(Λ, sp)-continuous functions

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Abstract: This paper deals with the concept of (Λ, sp)-continuous functions. Moreover, several characterizations of (Λ, sp)-continuous functions are investigated.

Key–Words: (Λ, sp)-open set, (Λ, sp)-closed set, (Λ, sp)-continuous function

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1 Introduction

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Stronger and weaker forms of open sets play an important role in the researches of generalizations of continuity. In 1968, Singal and Singal [15] introduced and studied the notion of almost continuous functions as a generalization of continuity. Levine [8] introduced the concept of weakly continuous functions as a generalization of almost continuity. In 1983, Abd El-Monsef et al. [1] introduced and investigated the concept of β-continuous functions as a generalization of semi-continuity [7] and percontinuity [11]. Borsík and Doboš [4] introduced the notion of almost quasi-continuity which is weaker than that of quasi-continuity [10] and obtained a decomposition theorem of quasi-continuity. Popa and Noiri [13] investigated some characterizations of β-continuity and showed that almost quasi-continuity is equivalent to β-continuity. The equivalence of almost quasi-continuity and β-continuity is also shown by Borsík [3] and Ewert [6]. In 2004, Noiri and Hatir [12] introduced the notion of Λ_{sp}-sets in terms of β-open sets and investigated the notion of Λ_{sp}-closed sets by using Λ_{sp}-sets. In [2], the present author introduced and studied the concepts of (Λ, sp)-closed sets and (Λ, sp)-open sets. The purpose of the present paper is to introduce the notion of (Λ, sp)-continuous functions. Moreover, several characterizations of (Λ, sp)-continuous functions are discussed.

2 Preliminaries

Throughout this paper, the spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ), $\text{Cl}(A)$ and $\text{Int}(A)$ represent the closure of A and the interior of A, respectively. A subset A of a topological space (X, τ) is called *β-open* [1] if $A \subseteq \text{Cl}(\text{Int}(\text{Cl}(A)))$. The complement of a β-open set is called *β-closed*. The family of all β-open sets in a topological space (X, τ) is denoted by $\beta(X, \tau)$. Let A be a subset of a topological space (X, τ). A subset $\Lambda_{sp}(A)$ [12] is defined as follows: $\Lambda_{sp}(A) = \cap\{U \mid A \subseteq U, U \in \beta(X, \tau)\}$.

Lemma 1. [12] *For subsets A, B and $A_\alpha (\alpha \in \nabla)$ of a topological space (X, τ), the following properties hold:*

(1) $A \subseteq \Lambda_{sp}(A)$.

(2) If $A \subseteq B$, then $\Lambda_{sp}(A) \subseteq \Lambda_{sp}(B)$.

(3) $\Lambda_{sp}(\Lambda_{sp}(A)) = \Lambda_{sp}(A)$.

(4) If $U \in \beta(X, \tau)$, then $\Lambda_{sp}(U) = U$.

(5) $\Lambda_{sp}(\cap\{A_\alpha|\alpha \in \nabla\}) \subseteq \cap\{\Lambda_{sp}(A_\alpha)|\alpha \in \nabla\}$.

(6) $\Lambda_{sp}(\cup\{A_\alpha|\alpha \in \nabla\}) = \cup\{\Lambda_{sp}(A_\alpha)|\alpha \in \nabla\}$.

A subset A of a topological space (X, τ) is called a Λ_{sp}-*set* [12] if $A = \Lambda_{sp}(A)$. The family of all Λ_{sp}-sets of (X, τ) is denoted by $\Lambda_{sp}(X, \tau)$ (or simply Λ_{sp}).

Lemma 2. [12] *For subsets A and $A_\alpha(\alpha \in \nabla)$ of a topological space (X, τ), the following properties hold:*

(1) $\Lambda_{sp}(A)$ *is a* Λ_{sp}*-set.*

(2) If A is β-open, then A is a Λ_{sp}-set.

(3) If A_α is a Λ_{sp}-set for each $\alpha \in \nabla$, then $\cap_{\alpha\in\nabla} A_\alpha$ is a Λ_{sp}-set.

(4) If A_α is a Λ_{sp}-set for each $\alpha \in \nabla$, then $\cup_{\alpha\in\nabla} A_\alpha$ is a Λ_{sp}-set.

