Improvements to the fixed point results by the use of a simulation function employing rational terms

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Abstract: -Inside this paper, we introduce two new partial b-metric contractions utilizing a rational expression simulation function. The following conclusions extend, generalize, and integrate the earlier findings in two ways: in contraction terms and in the abstract environment. We provide an example to establish the main theorem's validity.

Key-Words: Contraction, Fixed point, partial b-metric space, Simulation functions

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1 Introduction

Banach published his foundational work on fixed point theory approximately a century ago. Banach's basic yet deep theorem has been extended, enhanced, and generalized by researchers since his first study (see [1, 2, 3, 4, 5, 6, 7, 8, 9]). This was accomplished by examining the terms of the contraction inequality and Banach's theorem's abstract structure. We'll combine these two tendencies by employing simulation functions that include rational terms to create two new type contractions in partial b-metric space.

2 Preliminaries

Definition 2.1. [10] *Let $\mathcal{A}$ be a non-empty set, a function $p : \mathcal{A}\times\mathcal{A} \to \mathbb{R}_0^+$ is a partial metric if the following conditions:*

(p_1) $\alpha = \delta$ *if and only if* $p(\alpha,\alpha) = p(\alpha,\delta) = p(\delta,\delta)$;

(p_2) $p(\alpha,\alpha) \le p(\alpha,\delta)$;

(p_3) $p(\alpha,\delta) = p(\delta,\alpha)$;

(p_4) $p(\alpha,\delta) \le p(\alpha,\upsilon) + p(\upsilon,\delta) - p(\upsilon,\upsilon)$,
for all $\alpha,\upsilon,\delta \in \mathcal{A}$.

The pair $(\mathcal{A}, p)$ is called a partial-metric space.

Lemma 2.2. [11] *If $\{\alpha_\lambda\}$, $\{\delta_\lambda\}$ are two sequences in a partial-metric space $(\mathcal{A}, p)$ such that*

$$\lim_{\lambda\to\infty} p(\alpha_\lambda,\mu) = \lim_{\lambda\to\infty} p(\alpha_\lambda,\alpha_\lambda) = p(\mu,\mu),$$
$$\lim_{\lambda\to\infty} p(\delta_\lambda,\kappa) = \lim_{\lambda\to\infty} p(\delta_\lambda,\delta_\lambda) = p(\kappa,\kappa).$$

Then $\lim_{\lambda\to\infty} p(\alpha_\lambda,\delta_\lambda) = p(\mu,\kappa)$. *Moreover,* $\lim_{\lambda\to\infty} p(\alpha_\lambda,\sigma) = p(\mu,\sigma)$, *for each* $\sigma\in\mathcal{A}$.

Denote by $\mathcal{L}(\alpha_\lambda)$ the set of limit points (if there exist any),

$$\mathcal{L}(\alpha_\lambda) = \{\sigma\in\mathcal{A} : \lim_{\lambda\to\infty} p(\alpha_\lambda,\sigma) = p(\sigma,\sigma)\}.$$

Lemma 2.3. [12] *Let $\{\alpha_\lambda\}$ be a Cauchy sequence on a complete partial-metric space $(\mathcal{A}, p)$. If there exists $\mu\in\mathcal{L}(\{\alpha_\lambda\})$ with $p(\mu,\mu) = 0$, then $\mu\in\mathcal{L}(\{\alpha_{\lambda(\iota)}\})$, for every subsequence $\{\alpha_{\lambda(\iota)}\}$ of $\{\alpha_\lambda\}$.*

Lemma 2.4. [12] *On a complete partial-metric space $(\mathcal{A}, p)$, let $\mathcal{F}$ be a continuous mapping and $\{\alpha_\lambda\}$ be a Cauchy sequence in $\mathcal{A}$. If there exists $\mu\in\mathcal{L}(\{\alpha_\lambda\})$ with $p(\mu,\mu) = 0$. Then $\mathcal{F}\mu\in\mathcal{L}(\{\mathcal{F}\alpha_\lambda\})$.*

Definition 2.5. [13] *Let $\mathcal{A}$ be a non-empty set and $s\ge 1$. A function $p_b : \mathcal{A}\times\mathcal{A}\to\mathbb{R}_0^+$ is a partial b-metric with a coefficient s if the following conditions hold for all $\alpha,\delta,\upsilon\in\mathcal{A}$*

(p_{b1}) $\alpha = \delta$ *if and only if* $p_b(\alpha,\alpha) = p_b(\alpha,\delta) = p_b(\delta,\delta)$;

(p_{b2}) $p_b(\alpha,\alpha) \le p_b(\alpha,\delta)$;

(p_{b3}) $p_b(\alpha,\delta) = p_b(\delta,\alpha)$;

(p_{b4}) $p_b(\alpha,\delta) \le s(p_b(\alpha,\upsilon) + p_b(\upsilon,\delta)) - p_b(\upsilon,\upsilon)$.

In this case, we say that $(\mathcal{A}, p_b, s)$ *is a partial b-metric space.*

Lemma 2.6. [14] *Let* $(\mathcal{A}, p_b, s)$ *be a partial b-metric space. If* $p_b(\alpha, \delta) = 0$ *then* $\alpha = \delta$ *and* $p_b(\alpha, \delta) > 0$ *for all* $\alpha \neq \delta$.

