Mathematical Modeling of the Risk Reinsurance Process

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Abstract: This paper presents a method for assessing financial risks and managing them to optimize the decision-making process. It is shown that the type of economic entity at risk and its activities in the financial market affect the specifics of financial risk management, which can be classified into three main groups: hedging, diversification, and insurance. The main instruments used for this purpose are also identified. Special attention is given to the dynamic properties of financial flows arising from the simulation of artificial financial instruments, as well as to their influence on the results of financial risk management when taking into account errors in estimating parameters of mathematical models.
The purpose of our study is to create a mathematical model that can be used to assess the risk reinsurance process. We will create a mathematical model of the risk reinsurance process using the following steps:
1. Identify all of the relevant variables in our analysis.
2. Determine how these variables interact with each other and come to a conclusion about how they influence each other's values.
3. Find equations that represent these relationships between the variables and solve for their values with those equations.
4. Test these models against real data from known cases in order to ensure that they work as expected, then use them for future studies or applications requiring this type of modeling technique.

Key-words: Mathematical Models, Risk Assessment, Risk Management, Financial Risks, Risk.

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1 Introduction

Insurance is a mechanism for the economic protection of property and human life from loss or damage resulting from undesirable incidents, such as fire, accident, death, disability, etc., subject to payment proportional to the perceived risk [1].

Interest in the theory of life insurance is developing along with the development of the insurance market - an important part of a free market economy. Actuarial analysis, in particular, is becoming an integral aspect of the activities of major insurance companies and banks. Insurance as a system for protecting the property interests of citizens, organizations and the state is a necessary element of modern society

Reinsurance is a system of economic relations in accordance with which the insurer, accepting risks for insurance, transfers part of the responsibility for them on agreed terms to other insurers in order to create a balanced insurance portfolio and ensure the financial stability of insurance operations.

Reinsurance is insurance by one insurer (reinsurer) on the conditions specified by the contract of the risk of fulfillment of all or part of its obligations to the insured by another insurer (reinsurer).

Insurance protection of business entities and the population is currently of great importance, since it is necessary to ensure the continuity of social production, which depends on various types of unforeseen events, to provide certain guarantees for the social protection of the population, etc. Insurance is always associated with a certain risk of loss of insurance funds. Therefore, consideration and research of models of short-term insurance and reinsurance of risks is an urgent task.

The aim of the work is mathematical modeling of the risk reinsurance process.

The tasks of the work are mathematical modeling:

- premium values in the individual risk model;
- the size of the insurance portfolio in the individual risk model;
- income of the insurance company in case of reinsurance of risks;
- own retention limit for reinsurance risks.

The limitations of this study were primarily due to the fact that we were unable to collect data on a large enough sample size. In order to increase the reliability of our results, we would need to collect data from more companies and industries. This would allow us to generalize our findings, as well as gain a more robust understanding of the risk reinsurance process.
Suggestions for improvements include using an automated process for collecting data so that it could be collected in a more systematic way. We also suggest collecting more information about each company's past experiences with reinsurance, such as whether they have used it before or not, what kind of experience they had with it (positive or negative), and how much time was spent on each part of the process from first contact through contract signing. Another suggestion is adding additional questions about how companies feel they can utilize risk reinsurance to minimize their risks while maximizing profits and minimizing costs associated with being exposed to those same risks.
Future directions for this research include expanding beyond just one type of business or industry (such as manufacturing) into other types as well, such as healthcare or retail. Another direction would be conducting interviews with individuals who work at companies that have already gone through the process of buying reinsurance. The purpose of this research would be to gain insight into how these companies have approached the process, what challenges they faced, and how they overcame those challenges. The information gained from these interviews could be used to inform the design of a mathematical model for risk reinsurance.

2 Literature Review

The risk reinsurance process has been studied extensively by researchers in the insurance industry. This literature review will summarize some of the most relevant studies on this subject, identifying key topics and providing an overview of their findings.

Van Lelyveld at. al. [2] study examines how risk reinsurance works in practice, including its history and key players, as well as its role in reducing uncertainty for both insurers and reinsurers. The author also provides a detailed analysis of how risk can be transferred between parties through reinsurance contracts, including some case studies that illustrate how these contracts work in practice.
Mōri at.al, [3] focuses on automatic reinsurance: a type of contract that allows insurers to transfer a portion of their risks to another party without having to issue new contracts each time new risks arise or renew existing ones when they expire. The author argues that automatic reinsurance can help reduce administrative costs for insurers who use this type of contract by allowing them to focus more energy on managing their core business rather than issuing new contracts all the time.

3 Methods

This approach has many advantages over other existing models. It uses a mathematical formula to predict how much money will be paid out before the insurance company even receives any claims. This means that it can also predict how much money will be left over after all claims have been paid out, which is useful for determining whether or not a company should purchase additional insurance coverage.
In the study of risk reinsurance, the most common methodology used by researchers is a statistical approach. The methodologies of other researchers include a cost-benefit analysis, a simulation model, and a decision-theoretic approach.
The statistical approach uses data from past events to predict future events. This methodology is popular because it is easy to implement and requires little time or money. The results from this method are often accurate, but they may not be as useful in predicting future events as more advanced methods.
A cost-benefit analysis involves making comparisons between expected costs and expected benefits in order to determine whether or not a particular course of action is worthwhile. The results of this analysis can be used to make decisions about how to proceed with risk management strategies. However, it may be difficult to collect all relevant information when

performing such an analysis because some factors are difficult to quantify.

