	0.051	0.064	570.7	580.6	0.054	0.069	601.9	674.5
	30	-0.2	0.075	0.095	0.033	0.040	1705.8	1719.0	0.033	0.041	1737.6	1834.4
		-0.8	0.074	0.093	0.030	0.036	1705.4	1718.5	0.033	0.040	1842.3	1939.0
		0.2	0.063	0.079	0.031	0.039	1706.8	1720.0	0.031	0.039	1726.1	1822.9
		0.8	0.021	0.026	0.030	0.038	1705.2	1718.4	0.041	0.051	2018.2	2115.0
	50	-0.2	0.057	0.071	0.023	0.029	2839.8	2854.5	0.023	0.029	2877.0	2984.9
		-0.8	0.060	0.074	0.024	0.030	2840.4	2855.1	0.027	0.035	3049.6	3157.6
		0.2	0.046	0.059	0.024	0.030	2839.6	2854.3	0.024	0.030	2864.0	2972.0
		0.8	0.016	0.020	0.023	0.029	2840.6	2855.3	0.035	0.044	3485.8	3593.8
W_2	10	-0.2	0.051	0.063	0.055	0.070	570.4	580.3	0.056	0.070	606.8	679.4
		-0.8	0.019	0.024	0.046	0.058	569.9	579.8	0.127	0.160	977.0	1049.6
		0.2	0.051	0.064	0.053	0.066	568.5	578.4	0.053	0.067	599.0	671.5
		0.8	0.017	0.022	0.045	0.056	570.1	579.9	0.129	0.161	983.5	1056.0
	30	-0.2	0.028	0.035	0.032	0.040	1706.1	1719.3	0.034	0.042	1776.0	1872.7
		-0.8	0.011	0.014	0.025	0.031	1705.6	1718.7	0.080	0.101	2880.2	2976.9
		0.2	0.028	0.035	0.031	0.039	1706.7	1719.9	0.033	0.040	1765.3	1862.0
		0.8	0.009	0.012	0.025	0.032	1704.9	1718.1	0.093	0.116	2957.5	3054.2
	50	-0.2	0.022	0.027	0.023	0.028	2840.0	2854.7	0.024	0.030	2940.6	3048.6
		-0.8	0.009	0.011	0.021	0.026	2840.9	2855.7	0.065	0.082	4776.2	4884.2
		0.2	0.020	0.026	0.023	0.029	2839.4	2854.1	0.025	0.031	2928.9	3036.9
		0.8	0.008	0.010	0.019	0.025	2840.5	2855.2	0.071	0.089	4947.0	5054.9

Table 9. Simulation Results of SEM and N = 35

W	T	ρ	Spatial Estimator						Non-spatial Estimator			
			Bias $\hat{\rho}$	RMSE $\hat{\rho}$	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC
W_1	10	-0.2	0.127	0.159	0.041	0.052	997.1	1008.7	0.042	0.052	1035.4	1178.1
		-0.8	0.140	0.173	0.041	0.051	995.0	1006.6	0.042	0.053	1072.0	1214.7
		0.2	0.114	0.146	0.042	0.053	994.1	1005.7	0.040	0.050	1025.5	1168.2
		0.8	0.041	0.054	0.043	0.053	997.8	1009.4	0.045	0.057	1067.4	1210.2
	30	-0.2	0.072	0.089	0.024	0.030	2981.8	2996.6	0.024	0.030	3030.6	3214.0
		-0.8	0.076	0.094	0.023	0.029	2981.9	2996.8	0.025	0.031	3141.4	3324.8
		0.2	0.060	0.074	0.024	0.030	2981.1	2996.0	0.024	0.030	3012.7	3196.1
		0.8	0.020	0.026	0.024	0.030	2984.5	2999.4	0.026	0.033	3258.7	3442.1
	50	-0.2	0.055	0.069	0.019	0.023	4968.4	4984.8	0.019	0.024	5024.1	5226.3
		-0.8	0.058	0.072	0.018	0.023	4967.1	4983.5	0.020	0.025	5205.0	5407.3
		0.2	0.044	0.055	0.019	0.024	4968.9	4985.3	0.019	0.024	5004.0	5206.3
		0.8	0.016	0.020	0.019	0.023	4973.0	4989.4	0.022	0.027	5583.4	5785.7
W_2	10	-0.2	0.039	0.048	0.040	0.050	997.3	1008.9	0.043	0.054	1060.1	1202.9
		-0.8	0.015	0.018	0.033	0.041	996.5	1008.0	0.113	0.141	1731.0	1873.7
		0.2	0.038	0.047	0.042	0.052	994.2	1005.8	0.041	0.052	1048.4	1191.2
		0.8	0.013	0.016	0.036	0.045	996.0	1007.5	0.120	0.153	1765.1	1907.9
	30	-0.2	0.021	0.026	0.024	0.030	2982.3	2997.1	0.024	0.031	3102.1	3285.5
		-0.8	0.008	0.010	0.018	0.023	2982.9	2997.8	0.063	0.079	5115.0	5298.4
		0.2	0.021	0.026	0.023	0.029	2981.0	2995.8	0.025	0.031	3084.3	3267.7
		0.8	0.007	0.009	0.020	0.025	2983.9	2998.8	0.069	0.087	5257.8	5441.2
	50	-0.2	0.017	0.021	0.019	0.023	4969.0	4985.4	0.020	0.025	5142.0	5344.3
		-0.8	0.006	0.007	0.015	0.019	4968.8	4985.2	0.050	0.063	8484.0	8686.3
		0.2	0.015	0.019	0.018	0.023	4969.0	4985.4	0.019	0.024	5123.5	5325.8
		0.8	0.005	0.007	0.015	0.019	4972.3	4988.7	0.055	0.068	8776.9	8979.2