Definition 2.7. [15] *A sequence* $\{\alpha_\lambda\}$ *on a partial b-metric space* $(\mathcal{A}, p_b, s)$ *is* 0-p_b-*Cauchy if* $\lim_{\lambda,\rho\to\infty} p_b(\alpha_\lambda, \alpha_\rho) = 0$. *Moreover, the space* $(\mathcal{A}, p_b, s)$ *is said to be* 0-p_b-*complete if for each* 0-p_b-*Cauchy sequence in* $\mathcal{A}$, *there is* $\sigma \in \mathcal{A}$, *such that*

$$\lim_{\lambda,\rho\to\infty} p_b(\alpha_\lambda, \alpha_\rho) = \lim_{\lambda\to\infty} p_b(\alpha_\lambda, \sigma) = p_b(\sigma, \sigma) = 0.$$

Lemma 2.8. [15] *If the partial b-metric space* $(\mathcal{A}, p_b, s)$ *is* p_b-*complete, then it is* 0-p_b-*complete.*

Lemma 2.9. [16] *Let* $(\mathcal{A}, p_b, s \geq 1)$ *be a partial b-metric space,* $\mathcal{F} : \mathcal{A} \to \mathcal{A}$ *a mapping and a number* $\gamma \in [0, 1)$. *If* $\{\alpha_\lambda\}$ *is a sequence in* $\mathcal{A}$, *where* $\alpha_\lambda = \mathcal{F}\alpha_{\lambda-1}$ *and*

$$p_b(\alpha_\lambda, \alpha_{\lambda+1}) \leq \gamma p_b(\alpha_{\lambda-1}, \alpha_\lambda),$$

for each $\lambda \geq 1$, *then the sequence* $\{\alpha_\lambda\}$ *is* 0-p_b-*Cauchy.*

Let Υ be the set of all non-decreasing and continuous functions $\psi : [0, \infty) \to [0, \infty)$ such that $\psi(0) = 0$.

Definition 2.10. [17] *A function* $\eta : \mathbb{R}^+_0 \times \mathbb{R}^+_0 \to \mathbb{R}$ *is a* ψ-*simulation function if there exists* $\psi \in$ *such that the following conditions hold:*

(η_1) $\eta(\zeta, \xi) < \psi(\xi) - \psi(\zeta)$ *for all* $\zeta, \xi \in \mathbb{R}^+$;

(η_2) *if* $\{\zeta_\lambda\}$, $\{\xi_\lambda\}$ *are two sequences in* $[0, \infty)$ *such that* $\lim_{\lambda\to\infty} \zeta_\lambda = \lim_{\lambda\to\infty} \xi_\lambda > 0$, *then*

$$\limsup_{\lambda\to\infty} \eta(\zeta_\lambda, \xi_\lambda) < 0.$$

Denote by $\mathcal{Z}_\psi$ the family of all ψ-simulation functions (see [18, 19, 20, 21, 22]). It is clear, due to the axiom (η_1), that

$$\eta(\zeta, \zeta) < 0 \quad \text{for all } \zeta > 0.$$

3 Main Results

Definition 3.1. *Let* $(\mathcal{A}, p_b, s \geq 1)$ *be a partial b-metric space. A mapping* $\mathcal{F} : \mathcal{A} \to \mathcal{A}$ *is called* η-*rational contraction of type A, if there exists a function* $\eta \in \mathcal{Z}_\psi$ *such that*

$$\frac{1}{2s} \min\{p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta)\} \leq p_b(\alpha, \delta) \quad \textit{implies} \\ \eta(s^t p_b(\mathcal{F}\alpha, \mathcal{F}\delta), \mathcal{D}_A(\alpha, \delta) \geq 0, \tag{1}$$

for every $\alpha, \delta \in \mathcal{A}$, *where* $\mathcal{D}_A$ *is defined as*

$$\begin{aligned} &\mathcal{D}_A(\alpha, \delta) \\ &= \max\Big\{p_b(\alpha, \delta), p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta), \\ &\quad \frac{p_b(\delta, \mathcal{F}\delta)(1 + p_b(\alpha, \mathcal{F}\alpha))}{1 + p_b(\alpha, \delta)}, \\ &\quad \frac{p_b(\delta, \mathcal{F}\delta)(1 + p_b(\alpha, \mathcal{F}\alpha))}{1 + p_b(\mathcal{F}\alpha, \mathcal{F}\delta)}, \\ &\quad \frac{p_b(\alpha, \mathcal{F}\delta) + p_b(\delta, \mathcal{F}\alpha)}{2s}\Big\}. \end{aligned} \tag{2}$$

Theorem 3.2. *Let* $(\mathcal{A}, p_b, s > 1)$ *be a* p_b-*complete partial b-metric space and* $\mathcal{F} : \mathcal{A} \to \mathcal{A}$ *be a* η-*rational contraction of type A. Then* $\mathcal{F}$ *admits exactly one fixed point.*

Proof. Let $\alpha_0 \in \mathcal{A}$ be an arbitrary but fixed point and $\{\alpha_\lambda\}$ be the sequence defined in $\mathcal{A}$ as follows:

$$\alpha_\lambda = \mathcal{F}\alpha_{\lambda-1}, \quad \forall \lambda \geq 1. \tag{3}$$

Suppose that $\alpha_{\lambda-1} \neq \alpha_\lambda$ for every $\tau \geq 1$. Indeed, if we assume that there exists $\lambda_0 \in \mathbb{N}$ such that $\alpha_{\lambda_0-1} = \alpha_{\lambda_0}$. Taking (3) into consideration, we get $\alpha_{\lambda_0-1} = \mathcal{F}\alpha_{\lambda_0-1}$., that is, α_{λ_0-1} is a fixed point of $\mathcal{F}$. Hence, substituting $\alpha = \alpha_{\lambda-1}$ and $\delta = \alpha_\lambda$ in (2), we obtain

