Global Stability of Symbiotic Model of Commensalism and Parasitism with Harvesting in Commensal Populations

FENGDE CHEN, QIMEI ZHOU, SIJIA LIN
College of Mathematics and Statistics
Fuzhou University
No. 2, wulongjiang Avenue, Minhou County, Fuzhou
CHINA

Abstract: - This article revisit the stability property of symbiotic model of commensalism and parasitism with harvesting in the commensal population. The model was proposed by Nurmaini Puspitasari, Wuryansari Muharini Kusumawinahyu, Trisilowati (Dynamic analysis of the symbiotic model of commensalism and parasitism with harvesting in commensal populations, Jurnal Teori dan Aplikasi Matematika, 2021, 5(1): 193-204). By establishing three powerful Lemmas, sufficient conditions which ensure the global stability of the equilibria are obtained.

Key-Words: -Commensalism; Parasitism; Comparison theorem; Global attractivity

Received: August 17, 2021. Revised: May 9, 2022. Accepted: May 25, 2022. Published: June 17, 2022.

1 Introduction

The aim of this paper is to revisit the global stability property of the following symbiotic model of commensalism and parasitism with harvesting in the commensal population:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\Big(1-\frac{x}{k_1}+a\frac{y}{k_1}\Big) \\ &\quad -\frac{qEx}{m_1E+m_2x}, \\ \frac{dy}{dt} &= r_2 y\Big(1-\frac{y}{k_2}-b\frac{z}{k_2}\Big), \\ \frac{dz}{dt} &= r_3 z\Big(1-\frac{z}{k_3}+c\frac{y}{k_3}\Big), \end{aligned} \tag{1}$$

where $x(t), y(t)$ and $z(t)$ denote the commensal population, host population and parasite species, respectively. All parameters used in this model are positive. For the detail construction of model (1) and the interpret of the biological meaning of the coefficients, one could refer to Nurmaini Puspitasari, Wuryansari Muharini Kusumawinahyu, Trisilowati[25]).

During the lase decade, many scholars investigated the dynamic behaviors of the mutualism model or commensalism model ([1]-[30]), most of those works are concerned with the two species case, recently, Puspitasari, Kusumawinahyu and Trisilowati[25] began to study three species case. They proposed the system (1). The system has eight equilibria, which takes the form

$$T_0(0,0,0),\ \ T_1(0,0,k_3),\ \ T_2(0,k_2,0),$$

$$T_3(x_3^*,0,0),\ \ T_4\Big(0,\frac{k_2-bk_3}{1+bc},\frac{k_3+ck_2}{1+bc}\Big),$$

$$T_5(x_5^*,0,k_3),\ \ T_6(x_6^*,k_2,0),\ \ T_7(x_7^*,y_7^*,z_7^*).$$

Concerned with the local stability property of those equilibria, the authors gave a thoroughly study of the locally stability property of the eight equilibria, and finally, they declared `` Of the eight points, only two points are asymptotically stable if they meet certain conditions." Indeed, they showed that T_4 and T_7 is locally asymptotically stable while the other six equilibria are all unstable.

Now, one natural problem is that the conclusions of Puspitasari, Kusumawinahyu and Trisilowati[25] are all locally ones, whether we could obtain some sufficient conditions to ensure the globally stability property of the equilibria T_4 and T_7?

The aim of this paper is to give affirm answer to above issue. For more works on the ecosystem with Michaelis-Menten type harvesting, one could refer to [31]-[39] and the references cited therein.

The rest of the paper is arranged as follows. In next section, we will state the main results of this paper. We state and prove four useful Lemmas. We then prove the main results in Section 4. Numeric simulations are presented in Section 5 to show the feasibility of the main results. We end this paper by a briefly discussion.

2 Main Results

Following are the main results of this paper.

Theorem 2.1 *Assume that*

$$r_1\Big(1+\frac{ay^*}{k_1}\Big) < \frac{qE}{m_1E+m_2(k_1+ay^*)} \tag{2}$$

and

$$1 > \frac{bk_3}{k_2} \tag{3}$$

hold, then $T_4\Big(0,\frac{k_2-bk_3}{1+bc},\frac{k_3+ck_2}{1+bc}\Big)$ *is globally at-*

tractive, where

$$y^* = \frac{k_2 - bk_3}{1+bc}.$$

Theorem 2.2 *Assume that*

$$r_1\Big(1+\frac{ay_7^*}{k_1}\Big) > \frac{q}{m_1} \tag{4}$$

and

$$1 > \frac{bk_3}{k_2} \tag{5}$$

hold, then $T_7(x_7^*, y_7^*, z_7^*)$ is globally attractive, where

$$y_7^* = \frac{k_2 - bk_3}{1+bc}, z_7^* = \frac{k_3 + ck_2}{1+bc}.$$

3 Lemmas

To finish the proof of Theorem 2.1 and 2.2, we need several powerful Lemmas.