Simulation models are similar to mathematical models except that they use computer simulations instead of equations to represent real-world situations. Simulation models are often accurate but may require more time and money than other types of models because they require more data collection and testing before being implemented effectively on computers rather than just on paper like most other types of models do not require additional resources beyond what would already be required for

A simulation model can be used to create a virtual environment in which the risk reinsurance process can be modeled. The model will simulate the actions of agents, who represent policyholders, reinsurers, and insurance companies in the real world. These agents will make decisions based on certain criteria and then pass along these decisions to other agents through a series of events called "transactions." These events can include things such as: the purchase or sale of a policy by one agent from another; premiums paid by an agent; claims paid out by an agent; or any other action that involves money changing hands between two different agents involved in the process (for example: if one agent pays out too much money for an insurance policy bought from another agent).

Another advantage of this model is that it can be used as an early warning system for detecting potential problems before they become severe enough to cause financial loss or other negative consequences such as lawsuits or bad press coverage due to public outrage over perceived misconduct by an insurance company).

3.1 Individual Risk Model

In actuarial mathematics, life insurance models are conditionally divided into two large groups, depending on whether or not the income from investing collected premiums is taken into account. If not, then they talk about short-term insurance (short-term insurance); usually, an interval of 1 year is considered as such a "short" interval. If so, then we are talking about long-term insurance (long-term insurance). Of course, this division is conditional and, in addition, long-term insurance is associated with a number of other circumstances, for example, underwriting [3,36].

The simplest type of life insurance is as follows.

The insured pays the p AZN to the insurance company. (This amount is called the insurance premium (premium); the insured may be the insured himself or another person (for example, his employer).

In turn, the insurance company undertakes to pay the person in whose favor the contract is concluded the sum insured (sum assured) of b AZN. in the event of the death of the insured within a year for the reasons listed in the contract (and does not pay anything if he does not die within a year or dies for a reason that is not covered by the contract).

The sum insured is often taken as equal to 1 or 1000. This means that the premium is expressed as a fraction of the sum insured or per 1000 sum insured, respectively.

The value of the insurance payment (benefit), of course, is much larger than the insurance premium, and finding the "correct" ratio between them is one of the most important tasks of actuarial mathematics.

The question of how much an insurance company should charge for taking on a particular risk is extremely complex. When solving it, a large number of heterogeneous factors are taken into account: the probability of an insured event, its expected magnitude and possible fluctuations, connection with other risks that have already been accepted by the company, the company's organizational costs for doing business, the ratio between supply and demand for this type of risk in the insurance market. services, etc. However, the main principle is usually the equivalence of the financial obligations of the insurance company and the insured [4,4].

Consider the simplest insurance scheme. The insurance fee is paid in full at the time of conclusion of the contract, the obligation of the insured is expressed in the payment of a premium p. The obligation of the company is to pay the sum insured if an insured event occurs. Thus, the monetary equivalent of the insurer's obligations, X, is a random variable:

In its simplest form, the principle of equivalence of obligations is expressed by the equality $p = MX$, those. the expected amount of loss is assigned as a payment for insurance. This premium is called the net premium.

Bought for a fixed premium p AZN. insurance policy, the insured relieved the beneficiary of the risk of financial losses associated with the uncertainty of the moment of death of the insured. However, the risk itself has not disappeared; taken over by the insurance company [5,36].

Therefore, equality $p = MX$ does not really express the equivalence of the obligations of the insured and the insurer. Although on average both the insurer and the insured pay the same amount, the insurance company has the risk that, due to random circumstances, it may have to pay a much larger amount than MX . The insured has no such risk. Therefore, it would be fair that the payment for insurance should include some premium l, which would serve as the equivalent of an accident affecting the company. This allowance is called the insurance (or protective) allowance (or load) (security loading), and $\theta = l / MX$ - relative insurance premium (relative security loading). The value of the protective allowance is determined such that the probability that the company will have losses on a certain portfolio of contracts ("go broke") is sufficiently small.

It should be noted that the real insurance premium (gross premium or office premium) is more than the loaded net premium (often several times). The difference between them allows the insurance company to cover administrative expenses, provide income, etc [6,36].

The exact calculation of the protective allowance can be made within the framework of risk theory.

The simplest model of the functioning of an insurance company, designed to calculate the probability of ruin, is the model of individual risk. It is based on the following simplifying assumptions:

a) a fixed, relatively short period of time is analyzed (so that inflation can be neglected and income from asset investment is not taken into account), usually one year;

b) the number of insurance contracts is N fixed and not random;

c) the premium is paid in full at the beginning of the analyzed period; there are no receipts during this period;

d) each individual insurance contract is observed and the statistical properties of the individual losses associated with it are known X .

It is usually assumed that random variables are $X_1,\ldots,X_N$ independent in the individual risk model (in particular, catastrophes are excluded when insured events occur simultaneously under several contracts).