$$\begin{aligned} &\mathcal{D}_A(\alpha_{\lambda-1}, \alpha_\lambda) \\ &= \max\Big\{p_b(\alpha_{\lambda-1}, \alpha_\lambda), p_b(\alpha_{\lambda-1}, \mathcal{F}\alpha_{\lambda-1}), p_b(\alpha_\lambda, \mathcal{F}\alpha_\lambda), \\ &\quad \frac{p_b(\alpha_\lambda, \mathcal{F}\alpha_\lambda)(1 + p_b(\alpha_{\lambda-1}, \mathcal{F}\alpha_{\lambda-1}))}{1 + p_b(\alpha_{\lambda-1}, \alpha_\lambda)}, \\ &\quad \frac{p_b(\alpha_\lambda, \mathcal{F}\alpha_\lambda)(1 + p_b(\alpha_{\lambda-1}, \mathcal{F}\alpha_{\lambda-1}))}{1 + p_b(\mathcal{F}\alpha_{\lambda-1}, \mathcal{F}\alpha_\lambda)}, \\ &\quad \frac{p_b((\alpha_{\lambda-1}, \mathcal{F}\alpha_\lambda) + p_b(\alpha_\lambda, \mathcal{F}\alpha_{\lambda-1})}{2s}\Big\} \\ &= \max\Big\{p_b(\alpha_{\lambda-1}, \alpha_\lambda), p_b(\alpha_{\lambda-1}, \alpha_\lambda), p_b(\alpha_\lambda, \alpha_{\lambda+1}), \\ &\quad \frac{p_b(\alpha_\lambda, \alpha_{\lambda+1})(1 + p_b(\alpha_{\lambda-1}, \alpha_\lambda))}{1 + p_b(\alpha_{\lambda-1}, \alpha_\lambda)}, \\ &\quad \frac{p_b(\alpha_\lambda, \alpha_{\lambda+1})(1 + p_b(\alpha_{\lambda-1}, \alpha_\lambda))}{1 + p_b(\alpha_\lambda, \alpha_{\lambda+1})}, \\ &\quad \frac{p_b(\alpha_{\lambda-1}, \alpha_{\lambda+1}) + p_b(\alpha_\lambda, \alpha_\lambda)}{2s}\Big\} \\ &\leq \max\Big\{p_b(\alpha_{\lambda-1}, \alpha_\lambda), p_b(\alpha_{\lambda-1}, \alpha_\lambda), p_b(\alpha_\lambda, \alpha_{\lambda+1}), \\ &\quad \frac{p_b(\alpha_\lambda, \alpha_{\lambda+1})(1 + p_b(\alpha_{\lambda-1}, \alpha_\lambda))}{1 + p_b(\alpha_{\lambda-1}, \alpha_\lambda)}, \\ &\quad \frac{p_b(\alpha_\lambda, \alpha_{\lambda+1})(1 + p_b(\alpha_{\lambda-1}, \alpha_\lambda))}{1 + p_b(\alpha_\lambda, \alpha_{\lambda+1})}, \end{aligned}$$

$$\begin{aligned}&\frac{s(p_b(\alpha_{\lambda-1},\alpha_\lambda)+p_b(\alpha_\lambda,\alpha_{\lambda+1}))}{2s}\\&\left.-\frac{p_b(\alpha_\lambda,\alpha_\lambda)+p_b(\alpha_\lambda,\alpha_\lambda)}{2s}\right\}\\&=\max\{p_b(\alpha_{\lambda-1},\alpha_\lambda),p_b(\alpha_\lambda,\alpha_{\lambda+1})\}\end{aligned} \tag{4}$$

Furthermore, by (1), we get

$$\begin{aligned}&\frac{1}{2s}\min\{p_b(\alpha_{\lambda-1},\mathcal{F}\alpha_{\lambda-1}),p_b(\alpha_\lambda,\mathcal{F}\alpha_\lambda)\}\\&=\frac{1}{2s}\min\{p_b(\alpha_{\lambda-1},\alpha_\lambda),p_b(\alpha_\lambda,\alpha_{\lambda+1})\}\\&\leq p_b(\alpha_{\lambda-1},\alpha_\lambda)\end{aligned} \tag{5}$$

implies

$$\eta(s^t p_b(\mathcal{F}\alpha_{\lambda-1},\mathcal{F}\alpha_\lambda),\mathcal{D}_A(\alpha_{\lambda-1},\alpha_\lambda))\geq 0, \tag{6}$$

Taking (η_1) into account, the preceding inequality provides

$$0<\psi(\mathcal{D}_A(\alpha_{\lambda-1},\alpha_\lambda))-\psi(s^t p_b(\mathcal{F}\alpha_{\lambda-1},\mathcal{F}\alpha_\lambda)), \tag{7}$$

or, equivalently,

$$\begin{aligned}&\psi(s^t p_b(\alpha_\lambda,\alpha_{\lambda+1}))\\&<\psi(\mathcal{D}_A(\alpha_{\lambda-1},\alpha_\lambda))\\&=\psi(\max\{p_b(\alpha_{\lambda-1},\alpha_\lambda),p_b(\alpha_\lambda,\alpha_{\lambda+1})\}).\end{aligned} \tag{8}$$

As a result of the monotony of the function ψ, we obtain

$$s^t p_b(\alpha_\lambda,\alpha_{\lambda+1})<\max\{p_b(\alpha_{\lambda-1},\alpha_\lambda),p_b(\alpha_\lambda,\alpha_{\lambda+1})\}. \tag{9}$$

If there exists $\lambda_1\in\mathbb{N}$ such that

$$\max\{p_b(\alpha_{\lambda_1-1},\alpha_{\lambda_1}),p_b(\alpha_{\lambda_1},\alpha_{\lambda_1+1})\}=p_b(\alpha_{\lambda_1},\alpha_{\lambda_1+1}),$$

(9) becomes $s^t p_b(\alpha_{\lambda_1},\alpha_{\lambda_1+1}) < p_b(\alpha_{\lambda_1},\alpha_{\lambda_1+1})$, which is a contradiction (because $s>1$). Hence, for any $\lambda\in\mathbb{N}$ we obtain

$$s^t p_b(\alpha_\lambda,\alpha_{\lambda+1})<p_b(\alpha_{\lambda-1},\alpha_\lambda),$$

or

$$p_b(\alpha_\lambda,\alpha_{\lambda+1})<\frac{1}{s^t}p_b(\alpha_{\lambda-1},\alpha_\lambda). \tag{10}$$