As a direct corollary of Lemma 2.2 of Chen[40], we have

Lemma 3.1. *If $a > 0, b > 0$ and $\dot{x} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have*

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0, b > 0$ and $\dot{x} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}.$$

Consider the equation

$$\frac{dx}{dt} = x(a - bx) - \frac{cx}{d+ex}, \tag{6}$$

where a, b, c, d, e are all positive constants.

Lemma 3.2. *Assume that*

$$a > \frac{c}{d} \tag{7}$$

holds, then system (6) admits a unique positive equilibrium x^* which is globally stable, where

$$x^* = \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1}, \tag{8}$$

and

$$\begin{aligned} A_1 &= be > 0, \\ A_2 &= -ae + bd, \\ A_3 &= c - ad < 0. \end{aligned} \tag{9}$$

Proof. Since

$$\begin{aligned} F(x) &= a - bx - \frac{c}{d+ex} \\ &= -\frac{G(x)}{ex+d}, \end{aligned} \tag{10}$$

where

$$G(x) = A_1x^2 + A_2x + A_3.$$

Noting that $G(x)$ is the quadratic function, and under the assumption of Lemma 2.2, $G(0) = A_3 < 0$. Hence, from the properties of quadratic function, $G(x) = 0$ admits unique positive solution $x^* \in (0, +\infty)$. From (10) one could see that $F(x) = 0$ also admits unique positive solution $x^* \in (0, +\infty)$, $F(x) > 0$ for $x \in (0, x^*)$ and $F(x) < 0$ for $x \in (x^*, +\infty)$. Hence, it immediately follows from Theorem 2.1 in [32] that the unique positive equilibrium x^* of system (6) is globally stable.

The proof of Lemma 2.2 is finished.

Lemma 3.3. *Assume that*

$$c > a\Big(d + \frac{ea}{b}\Big) \tag{11}$$

holds, then in system (6), species x will finally be driven to extinction, i.e.,

$$\lim_{t\to+\infty} x(t) = 0. \tag{12}$$

Proof. From (11), for any enough small positive constant $\varepsilon > 0$, the inequality

$$a < \frac{c}{d + e\Big(\dfrac{a}{b} + \varepsilon\Big)} \tag{13}$$

holds. From (6) we have

$$\frac{dx}{dt} \leq x(a - bx). \tag{14}$$

Applying Lemma 2.1 to (14) leads to

$$\lim_{t\to+\infty} x(t) \leq \frac{a}{b}. \tag{15}$$

For $\varepsilon > 0$ enough small which satisfies (13), it follows from (15) that there exists an enough large $T_1 > 0$ such that

$$x(t) < \frac{a}{b} + \varepsilon \quad \text{for all} \quad t \geq T_1. \tag{16}$$

For $t \geq T_1$, from (6) and (16), one has

$$\frac{dx}{dt} \leq x(a-bx) - \frac{cx}{d+e\Big(\frac{a}{b}+\varepsilon\Big)}, \qquad (17)$$

and so,

$$x(t) \leq x(T_1)\exp\Big\{\Big(a - \frac{c}{d+e\Big(\frac{a}{b}+\varepsilon\Big)}\Big)(t-T_1)\Big\}. \qquad (18)$$

(18) together with (13) leads to

$$\lim_{t\to+\infty} x(t) = 0. \qquad (19)$$

This ends the proof of Lemma 2.3.

Now let's consider the system

$$\begin{aligned}\frac{dy}{dt} &= r_2 y\Big(1-\frac{y}{k_2}-b\frac{z}{k_2}\Big),\\ \frac{dz}{dt} &= r_3 z\Big(1-\frac{z}{k_3}+c\frac{y}{k_3}\Big).\end{aligned} \qquad (20)$$

Lemma 3.4. *Assume that*

$$1 > \frac{bk_3}{k_2} \qquad (21)$$

hold, then system (20) admits a unique positive equilibrium (y_7^*, z_7^*)*, which is globally attractive, where*

$$y_7^* = \frac{k_2 - bk_3}{1+bc}, z_7^* = \frac{k_3+ck_2}{1+bc}. \qquad (22)$$

Proof. One could easily check that under the assumption (21) holds, system (20) admits a unique positive equilibrium (y_7^*, z_7^*). The positive equilibrium of (20) satisfies the equation

$$\begin{aligned}1-\frac{y_7^*}{k_2}-b\frac{z_7^*}{k_2} &= 0,\\ 1-\frac{z_7^*}{k_3}+c\frac{y_7^*}{k_3} &= 0.\end{aligned} \qquad (23)$$

