Some open sets and related topics in topological spaces

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Abstract: Our purpose is to investigate some properties of (Λ, b)-open sets. Several characterizations of (Λ, b)-extremally disconnected spaces, (Λ, b)-hyperconnected spaces and (Λ, b)-submaximal spaces are established. Especially, several characterizations of (Λ, b)-continuous functions are discussed.

Key–Words: (Λ, b)-open set, (Λ, b)-extremally disconnected space, (Λ, b)-hyperconnected space, (Λ, b)-submaximal space, (Λ, b)-continuous function

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1 Introduction

Stronger and weaker forms of open sets play an important role in topological spaces. The concept of semi-open sets was first introduced by Levine [15]. Mashhour et al. [16] introduced and studied the notion of preopen sets. In 1986, Andrijević [3] introduced a new class of sets, called semi-preopen sets. The class of semi-preopen sets contains both the class of semi-open sets and the class of preopen sets. In 1996, Andrijević [2] introduced a new class of generalized open sets, so-called b-open sets. The class of b-open sets is contained in the class of semi-preopen sets and contains all semi-open sets and preopen sets. Caldas et al. [7] introduced the concept of Λ_b-sets which is the intersection of b-open sets and studied the fundamental properties of Λ_b-sets. In [6], the author introduced and investigated the concept of generalized (Λ, b)-closed sets in topological spaces. The concepts of maximality and submaximality of general topological spaces were introduced by Hewitt [12]. Moreover, Hewitt discovered a general way of constructing maximal topologies. The existence of a maximal space that is Tychonoff is nontrivial and due to van Douwen [9]. The first systematic study of submaximal spaces was undertaken in the paper of Arhangel'skiĭ and Collins [4]. They gave various necessary and sufficient conditions for a space to be submaximal and showed that every submaximal space is left-separated. This led to the question whether every submaximal space is σ-discrete [4].

The concept of extremally disconnected topological spaces was introduced by Gillman and Jerison [10]. Thompson [26] introduced the notion of S-closed spaces. Herrman [11] showed that every S-closed weakly Hausdorff space is extremally disconnected. Cameron [8] proved that every maximally S-closed space is extremally disconnected. Noiri [20] introduced the notion of locally S-closed spaces which is strictly weaker than that of S-closed spaces. Noiri [19] showed that every locally S-closed weakly Hausdorff space is extremally disconnected. Sivaraj [22] investigated some characterizations of extremally disconnected spaces by utilizing semi-open sets due to Levine [15]. In [18], the present author obtained several characterizations of extremally disconnected spaces by utilizing preopen sets and semi-preopen sets. Steen and Seebach [24] introduced the notion of hyperconnected spaces. Several concepts which are equivalent to hyperconnectedness were defined and investigated in the literature. Levine [14] called a topological space X a D-space if every nonempty open set of X is dense in X and showed that X is a D-space if and only if it is hyperconnected. Pipitone and Russo [21] defined a topological space X to be semi-connected if X is not the union of two disjoint nonempty semi-open sets of X and showed that X is semi-connected if and only if it is a D-space. Sharma [23] indicated that a space is a D-space if it is a hyperconnected space due to Steen and Seebach. Ajmal and Kohli [1] have investigated the further properties of hyperconnected spaces. Noiri [17] investigated several characterizations of hyperconnected spaces by using semi-preopen sets and almost feebly continuous functions. Hyperconnected spaces are also called irreducible in [25]. Janković and Long [13] introduced and investigated the notion of θ-irreducible spaces.

The purpose of the present paper is to investigate some properties of (Λ, b)-open sets. In Section 4, we introduce the notions of (Λ, b)-extremally disconnected spaces and (Λ, b)-hyperconnected spaces. Moreover, several interesting characterizations of (Λ, b)-extremally disconnected spaces and (Λ, b)-hyperconnected spaces are discussed. Section 5 is devoted to introducing and studying (Λ, b)-submaximal spaces. In Section 6, we introduce the notion of (Λ, b)-continuous functions and investigate some characterizations of (Λ, b)-continuous functions.

2 Preliminaries

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is called *b-open* [2] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(A)) \cup \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a b-open set is called *b-closed*. The family of all b-open sets of a topological space (X, τ) is denoted by $BO(X, \tau)$. The family of all b-closed sets of a topological space (X, τ) is denoted by $BC(X, \tau)$. Let A be a subset of a topological space (X, τ). The intersection of all b-closed sets containing A is called the *b-closure* of A and is denoted by $b\mathrm{Cl}(A)$. The union of all b-open sets contained in A is called the *b-interior* of A and is denoted by $b\mathrm{Int}(A)$.

Definition 1. [7] *Let A be a subset of a topological space (X, τ). A subset A^{Λ_b} is defined as follows: $A^{\Lambda_b} = \cap\{U \mid U \supseteq A,\, U \in BO(X, \tau)\}$.*

Definition 2. [7] *Let A be a subset of a topological space (X, τ). A subset A^{V_b} is defined as follows: $A^{V_b} = \cup\{F \mid F \subseteq A,\, X - F \in BO(X, \tau)\}$.*

Lemma 3. [7] *For subsets A, B and $A_\gamma(\gamma \in \Gamma)$ of a topological space (X, τ), the following properties hold:*

(1) $A \subseteq A^{\Lambda_b}$.

(2) If $A \subseteq B$, *then* $A^{\Lambda_b} \subseteq B^{\Lambda_b}$.

(3) $(A^{\Lambda_b})^{\Lambda_b} = A^{\Lambda_b}$.

(4) $[\underset{\gamma\in\Gamma}{\cup} A_\gamma]^{\Lambda_b} = \underset{\gamma\in\Gamma}{\cup} A_\gamma^{\Lambda_b}$.

(5) If $A \in BO(X, \tau)$, *then* $A = A^{\Lambda_b}$.

(6) $(X - A)^{\Lambda_b} = X - A^{V_b}$.

(7) $A^{V_b} \subseteq A$.

(8) If $A \in BC(X, \tau)$, *then* $A = A^{V_b}$.