Denoting $\frac{1}{s^t}$ by μ, we have

$$p_b(\alpha_\lambda,\alpha\lambda+1)<\mu p_b(\alpha_{\lambda-1},\alpha_\lambda),$$

with $0\leq\mu<1$. Using Lemma 2.9, we see that the sequence $\{\alpha_\rho\}$ is a 0-p_b-Cauchy sequence on the p_b-complete partial b-metric space. Since by Lemma 2.8, the space is also 0-p_b-complete, it follows that there exists $\sigma\in\mathcal{A}$ such that

$$\lim_{\lambda,\rho\to\infty}p_b(\alpha_\lambda,\alpha_\rho)=\lim_{\lambda\to\infty}p_b(\alpha_\lambda,\sigma)=p_b(\sigma,\sigma)=0. \tag{11}$$

We now assert that

$$\frac{1}{2s}p_b(\alpha_\lambda,\alpha_{\lambda+1})\leq p_b(\alpha_\lambda,\sigma)\quad\text{or}$$
$$\frac{1}{2s}p_b(\alpha_{\lambda+1},\alpha_{\lambda+2})\leq p_b(\alpha_{\lambda+1},\sigma).$$

On the other hand, assuming the opposite, we can obtain $\lambda_0\in\mathbb{N}$ such that

$$\begin{aligned}&p_b(\alpha_{\lambda_0},\alpha_{\lambda_0+1})\\&\leq s(p_b(\alpha_{\lambda_0},\sigma)+p_b(\sigma,\alpha_{\lambda_0+1}))-p_b(\sigma,\sigma)\\&<s\left(\frac{1}{2s}p_b(\alpha_{\lambda_0},\alpha_{\lambda_0+1})+\frac{1}{2s}p_b(\alpha_{\lambda_0+1},\alpha_{\lambda_0+2})\right)\\&=\frac{1}{2}\left(p_b(\alpha_{\lambda_0},\alpha_{\lambda_0+1})+p_b(\alpha_{\lambda_0+1},\alpha_{\lambda_0+2})\right)\\&<p_b(\alpha_{\lambda_0},\alpha_{\lambda_0+1}),\end{aligned}$$

which is a contradiction. Hence, there exists a subsequence $\{\alpha_{\lambda(\iota)}\}$ of $\{\alpha_\lambda\}$ such that

$$\begin{aligned}&\frac{1}{2s}\min\{p_b(\alpha_{\lambda(\iota)},\mathcal{F}\alpha_{\lambda(\iota)}),p_b(\sigma,\mathcal{F}\sigma)\}\\&=\frac{1}{2s}p_b(\alpha_{\lambda(\iota)},\alpha_{\lambda(\iota)+1})\\&\leq p_b(\alpha_{\lambda(\iota)},\sigma)\end{aligned}$$

implies

$$\eta(s^t p_b(\mathcal{F}\alpha_{\lambda(\iota)},\mathcal{F}\sigma),\mathcal{D}_A(\alpha_{\lambda(\iota)},\sigma))\geq 0,$$

where

$$\begin{aligned}&p_b(\sigma,\mathcal{F}\sigma)\\&\leq\mathcal{D}_A(\alpha_{\lambda(\iota)},\sigma)\\&=\max\Big\{p_b(\alpha_{\lambda(\iota)},\sigma),p_b(\alpha_{\lambda(\iota)},\mathcal{F}\alpha_{\lambda(\iota)}),p_b(\sigma,\mathcal{F}\sigma),\\&\quad\frac{p_b(\sigma,\mathcal{F}\sigma)(1+p_b(\alpha_{\lambda(\iota)},\mathcal{F}\alpha_{\lambda(\iota)}))}{1+p_b(\alpha_{\lambda(\iota)},\sigma)},\\&\quad\frac{p_b(\sigma,\mathcal{F}\sigma)(1+p_b(\alpha_{\lambda(\iota)},\mathcal{F}\alpha_{\lambda(\iota)}))}{1+p_b(\mathcal{F}\alpha_{\lambda(\iota)},\mathcal{F}\sigma)},\\&\quad\frac{p_b(\alpha_{\lambda(\iota)},\mathcal{F}\sigma)+p_b(\sigma,\mathcal{F}\alpha_{\lambda(\iota)})}{2s}\Big\}\\&=\max\Big\{p_b(\alpha_{\lambda(\iota)},\sigma),p_b(\alpha_{\lambda(\iota)},\alpha_{\lambda(\iota)+1}),p_b(\sigma,\mathcal{F}\sigma),\\&\quad\frac{p_b(\sigma,\mathcal{F}\sigma)(1+p_b(\alpha_{\lambda(\iota)},\alpha_{\lambda(\iota)+1}))}{1+p_b(\alpha_{\lambda(\iota)},\sigma)},\\&\quad\frac{p_b(\sigma,\mathcal{F}\sigma)(1+p_b(\alpha_{\lambda(\iota)},\alpha_{\lambda(\iota)+1}))}{1+p_b(\alpha_{\lambda(\iota)+1},\mathcal{F}\sigma)},\\&\quad\frac{p_b(\alpha_{\lambda(\iota)},\mathcal{F}\sigma)+p_b(\sigma,\alpha_{\lambda(\iota)+1})}{2s}\Big\}.\end{aligned}$$

Taking $\iota \to \infty$ and using (11) in mind, we arrive at

$$\lim_{\iota\to\infty} \mathcal{D}_A(\alpha_{\lambda(\iota)}, \sigma) = p_b(\sigma, \mathcal{F}\sigma). \tag{12}$$

On the one hand, we assume that $\alpha_\lambda \neq \sigma$ for an infinite number of $\lambda \in \mathbb{N}$ without sacrificing generality. So,

$$\eta(s^t p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\sigma), \mathcal{D}_A(\alpha_\lambda, \sigma)) \geq 0.$$

Thus, as a result of η_1, leads us to

$$\psi(s^t p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\sigma)) < \psi(\mathcal{D}_A(\alpha_\lambda, \sigma)).$$

Taking into account the fact that has a non-decreasing property of ψ,

$$s^t p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\sigma) < \mathcal{D}_A(\alpha_\lambda, \sigma).$$