Now let's consider the Lyapunov function

$$V(x,y) = l_1\Big(y-y_7^*-y_7^*\ln\frac{y}{y_7^*}\Big)+l_2\Big(z-z_7^*-z_7^*\ln\frac{z}{z_7^*}\Big). \qquad (24)$$

By computation, from (23) we have

$$\begin{aligned}&\frac{dV}{dt}\\ =\;& l_1 r_2(y-y_7^*)\Big(1-\frac{y}{k_2}-b\frac{z}{k_2}\Big)\\ &+l_2 r_3(z-z_7^*)\Big(1-\frac{z}{k_3}+c\frac{y}{k_3}\Big)\\ =\;& l_1 r_2(y-y_7^*)\Big(\frac{y_7^*}{k_2}+b\frac{z_7^*}{k_2}-\frac{y}{k_2}-b\frac{z}{k_2}\Big)\\ &+l_2 r_3(z-z_7^*)\Big(\frac{z_7^*}{k_3}-c\frac{y_7^*}{k_3}-\frac{z}{k_3}+c\frac{y}{k_3}\Big)\end{aligned} \qquad (25)$$

$$\begin{aligned}=\;& -\frac{l_1 r_2}{k_2}(y-y_7^*)^2 + l_1 r_2(y-y_7^*)\frac{b}{k_2}(z_7^*-z)\\ &-\frac{l_2 r_3}{k_3}(z-z_7^*)^2+\frac{l_2 r_3}{k_3}(z-z_7^*)(y-y_7^*)\end{aligned} \qquad (26)$$

By choosing the positive constants as: $l_1 = 1, l_2 = \frac{r_2 b k_3}{k_2 r_3 c}$, the following is obtained:

$$\frac{dV}{dt} = -\frac{r_2}{k_2}(y-y_7^*)^2 - \frac{r_2 b}{k_2 c}(z-z_7^*)^2. \qquad (27)$$

Obviously, $\frac{dV}{dt} < 0$ strictly for all $y, z > 0$ except the positive equilibrium (y_7^*, z_7^*), where $\frac{dV}{dt} = 0$. Thus, $V(x, y)$ satisfies Lyapunov's asymptotic stability theorem, and the positive equilibrium (y_7^*, z_7^*) of system (20) is globally stable. This ends the proof of Lemma 2.4.

4 Proof of the main results

Proof of Theorem 2.1. For $\varepsilon > 0$ enough small, condition (2) implies that

$$r_1\Big(1+\frac{a(y^*+\varepsilon)}{k_1}\Big) < \frac{qE}{m_1 E + m_2\Big(k_1 + a(y^*+\varepsilon)+\varepsilon\Big)}. \qquad (28)$$

Noting that in system (1) the second and third equations are independent of x, hence, under the assumption (3) hold, it follows from Lemma 3.4 that system (20) admits a unique positive equilibrium (y_7^*, z_7^*), which is globally attractive, i.e.,

$$\begin{aligned}\lim_{t\to+\infty} y(t) &= y_7^* = \frac{k_2-bk_3}{1+bc} = y^*,\\ \lim_{t\to+\infty} z(t) &= z_7^* = \frac{k_3+ck_2}{1+bc} = z^*.\end{aligned} \qquad (29)$$

For $\varepsilon > 0$ which satisfies (28), there exists a $T_1 > 0$ such that

$$y(t) < y^* + \varepsilon \quad for\ all\ \ t \geq T_1. \qquad (30)$$

From the first equation of system (1), we have

$$\begin{aligned}\frac{dx}{dt} &\le r_1x\Big(1-\frac{x}{k_1}+a\frac{y}{k_1}\Big)\\ &\le r_1x\Big(1-\frac{x}{k_1}+a\frac{y^*+\varepsilon}{k_1}\Big).\end{aligned} \tag{31}$$

Applying Lemma 3.1 to above inequality leads to

$$\limsup_{t\to+\infty} x(t) \le \Big(1+a\frac{y^*+\varepsilon}{k_1}\Big)k_1 = k_1 + a(y^*+\varepsilon). \tag{32}$$

It follows from (32) that there exists a $T_2 > T_1$ such that

$$x(t) < k_1 + a(y^*+\varepsilon) + \varepsilon \quad for\ all\ \ t \ge T_2. \tag{33}$$