(9) $[\underset{\gamma\in\Gamma}{\cap} A_\gamma]^{\Lambda_b} \subseteq \underset{\gamma\in\Gamma}{\cap} A_\gamma^{\Lambda_b}$.

(10) $[\underset{\gamma\in\Gamma}{\cup} A_\gamma]^{V_b} \supseteq \underset{\gamma\in\Gamma}{\cup} A_\gamma^{V_b}$.

Definition 4. [7] *A subset A of a topological space (X, τ) is said to be a Λ_b-set (resp. V_b-set) if $A = A^{\Lambda_b}$ (resp. $A = A^{V_b}$).*

The family of all Λ_b-sets (resp. V_b-sets) in a topological space (X, τ) is denoted by Λ_b (resp. V_b).

Lemma 5. [7] *For a topological space (X, τ), the following properties hold:*

(1) The subsets $\emptyset$ and X are Λ_b-sets and V_b-sets.

(2) Every union of Λ_b-sets (resp. V_b-sets) is a Λ_b-set (resp. V_b-set).

(3) Every intersection of Λ_b-sets (resp. V_b-sets) is a Λ_b-set (resp. V_b-set).

(4) A subset B is a Λ_b-set if and only if $X - B$ is a V_b-set.

Proposition 6. *Let (X, τ) be a topological space. Then $\Lambda_b = \Lambda_{\Lambda_b}$*

Proof. By Lemma 3, we have $BO(X, \tau) \subseteq \Lambda_b$. Let A be any subset of X. Then, we have

$$\begin{aligned}\Lambda_{\Lambda_b}(A) &= \cap\{U \mid A \subseteq U, U \in \Lambda_b\}\\ &\subseteq \{U \mid A \subseteq U, U \in BO(X, \tau)\}\\ &= \Lambda_b(A).\end{aligned}$$

Therefore, we obtain $\Lambda_{\Lambda_b}(A) \subseteq \Lambda_b(A)$. Now, we suppose that $x \notin \Lambda_{\Lambda_b}(A)$. Then, there exists $U \in \Lambda_b$ such that $A \subseteq U$ and $x \notin U$. Since $x \notin U$, there exists $V \in BO(X, \tau)$ such that $U \subseteq V$ and $x \notin V$ and hence $x \notin \Lambda_b(A)$. This shows that $\Lambda_{\Lambda_b}(A) \supseteq \Lambda_b(A)$ and hence $\Lambda_b(A) = \Lambda_{\Lambda_b}(A)$. Consequently, we obtain $\Lambda_b = \Lambda_{\Lambda_b}$. □

3 Properties of (Λ, b)-open sets

In this section, we investigate several properties of (Λ, b)-open sets.

Definition 7. [6] *A subset A of a topological space (X, τ) is called (Λ, b)-closed if $A = T \cap C$, where T is a Λ_b-set and C is a b-closed set. A subset A is said to be (Λ, b)-open if the complement of A is (Λ, b)-closed.*

The family of all (Λ, b)-closed (resp. (Λ, b)-open) sets in a topological space (X, τ) is denoted by $\Lambda_b C(X, \tau)$ (resp. $\Lambda_b O(X, \tau)$).

Lemma 8. [6] *The following properties are equivalent for a subset A of a topological space (X, τ).*

(1) A is (Λ, b)-closed.

(2) $A = T \cap bCl(A)$, where T is a Λ_b-set.

(3) $A = A^{\Lambda_b} \cap bCl(A)$.

Lemma 9. [3] *For a subset A of a topological space (X, τ), the following properties hold:*

(1) $sCl(A) = A \cup Int(Cl(A))$.

(2) $pCl(A) = A \cap Cl(Int(A))$.

Lemma 10. [2] *For a subset A of a topological space (X, τ), the following properties hold:*

(1) $bCl(A) = sCl(A) \cap pCl(A)$.

(2) $bInt(A) = sInt(A) \cup pInt(A)$.

Theorem 11. *For a subset A of a topological space (X, τ), the following are equivalent:*

(1) A is (Λ, b)-open.

(2) $A = T \cup G$, where T is a V_b-set and G is a b-open set.

(3) $A = T \cup bInt(A)$, where T is a V_b-set.

(4) $A = A^{V_b} \cup bInt(A)$.

Proof. (1) $\Rightarrow$ (2): Suppose that A is (Λ, b)-open. Then, $X - A$ is (Λ, b)-closed and $X - A = T \cap F$, where T is a Λ_b-set and F is a b-closed set. Hence, we have $A = (X - A) \cup (X - F)$, where $X - T$ is a V_b-set and $X - F$ is a b-open set.

(2) $\Rightarrow$ (3): Let $A = T \cup G$, where T is a V_b-set and G is a b-open set. Since $G \subseteq A$ and G is b-open, $G \subseteq b\text{Int}(A)$ and hence $A = T \cup G \subseteq T \cup b\text{Int}(A) \subseteq A$. Thus, $A = T \cup b\text{Int}(A)$.

(3) $\Rightarrow$ (4): Let $A = T \cup b\text{Int}(A)$, where T is a V_b-set. Since $T \subseteq A$, we have $A^{V_b} \supseteq T^{V_b}$ and hence $A \supseteq A^{V_b} \cup b\text{Int}(A) \supseteq T^{V_b} \cup b\text{Int}(A) = T \cup b\text{Int}(A) = A$. Consequently, we obtain $A = A^{V_b} \cup b\text{Int}(A)$.

(4) $\Rightarrow$ (1): Let $A = A^{V_b} \cup b\text{Int}(A)$. Then, we have

$$\begin{aligned} X - A &= [X - A^{V_b}] \cap [X - b\text{Int}(A)] \\ &= [X - A]^{\Lambda_b} \cap b\text{Cl}(X - A). \end{aligned}$$

and by Lemma 8, $X - A$ is (Λ, b)-closed. Therefore, A is (Λ, b)-open. □

Theorem 12. *A subset A of a topological space (X, τ) is b-open if and only if A is (Λ, b)-open.*

Proof. Suppose that A is a b-open set. Then, $X - A$ is b-closed and by Lemma 3.3 of [6], we have $X - A$ is (Λ, b)-closed. Thus, A is (Λ, b)-open.