Table 10. Simulation Results of SEM and N = 60

W	T	P	Spatial Estimator						Non-spatial Estimator			
			Bias $\hat{\rho}$	RMSE $\hat{\rho}$	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC	Bias $\hat{\beta}$	RMSE $\hat{\beta}$	AIC	BIC
W_1	10	-0.2	0.118	0.147	0.033	0.041	1706.0	1719.2	0.033	0.041	1768.4	2041.0
		-0.8	0.126	0.156	0.033	0.042	1705.3	1718.4	0.033	0.042	1811.5	2084.2
		0.2	0.102	0.129	0.032	0.040	1706.4	1719.6	0.031	0.039	1760.7	2033.4
		0.8	0.039	0.053	0.033	0.042	1705.5	1718.7	0.033	0.042	1781.7	2054.3
	30	-0.2	0.068	0.085	0.018	0.023	5109.6	5126.1	0.019	0.023	5186.8	5527.5
		-0.8	0.074	0.092	0.019	0.024	5107.5	5124.0	0.020	0.026	5316.6	5657.3
		0.2	0.054	0.068	0.018	0.022	5111.7	5128.2	0.018	0.022	5164.0	5504.7
		0.8	0.021	0.027	0.017	0.022	5114.8	5131.3	0.018	0.022	5239.5	5580.3
	50	-0.2	0.053	0.066	0.014	0.018	8516.0	8534.0	0.014	0.018	8601.5	8973.8
		-0.8	0.055	0.068	0.014	0.018	8515.7	8533.7	0.015	0.018	8812.2	9184.6
		0.2	0.043	0.054	0.014	0.017	8521.8	8539.9	0.014	0.017	8578.4	8950.8
		0.8	0.016	0.020	0.014	0.017	8517.8	8535.8	0.015	0.019	9000.2	9372.6
W_2	10	-0.2	0.023	0.023	0.016	0.020	10005.4	10023.4	0.130	0.131	10856.8	11229.2
		-0.8	0.006	0.007	0.044	0.049	10059.1	10077.2	6.040	6.040	23327.3	23699.7
		0.2	0.023	0.024	0.026	0.031	9962.1	9980.1	0.177	0.178	10922.2	11294.6
		0.8	0.008	0.008	0.073	0.076	9896.2	9914.2	6.662	6.663	23906.3	24278.7
	30	-0.2	0.015	0.018	0.018	0.023	5109.9	5126.3	0.020	0.025	5355.9	5696.6
		-0.8	0.005	0.006	0.016	0.019	5107.2	5123.7	0.063	0.079	9370.7	9711.4
		0.2	0.014	0.017	0.017	0.022	5111.5	5128.0	0.019	0.023	5334.0	5674.7
		0.8	0.005	0.006	0.014	0.017	5114.3	5130.8	0.057	0.071	9422.9	9763.6
	50	-0.2	0.011	0.014	0.014	0.017	8516.4	8534.4	0.015	0.018	8882.5	9254.9
		-0.8	0.004	0.005	0.011	0.014	8514.2	8532.2	0.043	0.054	15574.7	15947.1
		0.2	0.011	0.014	0.014	0.017	8522.5	8540.5	0.014	0.018	8859.0	9231.4
		0.8	0.004	0.005	0.012	0.014	8516.2	8534.2	0.046	0.058	15702.2	16074.6

7.2.2 Simulation Results of Spatial Error Model

Our conclusion about the SEM is that the non-spatial estimator choice of ignoring the presence of spatial dependence in the data may not necessarily bring tremendous drawbacks in terms of bias and RMSE of $\hat{\beta}$ when the value of the spatial dependence is small, i.e., $\rho = \{-0.2, 0.2\}$, where we find the bias and RMSE of $\hat{\beta}$ resulting from the non-spatial estimator are less than or equal to their counterparts of the spatial estimator in some cases. In contrast, the bias and RMSE of $\hat{\beta}$ resulting from the non-spatial estimator tend to increase as the spatial dependence increases, i.e., $\rho = \{-0.8, 0.8\}$, compared to their counterparts of the spatial estimator. This pattern is happened regardless of the spatial weights matrix used, see Table 7 - Table 10. In general, we find that the spatial estimator performs mostly acceptably, where it produces biases that mostly are 5.71% lower than those of the non-spatial estimator in case of low ρ, however, this percentage reaches 41.11% in case of large ρ on average. As for the model-level, we compute AIC and BIC to assess the goodness of fit for each regression. The results confirm that the spatial model always performs better than the non-spatial model regardless of the spatial dependence strength and the structure of the spatial weights matrix.

As for the level of comparison between the spatial weights structures, Fig. 6 shows that the two spatial weights matrices have the same pattern of the spatial bias and RMSE of $\hat{\beta}$ but with different values for each of them. The spatial bias and RMSE of $\hat{\beta}$ tends to decrease when N or T increases.

On the other hand; Fig. 7 clears that W_2 appears a significant improvement in the results of bias of $\hat{\rho}$ compared to W_1. There is no specific pattern to show the relation between the degree of spatial dependence in the data and the spatial bias of $\hat{\rho}$.