On the alternative,

$$\begin{aligned} & p_b(\sigma, \mathcal{F}\sigma) \\ & \leq s\,(p_b(\sigma, \mathcal{F}\alpha_\lambda) + p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\sigma)) - p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\alpha_\lambda) \\ & \leq s p_b(\sigma, \mathcal{F}\alpha_\lambda) + s^t p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\sigma) - p_b(\alpha_{\lambda+1}, \alpha_{\lambda+1}) \\ & < s p_b(\sigma, \mathcal{F}\alpha_\lambda) + \mathcal{D}_A(\alpha_\lambda, \sigma). \end{aligned}$$

Taking $\lambda \to \infty$ in the above inequality and using (11) and (12), we get

$$\begin{aligned} p_b(\sigma, \mathcal{F}\sigma) &\leq s^t \lim_{\lambda\to\infty} p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\sigma) \\ &< \lim_{\lambda\to\infty} \mathcal{D}_A(\alpha_\lambda, \sigma) \\ &= p_b(\sigma, \mathcal{F}\sigma). \end{aligned}$$

Hence, $s^t \lim_{\lambda\to\infty} p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\sigma) = p_b(\sigma, \mathcal{F}\sigma)$. Therefore, putting $\zeta_\lambda = p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\sigma)$ and $\xi_\lambda = \mathcal{D}_A(\alpha_\lambda, \sigma)$, using η_2 it follows $\limsup_{\lambda\to\infty} \eta(s^t\zeta_\lambda, \xi_\lambda) < 0$, which is a contradiction. Then $p_b(\sigma, \mathcal{F}\sigma) = 0 = p_b(\sigma, \sigma)$, that is, σ is a fixed point of $\mathcal{F}$.
Finally, we establish uniqueness of the fixed point. Indeed, if we can find another point, $\upsilon \in \mathcal{A}, \upsilon \neq \sigma$ such that $\upsilon = \mathcal{F}\upsilon$,

$$0 = \frac{1}{2s} \min\{p_b(\upsilon, \mathcal{F}\upsilon), p_b(\sigma, \mathcal{F}\sigma)\} \leq p_b(\upsilon, \sigma),$$

implies

$$\begin{aligned} 0 &\leq \eta(s^t p_b(\mathcal{F}\upsilon, \mathcal{F}\sigma), \mathcal{D}_A(\upsilon, \sigma)) \\ &< \psi(\mathcal{D}_A(\upsilon, \sigma)) - \psi(s^t p_b(\mathcal{F}\upsilon, \mathcal{F}\sigma)) \\ &= \psi(p_b(\upsilon, \sigma)) - \psi(s^t p_b(\upsilon, \sigma)), \end{aligned}$$

which is a contradiction. Hence, $\sigma = \upsilon$. □

Corollary 3.3. *Let $\mathcal{F} : \mathcal{A} \to \mathcal{A}$ be a mapping on a p_b-complete partial b-metric space $(\mathcal{A}, p_b, s > 1)$. Suppose that $\psi \in \Upsilon$ and $\phi : [0, \infty) \to [0, \infty)$ is a function such that $\liminf_{\xi\to\xi_0} \phi(\xi) > 0$, for $\xi_0 > 0$ and $\phi(\xi) = 0 \Leftrightarrow \xi = 0$. If for every $\zeta, \xi \in \mathcal{A}$*

$$\frac{1}{2s} \min\{p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta)\} \leq p_b(\alpha, \delta) \quad \textit{implies}$$
$$\psi(s^t p_b(\mathcal{F}\alpha, \mathcal{F}\delta)) \leq \psi(\mathcal{D}_A(\alpha, \delta)) - \phi(\mathcal{D}_A(\alpha, \delta)).$$

Then $\mathcal{F}$ admits a unique fixed point.

Proof. Taking $\eta(\zeta, \xi) = \psi(\xi) - \phi(\xi) - \psi(\zeta)$ in Theorem 3.2. □

Corollary 3.4. *Let $\mathcal{F} : \mathcal{A} \to \mathcal{A}$ be a mapping on a p_b-complete partial b-metric space $(\mathcal{A}, p_b, s > 1)$. Suppose that $\psi \in \Upsilon$ and $\varphi : [0, \infty) \to [0, 1)$ is a function such that $\limsup_{\xi\to\xi_0} \varphi(\xi) < 1$, for $\xi_0 > 0$ and $\varphi(\xi) = 0 \Leftrightarrow \xi = 0$. If for every $\zeta, \xi \in \mathcal{A}$*

$$\frac{1}{2s} \min\{p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta)\} \leq p_b(\alpha, \delta) \quad \textit{implies}$$
$$\psi(s^t p_b(\mathcal{F}\alpha, \mathcal{F}\delta)) \leq \varphi(\mathcal{D}_A(\alpha, \delta))\psi(\mathcal{D}_A(\alpha, \delta)).$$

Then $\mathcal{F}$ admits a unique fixed point.

Proof. Taking $\eta(\zeta, \xi) = \varphi(\xi)\psi(\xi) - \psi(\zeta)$ in Theorem 3.2. □

Definition 3.5. *Let $(\mathcal{A}, p_b, s > 1)$ be a partial b-metric space. A mapping $\mathcal{F} : \mathcal{A} \to \mathcal{A}$ is called η-rational contraction of type B, if there exists a function $\eta \in \mathcal{Z}_\psi$ such that*

$$\frac{1}{2s} \min\{p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta)\} \leq p_b(\alpha, \delta) \quad \textit{implies}$$
$$\eta(s^t p_b(\mathcal{F}\alpha, \mathcal{F}\delta), \mathcal{D}_B(\alpha, \delta) \geq 0, \tag{13}$$

for every $\alpha, \delta \in \mathcal{A}$, where $\mathcal{D}_B$ is defined as

$$\begin{aligned} & \mathcal{D}_B(\alpha, \delta) \\ & = \max\Big\{p_b(\alpha, \delta), p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta), \\ & \frac{p_b(\delta, \mathcal{F}\delta)p_b(\alpha, \mathcal{F}\alpha)}{1 + p_b(\alpha, \delta)}, \frac{p_b(\delta, \mathcal{F}\delta)p_b(\alpha, \mathcal{F}\alpha)}{1 + p_b(\mathcal{F}\alpha, \mathcal{F}\delta)}, \\ & \frac{p_b(\alpha, \mathcal{F}\alpha) + p_b(\delta, \mathcal{F}\delta)}{2s}\Big\}. \end{aligned} \tag{14}$$