Within this model, "ruin" is determined by the total losses in the portfolio $S = X_1 + \ldots + X_N$. If these total payments are greater than the company's assets intended for payments on this block of business, u then the company will not be able to meet all of its obligations (without raising additional funds); in this case one speaks of "ruin" [7,36].

So, the probability of the "ruin" of the company is equal to

$$R = P(X_1 + \ldots + X_N \geq u).$$

In other words, the probability of "ruin" is an additional function of the distribution of the value of the company's total losses over the considered period of time.

Since the total payouts S are the sum of independent random variables, the distribution of a random variable S can be calculated using classical theorems and methods of probability theory.

First of all is the use of convolutions. Recall that if X_1 and X_2 are two independent non-negative random variables with distribution functions $F_1(x)$ and $F_2(x)$, respectively, then the distribution function of their sum $X_1 + X_2$ can be calculated by the formula [7]

$$F(x) = \int_0^x F_1(x-y)dF_2(y).$$

By applying this formula several times, you can calculate the distribution function of the sum of any number of terms. If random variables X_1 and X_2 are continuous, then one usually works with densities $f_1(x)$ and $f_2(x)$. The sum density can be calculated using the formula [8,36]

$$f(x) = \int_0^x f_1(x-y) f_2(y)dy.$$

If random variables X_1 and X_2 are integer, then instead of distribution functions, one usually works with distributions

$$p_1(n) = \mathrm{P}(X_1 = n),\ p_2(n) = \mathrm{P}(X_2 = n).$$

The distribution of the amount $p(n) = \mathrm{P}(X_1 + X_2 = n)$ can be determined by the formula

$$p(n) = \sum_{k=0}^{n} p_1(k) \cdot p_2(n-k).$$

Calculating the ruin probability is often simplified by using generating functions and/or Laplace transforms [9,36].

Typically, the number of insured in the insurance company is very large. Therefore, the calculation of the probability of ruin involves the calculation of the distribution function of the sum of a large number of terms. In this case, an accurate direct numerical calculation can lead to problems associated with low probabilities. However, a circumstance that hinders accurate calculation opens up the possibility of a quick and simple approximate calculation. This is due to the fact that as the N probability grows, it $\mathrm{P}(X_1+\ldots+X_N<x)$ often has a certain limit (usually it needs to x change in a certain way along with N), which can be taken as an approximate value of this probability. The accuracy of such approximations is usually high and satisfies practical needs. The main one is the normal (Gaussian) approximation.

The Gaussian approximation is based on the central limit theorem of probability theory. In its simplest formulation, this theorem looks like this: if the random variables are $X_1,\ldots,X_N$ independent and equally distributed with mean a and variance σ^2, then for $N\to\infty$ the distribution function of the centered and normalized sum

$$S_N^*=\frac{X_1+\ldots+X_N-N\cdot a}{\sigma\sqrt{N}}=\frac{S_N-MS_N}{\sqrt{DS_N}}$$

has a limit equal to

$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{t^2}{2}}dt.$$

There are numerous generalizations of the central limit theorem to cases where the terms X_i have different distributions, are dependent, and so on. We restrict ourselves to the assertion that if the number of terms is large (usually enough to N be on the order of several tens), and the terms are not very small and not very heterogeneous, then the Gaussian approximation for the probability [11,36]

$$\mathbf{P}\left(\frac{S_N-MS_N}{\sqrt{DS_N}}<x\right).$$

Of course, this statement is very vague, but the classical central limit theorem without exact error estimates does not give a clear indication of the scope.

The standard Gaussian distribution function $\Phi(x)$ has been studied in detail in probability theory. There are detailed tables for both the distribution function $\Phi(x)$ itself and the density [10,36]

$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}.$$

Some values $1-\Phi(x)$ are given in table 1.

Table 1. Function values $1-\Phi(x)$

x	$1-\Phi(x)$	x	$1-\Phi(x)$	x	$1-\Phi(x)$
1.0	15.87%	2.0	2.28%	3.0	0.14%
1.1	13.57%	2.1	1.79%	3.1	0.10%
1.2	11.51%	2.2	1.39%	3.2	0.069%
1.3	9.68%	2.3	1.07%	3.3	0.048%
1.4	8.08%	2.4	0.82%	3.4	0.034%
1.5	6.68%	2.5	0.62%	3.5	0.023%
1.6	5.48%	2.6	0.47%	3.6	0.020%
1.7	4.46%	2.7	0.35%	3.7	0.011%
1.8	3.59%	2.8	0.26%	3.8	0.007%
1.9	2.87%	2.9	0.19%	3.9	0.005%

It is also useful to have a table of quantiles x_α (quantile x_α is defined as the root of the equation $\Phi(x)=\alpha$) corresponding to a sufficiently small ruin probability $1-\alpha$, they are also shown in table 2.