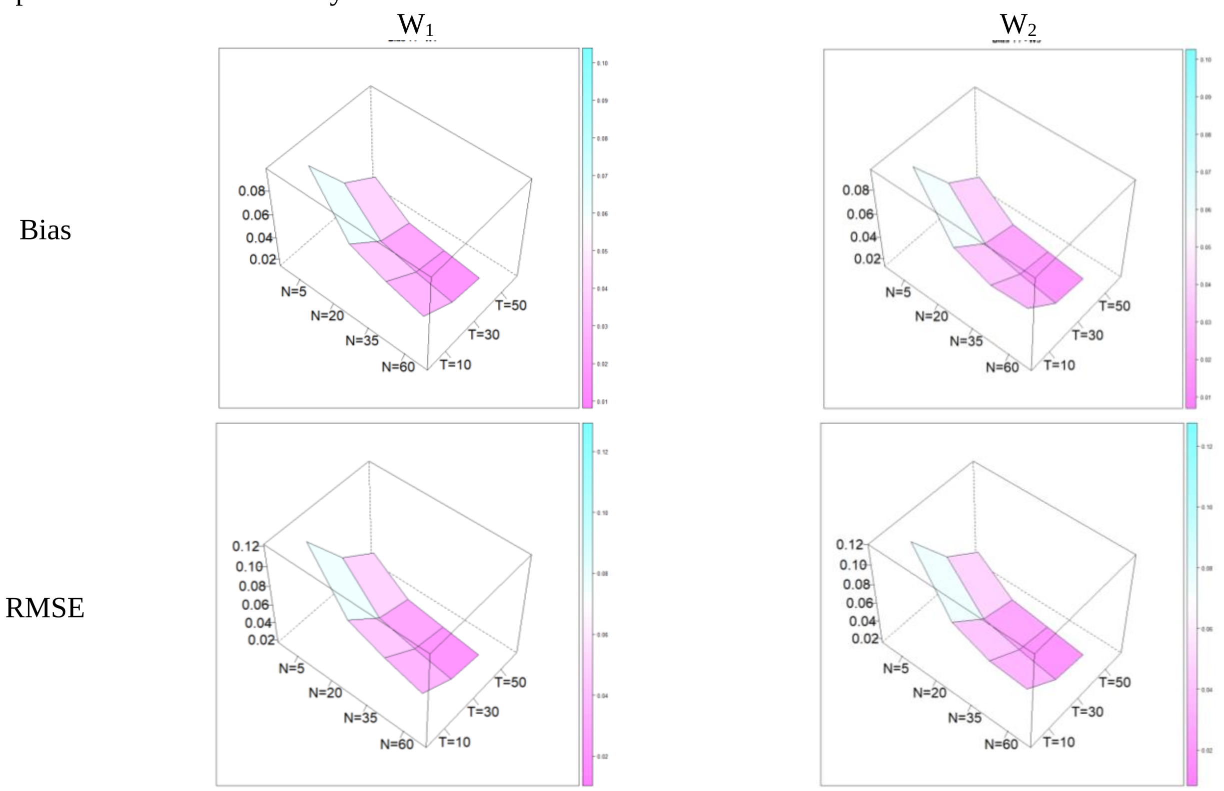


Fig. 6: The Spatial bias and RMSE of $\hat{\beta}$ in SEM at Different Values of N, T, and W for All ρ