Lemma 3. [12] *For a topological space (X, τ), put*

$$\tau^{\Lambda_{sp}} = \{G \mid G \in \Lambda_{sp}(X, \tau)\}.$$

Then, the pair $(X, \tau^{\Lambda_{sp}})$ is an Alexandroff space.

A subset A of a topological space (X, τ) is called (Λ, sp)-*closed* [2] if $A = T \cap C$, where T is a Λ_{sp}-set and C is a β-closed set. The complement of a (Λ, sp)-closed set is called (Λ, sp)-*open*. The collection of all (Λ, sp)-closed (resp. (Λ, sp)-open) sets in a topological space (X, τ) is denoted by $\Lambda_{sp}C(X, \tau)$ (resp. $\Lambda_{sp}O(X, \tau)$).

Lemma 4. *Every Λ_{sp}-set (resp. β-closed set) is (Λ, sp)-closed.*

Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, sp)-*cluster point* [2] of A if $A \cap U \neq \emptyset$ for every (Λ, sp)-open set U of X containing x. The set of all (Λ, sp)-cluster points of A is called the (Λ, sp)-*closure* of A and is denoted by $A^{(\Lambda,sp)}$.

Lemma 5. [2] *Let A and B be subsets of a topological space (X, τ). For the (Λ, sp)-closure, the following properties hold:*

(1) $A \subseteq A^{(\Lambda,sp)}$ *and* $[A^{(\Lambda,sp)}]^{(\Lambda,sp)} = A^{(\Lambda,sp)}$.

(2) If $A \subseteq B$, then $A^{(\Lambda,sp)} \subseteq B^{(\Lambda,sp)}$.

(3) $A^{(\Lambda,sp)}$ *is (Λ, sp)-closed.*

(4) A is (Λ, sp)-closed if and only if $A = A^{(\Lambda,sp)}$.

Let A be a subset of a topological space (X, τ). The union of all (Λ, sp)-open sets contained in A is called the (Λ, sp)-*interior* [2] of A and is denoted by $A_{(\Lambda,sp)}$.

Lemma 6. [2] *For subsets A and B of a topological space (X, τ), the following properties hold:*

(1) $A_{(\Lambda,sp)} \subseteq A$ *and* $[A_{(\Lambda,sp)}]_{(\Lambda,sp)} = A_{(\Lambda,sp)}$.

(2) If $A \subseteq B$, then $A_{(\Lambda,sp)} \subseteq B_{(\Lambda,sp)}$.

(3) $A_{(\Lambda,sp)}$ *is (Λ, sp)-open.*

(4) A is (Λ, sp)-open if and only if $A_{(\Lambda,sp)} = A$.

(5) $[X - A]^{(\Lambda,sp)} = X - A_{(\Lambda,sp)}$.

(6) $[X - A]_{(\Lambda,sp)} = X - A^{(\Lambda,sp)}$.

3 Some characterizations of (Λ, sp)-continuous functions

In this section, we introduce the concept of (Λ, sp)-continuous functions. Moreover, several characterizations of (Λ, sp)-continuous functions are discussed.

Definition 7. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be (Λ, sp)-continuous at a point $x \in X$ if, for each (Λ, sp)-open set V of Y containing $f(x)$, there exists a (Λ, sp)-open set U of X containing x such that $f(U) \subseteq V$. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be (Λ, sp)-continuous if f has this property at each point of X.*

Theorem 8. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is (Λ, sp)-continuous at $x \in X$;

(2) $x \in [f^{-1}(V)]_{(\Lambda,sp)}$ for every (Λ, sp)-open set V of Y containing $f(x)$;

(3) $x \in f^{-1}([f(A)]^{(\Lambda,sp)})$ for every subset A of X with $x \in A^{(\Lambda,sp)}$;

(4) $x \in f^{-1}(B^{(\Lambda,sp)})$ for every subset B of Y with $x \in [f^{-1}(B)]^{(\Lambda,sp)}$;

(5) $x \in [f^{-1}(B)]_{(\Lambda,sp)}$ for every subset B of Y with $x \in f^{-1}(B_{(\Lambda,sp)})$;

(6) $x \in f^{-1}(F)$ for every (Λ, sp)-closed set F of Y with $x \in [f^{-1}(F)]^{(\Lambda,sp)}$.