Conversely, suppose that A is a (Λ, b)-open set. Then, $X - A$ is (Λ, b)-closed and by Lemma 8,

$$X - A = (X - A)^{\Lambda_b} \cap b\text{Cl}(X - A).$$

By Lemma 9 and 10, we have

$$\begin{aligned} X - A &= (X - A)^{\Lambda_b} \cap [s\text{Cl}(X - A) \cap p\text{Cl}(X - A)] \\ &= s\text{Cl}(X - A) \cap [(X - A) \cap \text{Cl}(\text{Int}(X - A))] \\ &= s\text{Cl}(X - A) \cap p\text{Cl}(X - A) \\ &= b\text{Cl}(X - A) \end{aligned}$$

and hence $X - A$ is b-closed. Thus, A is b-open. □

Definition 13. [6] *Let A be a subset of a topological space (X, τ). A subset $\Lambda_{(\Lambda,b)}(A)$ is defined as follows: $\Lambda_{(\Lambda,b)}(A) = \cap\{U \in \Lambda_b O(X, \tau) \mid A \subseteq U\}$.*

Lemma 14. [6] *For subsets A, B of a topological space (X, τ), the following properties hold:*

(1) $A \subseteq \Lambda_{(\Lambda,b)}(A)$.

(2) If $A \subseteq B$, then $\Lambda_{(\Lambda,b)}(A) \subseteq \Lambda_{(\Lambda,b)}(B)$.

(3) $\Lambda_{(\Lambda,b)}[\Lambda_{(\Lambda,b)}(A)] = \Lambda_{(\Lambda,b)}(A)$.

(4) If A is (Λ, b)-open, then $\Lambda_{(\Lambda,b)}(A) = A$.

Definition 15. [6] *Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, b)-cluster point of A if $A \cap U \neq \emptyset$ for every (Λ, b)-open set U of X containing x. The set of all (Λ, b)-cluster points of A is called the (Λ, b)-closure of A and is denoted by $A^{(\Lambda,b)}$.*

Lemma 16. [6] *Let A and B be subsets of a topological space (X, τ). For the (Λ, b)-closure, the following properties hold:*

(1) $A \subseteq A^{(\Lambda,b)}$ *and* $[A^{(\Lambda,b)}]^{(\Lambda,b)} = A^{(\Lambda,b)}$.

(2) If $A \subseteq B$*, then* $A^{(\Lambda,b)} \subseteq B^{(\Lambda,b)}$.

(3) $A^{(\Lambda,b)} = \cap\{F | A \subseteq F$ *and* F *is* (Λ, b)*-closed*$\}$.

(4) $A^{(\Lambda,b)}$ *is* (Λ, b)*-closed.*

(5) A *is* (Λ, b)*-closed if and only if* $A = A^{(\Lambda,b)}$.

Proposition 17. *Let* (X, τ) *be a topological space and* $x, y \in X$*. Then,* $y \in \Lambda_{(\Lambda,b)}(\{x\})$ *if and only if* $x \in \{y\}^{(\Lambda,b)}$.

Proof. Suppose that $y \notin \Lambda_{(\Lambda,b)}(\{x\})$. There exists a (Λ, b)-open set V containing x such that $y \notin V$ and hence $x \notin \{y\}^{(\Lambda,b)}$. The converse is similarly shown. □

Theorem 18. *For any points* x *and* y *in a topological space* (X, τ)*, the following properties are equivalent:*

(1) $\Lambda_{(\Lambda,b)}(\{x\}) \neq \Lambda_{(\Lambda,b)}(\{y\})$*;*

(2) $\{x\}^{(\Lambda,b)} \neq \{y\}^{(\Lambda,b)}$.

Proof. (1) ⇒ (2): Suppose that $\Lambda_{(\Lambda,b)}(\{x\}) \neq \Lambda_{(\Lambda,b)}(\{y\})$. Then, there exists a point $z \in X$ such that $z \in \Lambda_{(\Lambda,b)}(\{x\})$ and $z \notin \Lambda_{(\Lambda,b)}(\{y\})$ or $z \in \Lambda_{(\Lambda,b)}(\{y\})$ and $z \notin \Lambda_{(\Lambda,b)}(\{x\})$. We prove only the first case being the second analogous. From $z \in \Lambda_{(\Lambda,b)}(\{x\})$ it follows that $\{x\} \cap \{z\}^{(\Lambda,b)} \neq \emptyset$ which implies $x \in \{z\}^{(\Lambda,b)}$. By $z \notin \Lambda_{(\Lambda,b)}(\{y\})$, we have $\{y\} \cap \{z\}^{(\Lambda,b)} = \emptyset$. Since $x \in \{z\}^{(\Lambda,b)}$, $\{x\}^{(\Lambda,b)} \subseteq \{z\}^{(\Lambda,b)}$ and $\{y\} \cap \{x\}^{(\Lambda,b)} = \emptyset$. Therefore, it follows that $\{x\}^{(\Lambda,b)} \neq \{y\}^{(\Lambda,b)}$. Thus, $\Lambda_{(\Lambda,b)}(\{x\}) \neq \Lambda_{(\Lambda,b)}(\{y\})$ implies that $\{x\}^{(\Lambda,b)} \neq \{y\}^{(\Lambda,b)}$.

(2) ⇒ (1): Suppose that $\{x\}^{(\Lambda,b)} \neq \{y\}^{(\Lambda,b)}$. There exists a point $z \in X$ such that $z \in \{x\}^{(\Lambda,b)}$ and $z \notin \{y\}^{(\Lambda,b)}$ or $z \in \{y\}^{(\Lambda,b)}$ and $z \notin \{x\}^{(\Lambda,b)}$. We prove only the first case being the second analogous. It follows that there exists a (Λ, b)-open set containing z and therefore x but not y, namely, $y \notin \Lambda_{(\Lambda,b)}(\{x\})$ and thus $\Lambda_{(\Lambda,b)}(\{x\}) \neq \Lambda_{(\Lambda,b)}(\{y\})$. □

Theorem 19. *For any points* x *and* y *in a topological space* (X, τ)*, the following properties hold:*

(1) $y \in \Lambda_{(\Lambda,b)}(\{x\})$ *if and only if* $x \in \{y\}^{(\Lambda,b)}$.