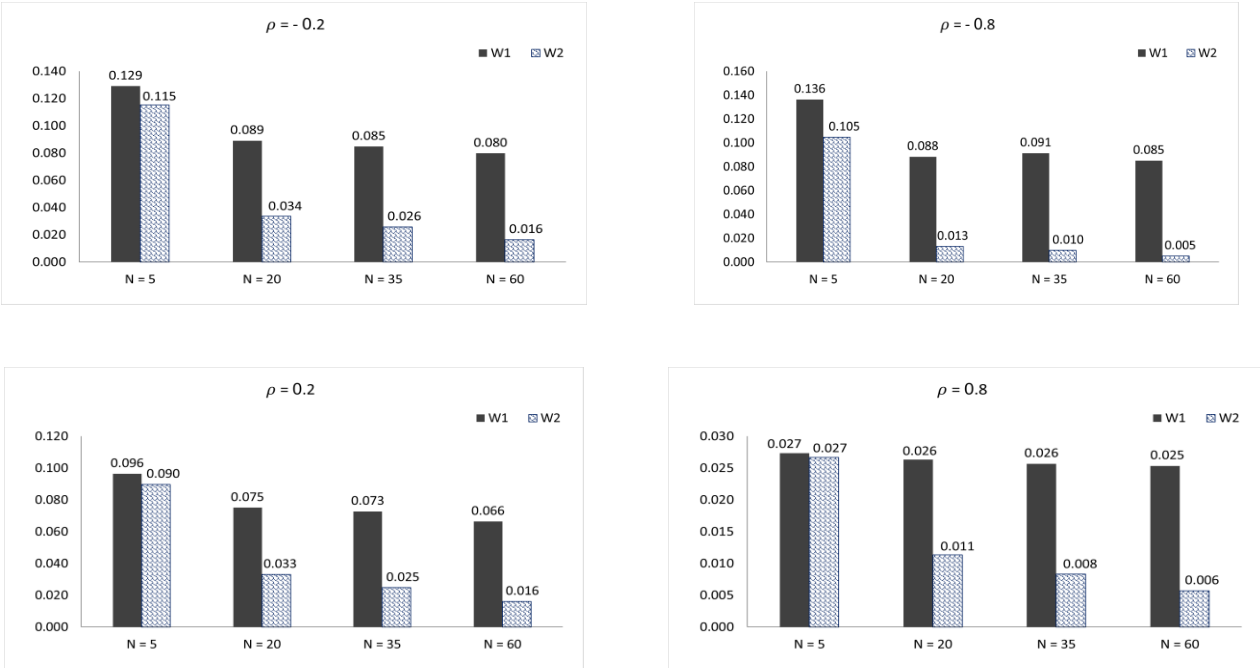


Fig. 7: $\hat{\rho}$ Bias in SEM at Different Values of N, ρ, and W for All T

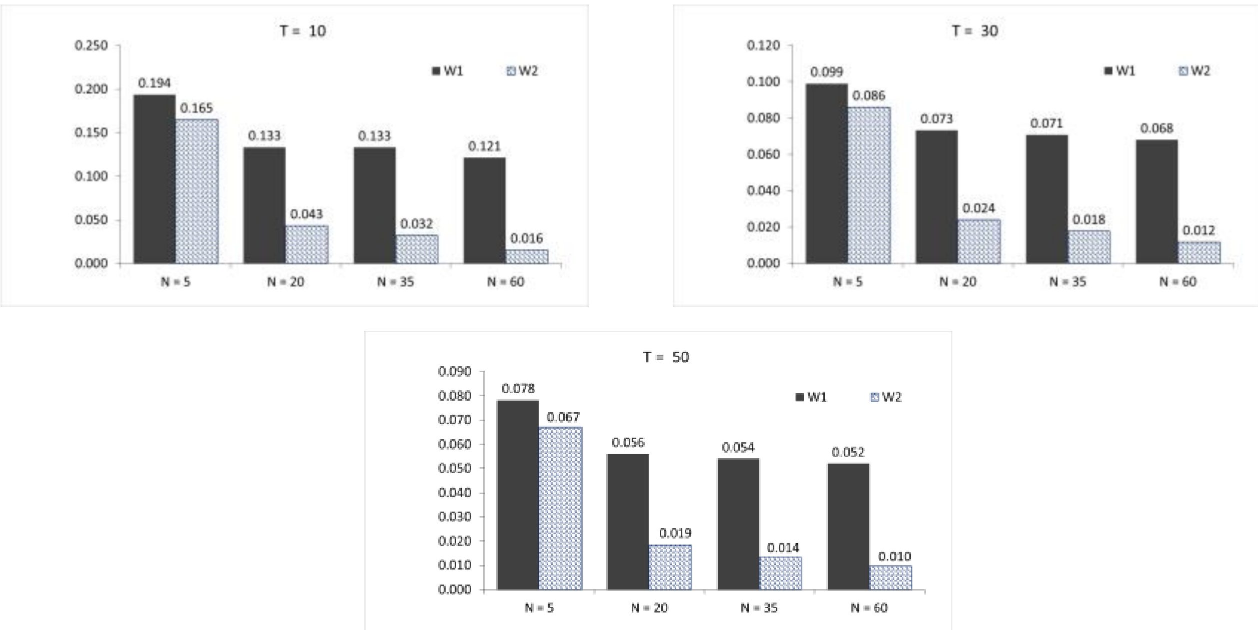


Fig. 8: $\hat{\rho}$ RMSE in SEM at Different Values of N, T, and W for All ρ

Fig. 8 shows that the results of RMSE of $\hat{\rho}$ are always in favor of W_2 across all combinations of N and T. In addition to, the RMSE of $\hat{\rho}$ gradually decreases when N or T increases in the two structures of weights.

For a comparison between the two structures of weights matrix in terms of the spatial AIC and BIC. The results show that the two structures of weights matrix seem to provide very similar levels of spatial AIC and BIC except for N is large and T is small, where the SEM based on W2 returns much higher value of spatial AIC and BIC in this case, see Table 7 - Table 10.

8 Application to Personal Income in U.S. States

This paper is an attempt to use the SPD approach for investigating the determinants of PCPI in U.S. States. As we know, panel data models have played an important role in the literature of analysing determinants of PCPI. So our contribution in this paper focuses on adding the spatial dimension to the analysis to enable us to model the spatial dependence.

Annual data are collected from U.S. census bureau and U.S. bureau of economic analysis (BEA) for 48 U.S. states over 11 periods from 2009 to 2019. The total sample size NT equals 528 without any missing data. Four model specifications; non-SPD models, SLM, SEM, and SAC, are utilized with individual FE and RE setting under different structures of spatial weights matrix. Additionally, attention was paid to direct and indirect effects estimates of the independent variables. The second objective of this application is to show how to select the appropriate model to fit the data. Today, the researcher in the spatial econometrics has the possibility to choose from many models. First, he should ask himself whether there are spatial effects, or not, and, if so, which type of spatial interaction effects should be accounted for a (1) spatially lagged dependent variable, (2) spatially autocorrelated error term, or (3) combination of them. Second, he asks himself whether they should be treated as FE or RE.