For $t \ge T_2$, from (30), (33) and the first equation of system (1), we have

$$\begin{aligned}\frac{dx}{dt} &\le r_1x\Big(1-\frac{x}{k_1}+a\frac{y^*+\varepsilon}{k_1}\Big)\\ &\quad -\frac{qEx}{m_1E+m_2\Big(k_1+a(y^*+\varepsilon)+\varepsilon\Big)}.\end{aligned} \tag{34}$$

Now let's consider the equation

$$\begin{aligned}\frac{du}{dt} &= r_1u\Big(1-\frac{u}{k_1}+a\frac{y^*+\varepsilon}{k_1}\Big)\\ &\quad -\frac{qEu}{m_1E+m_2\Big(k_1+a(y^*+\varepsilon)+\varepsilon\Big)}.\end{aligned} \tag{35}$$

It follows from (28) and Lemma 3.3 that

$$\lim_{t\to+\infty} u(t) = 0. \tag{36}$$

By the comparison theorem of differential equation, (35) and (36), it immediately follows that

$$\lim_{t\to+\infty} x(t) = 0. \tag{37}$$

(29) and (37) show that $T_4\Big(0, \dfrac{k_2-bk_3}{1+bc}, \dfrac{k_3+ck_2}{1+bc}\Big)$ is globally attractive. This ends the proof of Theorem 2.1.

Proof of Theorem 2.2. For $\varepsilon > 0$ enough small, condition (43) implies that

$$r_1\Big(1+\frac{a(y_7^*-\varepsilon)}{k_1}\Big) > \frac{q}{m_1}. \tag{38}$$

Noting that in system (1) the second and third equations are independent of x, hence, under the assumption (5) hold, it follows from Lemma 3.4 that system (20) admits a unique positive equilibrium (y_7^*, z_7^*), which is globally attractive, i.e.,

$$\begin{aligned}\lim_{t\to+\infty} y(t) = y_7^* = \frac{k_2-bk_3}{1+bc},\\ \lim_{t\to+\infty} z(t) = z_7^* = \frac{k_3+ck_2}{1+bc}.\end{aligned} \tag{39}$$

For $\varepsilon > 0$ which satisfies (38), without loss of generality, we may assume that $\varepsilon < \frac{1}{2}y^*$, there exists a $T_1 > 0$ such that

$$y_7^* - \varepsilon < y(t) < y_7^* + \varepsilon \quad for\ all\ \ t \ge T_1. \tag{40}$$

From the first equation of system (1) and (40), we have

$$\begin{aligned}\frac{dx}{dt} &= r_1x\Big(1-\frac{x}{k_1}+a\frac{y}{k_1}\Big)\\ &\quad -\frac{qEx}{m_1E+m_2x}\\ &\le r_1x\Big(1-\frac{x}{k_1}+a\frac{y^*+\varepsilon}{k_1}\Big)\\ &\quad -\frac{qEx}{m_1E+m_2x}.\end{aligned} \tag{41}$$

Now let's consider the equation

$$\begin{aligned}\frac{dw_1}{dt} &= r_1w_1\Big(1-\frac{w_1}{k_1}+a\frac{y^*+\varepsilon}{k_1}\Big)\\ &\quad -\frac{qEw_1}{m_1E+m_2w_1}.\end{aligned} \tag{42}$$

It follows from (43) that

$$r_1\Big(1+\frac{a(y_7^*+\varepsilon)}{k_1}\Big) > \frac{q}{m_1}. \tag{43}$$

Hence, from Lemma 2.2 system (42) admits a unique positive equilibrium $w_1(\varepsilon)$ which is globally attractive, where

$$w_1(\varepsilon) = \frac{-B_2+\sqrt{B_2^2-4B_1B_3}}{2B_1}, \tag{44}$$

and

$$\begin{aligned}B_1 &= m_2r_1 > 0,\\ B_2 &= r_1\Big(m_1E - k_1m_2 - am_2(y_7^*+\varepsilon)\Big),\\ B_3 &= -E\Big(k_1m_1r_1 - k_1q + am_1r_1(y_7^*+\varepsilon)\Big) < 0.\end{aligned} \tag{45}$$

It follows from (41)-(45) that

$$\limsup_{t\to+\infty} x(t) \le w_1(\varepsilon). \tag{46}$$

From the first equation of system (1) and (40), we also have

$$\begin{aligned}\frac{dx}{dt} &= r_1x\Big(1-\frac{x}{k_1}+a\frac{y}{k_1}\Big)\\ &\quad -\frac{qEx}{m_1E+m_2x}\\ &\geq r_1x\Big(1-\frac{x}{k_1}+a\frac{y^*-\varepsilon}{k_1}\Big)\\ &\quad -\frac{qEx}{m_1E+m_2x}.\end{aligned} \tag{47}$$