Theorem 3.6. *Let $(\mathcal{A}, p_b, s > 1)$ be a p_b-complete partial b-metric space and $\mathcal{F} : \mathcal{A} \to \mathcal{A}$ be a η-rational contraction of type B. Then $\mathcal{F}$ admits exactly one fixed point.*

Proof. Let the sequence $\{\alpha_\lambda\}$ be defined by (3). Because $\alpha_{\lambda-1} \neq \alpha_\lambda$, for each $\lambda \in \mathbb{N}$, using logic similar to that used to prove Theorem 3.2, we have

$$\begin{aligned} & \frac{1}{2s} \min\{p_b(\alpha_\lambda, \mathcal{F}\alpha_\lambda), p_b(\alpha_{\lambda+1}, \mathcal{F}\alpha_{\lambda+1})\} \\ & = \frac{1}{2s} \min\{p_b(\alpha_\lambda, \alpha_{\lambda+1}), p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})\} \\ & \leq p_b(\alpha_\lambda, \alpha_{\lambda+1}) \end{aligned}$$

implies

$$0 \leq \eta(s^t p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\alpha_{\lambda+1}), \mathcal{D}_B(\alpha_\lambda, \alpha_{\lambda+1})) < \psi(\mathcal{D}_B(\alpha_\lambda, \alpha_{\lambda+1})) - \psi(s^t p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\alpha_{\lambda+1})), \tag{15}$$

where

$$\begin{aligned}
&\mathcal{D}_B(\alpha_\lambda, \alpha_{\lambda+1}) \\
&= \max\Big\{p_b(\alpha_\lambda, \alpha_{\lambda+1}), p_b(\alpha_\lambda, \mathcal{F}\alpha_\lambda), p_b(\alpha_{\lambda+1}, \mathcal{F}\alpha_{\lambda+1}), \\
&\quad \frac{p_b(\alpha_{\lambda+1}, \mathcal{F}\alpha_{\lambda+1})p_b(\alpha_\lambda, \mathcal{F}\alpha_\lambda)}{1+p_b(\alpha_\lambda, \alpha_{\lambda+1})}, \\
&\quad \frac{p_b(\alpha_{\lambda+1}, \mathcal{F}\alpha_{\lambda+1})p_b(\alpha_\lambda, \mathcal{F}\alpha_\lambda)}{1+p_b(\mathcal{F}\alpha_\lambda, \mathcal{F}\alpha_{\lambda+1})}, \\
&\quad \frac{p_b(\alpha_\lambda, \mathcal{F}\alpha_\lambda)+p_b(\alpha_{\lambda+1}, \mathcal{F}\alpha_{\lambda+1})}{2s}\Big\} \\
&= \max\Big\{p_b(\alpha_\lambda, \alpha_{\lambda+1}), p_b(\alpha_\lambda, \alpha_{\lambda+1}), p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2}), \\
&\quad \frac{p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})p_b(\alpha_\lambda, \alpha_{\lambda+1})}{1+p_b(\alpha_\lambda, \alpha_{\lambda+1})}, \\
&\quad \frac{p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})p_b(\alpha_\lambda, \alpha_{\lambda+1})}{1+p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})}, \\
&\quad \frac{p_b(\alpha_\lambda, \alpha_{\lambda+1})+p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})}{2s}\Big\} \\
&= \max\{p_b(\alpha_\lambda, \alpha_{\lambda+1}), p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})\}.
\end{aligned}$$

Hence,

$$\begin{aligned}
&\psi(s^t p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})) \\
&< \psi(\mathcal{D}_B(\alpha_\lambda, \alpha_{\lambda+1})) \\
&\leq \psi(\max\{p_b(\alpha_\lambda, \alpha_{\lambda+1}), p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})\}.
\end{aligned}$$

Taking into account the fact that has a non-decreasing property of ψ,

$$\begin{aligned}
&s^t p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2}) \\
&< \max\{p_b(\alpha_\lambda, \alpha_{\lambda+1}), p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})\}.
\end{aligned}$$

If $\max\{p_b(\alpha_\lambda, \alpha_{\lambda+1}), p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})\} = p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2})$, we get a contradiction, and then it follows that

$$p_b(\alpha_{\lambda+1}, \alpha_{\lambda+2}) < \frac{1}{s^t} p_b(\alpha_\lambda, \alpha_{\lambda+1}).$$

Using Lemma 2.9, we conclude that $\{\alpha_\lambda\}$ is a 0-p_b-Cauchy on a p_b-complete b-partialmetric space, and there exists $\sigma \in \mathcal{A}$ such that $\lim_{\lambda\to\infty} \alpha_\lambda = \sigma$.
Taking into account the continuity of the mapping $\mathcal{F}$, we have

$$\sigma = \lim_{\lambda\to\infty} \alpha_{\lambda+1} = \lim_{\lambda\to\infty} \mathcal{F}\left(\lim_{\lambda\to\infty} \alpha_\lambda\right) = \mathcal{F}\sigma,$$

that is, σ is a fixed point of the mapping $\mathcal{F}$.
Finally, we establish uniqueness of the fixed point. Indeed, if we can find another point, $\upsilon \in \mathcal{A}, \upsilon \neq \sigma$ such that $\upsilon = \mathcal{F}\upsilon$,

$$0 = \frac{1}{2s}\min\{p_b(\upsilon, \mathcal{F}\upsilon), p_b(\sigma, \mathcal{F}\sigma)\} \leq p_b(\upsilon, \sigma),$$

implies

$$\begin{aligned}
0 &\leq \eta(s^t p_b(\mathcal{F}\upsilon, \mathcal{F}\sigma), \mathcal{D}_B(\upsilon, \sigma)) \\
&< \psi(\mathcal{D}_B(\upsilon, \sigma)) - \psi(s^t p_b(\mathcal{F}\upsilon, \mathcal{F}\sigma)) \\
&= \psi(p_b(\upsilon, \sigma)) - \psi(s^t p_b(\upsilon, \sigma)),
\end{aligned}$$

which is a contradiction. Hence, $p_b(\sigma, \upsilon) = 0$, that is $\sigma = \upsilon$. □