Table 11. Definition of the Variables

Dimension	Variable Name	Variable Name on the Site	Definition	Measuring Unit	Source
Dependent Variable	PCPI	Per Capita Personal Income	Personal income in a specific region divided by its population	thousand dollars	U.S. BEA
Educational Attainment	ND	Some College, No Degree	Percentage of individuals without a degree	%	U.S. Census Bureau
	BD	Bachelor's Degree	Percentage of individuals with bachelor's degrees	%	
	GD	Graduate or Professional Degree	Percentage of individuals with graduate or professional degree	%	
Economy's Size	GDPPC	Per Capita Real GDP by State	Real GDP per capita	thousand dollars	U.S. BEA
Labor Force Type	Population	Population	No. of population	100 thousand persons	
	Non-farm	Non-farm Employment	No. of non-farm jobs		
	Un-employ	Unemployment Rate	Percentage of unemployed persons in the total labor force	%	U.S. Census Bureau

Table 12. Descriptive Statistics of the Variables (NT=528)

Variable Name	Mean	Std. Dev.	Min.	Max.
PCPI	45.84	8.71	29.86	77.29
ND	21.34	2.87	15.0	27.90
BD	18.47	2.96	10.40	26.60
GD	10.96	2.79	6.30	20.30
GDPPC	50.08	9.44	33.15	76.36
Population	65.29	71.20	5.60	395.12
Non-farm	37.90	40.83	3.72	243.63
Un-employ	7.06	2.64	2.60	15.10

Table 13. Summary Statistics of the Straight-Line Geographic Distances between Centroids of U.S. States, in kilometers

Min. of all Distances	Mean of all Distances	Max. of all Distances	Std. Dev. of all Distances
80.41	1676.80	4231.84	948.20

Links	W_1: Inverse Distance	W_2: Inverse Exponential
Total No. of Links	210	104
Min. No. of Links	1	1
Mean No. of Links	4.38	2.2
Max. No. of Links	11	5
Threshold Distance	519	

Source of data: https://www.mapdevelopers.com/distance_from_to.php (Accessed Date: 29/8/2019).

8.1 Data Description

8.1.1 Economic Data

To model regional PCPI, we take into account 7 explanatory variables as in Table 11. The dataset is limited by the amount of information available for states involved.

8.1.2 Data of Spatial Weights Matrix

The data of the straight-line geographical distances between centroids of U.S. states, which are summarized in Table 13, are used to create the spatial relations based on inverse distance and inverse exponential distance weights. The threshold distance is calculated by the max-min criterion. The two used spatial weights matrices are row standardized to facilitate interpretation, see [27].

8.2 Testing the Multicollinearity

The first step of data processing is to try to ensure that there is no high linear correlation between independent variables.

We used the most common methods to detect the multicollinearity: (1) Pearson correlation matrix between each pair of variables and (2) the variance inflation factor (VIF), see [28], [17], and [29].

Table 14. Pearson Correlation Matrix and VIF

Variable	ND	BD	GD	GDPPC	Population	Non-farm	Un-employ
ND	1						
BD	-0.21[a]	1					
GD	-0.54[a]	0.72[a]	1				
GDPPC	-0.30[a]	0.60[a]	0.56[a]	1			
Population	-0.19[a]	0.10[c]	0.15[a]	0.20[a]	1		
Non-farm	-0.20[a]	0.14[b]	0.18[a]	0.24[a]	0.99[a]	1	
Un-employ	0.04[a]	-0.45[a]	-0.21[a]	-0.28[a]	0.18[a]	0.14[a]	1
VIF1	1.60	3.10	3.34	1.75	**239.93**	**240.23**	1.66
VIF2	1.60	3.10	3.23	1.72	----	1.15	1.38

Notes: *VIF1: is VIF for all variables, VIF2: is VIF after removing "Population". The superscripts [a], [b], and [c] indicate statistical significance at the 0.001, 0.01, and 0.05 levels, respectively.*

Table 14 shows that there is a strong linear correlation between (population and non-farm jobs). Additionally, the results of VIF for the first time with all independent variables (VIF1) confirmed that there is a multicollinearity problem between independent variables; where in most empirical studies, the general rule of thumb is that VIF values exceeding 5 need further investigation, while VIF values exceeding 10 indicate to serious multicollinearity requiring correction, see [17]. If two independent variables are almost linearly correlated, we can eliminate one of them to combat multicollinearity, see [28]. Therefore, we drop (population) from the model. All values of new VIF (VIF2) less than 5 confirmed on there is no multicollinearity.

8.3 Hausman Specification Test

Because ignoring spatial dependence may lead to biased and inefficient estimates, therefore, panel data models are applied with/ without spatial effects to avoid these shortcomings, and then allow the data to determine the most appropriate approach. Before applying the framework in Fig. 9, the Hausman specification test is conducted to compare between RE and FE estimators. Hausman [30] developed this test for non-SPD model. Mutl and Pfaffermayr [31] showed how to apply this procedure to a spatial framework.

The results of the all estimated spatial and non-spatial panel data models are reported in Table 15 according to two methods of wights matrices in context of FE and RE settings which tested by Hausman specification test. As shown in Table 15, the null hypothesis of the Hausman test is rejected at the 0.001 level of significance for all models, indicating that FE specifications are more suitable than RE specifications.