Now let's consider the equation

$$\begin{aligned}\frac{dw_2}{dt} &= r_1w_2\Big(1-\frac{w_2}{k_1}+a\frac{y^*+\varepsilon}{k_1}\Big)\\ &\quad -\frac{qEw_2}{m_1E+m_2w_2}.\end{aligned} \tag{48}$$

It follows from (38) and Lemma 3.2 that system (48) admits a unique positive equilibrium $w_2(\varepsilon)$ which is globally attractive, where

$$w_2(\varepsilon)=\frac{-C_2+\sqrt{C_2^2-4C_1C_3}}{2C_1}, \tag{49}$$

and

$$\begin{aligned}C_1 &= m_2r_1>0,\\ C_2 &= r_1\Big(m_1E-k_1m_2-am_2(y_7^*-\varepsilon)\Big),\\ C_3 &= -E\Big(k_1m_1r_1-k_1q+am_1r_1(y_7^*-\varepsilon)\Big)<0.\end{aligned} \tag{50}$$

It follows from (47)-(50) that

$$\liminf_{t\to+\infty} x(t)\geq w_2(\varepsilon). \tag{51}$$

(46) and (51) show that

$$w_2(\varepsilon)\leq \liminf_{t\to+\infty} x(t)\leq \limsup_{t\to+\infty} x(t)\leq w_1(\varepsilon). \tag{52}$$

Noting that

$$w_i(\varepsilon)\to x_7^* \ \text{ as } \ \varepsilon\to 0,\ i=1,2. \tag{53}$$

Since ε is enough small positive constant, setting $\varepsilon\to 0$ in (52) leads to

$$\lim_{t\to+\infty} x(t)=x_7^*. \tag{54}$$

(39) and (54) show that $T_7\Big(x_7^*,\dfrac{k_2-bk_3}{1+bc},\dfrac{k_3+ck_2}{1+bc}\Big)$ is globally attractive. This ends the proof of Theorem 2.2.

5 Numeric simulations

Now let us consider the following two examples.

Example 5.1 Consider the following system

$$\begin{aligned}\frac{dx}{dt} &= x\Big(1-\frac{x}{1}+\frac{y}{1}\Big)\\ &\quad -\frac{7x}{2+x},\\ \frac{dy}{dt} &= y\Big(1-\frac{y}{2}-\frac{z}{2}\Big),\\ \frac{dz}{dt} &= z\Big(1-\frac{z}{1}+\frac{y}{1}\Big).\end{aligned} \tag{55}$$

Here, corresponding to system (1.1), we choose $r_1=r_2=r_3=k_1=k_3=b=c=a=E=m_2=1, q=7, m_1=2$, then by simple computation, we have

$$r_1\Big(1+\frac{ay^*}{k_1}\Big)=\frac{3}{2}<2=\frac{qE}{m_1E+m_2(k_1+ay^*)} \tag{56}$$

and

$$1>\frac{1}{2}=\frac{bk_3}{k_2} \tag{57}$$

hold, then it follows from Theorem 2.1 that $T_4\Big(0,0.5,1.5\Big)$ is globally attractive. Figure 1 shows that the first component x in system (55) is approach to zero as t approach to infinite. Figure 2 shows that the second and third components y and z approach to 0.5 and 1.5, respectively, as t approach to infinite.

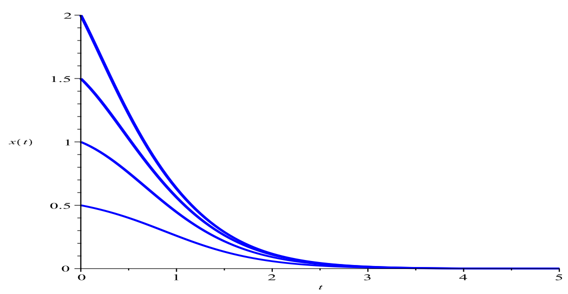

Figure 1: Dynamic behaviors of the first component x in system (55) with the initial condition $(x(0),y(0),z(0)) = (0.5,2,0.5)$, $(1,2,1)$, $(1.5,2,1.5)$ and $(2,2,2)$, respectively.

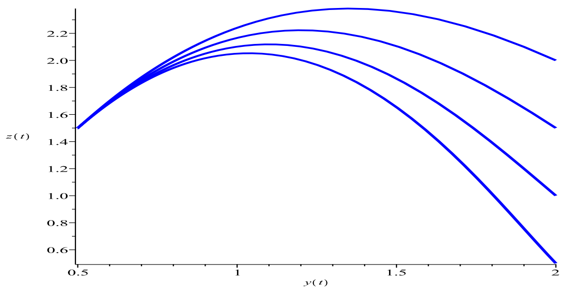

Figure 2: Phase portrait of the second and third component y and z in system (55) with the initial condition $(x(0), y(0), z(0)) = (0.5, 2, 0.5)$, $(1, 2, 1)$, $(1.5, 2, 1.5)$ and $(2, 2, 2)$, respectively.