Corollary 3.7. *Let $\mathcal{F} : \mathcal{A} \to \mathcal{A}$ be a continuous mapping on a p_b-complete partial b-metric space $(\mathcal{A}, p_b, s > 1)$. Suppose that $\psi \in \Upsilon$ and $\phi : [0, \infty) \to [0, \infty)$ is a function such that $\liminf_{\xi\to\xi_0} \phi(\xi) > 0$, for $\xi_0 > 0$ and $\phi(\xi) = 0 \Leftrightarrow \xi = 0$. If for every $\zeta, \xi \in \mathcal{A}$*

$$\frac{1}{2s}\min\{p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta)\} \leq p_b(\alpha, \delta) \quad \textit{implies} \quad \psi(s^t p_b(\mathcal{F}\alpha, \mathcal{F}\delta)) \leq \psi(\mathcal{D}_B(\alpha, \delta)) - \phi(\mathcal{D}_B(\alpha, \delta)).$$

Then $\mathcal{F}$ admits a unique fixed point.

Proof. Taking $\eta(\zeta, \xi) = \psi(\xi) - \phi(\xi) - \psi(\zeta)$ in Theorem 3.6. □

Corollary 3.8. *Let $\mathcal{F} : \mathcal{A} \to \mathcal{A}$ be a continuous mapping on a p_b-complete partial b-metric space $(\mathcal{A}, p_b, s > 1)$. Suppose that $\psi \in \Upsilon$ and $\varphi : [0, \infty) \to [0, 1)$ is a function such that $\limsup_{\xi\to\xi_0} \varphi(\xi) < 1$, for $\xi_0 > 0$ and $\varphi(\xi) = 0 \Leftrightarrow \xi = 0$. If for every $\zeta, \xi \in \mathcal{A}$*

$$\frac{1}{2s}\min\{p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta)\} \leq p_b(\alpha, \delta) \quad \textit{implies} \quad \psi(s^t p_b(\mathcal{F}\alpha, \mathcal{F}\delta)) \leq \varphi(\mathcal{D}_B(\alpha, \delta))\psi(\mathcal{D}_B(\alpha, \delta)).$$

Then $\mathcal{F}$ admits a unique fixed point.

Proof. Taking $\eta(\zeta, \xi) = \varphi(\xi)\psi(\xi) - \psi(\zeta)$ in Theorem 3.6. □

4 Illustrative example

Example 4.1. *Let the set $\mathcal{A} = \{7, 8, 9, 10\}$ and ρb be the partial b-metric on $\mathcal{A}$ $(s = 2)$, where*

$$p_b(\alpha, \delta) = \begin{cases} 0.000002, & \textit{for } \alpha = \delta = 10 \\ |\alpha - \delta|^2, & \textit{otherwise}. \end{cases}$$

Define the mapping $\mathcal{F}:\mathcal{A}\to\mathcal{A}$,

$$\mathcal{F}\alpha=\begin{cases}7, & \textit{for } \alpha\in\{7,8,9\},\\ 8, & \textit{for } \alpha=10.\end{cases}$$

We choose $\phi\in\Upsilon,\phi(\xi)=\frac{\xi}{2}$ and $\eta(\zeta,\xi)=\frac{\frac{7}{8}\xi-\zeta}{2}$. It is easy to see that $\eta\in\mathcal{Z}_\psi$. We have

α	$\mathcal{F}\alpha$	$p_b(\alpha,\mathcal{F}\alpha)$
7	7	0
8	7	1
9	7	4
10	8	4

and will take into account the following scenarios:
1. For $\alpha,\delta\in\{7,8,9\}$, we have $p_b(\mathcal{F}\alpha,\mathcal{F}\delta)=0$, and then

$$\frac{1}{4}\min\{p_b(\alpha,\mathcal{F}\alpha),p_b(\delta,\mathcal{F}\delta)\}\le 1\le p_b(\alpha,\delta)$$

implies

$$2p_b(\mathcal{F}\alpha,\mathcal{F}\delta)=0\le\frac{12}{13}\mathcal{D}_A(\alpha,\delta)\ge 0,$$

2. For $\alpha=7,\ \delta=10$ we have

$$p_b(\alpha,\delta)=9,\ p_b(7,\mathcal{F}7)=0,$$
$$p_b(10,\mathcal{F}10)=p_b(10,8)=4,$$
$$p_b(\mathcal{F}7,\mathcal{F}10)=p_b(7,8)=1$$

and then

$$\frac{1}{4}\min\{p_b(7,\mathcal{F}7),p_b(10,\mathcal{F}10)\}=0\le 9=p_b(\alpha,\delta)$$

implies

$$2p_b(\mathcal{F}7,\mathcal{F}10)=2\le\frac{63}{8}=\frac{7}{8}p_b(7,10).$$

3. For $\alpha=8,\delta=10$ we have

$$p_b(\alpha,\delta)=4,$$
$$p_b(8,\mathcal{F}8)=1,$$
$$p_b(10,\mathcal{F}10)=p_b(10,8)=4,$$
$$p_b(\mathcal{F}8,\mathcal{F}10)=p_b(7,8)=1$$

and then

$$\frac{1}{4}\min\{p_b(8,\mathcal{F}8),p_b(10,\mathcal{F}10)\}=\frac{1}{4}<4=p_b(\alpha,\delta)$$

implies

$$2p_b(\mathcal{F}8,\mathcal{F}10)=2\le\frac{7}{2}=\frac{7}{8}p_b(8,10).$$

4. For $\alpha=9,\ \delta=10$ we have

$$p_b(\alpha,\delta)=1,$$
$$p_b(9,\mathcal{F}9)=4,$$
$$p_b(10,\mathcal{F}10)=p_b(10,8)=4,$$
$$p_b(\mathcal{F}9,\mathcal{F}10)=p_b(7,8)=1$$

and then

$$\frac{1}{4}\min\{p_b(9,\mathcal{F}9),p_b(10,\mathcal{F}10)\}=1=p_b(\alpha,\delta)$$

implies

$$\begin{aligned}2p_b(\mathcal{F}9,\mathcal{F}10)&=2\\ &\le\frac{70}{8}\\ &=\frac{7}{8}\frac{p_b(9,\mathcal{F}9)(1+p_b(10,\mathcal{F}10)}{1+p_b(\mathcal{F}9,\mathcal{F}10)}\\ &\le\frac{7}{8}\mathcal{D}_A(9,10).\end{aligned}$$