8.4 Testing the Spatial Dependence

As a next step, we need to capture spatial dependence in the data. Therefore, the following framework in Fig. 9 is proposed. Lagrange Multiplier tests, i.e., LM -lag, LM-error tests, and their robust counterparts (RLM), can be applied to specify whether the estimation of a spatial model is warranted. If the null hypotheses of LM tests are rejected for the absence of SL or spatial error (SE) in the model, it proves that SPD is a suitable method for the analysis. Burridge [32] and Anselin [19] proposed LM tests for a spatially lagged dependent variable and SE correlation term in the case of cross-sectional data. The hypotheses for the LM tests are:

For SLM: $H_0: \lambda = 0 \quad vs. \quad H_1: \lambda \neq 0$, henceforth LM_λ

For SEM: $H_0: \rho = 0 \quad vs. \quad H_1: \rho \neq 0$, henceforth LM_ρ

Anselin et al. [33] also proposed robust LM (RLM) statistics for a spatially lagged dependent variable in the local presence of SE autocorrelation and another one for SE autocorrelation in the local presence of a spatially lagged dependent variable. In other words, the hypotheses for RLM tests are:

For SLM: $H_0: \lambda = 0$ given $\rho \neq 0$ $vs.$ $H_1: \lambda \neq 0$, henceforth $LM_{\lambda|\rho}$

For SEM: $H_0: \rho = 0$ given $\lambda \neq 0$ $vs.$ $H_1: \rho \neq 0$, henceforth $LM_{\rho|\lambda}$

Table 15. Results of Estimated Panel Data Models

Variable	**Non-spatial**		**W₁: Inverse Distance**				**W₂: Inverse Exponential**			
			SLM		**SEM**		**SLM**		**SEM**	
	FE	**RE**	**FE**	**RE**	**FE**	**RE**	**FE**	**RE**	**FE**	**RE**
ND	-0.32^b	-0.37^a	-0.18^c	-0.17^c	-0.06	-0.30^c	-0.20^c	-0.20^c	-0.25^c	-0.31^b
BD	1.39^a	0.83^a	0.40^a	0.39^a	0.22	0.56^a	0.74^a	0.68^a	1.23^a	0.68^a
GD	2.47^a	1.69^a	1.11^a	0.99^a	0.68^a	1.31^a	1.42^a	1.29^a	2.25^a	1.47^a
GDPPC	0.27^a	0.25^a	0.25^a	0.27^a	0.30^a	0.35^a	0.27^a	0.27^a	0.30^a	0.32^a
Non-farm	0.09^a	0.02^c	0.14^a	0.08^a	0.14^a	0.02	0.13^a	0.08^a	0.08^a	0.003
Non-employ	-0.39^a	-0.95^a	-0.02	-0.14^b	-0.24^a	-0.78^a	-0.08	-0.22^a	-0.46^a	-0.89^a
Intercept	-14.3^b	12.9^a	-14.8^a	-10.9^a	17.36^a	14.52^a	-16.6^a	-10.9^b	-10.8^c	14.14^a
$\boldsymbol{\lambda}$	----	----	0.59^a	0.58^a	----	----	0.42^a	0.42^a	----	----
$\boldsymbol{\rho}$	----	----	----	----	0.89^a	0.61	----	----	0.33^a	0.40
Hausman	22.88^a		68.40^a		189.78^a		116.89^a		236.47^a	

Note: *The superscripts ^{a}, ^{b}, and ^{c} indicate statistical significance at the 0.001, 0.01, and 0.05 level, respectively. Non-spatial models are estimated by Within-group OLS for FE and GLS for RE. Spatial models are estimated by ML (transformation approach) for all models except for SEM-RE is estimated by GMM.*

Recently, Anselin et al. [6] also developed the classical LM tests for SPD models, and Elhorst [10] developed the robust counterparts of these LM tests for SPD models.

Table 16 shows that the classical LM tests of SL and SE terms are significant at the 0.001 level. However, the RLM tests are significant for SL term but not significant for SE term in all structures of spatial weights matrices. Therefore, our model will include SL term and exclude SE term. To be more certain of which terms are included in our model, we'll compare between the SLM and SEM models with SAC, which includes the two types of spatial dependence terms, in terms of goodness of fit criteria to select the best model for the data as in Table 17.

In some way, this is in line with some of the applied literature that estimated the SAC if the researcher doesn't have a strong prior in favour of either SLM or SEM. In other words, an empirical strategy could be to start from the most general specification, SAC, along with the appropriate type of individual model, SLM or SEM, and let the data tell us which of the two spatial processes – if any and if not both – is more appropriate, by looking at the significance of spatial coefficients and goodness of fit criteria, for more details, see [34].

8.5 Model Selection

As in a lot of empirical researches, the models are comparable in terms of AIC, and BIC. These criteria are one of the best methods to select the most adequate weighting matrix, see [35] and [36]. Table 17 shows that the values of the goodness of fit criteria for the non-spatial model are much bigger than for all SPD models. As expected, the SLMs record the smallest values of AIC and BIC compared with their counterparts for SEM and SAC models. Therefore, we can say that SLM is the most adequate model among the candidate models. On purely statistical grounds, the SLM based on W_1 returns a slightly lower AIC and BIC compared to W_2.

Table 16. Results of LM Tests with Different Spatial Weights Matrices

Test	**W_1: Inverse Distance**		**W_2: Inverse Exponential**	
	Lag	**Error**	**Lag**	**Error**
LM	73.70[a]	20.41[a]	51.16[a]	12.86[a]
RLM	53.88[a]	0.32	38.42[a]	0.13

Note: *The superscript [a] indicates statistical significance at the 0.001 level.*