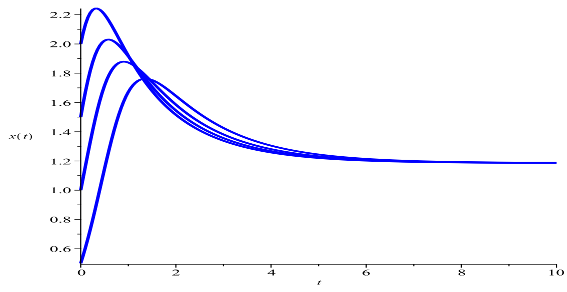

Figure 3: Dynamic behaviors of the first component x in system (58) with the initial condition $(x(0), y(0), z(0)) = (0.5, 2, 0.5)$, $(1, 2, 1)$, $(1.5, 2, 1.5)$ and $(2, 2, 2)$, respectively.

Example 5.2 Consider the following system

$$\begin{aligned} \frac{dx}{dt} &= x\Big(1 - \frac{x}{1} + \frac{y}{1}\Big) \\ &\quad - \frac{1x}{2+x}, \\ \frac{dy}{dt} &= y\Big(1 - \frac{y}{2} - \frac{z}{2}\Big), \\ \frac{dz}{dt} &= z\Big(1 - \frac{z}{1} + \frac{y}{1}\Big). \end{aligned} \tag{58}$$

Here, corresponding to system (1), we choose $r_1 = r_2 = r_3 = k_1 = k_3 = b = c = a = E = m_2 = 1, q = 1, m_1 = 2$, then by simple computation, we have

$$r_1\Big(1 + \frac{ay^*}{k_1}\Big) = \frac{3}{2} > 1 = \frac{q}{m_1} \tag{59}$$

and

$$1 > \frac{1}{2} = \frac{bk_3}{k_2} \tag{60}$$

hold, then it follows from Theorem 2.2 that $T_7\Big(1.186, 0.5, 1.5\Big)$ is globally attractive. Figure 3 shows that the first component x in system (58) is approach to 1.186 as t approach to infinite. Figure 4 shows that the second and third components y and z approach to 0.5 and 1.5, respectively, as t approach to infinite.

6 Conclusion

Puspitasari, Kusumawinahyu and Trisilowati [25] proposed the system (1.1). The system have eight equilibria. By computation, they showed that T_4 and T_7 is locally asymptotically stable while the other six equilibria are all unstable. In this paper, by introducing three powerful Lemmas, we are able to obtain sufficient conditions to ensure the globally attractive of

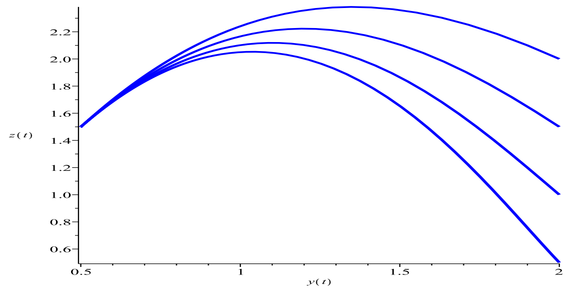

Figure 4: Phase portrait of the second and third component y and z in system (58) with the initial condition $(x(0), y(0), z(0)) = (0.5, 2, 0.5)$, $(1, 2, 1)$, $(1.5, 2, 1.5)$ and $(2, 2, 2)$, respectively.

these two equilibrium.

It is well known that a more plausible system should consider the past state of the species, this will lead to the system with delay, whether our method could be applied to the delay system or not is still unknown, we will leave this for future investigation.

We also notice that the nonautonomous system is more appropriate ([39]-[43]), for such kind of model, the existence of positive periodic solution or almost periodic solution is main topic, we will try to do some works on this direction.

References:

[1] Chen F. D., Xie X. D. and Chen X. F. , Dynamic behaviors of a stage-structured cooperation model, *Commun. Math. Biol. Neurosci.*, Vol. 2015, 2015, 19 pages.

[2] Yang K. , Miao Z., Chen F., et al, Influence of single feedback control variable on an au-

tonomous Holling-II type cooperative system, *Journal of Mathematical Analysis and Applications*, Vol.435, No.1, 2016, pp. 874-888.