(2) $\Lambda_{(\Lambda,b)}(\{x\}) = \Lambda_{(\Lambda,b)}(\{y\})$ *if and only if* $\{x\}^{(\Lambda,b)} = \{y\}^{(\Lambda,b)}$.

Proof. (1) Let $x \notin \{y\}^{(\Lambda,b)}$. Then, there exists a (Λ, b)-open set $U$of X such that $x \in U$ and $y \notin U$. Thus, $y \notin \Lambda_{(\Lambda,b)}(\{x\})$. The converse is similarly shown.

(2) Suppose that $\Lambda_{(\Lambda,b)}(\{x\}) = \Lambda_{(\Lambda,b)}(\{y\})$ for any $x, y \in X$. Since $x \in \Lambda_{(\Lambda,b)}(\{x\})$, we have

$$x \in \Lambda_{(\Lambda,b)}(\{y\})$$

and by (1), $y \in \{x\}^{(\Lambda,b)}$. By Lemma 16, $\{y\}^{(\Lambda,b)} \subseteq \{x\}^{(\Lambda,b)}$. Similarly, we have $\{x\}^{(\Lambda,b)} \subseteq \{y\}^{(\Lambda,b)}$ and hence $\{x\}^{(\Lambda,b)} = \{y\}^{(\Lambda,b)}$.

Conversely, suppose that $\{x\}^{(\Lambda,b)} = \{y\}^{(\Lambda,b)}$. Since $x \in \{x\}^{(\Lambda,b)}$, we have $x \in \{y\}^{(\Lambda,b)}$ and by (1), $y \in \Lambda_{(\Lambda,p)}(\{x\})$. By Lemma 14, $\Lambda_{(\Lambda,b)}(\{y\}) \subseteq \Lambda_{(\Lambda,b)}[\Lambda_{(\Lambda,b)}(\{x\})] = \Lambda_{(\Lambda,b)}(\{x\})$. Similarly, we have $\Lambda_{(\Lambda,b)}(\{x\}) \subseteq \Lambda_{(\Lambda,b)}(\{y\})$ and hence $\Lambda_{(\Lambda,b)}(\{x\}) = \Lambda_{(\Lambda,b)}(\{y\})$. □

4 On (Λ, b)-extremally disconnected spaces and (Λ, b)-hyperconnected spaces

In this section, we introduce the notions of (Λ, b)-extremally disconnected spaces and (Λ, b)-hyperconnected spaces. Several characterizations of (Λ, b)-extremally disconnected spaces and (Λ, b)-hyperconnected spaces are discussed.

Lemma 20. [6] *Let* A *and* B *be subsets of a topological space* (X, τ)*. For the* (Λ, b)*-interior, the following properties hold:*

(1) $A_{(\Lambda,b)} \subseteq A$ *and* $[A_{(\Lambda,b)}]_{(\Lambda,b)} = A_{(\Lambda,b)}$.

(2) If $A \subseteq B$*, then* $A_{(\Lambda,b)} \subseteq B_{(\Lambda,b)}$.

(3) $A_{(\Lambda,b)}$ *is* (Λ, b)*-open.*

(4) A *is* (Λ, b)*-open if and only if* $A_{(\Lambda,b)} = A$.

Definition 21. *A topological space* (X, τ) *is called* (Λ, b)*-extremally disconnected if* $U^{(\Lambda,b)}$ *is* (Λ, b)*-open in* X *for every* (Λ, b)*-open set* U *of* X.

Theorem 22. *For a topological space* (X, τ)*, the following properties are equivalent:*

(1) (X, τ) *is* (Λ, b)*-extremally disconnected.*

(2) $F_{(\Lambda,b)}$ *is* (Λ, b)*-closed for every* (Λ, b)*-closed set* F *of* X.

(3) $[A_{(\Lambda,b)}]^{(\Lambda,b)} \subseteq [A^{(\Lambda,b)}]_{(\Lambda,b)}$ *for every subset* A *of* X.

Proof. (1) $\Rightarrow$ (2): Let F be any (Λ, b)-closed set. Then, $X-F$ is (Λ, b)-open and by (1), $[X-F]^{(\Lambda,b)} = X - F_{(\Lambda,b)}$ is (Λ, b)-open. Thus, $F_{(\Lambda,b)}$ is (Λ, b)-closed.

(2) $\Rightarrow$ (3): Let A be any subset of X. Then, we have $X - A_{(\Lambda,b)}$ is (Λ, b)-closed in X. By (2),

$$[X - A_{(\Lambda,b)}]_{(\Lambda,b)}$$

is (Λ, b)-closed and hence $[A_{(\Lambda,b)}]^{(\Lambda,b)}$ is (Λ, b)-open. Consequently, we obtain $[A_{(\Lambda,b)}]^{(\Lambda,b)} \subseteq [A^{(\Lambda,b)}]_{(\Lambda,b)}$.

(3) $\Rightarrow$ (1): Let U be any (Λ, b)-open set. By (3), we have $U^{(\Lambda,b)} = [U_{(\Lambda,b)}]^{(\Lambda,b)} \subseteq [U^{(\Lambda,b)}]_{(\Lambda,b)}$ and hence $U^{(\Lambda,b)}$ is (Λ, b)-open. This shows that (X, τ) is (Λ, b)-extremally disconnected. $\square$

Theorem 23. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, b)-extremally disconnected.

(2) For every (Λ, b)-open sets U and V such that

$$U \cap V = \emptyset,$$

there exist disjoint (Λ, b)-closed sets F and H such that $U \subseteq F$ and $V \subseteq H$.

(3) $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$ for every (Λ, b)-open sets U and V such that $U \cap V = \emptyset$.

(4) $[[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)} \cap U^{(\Lambda,b)} = \emptyset$ for every subset A of X and every (Λ, b)-open set U such that $A \cap U = \emptyset$.