Proof. (1) $\Rightarrow$ (2): Let V be any (Λ, sp)-open set of Y containing $f(x)$. By (1), there exists a (Λ, sp)-open set U of X containing x such that $f(U) \subseteq V$. Thus, $U \subseteq f^{-1}(V)$ and hence $x \in [f^{-1}(V)]_{(\Lambda,sp)}$.

(2) $\Rightarrow$ (3): Let A be any subset of X, $x \in A^{(\Lambda,sp)}$ and let V be any (Λ, sp)-open set of Y containing $f(x)$. By (2), we have $x \in [f^{-1}(V)]_{(\Lambda,sp)}$ and there exists a (Λ, sp)-open set U of X such that $x \in U \subseteq f^{-1}(V)$. Since $x \in A^{(\Lambda,sp)}$, we have $U \cap A \neq \emptyset$ and

$$\emptyset \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A).$$

Thus, $f(x) \in [f(A)]^{(\Lambda,sp)}$ and hence

$$x \in f^{-1}([f(A)]^{(\Lambda,sp)}).$$

(3) $\Rightarrow$ (4): Let B be any subset of Y and let

$$x \in [f^{-1}(B)]^{(\Lambda,sp)}.$$

By (3), we have $x \in f^{-1}([f(f^{-1}(B))]^{(\Lambda,sp)}) \subseteq f^{-1}(B^{(\Lambda,sp)})$ and hence $x \in f^{-1}(B^{(\Lambda,sp)})$.

(4) $\Rightarrow$ (5): Let B be any subset of Y such that $x \notin [f^{-1}(B)]_{(\Lambda,sp)}$. Then, $x \in X - [f^{-1}(B)]_{(\Lambda,sp)} = [X - f^{-1}(B)]^{(\Lambda,sp)} = [f^{-1}(Y - B)]^{(\Lambda,sp)}$. By (4),

$$\begin{aligned} x \in f^{-1}([Y - B]^{(\Lambda,sp)}) &= f^{-1}(Y - B_{(\Lambda,sp)}) \\ &= X - f^{-1}(B_{(\Lambda,sp)}). \end{aligned}$$

Thus, $x \notin f^{-1}(B_{(\Lambda,sp)})$.

(5) $\Rightarrow$ (6): Let F be any (Λ, sp)-closed set of Y such that $x \notin f^{-1}(F)$. Then, $x \in X - f^{-1}(F) = f^{-1}(Y - F) = f^{-1}([Y - F]_{(\Lambda,sp)})$, by (5),

$$\begin{aligned} x \in [f^{-1}(Y - F)]_{(\Lambda,sp)} &= [(X - f^{-1}(F)]_{(\Lambda,sp)} \\ &= X - [f^{-1}(F)]^{(\Lambda,sp)} \end{aligned}$$

and hence $x \notin [f^{-1}(F)]^{(\Lambda,sp)}$.

(6) $\Rightarrow$ (2): Let V be any (Λ, sp)-open set of Y containing $f(x)$. Suppose that $x \notin [f^{-1}(V)]_{(\Lambda,sp)}$. Then,

$$\begin{aligned} x \in X - [f^{-1}(V)]_{(\Lambda,sp)} &= [X - f^{-1}(V)]^{(\Lambda,sp)} \\ &= [f^{-1}(Y - V)]^{(\Lambda,sp)}. \end{aligned}$$

By (6), $x \in f^{-1}(Y - V) = X - f^{-1}(V)$ and hence $x \notin f^{-1}(V)$. This contraries to the hypothesis.

(2) $\Rightarrow$ (1): Let $x \in X$ and let V be any (Λ, sp)-open set of Y containing $f(x)$. By (2), we have

$$x \in [f^{-1}(V)]_{(\Lambda,sp)}$$

and there exists a (Λ, sp)-open set U of X containing x such that $U \subseteq f^{-1}(V)$. Thus, $f(U) \subseteq V$ and hence f is (Λ, sp)-continuous at x. □

Theorem 9. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is (Λ, sp)-continuous;

(2) $f^{-1}(V)$ is (Λ, sp)-open in X for every (Λ, sp)-open set V of Y;

(3) $f(A^{(\Lambda,sp)}) \subseteq [f(A)]^{(\Lambda,sp)}$ for every subset A of X;

(4) $[f^{-1}(B)]^{(\Lambda,sp)} \subseteq f^{-1}(B^{(\Lambda,sp)})$ for every subset B of Y;

(5) $f^{-1}(B_{(\Lambda,sp)}) \subseteq [f^{-1}(B)]_{(\Lambda,sp)}$ for every subset B of Y;

(6) $f^{-1}(F)$ is (Λ, sp)-closed in X for every (Λ, sp)-closed set F of Y.