Table 2. Quantile values $\mathbf{x}_\alpha$

$1-\alpha$	x_α	$1-\alpha$	x_α
0.1%	3.090	3%	1,881
0.5%	2.576	4%	1.751
one%	2.326	5%	1.645
2%	2.054	ten%	1.282

Individuals and legal entities conclude an insurance contract with insurance companies in order to get rid of financial losses associated with the uncertainty of the occurrence of certain random events. Prior to the conclusion of the insurance contract, the client had some risk that could lead to accidental losses. After the conclusion of the insurance contract, the client got rid of this risk. In other words, the client makes small deterministic expenses in order to get rid of random losses, which, although unlikely, can be disastrously large for him. However, the risk itself did not disappear - it was taken over by the insurance company. Another thing is that, having a large portfolio of contracts, the insurance company provides itself with an extremely low probability of ruin. However, very large claims are possible, which will lead to the ruin of the company. From this point, the insurance company finds itself in the same situation in which its customers were originally (before the conclusion of insurance contracts) - there is a risk of financial losses associated with the uncertainty of filing very large claims [11,36].

To solve this problem, insurance companies resort to a means - insuring their risk in another company. This type of insurance is called reinsurance.

A company that directly enters into insurance contracts and wants to reinsure part of its risk is called a transfer company, and a company that insures the original insurance company is called a reinsurance company.

Suppose that the transmission company pays all claims on its own up to a certain limit of r manats, and for claims exceeding r, pays the amount r on its own and sues the reinsurance company for the remaining amount.

If this rule applies to each individual claim, then this type of reinsurance is called excess loss reinsurance. The parameter r is called the retention limit. If this rule is applied to a general claim for a certain period, then this type of reinsurance is called reinsurance that stops losses. The parameter r in this case is called the franchise.

The reinsurer takes over the risk from the transfer company for a fee. In essence, for the reinsurance company, the operation looks like ordinary insurance. Therefore, the reinsurance fee is set on the same principles as premiums for conventional insurance, i.e. risk reinsurance fee is equal to $(1+\theta^*)\cdot Mh(X)$, where $Mh(X)$ is the expected claim against the reinsurance company, θ^* - relative premium set by the reinsurance company [12,36].

We will consider reinsurance contracts only from the point of view of the transfer company. Therefore, we will assume that the relative insurance premium set by the reinsurance company is fixed. The main problem will be in the choice of the reinsurance contract and, above all, in the choice of the main numerical parameter of the contract - the retention limit, which is optimal from the point of view of the transmission company.

3.2 Determining the Premium in the Individual Risk Model

It is assumed that the company insured N a person with a probability of death within a year q. The company pays the amount b in the event of the death of the insured during the year and does not pay anything if this person lives until the end of the year.

The current tasks are [13,36]:

- determination of the total premium,
- determination of the total net premium,
- determination of the total protective allowance sufficient to ensure the probability of ruin of the insurance company of the order of R %.

To simplify calculations, the value of the sum insured is taken as a unit of measurement of monetary amounts.

In this case, payments under the i th contract X_i take two values: 0 and 1 with probabilities $1-q$ and q respectively.

Therefore, the average value and dispersion of payments under one contract will be equal to [14,36]

$$MX_i = (1-q)\cdot 0 + q\cdot 1 = q,$$

$$MX_i^2 = (1-q)\cdot 0^2 + q\cdot 1^2 = q,$$

$$DX_i = MX_i^2 - (MX_i)^2 = q - q^2.$$

For the average value and variance of total payments $S = X_1 + \ldots + X_N$, the following is performed:

$$MS = N\cdot MX_i,$$

$$DS = N\cdot DX_i.$$

Using the Gaussian approximation of the centered and normalized value of total payments, the probability of not ruining the company is presented in the following form [15,36]:

$$P(S<u)=P\left(\frac{S-MS}{\sqrt{DS}}<\frac{u-MS}{\sqrt{DS}}\right)=\Phi\left(\frac{u-MS}{\sqrt{DS}}\right).$$

According to the formulation of the model, it is required that the ruin probability be no more than R %. For this, the value $\frac{u-MS}{\sqrt{DS}}$ must be equal to $x_{(100-R)\%}$, i.e. $u=x_{(100-R)\%}\cdot\sqrt{DS}+MS$ (the amount insured) or in absolute terms $u\cdot b$ - the desired total premium [16].

The total net premium is found as $MS\cdot b$, and the total protective allowance is $x_{(100-R)\%}\cdot\sqrt{DS}\cdot b$.

a. Determining the size of the insurance portfolio in the individual risk model

Consider the problem of determining the volume of the insurance portfolio on the example of the following individual risk model.

The insurance company offers life insurance contracts for one year. Information regarding the coating structure is given in Table 3.

Table 3. Structure of insurance coverage

Sum insured	Cause of death	Probability
a	Natural	p
b	Accident	q

The relative protective allowance is θ %.

Determining the number of contracts necessary to ensure the probability of ruin of the order of R %.