Proof. The proof follows from Theorem 19 of [5]. $\square$

Definition 24. *A subset A of a topological space (X, τ) is called $r(\Lambda, b)$-open (resp. $r(\Lambda, b)$-closed) if $A = [A^{(\Lambda,b)}]_{(\Lambda,b)}$ (resp. $A = [A_{(\Lambda,b)}]^{(\Lambda,b)}$).*

Theorem 25. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, b)-extremally disconnected.

(2) Every $r(\Lambda, b)$-open set of X is (Λ, b)-closed.

(3) Every $r(\Lambda, b)$-closed set of X is (Λ, b)-open.

Proof. (1) $\Rightarrow$ (2): Suppose that (X, τ) is (Λ, b)-extremally disconnected. Let U be any $r(\Lambda, b)$-open set of X. Then, $U = [U^{(\Lambda,b)}]_{(\Lambda,b)}$ and U is (Λ, b)-open. By (1), $U^{(\Lambda,b)}$ is (Λ, b)-open and hence $U = [U^{(\Lambda,b)}]_{(\Lambda,b)} = U^{(\Lambda,b)}$. Thus, U is (Λ, b)-closed.

(2) $\Rightarrow$ (1): Suppose that for every $r(\Lambda, b)$-open set of X is (Λ, b)-closed. Let U be any (Λ, b)-open set. Since $[U^{(\Lambda,b)}]_{(\Lambda,b)}$ is $r(\Lambda, b)$-open, we have $[U^{(\Lambda,b)}]_{(\Lambda,b)}$ is (Λ, b)-closed and hence $U^{(\Lambda,b)} \subseteq [[U^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)} = [U^{(\Lambda,b)}]_{(\Lambda,b)}$. Thus, $U^{(\Lambda,b)}$ is (Λ, b)-open. This shows that (X, τ) is (Λ, b)-extremally disconnected.

(2) $\Leftrightarrow$ (3): The proof is obvious. $\square$

Definition 26. *A subset A of a topological space (X, τ) is said to be:*

(i) (Λ, b)-dense if $A^{(\Lambda,b)} = X$.

(ii) (Λ, b)-codense if its complement is (Λ, b)-dense.

(iii) (Λ, b)-nowhere dense if $[A^{(\Lambda,b)}]_{(\Lambda,b)} = \emptyset$.

Definition 27. *A topological space (X, τ) is called (Λ, b)-hyperconnected if U is (Λ, b)-dense for every nonempty (Λ, b)-open set U of X.*

Definition 28. *A subset A of a topological space (X, τ) is called $s(\Lambda, b)$-open if $A \subseteq [A_{(\Lambda,sp)}]^{(\Lambda,sp)}$.*

Lemma 29. *A subset A of a topological space (X, τ) is $s(\Lambda, b)$-open if and only if there exists a (Λ, b)-open set U such that $U \subseteq A \subseteq U^{(\Lambda,b)}$.*

Proof. Suppose that A is a $s(\Lambda, b)$-open set. Then, we have $A \subseteq [A_{(\Lambda,b)}]^{(\Lambda,b)}$. Put $U = A_{(\Lambda,b)}$. Then U is a (Λ, b)-open set such that $U \subseteq A \subseteq U^{(\Lambda,b)}$.

Conversely, suppose that there exists a (Λ, b)-open set U such that $U \subseteq A \subseteq U^{(\Lambda,b)}$. Then $U \subseteq A_{(\Lambda,b)}$ and hence $U^{(\Lambda,b)} \subseteq [A_{(\Lambda,b)}]^{(\Lambda,b)}$. Since $A \subseteq U^{(\Lambda,b)}$, we have $A \subseteq [A_{(\Lambda,b)}]^{(\Lambda,b)}$. Thus, A is $s(\Lambda, b)$-open. $\square$

Theorem 30. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, b)-hyperconnected.

(2) A is (Λ, b)-dense or (Λ, b)-nowhere dense for every subset A of X.

(3) $U \cap V \neq \emptyset$ for every nonempty (Λ, b)-open sets U and V of X.

(4) $U \cap V \neq \emptyset$ for every nonempty $s(\Lambda, b)$-open sets U and V of X.

Proof. The proof follows from Theorem 34 of [5]. $\square$

Theorem 31. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X,τ) *is* (Λ,b)*-hyperconnected;*

(2) V *is* (Λ,b)*-dense for every nonempty* $s(\Lambda,b)$*-open set* V *of* X*;*

(3) $V \cup [V^{(\Lambda,b)}]_{(\Lambda,b)} = X$ *for every nonempty* $s(\Lambda,b)$*-open set* V *of* X.

Proof. (1) $\Rightarrow$ (2): Suppose that (X,τ) is (Λ,b)-hyperconnected. Let V be a nonempty $s(\Lambda,b)$-open set. It follows that $V_{(\Lambda,b)} \neq \emptyset$ and hence

$$X = [V_{(\Lambda,b)}]^{(\Lambda,b)} = V^{(\Lambda,b)}.$$

Thus, V is (Λ,b)-dense.

(2) $\Rightarrow$ (3): Let V be a nonempty $s(\Lambda,b)$-open set. Then by (2), we have

$$V \cup [V^{(\Lambda,b)}]_{(\Lambda,b)} = V \cup X_{(\Lambda,b)} = X.$$

(3) $\Rightarrow$ (1): Let V be a nonempty (Λ,b)-open set. It follows (3) that $V^{(\Lambda,b)} \supseteq V \cup [V^{(\Lambda,b)}]_{(\Lambda,b)} = X$ and hence $V^{(\Lambda,b)} = X$. This shows that (X,τ) is (Λ,b)-hyperconnected. □

5 On (Λ,b)-submaximal spaces

In this section, we introduce the notion of (Λ,b)-submaximal spaces and investigate some characterizations of (Λ,b)-submaximal spaces.

Definition 32. *A topological space* (X,τ) *is said to be* (Λ,b)*-submaximal if, for each* (Λ,b)*-dense subset of* X *is* (Λ,b)*-open.*

Lemma 33. [6] *For a subset* A *of a topological space* (X,τ)*, the following properties are equivalent:*

(1) A *is locally* (Λ,b)*-closed.*

(2) $A = U \cap A^{(\Lambda,b)}$ *for some* $U \in \Lambda_bO(X,\tau)$.