Proof. (1) $\Rightarrow$ (2): Let V be any (Λ, sp)-open set of Y such that $x \in f^{-1}(V)$. Then, $f(x) \in V$ and there exists a (Λ, sp)-open set U of X containing x such that $f(U) \subseteq V$. Thus, $U \subseteq f^{-1}(V)$ and hence

$$x \in [f^{-1}(V)]_{(\Lambda,sp)}.$$

(2) $\Rightarrow$ (3): Let A be any subset of X, $x \in A^{(\Lambda,sp)}$ and let V be any (Λ, sp)-open set of Y containing $f(x)$. Then, $x \in [f^{-1}(V)]_{(\Lambda,sp)}$ and there exists a (Λ, sp)-open set U of X such that $x \in U \subseteq f^{-1}(V)$. Since $x \in A^{(\Lambda,sp)}$, we have $U \cap A \neq \emptyset$ and

$$\emptyset \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A).$$

Thus, $f(x) \in [f(A)]^{(\Lambda,sp)}$.

(3) $\Rightarrow$ (4): Let B be any subset of Y. By (3), $f([f^{-1}(B)]^{(\Lambda,sp)}) \subseteq [f(f^{-1}(B))]^{(\Lambda,sp)}$. Thus, $[f^{-1}(B)]^{(\Lambda,sp)} \subseteq f^{-1}(B^{(\Lambda,sp)})$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. By (4), we have

$$\begin{aligned} X - [f^{-1}(B)]_{(\Lambda,sp)} &= [X - f^{-1}(B)]^{(\Lambda,sp)} \\ &= [f^{-1}(Y - B)]^{(\Lambda,sp)} \\ &\subseteq f^{-1}([Y - B]^{(\Lambda,sp)}) \\ &= f^{-1}(Y - B_{(\Lambda,sp)}) \\ &= X - f^{-1}(B_{(\Lambda,sp)}) \end{aligned}$$

and hence $f^{-1}(B_{(\Lambda,sp)}) \subseteq [f^{-1}(B)]_{(\Lambda,sp)}$.

(5) $\Rightarrow$ (6): Let F be any (Λ, sp)-closed set of Y. Then, $Y - F = [Y - K]_{(\Lambda,sp)}$. By (5),

$$\begin{aligned} X - f^{-1}(F) &= f^{-1}(Y - F) \\ &= f^{-1}([Y - F]_{(\Lambda,sp)}) \\ &\subseteq [f^{-1}(Y - F)]_{(\Lambda,sp)} \\ &= [X - f^{-1}(F)]_{(\Lambda,sp)} \\ &= X - [f^{-1}(F)]^{(\Lambda,sp)}. \end{aligned}$$

Thus, $[f^{-1}(F)]^{(\Lambda,sp)} \subseteq f^{-1}(F)$.

(6) $\Rightarrow$ (2): The proof is obvious.

(2) $\Rightarrow$ (1): Let $x \in X$ and let V be any (Λ, sp)-open set of Y containing $f(x)$. By (2), we have

$$x \in [f^{-1}(V)]_{(\Lambda,sp)}$$

and there exists a (Λ, sp)-open set U of X containing x such that $U \subseteq f^{-1}(V)$. Thus, $f(U) \subseteq V$ and hence f is (Λ, sp)-continuous at x. This shows that f is (Λ, sp)-continuous. $\square$

A function $f : (X, \tau) \to (Y, \sigma)$ is called *β-irresolute* [9] if $f^{-1}(V)$ is β-open in X for each β-open set V of Y.