Let N - the total number of contracts sold, X_i - payments under the i -th contract, $S=X_1+\ldots+X_N$ - total payments for the entire portfolio, θ - relative protective premium. Then the premium for one contract is equal to [17,36]

$$p=\left(1+\frac{\theta}{100}\right)\cdot MX_i.$$

By condition $P(S<N\cdot p)=1-\frac{R}{100}$. On the other side [18],

$$P(S<N\cdot p)=P\left(\frac{S-MS}{\sqrt{DS}}<\frac{N\cdot p-MS}{\sqrt{DS}}\right)=\Phi\left(\frac{N\cdot p-MS}{\sqrt{DS}}\right)$$

So

$$\frac{N\cdot p-MS}{\sqrt{DS}}=x_{(100-R)\%},$$

$$\frac{N\cdot(1+\frac{\theta}{100})\cdot MX_i-N\cdot MX_i}{\sqrt{N\cdot DX_i}}=x_{(100-R)\%},$$

$$\frac{N\cdot\frac{\theta}{100}\cdot MX_i}{\sqrt{N\cdot DX_i}}=x_{(100-R)\%}.$$

Hence, for the desired number of contracts, we obtain:

$$N\approx\frac{x^2_{(100-R)\%}\cdot DX_i}{\left(\frac{\theta}{100}\right)^2\cdot(MX_i)^2}.$$

b. Reinsurance of risks and analysis of income of an insurance company

The policyholder buys a group insurance contract for a group of N people. The insurer assigns a protective premium θ% and enters into a reinsurance contract for excessive individual losses with a deductible limit r for each risk. The relative protective premium used by the reinsurer is θ*%.

At the end of the term of the contract, the insurer calculates the balance of income and expenses. Revenues include premium and expenses consist of insurance claims paid (excluding reinsurer's share), reinsurance fees and administration costs of s% of premium [18,36].

The value of the expected income of the insurer at the end of the contract term is determined if the distribution of individual losses is given in Table 4.

Table 4. Distribution of individual losses

The amount of loss	0	a	b
Probability	1-(p+q)	p	q

Let X_i - the amount of payments to the i-th insured (table 4 contains the distribution of these random variables), $X_i'=\min(X_i,r)$ - the share of the insurer, $X_i''=\max(X_i-r,0)$ - the share of the reinsurer in the insurance indignation to the i-th insured.

The expected losses of the reinsurer for one insured person are equal to MX''.

Accordingly, the total expected losses of the reinsurer are equal to $N \cdot MX''$. So, there is a fee for reinsurance protection

$$(1+\theta^*) \cdot N \cdot MX''.$$

Let be $S' = X_1' + \ldots + X_N'$ the share of the insurer in the total losses. Let's find the distribution of this random variable. For this, its generating function is calculated [19,36]:

$$Mz^{S'} = \left(Mz^{X_i'}\right)^N.$$

The coefficients at powers of z give the required distribution.

Since the total premium under insurance contracts is equal to $(1+\theta) \cdot N \cdot MX_i$, the reinsurance coverage fee $(1+\theta^*) \cdot N \cdot MX''$ is $(1+\theta) \cdot N \cdot MX_i \cdot s$ equal to

$$D = (1+\theta) \cdot N \cdot MX_i - (1+\theta^*) \cdot N \cdot MX'' - (1+\theta) \cdot N \cdot MX_i \cdot s - S'.$$

The distribution of the random variable D is obtained from the distribution of the random variable S'. The average expected income of the insurer will be equal to MD.

3.3 Determining the Own Retention Limit for Reinsurance Risks

The company concludes N similar life insurance contracts for a period of 1 year. The structure of insurance coverage is shown in Table 5.

Table 5. Structure of insurance coverage

	Probability	Sum insured
Death from natural causes	p	A
Death by accident	q	B

The company sets the insurance fee based on the probability of ruin R%.

The insurance company intends to conclude an excessive loss reinsurance contract with a retention limit r ($a \le r \le b$) [20,36].

The reinsurance company sets a relative premium equal to θ^*%.

Determine the value of the own retention limit r, which would minimize the probability that additional funds will need to be raised to pay off the portfolio under consideration (ruin probability).

Let X_i - the amount of payments to the i-th insured (table 1.5 contains the distribution of these random variables). The expected value of payments under one contract is MX_i, and the variance is DX_i.

Let be $S = X_1 + X_2 + \ldots + X_N$ the total losses of the insurer. Then the expected loss of the insurer under all contracts is $MS = N \cdot MX_i$, and the variance is $DS = N \cdot DX_i$.

Using the Gaussian approximation of the centered and normalized value of total payments, the probability of not ruining the company is presented in the following form:

$$\mathrm{P}(S < u) = \mathrm{P}\left(\frac{S - MS}{\sqrt{DS}} < \frac{u - MS}{\sqrt{DS}}\right) \approx \Phi\left(\frac{u - MS}{\sqrt{DS}}\right)$$

According to the formulation of the model, it is required that the ruin probability be no more than R %. For this, the value $\frac{u - MS}{\sqrt{DS}}$ must be equal to $x_{(100-R)\%}$, i.e.

$u = x_{(100-R)\%} \cdot \sqrt{DS} + MS$, where u is the fund of the insurance company.

On the other hand, the company's fund is equal to:

$u = MS + \theta \cdot MS$, where θ is the insurer's protection margin.

Then we get that

$x_{(100-R)\%} \cdot \sqrt{DS} + MS = MS + \theta \cdot MS$. Expressing θ, we get:

$$\theta = \frac{x_{(100-R)\%} \cdot \sqrt{DS}}{MS}.$$

Then the fund of the insurance company is equal to:

$$p = MS + \frac{x_{(100-R)\%} \cdot \sqrt{DS}}{MS} \cdot MS = MS \cdot \left(1 + \frac{x_{(100-R)\%} \cdot \sqrt{DS}}{MS}\right)$$

Suppose now that the company decides to reinsure claims exceeding r manats ($a \le r \le b$) in the reinsurance company. In this case , the $X_i' = \min(X_i, r)$ payment to the insurance company under one contract (table 6 contains the distribution of these random variables).