Hence, the hypothesis of Theorem 3.2 are satisfied and $\alpha=10$ is the fixed point of the mapping $\mathcal{F}$.

Example 4.2. *Let the set $\mathcal{A}=[0,1]$, and $p_b:\mathcal{A}\times\mathcal{A}\to[0,\infty)$, $p_b(\alpha,\delta)=(\max\{\alpha,\delta\})^2$ be a partial b-metric on $\mathcal{A}$. Let the continuous mapping $\mathcal{F}:\mathcal{A}\to\mathcal{A}$ be defined by*

$$\mathcal{F}\alpha=\begin{cases}\alpha^2, & \textit{for } \alpha\in\left[0,\frac{2}{5}\right],\\ \\ \frac{4}{25}, & \textit{for } \alpha\in\left(\frac{2}{5},1\right],\end{cases}$$

and the functions $\psi\in\Upsilon$, $\eta\in\mathcal{Z}_\psi$, where $\psi(\xi)=\frac{\xi}{2}$ and $\eta(\zeta,\xi)=\frac{\frac{8}{25}\xi-\zeta}{2}$.
We verify that $\mathcal{F}$ is a η-rational contraction of type B.
1. For $\alpha,\delta\in[0,\frac{2}{5}]$, if $\alpha>\delta$, (the case $\alpha\le\delta$ is similar), we have

$$p_b(\alpha,\delta)=(\max\{\alpha,\delta\})^2=\alpha^2,$$
$$p_b(\alpha,\mathcal{F}\alpha)=(\max\{\alpha,\alpha^2)^2=\alpha^2,$$
$$p_b(\delta,\mathcal{F}\delta)=\delta^2,$$
$$p_b(\mathcal{F}\alpha,\mathcal{F}\delta)=(\max\{\alpha^2,\delta^2\})^2=\alpha^4,$$

and then

$$\frac{1}{4}\min\{p_b(\alpha,\mathcal{F}\alpha),p_b(\delta,\mathcal{F}\delta)\}\le\frac{1}{4}\le\alpha^2\le p_b(\alpha,\delta)$$

implies

$$2p_b(\mathcal{F}\alpha,\mathcal{F}\delta)=2\alpha^4\le\frac{8}{25}\alpha^2\le\frac{8}{25}\mathcal{D}_B(\alpha,\delta).$$

2. For $\alpha, \delta \in (2/5, 1]$, if $\alpha > \delta$, (the case $\alpha \leq \delta$ is similar), we have

$$\begin{aligned} p_b(\alpha, \delta) &= (\max\{\alpha, \delta\})^2 = \alpha^2, \\ p_b(\alpha, \mathcal{F}\alpha) &= (\max\{\alpha, \frac{4}{25}\})^2 = \alpha^2, \\ p_b(\delta, \mathcal{F}\delta) &= \delta^2, \\ p_b(\mathcal{F}\alpha, \mathcal{F}\delta) &= \frac{16}{625}, \end{aligned}$$

and then

$$\frac{1}{4}\min\{p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta)\} \leq \frac{1}{4}\alpha^2 \leq \alpha^2 \leq p_b(\alpha, \delta)$$

implies

$$2p_b(\mathcal{F}\alpha, \mathcal{F}\delta) = \frac{32}{625} \leq \frac{8}{25}\alpha^2 \leq \frac{8}{25}\mathcal{D}_B(\alpha, \delta).$$

3. For $\alpha \in [0, 2/5]$, $\delta \in (2/5, 1]$, we have

$$\begin{aligned} p_b(\alpha, \delta) &= (\max\{\alpha, \delta\})^2 = \delta^2, \\ p_b(\alpha, \mathcal{F}\alpha) &= \alpha^2, \\ p_b(\delta, \mathcal{F}\delta) &= \delta^2, \\ p_b(\mathcal{F}\alpha, \mathcal{F}\delta) &= \frac{16}{625}, \end{aligned}$$

and then

$$\frac{1}{4}\min\{p_b(\alpha, \mathcal{F}\alpha), p_b(\delta, \mathcal{F}\delta)\} \leq \frac{1}{4}\delta^2 \leq \delta^2 \leq p_b(\alpha, \delta)$$

implies

$$\begin{aligned} 2p_b(\mathcal{F}\alpha, \mathcal{F}\delta) &= \frac{32}{625} \\ &\leq \frac{8}{25}\delta^2 \\ &= \frac{8}{25}p_b(\delta, \mathcal{F}\delta) \\ &\leq \frac{8}{25}\mathcal{D}_B(\alpha, \delta). \end{aligned}$$

Hence, all the hypotheses of Theorem (3.6) *are satisfied and $\alpha = 0$ is the unique fixed point of $\mathcal{F}$.*

5 Conclusion

The search for fixed point theorems involving contractive type conditions has received much interest in recent decades. In this context, we analyzed convergence point results for such mappings and illustrative for support theorem based on the new idea of fixed point results by the use of a simulation function employing rational terms in partial b-metric space metric spaces. The fresh ideas inspire more research and applications. It will be fascinating to apply these principles, for example, in metric spaces that do not involve the entire form of triangle inequality, such as partial order b-metric spaces.

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