(3) $A^{(\Lambda,b)} - A$ *is* (Λ,b)*-closed.*

(4) $A \cup [X - A^{(\Lambda,b)}] \in \Lambda_bO(X,\tau)$.

(5) $A \subseteq [A \cup [X - A^{(\Lambda,b)}]]_{(\Lambda,b)}$.

Theorem 34. *For a topological space* (X,τ)*, the following properties are equivalent:*

(1) (X,τ) *is* (Λ,b)*-submaximal;*

(2) Every subset of X *is a locally* (Λ,b)*-closed set.*

(3) Every subset of X *is the union of a* (Λ,b)*-open set and a* (Λ,b)*-closed set.*

(4) Every (Λ,b)*-dense set of* X *is the intersection of a* (Λ,b)*-closed set and a* (Λ,b)*-open set.*

(5) Every (Λ,b)*-codense set of* X *is the union of a* (Λ,b)*-open set and a* (Λ,b)*-closed set.*

Proof. The proof follows from Theorem 27 of [5]. □

Definition 35. *A subset* A *of a topological space* (X,τ) *is said to be:*

(i) a $t(\Lambda,b)$*-set if* $A_{(\Lambda,b)} = [A^{(\Lambda,b)}]_{(\Lambda,b)}$.

(ii) a $\mathcal{B}(\Lambda,b)$*-set if* $A = U \cap V$*, where*

$$U \in \Lambda_bO(X,\tau)$$

and V *is a* $t(\Lambda,b)$*-set.*

Theorem 36. *For a topological space* (X,τ)*, the following properties are equivalent:*

(1) (X,τ) *is* (Λ,b)*-submaximal.*

(2) $A^{(\Lambda,b)} - A$ *is* (Λ,b)*-closed for every subset* A *of* X.

(3) Every subset of X *is locally* (Λ,b)*-closed.*

(4) Every subset of X *is a* $\mathcal{B}(\Lambda,b)$*-set.*

(5) Every (Λ,b)*-dense set of* X *is a* $\mathcal{B}(\Lambda,b)$*-set.*

Proof. The proof follows from Theorem 29 of [5]. □

6 Some characterizations of (Λ,b)-continuous functions

In this section, we introduce the notion of (Λ,b)-continuous functions. Moreover, some characterizations of (Λ,b)-continuous functions are investigated.

Definition 37. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is said to be* (Λ,b)*-continuous at a point* $x \in X$ *if for each* (Λ,b)*-open set* V *of* Y *containing* $f(x)$*, there exists a* (Λ,b)*-open set* U *of* X *containing* x *such that* $f(U) \subseteq V$*. A function* $f : (X,\tau) \to (Y,\sigma)$ *is said to be* (Λ,b)*-continuous if* f *has this property at each point* $x \in X$.

Theorem 38. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is* (Λ,b)*-continuous at* $x \in X$.

(2) $x \in [f^{-1}(V)]_{(\Lambda,b)}$ *for every* (Λ,b)*-open set* V *of* Y *containing* $f(x)$.

(3) $x \in f^{-1}([f(A)]^{(\Lambda,b)})$ *for every subset* A *of* X *such that* $x \in A^{(\Lambda,b)}$.

(4) $x \in f^{-1}(B^{(\Lambda,b)})$ *for every subset* B *of* Y *such that* $x \in [f^{-1}(B)]^{(\Lambda,b)}$.

(5) $x \in [f^{-1}(B)]_{(\Lambda,b)}$ *for every subset* B *of* Y *such that* $x \in f^{-1}(B_{(\Lambda,b)})$.

(6) $x \in f^{-1}(K)$ *for every* (Λ, b)*-closed set* K *of* Y *such that* $x \in [f^{-1}(K)]^{(\Lambda,b)}$.

Proof. (1) $\Rightarrow$ (2): Let V be any (Λ, b)-open set of Y containing $f(x)$. Then, there exists a (Λ, sp)-open subset U of X containing x such that $f(U) \subseteq V$ and hence $U \subseteq f^{-1}(V)$. Since $U \in \Lambda_bO(X,\tau)$, we have $x \in [f^{-1}(V)]_{(\Lambda,b)}$.

(2) $\Rightarrow$ (3): Let A be any subset of X. Let $x \in A^{(\Lambda,b)}$ and $V \in \Lambda_bO(Y,\sigma)$ containing $f(x)$. By (2), we have $x \in [f^{-1}(V)]_{(\Lambda,b)}$ and there exists $U \in \Lambda_bO(X,\tau)$ such that $x \in U \subseteq f^{-1}(V)$. Since $x \in A^{(\Lambda,sp)}$, $U \cap A \neq \emptyset$ and $\emptyset \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. Thus, $f(x) \in [f(A)]^{(\Lambda,b)}$ and hence $x \in f^{-1}([f(A)]^{(\Lambda,b)})$.

(3) $\Rightarrow$ (4): Let B be any subset of Y and let $x \in [f^{-1}(B)]^{(\Lambda,b)}$. By (3),

$$x \in f^{-1}([f(f^{-1}(B))]^{(\Lambda,b)}) \subseteq f^{-1}(B^{(\Lambda,b)})$$

and hence $x \in f^{-1}(B^{(\Lambda,b)})$.

(4) $\Rightarrow$ (5): Let B be any subset of Y such that $x \notin [f^{-1}(B)]_{(\Lambda,b)}$. Then, $x \in X - [f^{-1}(B)]_{(\Lambda,b)} = [X - f^{-1}(B)]^{(\Lambda,b)} = [f^{-1}(Y - B)]^{(\Lambda,b)}$. By (4), we have $x \in f^{-1}([Y - B]^{(\Lambda,b)}) = f^{-1}(Y - B_{(\Lambda,b)}) = X - f^{-1}(B_{(\Lambda,b)})$ and hence $x \notin f^{-1}(B_{(\Lambda,b)})$.