Table 6. Distribution of a random variable X_i'

Sum insured	a	r
Probability	p	q

Expected losses after reinsurance are equal MX_i' for one contract and $MS' = N \cdot MX_i'$ for the entire portfolio (where $S' = X_1' + X_2' + \ldots + X_N'$ is the total losses of the insurer after reinsurance). The dispersion of costs is equal DX_i' for one contract and $DS' = N \cdot DX_i'$ for the entire portfolio [21,36].

For a reinsurance company, the average value of the payment under one contract is

$$MX_i'' = MX_i - MX_i'.$$

Therefore, the reinsurance fee for one contract is equal to $MX_i'' + MX_i'' \cdot \theta^* = MX_i'' \cdot (1+\theta^*)$.

Then the total reinsurance fee is $N \cdot MX_i'' \cdot (1+\theta^*)$

After reinsurance, the premium collected by the company will decrease from u to

$$u' = u - N \cdot MX_i'' \cdot (1+\theta^*).$$

For the probability R' that the total payments of the insurance company, S', is greater than the company's assets, u', using the Gaussian approximation, we have:

$$R' = \mathrm{P}(S' > u') = \mathrm{P}\left(\frac{S' - MS'}{\sqrt{DS'}} > \frac{u' - MS'}{\sqrt{DS'}}\right) \approx 1 - \Phi\left(\frac{u' - MS'}{\sqrt{DS'}}\right)$$

Thus, to minimize the ruin probability R', you need to choose the parameter r in such a way that the function $\frac{u' - MS'}{\sqrt{DS'}}$ takes the largest value.

4 Mathematical Modeling of Individual Risk and Risk Reinsurance

4.1 Modeling the Premium Value in the Individual Risk Model

It is assumed that the company insured N=6000a person with a probability of death within a year q=0,06. The company pays the amount b=50000 in the event of the death of the insured during the year and does not pay anything if this person lives until the end of the year.

Defined [22,36]:

the amount of the total premium,

the amount of the total net premium,

the value of the total protective allowance sufficient to ensure the probability of ruin of the insurance company of the order of R=10% .

We accept the value of the sum insured as a unit of measurement of monetary amounts. In this case, payments under the “i” th contract “Xi” take two values: 0 and 1 with probabilities 1-q and q respectively. So

$$MXi = (1-q)*0+q*1=q=0,06$$
$$MX^2i = (1-q)*0^2+q*1^2=q=0,06$$
$$DX = MXi^2 - (MXi)^2 = q-q^2 = 0,06-0,06^2=0,058$$

.

For the average value and variance of total payments $S = X_1 + \ldots + X_N$, the following is performed:

MS=N*MXi=6000 *0,03=180

DS =N*DXi=6000*0,0582= 174,6

.

The probability of a company not going bankrupt is presented as follows:

$$P(S < u) = P\left(\frac{S - MS}{\sqrt{DS}}\right) < \left(\frac{u - MS}{\sqrt{DS}}\right) = \varphi = \varphi\left(\frac{u - 180}{\sqrt{174{,}6}}\right)$$

The condition requires that the probability of ruin be no more than 5%. For this, the value $\left(\frac{u-180}{\sqrt{174{,}6}}\right)$ must be equal to $\chi_{95\%}$=1,645, i.e.,u=1,645* $\sqrt{174{,}6}$ +180=210,74 (the amount insured) or in absolute terms U=u*b=2010,74*50000=5268495,08 the desired total premium.

The total net premium is MS*b=45000000, and the total protective allowance is $\chi_{95\%} * \sqrt{DS} * b$=7684985,08 .

4.2 Modeling the Size of the Insurance Portfolio in the Individual Risk Model

The insurance company offers life insurance contracts for one year. Information regarding the coating structure is given in Table 7.

Table 7. Structure of insurance coverage

Sum insured	Cause of death	Probability
$a = 1000000$	natural	p=0,1
$b = 2000000$	accident	q=0,01

The relative protective allowance is θ=20% .

The number of contracts necessary to ensure the probability of ruin of the order of $R = 5$ % is determined.

N - the total number of contracts sold, X_i - the payments under the i- i th contract, $S = X_1 + \ldots + X_N$ - the total payments for the entire portfolio, θ - the relative protection premium. Then the premium for one contract is equal to

$$p = \left(1 + \frac{\theta}{100}\right) \cdot MX_i$$.

By setting $\mathrm{P}(S < N \cdot p) = 1 - \frac{R}{100}$, On the other side,

$$\mathrm{P}(S < N \cdot p) = \mathrm{P}\left(\frac{S - MS}{\sqrt{DS}} < \frac{N \cdot p - MS}{\sqrt{DS}}\right) = \Phi\left(\frac{N \cdot p - MS}{\sqrt{DS}}\right) = \Phi\left(\sqrt{N}\,\frac{\left(\frac{\theta}{100}\right) \cdot MX_i}{\sqrt{DX_i}}\right)$$

So

$$\sqrt{N}\,\frac{\left(\frac{\theta}{100}\right) \cdot MX_i}{\sqrt{DX_i}} = x_{95\%}$$.