Lemma 10. *Let $f : (X, \tau) \to (Y, \sigma)$ be β-irresolute. Then, $f : (X, \tau^{\Lambda_{sp}}) \to (Y, \sigma^{\Lambda_{sp}})$ is continuous.*

Proof. The proof follows from Theorem 2.8 of [5]. $\square$

Theorem 11. *If $f : (X, \tau) \to (Y, \sigma)$ is β-irresolute, then f is (Λ, sp)-continuous.*

Proof. Let F be any (Λ, sp)-closed set of Y. Then, there exist a Λ_{sp}-set T and a β-closed set C such that $F = T \cap C$. Since f is β-irresolute, $f^{-1}(C)$ is β-closed and $f^{-1}(T)$ is a Λ_{sp}-set of X by Lemma 10. Thus, $f^{-1}(F) = f^{-1}(T) \cap f^{-1}(C)$ is (Λ, sp)-closed in X, by Theorem 9, f is (Λ, sp)-continuous. $\square$

Definition 12. *Let (X, τ) be a topological space, $x \in X$ and let $\{x_\gamma\}_{\gamma\in\Gamma}$ be a net in (X, τ). A net $\{x_\gamma\}_{\gamma\in\Gamma}$ is called Λ_{sp}-converges to x if, for each (Λ, sp)-open set U containing x, there exists $\gamma_0 \in \Gamma$ such that $\gamma \geq \gamma_0$ implies $x_\gamma \in U$.*

Lemma 13. *Let A be a subset of a topological space (X, τ). A point $x \in A^{(\Lambda,sp)}$ if and only if there exists a net $\{x_\gamma\}_{\gamma\in\Gamma}$ of A which Λ_{sp}-converges to x.*

Definition 14. *Let (X, τ) be a topological space, $\mathcal{F} = \{F_\gamma \mid \gamma \in \Gamma\}$ be a filterbase of X and $x \in X$. A filterbase $\mathcal{F}$ is called Λ_{sp}-convergent to x if, for each (Λ, sp)-open set U of X containing x, there exists $F_{\gamma_0} \in \mathcal{F}$ such that $F_{\gamma_0} \subseteq U$.*

Theorem 15. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is (Λ, sp)-continuous;

(2) $f^{-1}(V)$ is (Λ, sp)-open in X for every (Λ, sp)-open set V of Y;

(3) $f(A^{(\Lambda,sp)}) \subseteq [f(A)]^{(\Lambda,sp)}$ for every subset A of X;

(4) $[f^{-1}(B)]^{(\Lambda,sp)} \subseteq f^{-1}(B^{(\Lambda,sp)})$ for every subset B of Y;

(5) $f^{-1}(B_{(\Lambda,sp)}) \subseteq [f^{-1}(B)]_{(\Lambda,sp)}$ for every subset B of Y;

(6) For each $x \in X$ and each filterbase $\mathcal{F}$ which Λ_{sp}-converges to x, $f(\mathcal{F})$ Λ_{sp}-converges to $f(x)$;

(7) For each $x \in X$ and each net $\{x_\gamma\}_{\gamma\in\Gamma}$ in X which Λ_{sp}-converges to x, the net $\{f(x_\gamma)\}_{\gamma\in\Gamma}$ of Y Λ_{sp}-converges to $f(x)$.

Proof. The proof follows from Theorem 3.2 of [5]. $\square$

Definition 16. *A topological space (X, τ) is said to be:*

(i) Λ_{sp}-compact if every cover of X by (Λ, sp)-open sets of X has a finite subcover;

(ii) nearly compact [14] *if every regular open cover of X has a finite subcover.*

Lemma 17. *A topological space (X, τ) is Λ_{sp}-compact if and only if for every family $\{F_\gamma \mid \gamma \in \Gamma\}$ of (Λ, sp)-closed sets in X satisfying $\bigcap_{\gamma\in\Gamma} F_\gamma = \emptyset$, there is a finite subfamily $\{F_{\gamma_i} \mid i = 1, 2, ..., n\}$ with $\bigcap_{i=1}^{n} F_{\gamma_i} = \emptyset$.*

Theorem 18. *For a topological space (X, τ), the following properties hold:*

(1) If $(X, \tau^{\Lambda_{sp}})$ is compact, then (X, τ) is nearly compact.

(2) If (X, τ) is Λ_{sp}-compact, then (X, τ) is nearly compact.