[3] Xie X. D., Chen F. D. and Xue Y. L., Note on the stability property of a cooperative system incorporating harvesting, *Discrete Dyn. Nat. Soc.*, Vol. 2014, 2014, 5 pages.

[4] Xue Y. L., Chen F. D. and Xie X. D. , et al. Dynamic behaviors of a discrete commensalism system, *Annals of Applied Mathematics*, Vol.31, No. 4, 2015, pp. 452-461.

[5] Xue Y. L. , Xie X. D. and Chen F. D., et al. Almost periodic solution of a discrete commensalism system, *Discrete Dynamics in Nature and Society*, Volume 2015, Article ID 295483, 11 pages.

[6] Miao Z. S., Xie X. D. and Pu L. Q., Dynamic behaviors of a periodic Lotka-Volterra commensal symbiosis model with impulsive, *Commun. Math. Biol. Neurosci.*, Vol. 2015, 2015, 15 pages.

[7] Wu R. X., Lin L. and Zhou X. Y., A commensal symbiosis model with Holling type functional response, *J. Math. Computer Sci.*, Vol. 16, No.1, 2016, pp. 364-371.

[8] Xie X. D., Miao Z. S. and Xue Y. L., Positive periodic solution of a discrete Lotka-Volterra commensal symbiosis model, *Commun. Math. Biol. Neurosci.*, Vol. 2015 , 2015, 10 pages.

[9] Chen B., The influence of commensalism on a Lotka-Volterra commensal symbiosis model with Michaelis-Menten type harvesting, *Advances in Difference Equations*, Vol. 2019, 2019, Article ID 43.

[10] Liu Y., Xie X. and Lin Q., Permanence, partial survival, extinction, and global attractivity of a nonautonomous harvesting Lotka-Volterra commensalism model incorporating partial closure for the populations, *Advances in Difference Equations*, 2018, Article ID 211.

[11] Deng H. and Huang X., The influence of partial closure for the populations to a harvesting Lotka-Volterra commensalism model, *Commun. Math. Biol. Neurosci.*, Vol. 2018, 2018, Article ID 10.

[12] Xue Y. , Xie X. and Lin Q. , Almost periodic solutions of a commensalism system with Michaelis-Menten type harvesting on time scales, *Open Mathematics*, Vol. 17, No. 1, 2019, pp. 1503-1514.

[13] Lei C., Dynamic behaviors of a stage-structured commensalism system, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 301.

[14] Lin Q., Allee effect increasing the final density of the species subject to the Allee effect in a Lotka-Volterra commensal symbiosis model, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 196.

[15] Chen B., Dynamic behaviors of a commensal symbiosis model involving Allee effect and one party can not survive independently, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 212.

[16] Wu R., Li L. and Lin Q., A Holling type commensal symbiosis model involving Allee effect, *Commun. Math. Biol. Neurosci.*, Vol. 2018, 2018, Article ID 6.

[17] Lei C. , Dynamic behaviors of a Holling type commensal symbiosis model with the first species subject to Allee effect, *Commun. Math. Biol. Neurosci.*, Vol. 2019, 2019, Article ID 3.

[18] Vargas-De-Leon C. and Gomez-Alcaraz G., Global stability in some ecological models of commensalism between two species, *Biomatematica*, Vol.23, No.1, 2013, pp. 139-146.

[19] Chen F., Xue Y. , Lin Q., et al, Dynamic behaviors of a Lotka-Volterra commensal symbiosis model with density dependent birth rate, *Advances in Difference Equations*, Vol.2018, 2018, Article ID 296.

[20] Han R. and Chen F., Global stability of a commensal symbiosis model with feedback controls, *Commun. Math. Biol. Neurosci.*, Vol. 2015, 2015, Article ID 15.

[21] Chen F., Pu L. and Yang L., Positive periodic solution of a discrete obligate Lotka-Volterra model, *Commun. Math. Biol. Neurosci.*, Vol. 2015, 2015, Article ID 14.

[22] Guan X. and Chen F., Dynamical analysis of a two species amensalism model with Beddington-DeAngelis functional response and Allee effect on the second species, *Nonlinear Analysis: Real World Applications*, Vol. 48, No.1, 2019, pp. 71-93.

[23] Li T., Lin Q. and Chen J., Positive periodic solution of a discrete commensal symbiosis model with Holling II functional response, *Commun. Math. Biol. Neurosci.*, Vol. 2016, 2016, Article ID 22.

[24] Ji W. and Liu M., Optimal harvesting of a stochastic commensalism model with time delay, *Physica A: Statistical Mechanics and its Applications*, Vol. 527, 2019, 121284.