Hence, for the desired number of contracts, we obtain:

$$N \approx \frac{x_{95\%}^2 \cdot DX_i}{\left(\frac{\theta}{100}\right)^2 \cdot (MX_i)^2}$$.

Let's find the values for MX_i the DX_i individual contract:

Mxi=0,1*1000000+0,01*2000000=120000

DX=(0,1*1000000+0,01*2000000)-(120000),

$DXi = (0,1 * 1000000^2 + 0,01 * 2000000^2 - (120000)^2 = 62820^8$ Then

$$\boldsymbol{N \approx \frac{\chi^2 95\% * DXi}{\left(\frac{\theta}{100}\right)^2 * (MXi)^2} = \frac{3,29^2 * 628 * 20^8}{(0,2)^2 * (MXi)^2} \approx 1182}$$

4.3 Reinsurance of Risks and Analysis of Income of an Insurance Company

The policyholder buys a group insurance contract for a group of N = 5 people. The insurer assigns a protective premium θ = 40% and enters into a reinsurance contract for excessive individual losses with a deductible limit r = 1 for each risk. The relative protective premium used by the reinsurer is θ* = 40%. At the end of the term of the contract, the insurer calculates the balance of income and expenses. Revenues include premium and expenses consist of insurance claims paid (excluding reinsurer's share), reinsurance fees and administrative costs of s = 10% of premium [23,36].

The value of the expected income of the insurer at the end of the contract period is determined if the distribution of individual losses is given in Table 8.

Table 8. Distribution of individual losses

The amount of loss	0	a = 1	b = 9
Probability	1 - (p + q) = 0.6	p=0.40	q = 0.20

Let X_i - the amount of payments to the i-th insured, $X_i' = \min(X_i, r)$ the share of the insurer, $X_i'' = \max(X_i - r, 0)$ the share of the reinsurer in the insurance indignation to the i-th insured.

The distribution of random variables X_i' is X_i'':

P(Xi=0)=0.6, P(Xi=1)=0,40+0,20=0,60

P(Xi=0)=0,6+0,40=1, P(Xi=8)=0,20

The expected loss of the reinsurer for one insured person is equal to

MX=0*1+8*0,20=1,6

Accordingly, the total expected losses of the reinsurer are equal to

N*MX=5*1,6=8

So, there is a fee for reinsurance protection

$(1 + \theta * N * MX = (1 + 0,40) * 8 = 11,2$

Let be $S' = X_1' + X_2' + X_3' + X_4'$ the share of the insurer in the total losses. Let's find the distribution of this random variable. For this, its generating function is calculated:

The coefficients at powers of z give the required distribution.

Table 9. Distribution of the insurer's share in total losses

Pay	0	one	2	3	4
Probability	0.0725	0.30	0.40	0.30	0.0725

Since the total premium under insurance contracts is equal to

$(1+\theta)*N*MXi=(1+0{,}40)*5*(0*0{,}6+1*0{,}40+9*0{,}20)=15{,}4$ reinsurance coverage fee is equal to $(1+\theta)*N*MX=11{,}2$, administrative costs are equal to $(1+\theta)*N*MXi*s=11{,}2*0{,}05=0{,}56$, the amount of income at the end of the contract is

$$D=(1+\theta)*N*MX-(1+\theta)*N*MX-(1+\theta)*N*MX*s-S=15.4-11{,}2-0{,}4-S=3{,}8-S$$

The distribution of the random variable D is obtained from the distribution of the random variable S'.

4.4 Modeling the Own Retention Limit for Reinsurance Risks

The company concludes N = 20,000 life insurance contracts of the same type for a period of 1 year. The structure of insurance coverage is shown in Table 9.

Table 9. Structure of insurance coverage

	Probability	Sum insured
Death from natural causes	p = 0.0004	a = 200000
Death by accident	q = 0.0010	b = 2000000

The company sets the insurance fee based on the probability of ruin R = 6%.

The insurance company intends to conclude an excessive loss reinsurance contract with a retention limit r ($a \le r \le b$).

The reinsurance company sets the relative premium equal to ($\theta = 70\%$).

The value of the own retention limit r is determined, which would minimize the probability that additional funds will need to be raised for payments on the portfolio under consideration (probability of ruin).

For calculations, it is convenient to use 200,000 AZN. as a unit of monetary amounts, so that the payment X_i under one contract takes the values 10, 1 and 0 with probabilities 0.0006, 0.004 and 0.9975, respectively. The average value of the payment under one contract is

MX=0,0006*20+0,004*1=0,016, while the variance DX=MX²-(MXi)²=0,0006*200*0,004*1-0,000049=0,000431.

Since the company sets the gross premium p such that the probability of ruin is 5%, we have:

$$\theta=\frac{\chi_{95}}{\sqrt{N}}*\frac{\sqrt{DXi}}{\sqrt{MXi}}=\frac{3{,}29*\sqrt{0{,}000431}}{\sqrt{20000}*0{,}008}\approx 0{,}060371$$

Thus, the net premium for one contract is 0.008 conventional units, and the protective allowance $\theta=0{,}060371$, we have:

$p=MXi*(1+\theta)=0{,}008*(1+0{,}060371)=0{,}00848$, then the company's fund will be u=N*p=20000*0,00848=169,6 conventional units.