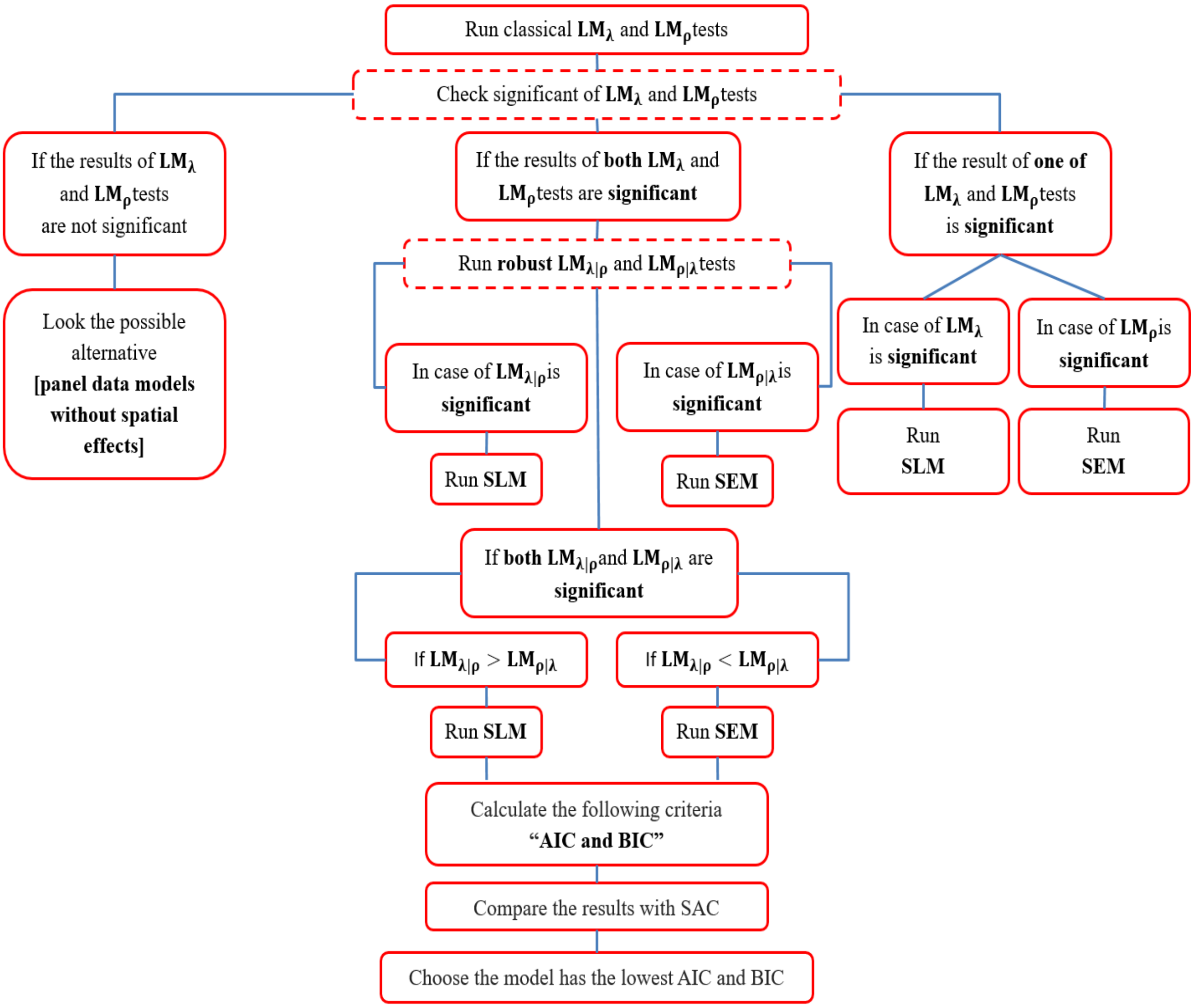


Fig. 9: Our Strategy for Selecting the Most Appropriate SPD Model

Note: *LM_λ and LM_ρ: The Lagrange Multiplier Tests for a Spatially Lagged Dependent Variable and Spatial Error Correlation Respectively - $LM_{\lambda|\rho}$ and $LM_{\rho|\lambda}$: The Robust Counterparts of these Tests - SLM: Spatial Lag Model - SEM: Spatial Error Model - SAC: Spatial Autoregressive Combined Model - AIC: Akaike Information Criterion - BIC: Bayesian Information Criterion*.

Table 17. Results of Estimated SPD Models with Spatial FE

Criterion	**Non-spatial**	**W_1: Inverse Distance**			**W_2: Inverse Exponential**		
		SLM	**SEM**	**SAC**	**SLM**	**SEM**	**SAC**
AIC	1813.19	1390.42	1525.50	1392.42	1463.27	1621.76	1465.27
BIC	1843.08	1423.81	1558.89	1429.98	1496.66	1655.15	1502.83

8.6 Interpretation of the Results

The coefficients interpretation in the linear regression model is not complicated. Since the model is linear in parameters and assumes that the observations are independent, the parameter can be explained as the partial derivative of the response variable with respect to the independent variable. When we consider SPD models, the interpretation needs more proper considerations to fully interpret the effect of changes, direct and spillover effects, must be obtained and interpreted as the coefficients of model, see [37].

In SLM, the direct effect is the average of main diagonal elements of the matrix in (32), and the spillover effect is the average of row off-diagonal elements in the same matrix, see [38] and [39].

$$M_{SLM} = S_N^{-1}\beta_k \quad \textbf{(32)}$$

Table 18 provides the measures of direct, spillover, and the total effect of each regressor to assess the magnitudes of impacts arising from changes in the 6 independent variables under the study.

The first column of Table 18 express about the direct effects, which measure how much the dependent variable, PCPI, changes in a state when a given independent variable changes in that same state. The second column refers to the spillover (indirect) effects of changes in our independent variables; Finally, the last column shows the point estimates of the total effects that are defined as the sum of the direct and indirect effects. We note that all effects of four explanatory variables are statistically significant at the 0.001 level and one variable is significant at 0.05.