(5) $\Rightarrow$ (6): Let K be any (Λ, b)-closed set of Y such that $x \notin f^{-1}(K)$. Then, $x \in X - f^{-1}(K) = f^{-1}(Y - K) = f^{-1}([Y - K]_{(\Lambda,b)})$ and by (5),

$$\begin{aligned} x \in [f^{-1}(Y - K)]_{(\Lambda,b)} &= [X - f^{-1}(K)]_{(\Lambda,b)} \\ &= X - [f^{-1}(K)]^{(\Lambda,b)}. \end{aligned}$$

Thus, $x \notin [f^{-1}(K)]^{(\Lambda,b)}$.

(6) $\Rightarrow$ (2): Let $x \in X$ and $V \in \Lambda_bO(Y,\sigma)$ containing $f(x)$. Let $x \notin [f^{-1}(V)]_{(\Lambda,b)}$. Then,

$$\begin{aligned} x \in X - [f^{-1}(V)]_{(\Lambda,b)} &= [X - f^{-1}(V)]^{(\Lambda,b)} \\ &= [f^{-1}(Y - V)]^{(\Lambda,b)}. \end{aligned}$$

By (6), we have $x \in f^{-1}(Y - V) = X - f^{-1}(V)$ and hence $x \notin f^{-1}(V)$. This contraries to the hypothesis.

(2) $\Rightarrow$ (1): Let $V \in \Lambda_bO(Y,\sigma)$ containing $f(x)$. By (2), $x \in [f^{-1}(V)]_{(\Lambda,b)}$ and so there exists $U \in \Lambda_bO(X,\tau)$ containing x such that $x \in U \subseteq f^{-1}(V)$; hence $f(U) \subseteq V$. This shows that f is (Λ, b)-continuous at x. □

Theorem 39. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is* (Λ, b)*-continuous.*

(2) $f^{-1}(V)$ *is* (Λ, b)*-open in* X *for every* (Λ, b)*-open set* V *of* Y.

(3) $f(A^{(\Lambda,b)}) \subseteq [f(A)]^{(\Lambda,b)}$ *for every subset* A *of* X.

(4) $[f^{-1}(B)]^{(\Lambda,b)} \subseteq f^{-1}(B^{(\Lambda,b)})$ *for every subset* B *of* Y.

(5) $f^{-1}(B_{(\Lambda,b)}) \subseteq [f^{-1}(B)]_{(\Lambda,b)}$ *for every subset* B *of* Y.

(6) $f^{-1}(K)$ *is* (Λ, b)*-closed in* X *for every* (Λ, b)*-closed set* K *of* Y.

Proof. (1) $\Rightarrow$ (2): Let V be any (Λ, b)-open set of Y and $x \in f^{-1}(V)$. Then, $f(x) \in V$ and there exists a (Λ, b)-open set U of X containing x such that $f(U) \subseteq V$. Since $U \in \Lambda_bO(X,\tau)$, we have $x \in [f^{-1}(V)]_{(\Lambda,b)}$ and hence $f^{-1}(V) \subseteq [f^{-1}(V)]_{(\Lambda,b)}$. This shows that $f^{-1}(V)$ is (Λ, b)-open.

(2) $\Rightarrow$ (3): Let A be any subset of X. Let $x \in A^{(\Lambda,b)}$ and $V \in \Lambda_bO(Y,\sigma)$ containing $f(x)$. By (2), we have $x \in [f^{-1}(V)]_{(\Lambda,b)}$ and there exists $U \in \Lambda_bO(X,\tau)$ such that $x \in U \subseteq f^{-1}(V)$. Since $x \in A^{(\Lambda,b)}$, $U \cap A \neq \emptyset$ and $\emptyset \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. Thus, $f(x) \in [f(A)]^{(\Lambda,b)}$. Consequently, we obtain $f(A^{(\Lambda,b)}) \subseteq [f(A)]^{(\Lambda,b)}$.

(3) $\Rightarrow$ (4): Let B be any subset of Y. By (3),

$$f([f^{-1}(B)]^{(\Lambda,b)}) \subseteq [f(f^{-1}(B))]^{(\Lambda,b)} \subseteq B^{(\Lambda,b)}$$

and hence $[f^{-1}(B)]^{(\Lambda,b)} \subseteq f^{-1}(B^{(\Lambda,b)})$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. By (4), we have

$$\begin{aligned} X - [f^{-1}(B)]_{(\Lambda,b)} &= [X - f^{-1}(B)]^{(\Lambda,b)} \\ &= [f^{-1}(Y - B)]^{(\Lambda,b)} \\ &\subseteq f^{-1}([Y - B]^{(\Lambda,b)}) \\ &= f^{-1}(Y - B_{(\Lambda,b)}) \\ &= X - f^{-1}(B_{(\Lambda,b)}) \end{aligned}$$

and hence $f^{-1}[B_{(\Lambda,b)}] \subseteq [f^{-1}(B)]_{(\Lambda,b)}$.

(5) $\Rightarrow$ (6): Let K be any (Λ, b)-closed set of Y. Then, $Y - K = [Y - K]_{(\Lambda,b)}$ and by (5),

$$\begin{aligned} X - f^{-1}(K) &= f^{-1}(Y - K) \\ &= f^{-1}([Y - K]_{(\Lambda,b)}) \\ &\subseteq [f^{-1}(Y - K)]_{(\Lambda,b)} \\ &= [X - f^{-1}(K)]_{(\Lambda,b)} \\ &= X - [f^{-1}(K)]^{(\Lambda,b)}. \end{aligned}$$

Thus, $[f^{-1}(K)]^{(\Lambda,b)} \subseteq f^{-1}(K)$ and hence $f^{-1}(K)$ is (Λ,b)-closed.

(6) ⇒ (2): This is obvious.