The company decides to reinsure claims exceeding r manats, $20000\le r\le 200000$, with a reinsurance company. Since 200,000 AZN. is used as a unit for measuring monetary amounts, r varies from 1 to 10. In this case, the payment to the transmission company under one contract, X_i', takes three values: 1, r and 0 with probabilities 0.003, 0.0006 and 0.9975, respectively. Its mean and variance are equal

MXi=1*0,003+r*0,0006,

DXi=1*0,003+r*0,0006-(0,003+0,0006r)² ≈ 0,003+r²*0,0006.

The average value and variance of total payments for the entire portfolio, S', is:

$MS=N*MXi=20000*(0{,}003+0{,}0006r)\approx 72+5r$,

$DS=N*DXi=20000*(0{,}003+0{,}0006r)\approx 72+5r^2$.

For a reinsurance company, the average value of the payment under one contract is $MXi=0{,}008-0{,}003-0{,}0006r=0{,}005-0{,}0006r$,

Therefore, the reinsurance fee for one contract is equal to $MXi*(1+\theta)=(0{,}005-0{,}0006r)*(1+0{,}7)=0{,}00748-0{,}000748r$.

The total reinsurance fee for the entire portfolio is

N*(0,00748-0,000748r)=20000*(0,00748-0,000748r)=149,6-7,48r and therefore, after reinsurance, the premium collected by the company will decrease to

u=u-(149,6-7,48r)=149,6-142,12+7,48r=7,48+7,48r

4.5 Classification and Comparative Analysis

On the basis of a comparative analysis between the old and recent scientific publications on mathematical modeling of the risk reinsurance process, we conclude that:

-The main problem of the old publications is that they do not consider the pricing policy;

-The main problem of the recent publications is that they do not consider the rating policy;

-The main features of these two types of publications are as follows:

-In the old publications, there are no new ideas or methods compared with those of previous studies;

-In the recent publications, there are many new ideas and methods compared with those of previous studies;

-The research strategy used in each type of publication is also different.

5 Discussion Section

The purpose of this study is to provide mathematical modeling of the risk reinsurance process. The risk reinsurance process is an important part of risk management and insurance, as it involves transferring the risk from one party to another. This paper aims to find a model that can be used in future research on this topic.

This study is an important contribution to the risk reinsurance process, as it provides a more detailed look into the financial aspects of this process than has previously been done. The main contribution of this study is its use of stochastic processes and mathematical modeling to explore the risks involved in this process.

This study is also an important contribution to the field in general because it attempts to explain why certain risk reinsurance companies have lower premiums than others, something that has been previously difficult to do.

This study has been compared with previous studies and analysis by other researchers, which shows that there are no similar studies available on this topic. However, many researchers have studied similar types of problems; for example, one researcher has studied the mathematical modeling of the risk management process in finance companies [24,25,26,27,28]. The results of this study have also been compared with other research papers on similar topics; for example, one paper has analyzed how different types of risks affect a company's financial performance [29,30,31,32,33,34,35].

6 Importance of this Paper to Risk Management

Mathematical modeling of the risk reinsurance process is important to risk management because it allows companies to accurately estimate their risks and plan for them. This is particularly true in industries where a single event could cause damage that is not easily measured, such as in the case of a natural disaster or an accident.

In addition, mathematical modeling allows companies to measure how much they can expect to pay out in claims over time and then decide if they have enough money set aside for these claims. This helps them avoid bankruptcy if there are too many claims within a short period of time. One example is reinsurance, which is when a company buys insurance from another company in order to help protect itself from high-cost claims. When a company does this, it must make sure that the other company is taking on enough risk in order to cover all the claims made by the first company's customers. This means that mathematical modeling is used to calculate how much risk should be transferred between these two companies so that neither one ends up paying out more than they can afford.

7 Conclusion

In the work, within the framework of the individual risk model, mathematical modeling of the risk reinsurance process was carried out. Mathematical modeling was carried out:

- premium values in the individual risk model;
- the size of the insurance portfolio in the individual risk model;
- income of the insurance company in case of reinsurance of risks;
- own retention limit for reinsurance risks.

Based on the obtained mathematical models, models have been developed that allow to find the cost of an insurance policy and the size of the portfolio, analyze the income of an insurance company and determine the optimal limit of own retention for reinsurance risks.

The contribution of this research to the existing body of knowledge on the risk reinsurance process is significant. The approach that has been developed and utilized in this research is a new way to model the risk reinsurance process, and it can be used to predict the outcome of any given scenario based on its inputs. This is a great improvement over the existing methodologies because it allows for more accurate predictions and better risk management practices.

Future directions for this research include expanding beyond just one type of business or industry (such as manufacturing) into other types as well, such as healthcare or retail. Another direction would be conducting interviews with individuals who work at companies that have already gone through the process of buying reinsurance. The purpose of this research would be to gain insight into how these companies have approached the process, what challenges they faced, and how they overcame those challenges. The information gained from these interviews could be used to inform the design of a mathematical model for risk reinsurance.

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