[25] Puspitasari N., Kusumawinahyu W. M. , Trisilowati T., Dynamic analysis of the symbiotic model of commensalism and parasitism with harvesting in commensal populations, *JTAM (Jurnal Teori dan Aplikasi Matematika)*, Vol.5, No.1, 2021, pp. 193-204.

[26] Jawad S., Study the dynamics of commensalism interaction with Michaels-Menten type prey harvesting, *Al-Nahrain Journal of Science*, Vol. 25, No.1, 2022, pp. 45-50.

[27] Kumar G. B. and Srinivas M.N., Influence of spatiotemporal and noise on dynamics of a two species commensalism model with optimal harvesting, *Research Journal of Pharmacy and Technology*, Vol.9, No.10, 2016, pp. 1717-1726.

[28] Li T., Wang Q., Stability and Hopf bifurcation analysis for a two-species commensalism system with delay, *Qualitative Theory of Dynamical Systems*, Vol. 20, No. 3, 2021, pp. 1-20.

[29] Chen L., Liu T., et al, Stability and bifurcation in a two-patch model with additive Allee effect, *AIMS Mathematics*, Vol. 7, No. 1, 2022, pp. 536-551.

[30] Zhu Z., Chen Y., et al. Stability and bifurcation in a Leslie-Gower predator-prey model with Allee effect, *International Journal of Bifurcation and Chaos*, Vol. 32, No. 03, 2022, 2250040.

[31] Z. Wei, Y. Xia, T. Zhang, Stability and bifurcation analysis of a commensal model with additive Allee effect and nonlinear growth rate, *International Journal of Bifurcation and Chaos*, Vol.31, No.13, 2021, 2150204.

[32] Chen L. S., Mathematical Models and Methods in Ecology, Science Press, Beijing (1988), (in Chinese).

[33] Lin Q., Xie X., et al, Dynamical analysis of a logistic model with impulsive Holling type-II harvesting, *Advances in Difference Equations*, Vol. 2018, No.1, 2018, Article ID: 112.

[34] Yu X., Zhu Z., et al, Stability and bifurcation analysis in a single-species stage structure system with Michaelis-Menten-type harvesting, *Advances in Difference Equations*, Vol. 2020, 2020, Article ID: 238.

[35] Zhu Z., Chen F., et al, Dynamic behaviors of a discrete May type cooperative system incorporating Michaelis-Menten type harvesting, *IAENG International Journal of Applied Mathematics*, Vol. 50, No. 3, 2020, pp. 1-10.

[36] Yu X., Zhu Z., et al, Dynamic behaviors of a single species stage structure model with Michaelis-Menten-type juvenile population harvesting, *Mathematics*, Vol.8. No. 8, 2020, 1281.

[37] Zhu Z., Wu R., et al, Dynamic behaviors of a Lotka-Volterra commensal symbiosis model with non-selective Michaelis-Menten type harvesting, *IAENG International Journal of Applied Mathematics*, Vol. 50, No. 2, 2020, pp. 1-10.

[38] Yu X., Chen F. and Lai L., Dynamic behaviors of May type cooperative system with Michaelis¨CMenten type harvesting, *Ann. Appl. Math*, Vol. 35, No.4, 2019, pp. 374-391.

[39] Liu Y., Zhao L. , Huang X., et al, Stability and bifurcation analysis of two species amensalism model with Michaelis-Menten type harvesting and a cover for the first species, *Advances in Difference Equations*, Vol. 2018, 2018, Article number: 295

[40] Chen F. D., On a nonlinear non-autonomous predator-prey model with diffusion and distributed delay, *Journal of Computational and Applied Mathematics*, Vol. 180, No.1, 2005, pp. 33-49.

[41] Xie Q. J., He Z. R., Qiu Z. Y., et al, Periodic solutions for three-species diffusive systems with Beddington-Deangelis and Holling-type Iii schemes, *WSEAS Transactions on Mathematics*, Vol. 14, 2015, pp. 47-56.

[42] Hu M., Wang L. L., Dynamic behaviors of N-Species cooperation system with distributed delays and feedback controls on time scales, *WSEAS Transactions on Systems and Control*, Vol. 9, 2014, pp. 291-301.

[43] Li Z. H., Four positive almost periodic solutions to two species parasitical model with impulsive effects and harvesting terms, *WSEAS Transactions on Mathematics*, Vol. 13, 2014, pp. 932-940.

Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

All authors reviewed the literature, formulated the problem, provided independent analysis, and jointly wrote and revised the manuscript

Sources of funding for research presented in a scientific article or scientific article itself

This work is supported by the Natural Science Foundation of Fujian Province(2020J01499).