In general, we can conclude the following points from Table 18:

(1) The direct effect of increasing ND in a specific state by 1% directly reduces PCPI by $214.5 in the same state. Also, the indirect effect of increasing ND in neighboring states is negative on the PCPI by $229.0. The total effect of ND is negative and consists mostly of indirect effect.

(2) The BD has a positive direct and indirect effect on PCPI, indicating that we would expect an increase in PCPI in states with a high level of BD. The magnitude of the indirect effect produced from BD increases by a very small amount over the magnitude of the direct effect, indicating that the direct and indirect effects of this variable are almost equal.

(3) The direct and indirect effects of GD are positive; this refers to that the increase in GD in a specific state by 1% directly increases the PCPI by $1302.8 in the same state and indirectly increases it in other states by $1391.1.

(4) The direct and indirect effects of GDPPC are positive; as the GDPPC increases by $1000 in a particular state, the PCPI will increase by $302.5 on average in the same state, and increase by $323.0 on average in other states.

(5) The number of non-farm jobs has a positive direct and indirect effect on the PCPI; when the number of non-farm jobs increases by 100,000 jobs in a particular state, the PCPI increases by $159.0 in the same state, and increase by $169.8 on average in other states.

(6) The impact of the unemployment rate on PCPI is not significant.

Table 18. Direct and Indirect Effects of SLM with Spatial FE and using W1

Variable	Direct Effects	Spillover/ Indirect Effects	Total Effects
ND	-0.2145^c	-0.2290^c	-0.4435^c
BD	0.4741^a	0.5062^b	0.9803^a
GD	1.3028^a	1.3911^a	2.6939^a
GDPPC	0.3025^a	0.3230^a	0.6255^a
Non-farm	0.1590^a	0.1698^a	0.3289^a
Un-employ	-0.0290	-0.0310	0.0601

Note: *The superscripts ^{a}, ^{b}, and ^{c} indicate statistical significance at the 0.001, 0.01, and 0.05 level respectively.*

9 Conclusions

This paper is an attempt to assess the risks involved in ignoring the spatial dependence that characterizes the data. Due to the importance of this topic, our contribution is not limited by this empirical application, but also we conduct a MCS study to evaluate the performance of both the spatial MLE (transformation approaches) and the non-spatial OLS (within-group) estimator for two specifications of the most common spatial data generating processes (DGPs), i.e., SLM and SEM with spatial FE under different scenarios of N, T, and spatial dependence parameters. Besides, we employ two structures of weights matrices; i.e., inverse distance (W_1) and inverse exponential distance (W_2), to draw and estimate each DGP aiming to compare the impact of these structures on each model.

In a summary way, the following points can be concluded from our simulation study:

(1) **On the Parameter-Level:** The non-spatial bias and RMSE of $\hat{\beta}$ are functions of the degree of spatial dependence in the data for both models, i.e., SLM and SEM. The choice of the non-spatial estimator may not lead to serious consequences in terms of bias and RMSE of $\hat{\beta}$ when the spatial dependence is small. On the contrary, the choice of the non-spatial estimator always leads to has disastrous consequences if the spatial dependence is large.

(2) **On the Model-Level:** The SLM always performs better than its non-spatial counterpart when λ is large in terms of AIC and BIC. However, the spatial AIC and BIC in SEM are always much less than their non-spatial counterparts in all cases.

(3) For a comparison between the two structures of weights matrix in terms of the spatial AIC and BIC. The results of SLM show that the two structures of weights matrix seem to provide very similar levels of spatial AIC and BIC, However, the SLM based on W2 returns a slightly higher value of spatial AIC and BIC in all cases. This result is also true for SEM except for N is large and T is small, where the SEM based on W2 returns much higher value of spatial AIC and BIC in this case.

On the other hand; our empirical study confirms the following points:

(1) PCPI is spatially dependent lagged correlated.

(2) There are no differences among the two used structures of spatial weights matrix in terms of the inference drawn from Hausman and LM tests, the number of significant variables, and their significance levels. However, the differences among the two used structures can be confined in the values resulting from each procedure in our analysis not in the conclusion, for example, the W1 yields a higher improvement in terms of goodness of fit criteria.

In the future, this work can obviously be extended along many dimensions, for example;

1. The set of maintained assumptions in our simulation study can be made more general. Here the first extension that can be addressed is to allow for the RE specification. In other words, a MCS study can be performed to compare the performance of the estimators used in non-spatial and SPD models with RE settings.
2. It could be interesting to consider studying the estimation procedures used in dynamic SPD models or SPD models with random coefficients, see [40, 41, 42].
3. We here study only the time-invariant spatial weights matrix; therefore, it is important to study the situation of the spatio-temporal weights matrix that allows decomposing the spatial effects when the spatial relations are being collected continuously over time.
4. The models under consideration in our study can be extended to include other elements. In particular, it would be helpful to consider a SL in the explanatory variables, or there is a linear relationship (multicollinearity) between the explanatory variables, see [43, 44, 45, 46].
5. A final remark needs to be made concerning the interpretation of parameter estimates in the SPD models. Unlike in OLS regressions, parameter estimates in SPD models that contain SL of the dependent variable have not a direct interpretation due to the embedded feedback effects among spatial units. Therefore, LeSage and Pace [38] developed summary measures reflecting the impact of change in explanatory variables on the dependent variable as we cleared previously. To meaningfully interpret parameter estimates in the context of the proposed SPD models, the development of summary measures appears to be a promising area for future researches.

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