(2) ⇒ (1): Let $x \in X$ and $V \in \Lambda_b O(Y,\sigma)$ containing $f(x)$. By (2), $x \in [f^{-1}(V)]_{(\Lambda,b)}$ and so there exists $U \in \Lambda_b O(X,\tau)$ containing x such that $x \in U \subseteq f^{-1}(V)$. Therefore, $f(U) \subseteq V$ and hence f is (Λ, sp)-continuous at x. This shows that f is (Λ, sp)-continuous. □

Definition 40. *A topological space (X,τ) is said to be (Λ,b)-connected if X cannot be written as a disjoint union of two nonempty (Λ,b)-open sets of X.*

Proposition 41. *If $f : (X,\tau) \to (Y,\sigma)$ is a (Λ,b)-continuous surjection and (X,τ) is (Λ,b)-connected, then (Y,σ) is (Λ,b)-connected.*

Proof. Suppose that (Y,σ) is not (Λ,b)-connected. There exist nonempty (Λ,b)-open sets U and V of Y such that $U \cap V = \emptyset$ and $U \cup V = Y$. Then, we have $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ and $f^{-1}(U) \cup f^{-1}(V) = X$. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty (Λ,b)-open sets of X. This shows that (X,τ) is not (Λ,b)-connected. Thus, (Y,σ) is (Λ,b)-connected. □

Definition 42. *A topological space (X,τ) is said to be (Λ,b)-compact if every cover of X by (Λ,b)-open sets of X has a finite subcover.*

Proposition 43. *If $f : (X,\tau) \to (Y,\sigma)$ is a (Λ,b)-continuous surjection and (X,τ) is a (Λ,b)-compact space, then (Y,σ) is (Λ,b)-compact.*

Proof. Let $\{V_\gamma \mid \gamma \in \Gamma\}$ be any cover of Y. Since f is (Λ,b)-continuous, by Theorem 39,

$$\{f^{-1}(V_\gamma) \mid \gamma \in \Gamma\}$$

is a cover of X by (Λ,b)-open sets of X. Thus, there exists a finite subset Γ_0 of Γ such that $X = \cup\{f^{-1}(V_\gamma) \mid \gamma \in \Gamma_0\}$. Since f is surjective, $Y = f(X) = \cup\{V_\gamma \mid \gamma \in \Gamma_0\}$. This shows that (Y,σ) is (Λ,b)-compact. □

Definition 44. *A subset A of a topological space (X,τ) is said to be a (Λ,b)-neighbourhood of x, if there exists a (Λ,b)-open set U of X such that $x \in U \subseteq A$.*

Lemma 45. *Let A be a subset of a topological space (X,τ) and $x \in X$. Then, $x \in \Lambda_{(\Lambda,b)}(A)$ if and only if $A \cap F \neq \emptyset$ for every (Λ,b)-closed set F of X with $x \in F$.*

Theorem 46. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is (Λ,b)-continuous.

(2) For each $x \in X$ and each (Λ,b)-open set V of Y such that $f(x) \in V$, $f^{-1}(V)$ is a (Λ,b)-neighbourhood of x.

(3) $f(A_{(\Lambda,b)}) \subseteq \Lambda_{(\Lambda,b)}[f(A)]$ for every subset A of X.

(4) $[f^{-1}(B)]_{(\Lambda,b)} \subseteq f^{-1}(\Lambda_{(\Lambda,b)}(B))$ for every subset B of Y.

Proof. (1) ⇒ (2): Let $x \in X$ and V be any (Λ,b)-open set of Y such that $f(x) \in V$. Since f is (Λ,b)-continuous, there exists a (Λ,b)-open set U of X containing x such that $f(U) \subseteq V$. Thus, $x \in U \subseteq f^{-1}(V)$ and hence $f^{-1}(V)$ is a (Λ,b)-neighbourhood of x.

(2) ⇒ (1): Let $x \in X$ and $V \in \Lambda_b O(Y,\sigma)$ containing $f(x)$. By (2), $f^{-1}(V)$ is a (Λ,b)-neighbourhood of x and there exists a (Λ,b)-open set U of X such that $x \in U \subseteq f^{-1}(V)$. Thus, $f(U) \subseteq V$ and hence f is (Λ,b)-continuous.

(1) ⇒ (3): Let A be any subset of X and let $y \notin \Lambda_{(\Lambda,b)}[f(A)]$. By Lemma 45, there exists a (Λ,b)-closed set F of Y such that $y \in F$ and $f(A) \cap F = \emptyset$. Thus, $A \cap f^{-1}(F) = \emptyset$ and hence $f^{-1}(F) \cap A_{(\Lambda,b)} = \emptyset$. Therefore, $f(A_{(\Lambda,b)}) \cap F = \emptyset$. This shows that $y \notin f(A_{(\Lambda,b)})$. Consequently, we obtain $f(A_{(\Lambda,b)}) \subseteq \Lambda_{(\Lambda,b)}(f(A))$.

(3) ⇒ (4): Let B be any subset of Y. By (3) and Lemma 14, we have

$$f([f^{-1}(B)]_{(\Lambda,b)}) \subseteq \Lambda_{(\Lambda,b)}(f(f^{-1}(B))) \subseteq \Lambda_{(\Lambda,b)}(B)$$

and hence $[f^{-1}(B)]_{(\Lambda,b)} \subseteq f^{-1}(\Lambda_{(\Lambda,b)}(B))$.

(4) ⇒ (1): Let V be any (Λ,b)-open set of Y. By (4) and Lemma 14,

$$[f^{-1}(V)]_{(\Lambda,b)} \subseteq f^{-1}(\Lambda_{(\Lambda,b)}(V)) = f^{-1}(V)$$

and hence $[f^{-1}(V)]_{(\Lambda,b)} = f^{-1}(V)$. Thus, $f^{-1}(V)$ is (Λ,b)-open, by Theorem 39, f is (Λ,b)-continuous. □

7 Conclusion

The concepts of openness and continuity are fundamental with respect to the investigation of general topology. The study of openness and continuity have been found to be useful in computer science and digital topology. This paper is dealing with the concept of (Λ,b)-open sets which is the union of a V_b-set and a b-open set. Moreover, several properties of (Λ,b)-open sets are considered. Some characterizations of (Λ,b)-extremally disconnected spaces, (Λ,b)-hyperconnected spaces and (Λ,b)-submaximal spaces are explored. Additionally, several characterizations of (Λ, b)-continuous functions are obtained. The ideas and results of this paper may motivate further research.

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