Proof. (1) Let $\{G_\gamma \mid \gamma \in \Gamma\}$ be any regular open cover of X. Since every regular open set is β-open, by Lemma 2(2), G_γ is a Λ_{sp}-set for each $\gamma \in \Gamma$. Moreover, by the compactness of $(X, \tau^{\Lambda_{sp}})$, there exists a finite subset Γ_0 of Γ such that $X = \bigcup_{\gamma\in\Gamma_0} G_\gamma$. This shows that (X, τ) is nearly compact.

(2) Let $\{F_\gamma \mid \gamma \in \Gamma\}$ be a family of regular closed sets of X such that $\bigcap_{\gamma\in\Gamma} F_\gamma = \emptyset$. Since every regular closed set is β-closed and by Lemma 4, F_γ is a (Λ, sp)-closed set for each $\gamma \in \Gamma$. By Lemma 17, there exists a finite subset Γ_0 of Γ such that $\bigcap_{\gamma\in\Gamma_0} F_\gamma = \emptyset$. It follows from Theorem 3.1 of [14] that (X, τ) is nearly compact. $\square$

Theorem 19. *If $f : (X, \tau) \to (Y, \sigma)$ is a (Λ, sp)-continuous surjection and (X, τ) is a Λ_{sp}-compact space, then (Y, σ) is Λ_{sp}-compact.*

Proof. Let $\{V_\gamma \mid \gamma \in \Gamma\}$ be any cover of Y by (Λ, sp)-open sets of Y. Since f is (Λ, sp)-continuous, by Theorem 9, $\{f^{-1}(V_\gamma) \mid \gamma \in \Gamma\}$ is a cover of X by (Λ, sp)-open sets of X. Thus, there exists a finite subset Γ_0 of Γ such that $X = \underset{\gamma\in\Gamma_0}{\cup} f^{-1}(V_\gamma)$. Since f is surjective, $Y = f(X) = \underset{\gamma\in\Gamma_0}{\cup} V_\gamma$. This shows that (Y, σ) is Λ_{sp}-compact. □

Corollary 20. *The Λ_{sp}-compactness is preserved by β-irresolute surjections.*

Proof. This is an immediate consequence of Lemma 10 and Theorem 19. □

Definition 21. *A topological space (X, τ) is called Λ_{sp}-connected if X cannot be written as a disjoint union of two nonempty (Λ, sp)-open sets.*

Theorem 22. *For a topological space (X, τ), the following properties hold:*

(1) If $(X, \tau^{\Lambda_{sp}})$ is connected, then (X, τ) is connected.

(2) If (X, τ) is Λ_{sp}-connected, then (X, τ) is connected.

Proof. (1) Suppose that (X, τ) is not connected. There exist nonempty open sets U, V of X such that $U \cap V = \emptyset$ and $U \cup V = X$. Every open set is β-open and U, V are Λ_{sp}-sets by Lemma 2(2). This shows that $(X, \tau^{\Lambda_{sp}})$ is not connected.

(2) Suppose that $(X, \tau^{\Lambda_{sp}})$ is not connected. There exist nonempty Λ_{sp}-sets U, V of X such that $U \cap V = \emptyset$ and $U \cup V = X$. By Lemma 4, U and V are (Λ, sp)-closed sets. This shows that (X, τ) is not connected. □

Theorem 23. *If $f : (X, \tau) \to (Y, \sigma)$ is a (Λ, sp)-continuous surjection and (X, τ) is Λ_{sp}-connected, then (Y, σ) is Λ_{sp}-connected.*

Proof. Suppose that (Y, σ) is not Λ_{sp}-connected. There exist nonempty (Λ, sp)-open sets U and V of Y such that $U \cap V = \emptyset$ and $U \cup V = Y$. Then, $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty (Λ, sp)-open sets of X. This shows that (X, τ) is not Λ_{sp}-connected. Thus, (Y, σ) is Λ_{sp}-connected. □

Corollary 24. *The Λ_{sp}-connectedness is preserved by β-irresolute surjections.*

Proof. This is an immediate consequence of Lemma 10 and Theorem 23. □

4 Conclusion

Continuity is a basic concept for the study and investigation in topological spaces. Generalization of this concept by using weaker and stronger forms of open sets. This paper is dealing with the notion of (Λ, sp)-continuous functions in topological spaces. In particular, some characterizations of (Λ, sp)-continuous functions are established. The ideas and results of this paper may motivate further research.

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