# Bootstrap Methods for Claims Reserving: R Language Approach

ORIANA ZAÇAJ, ENDRI RAÇO, KLEIDA HAXHI, ETLEVA LLAGAMI, KOSTAQ HILA
Faculty of Mathematical Engineering and Physics Engineering,
Polytechnic University of Tirana,
Tirana, 1069,
ALBANIA

*Abstract* Bootstrap methods have been used by actuaries for a long time to predict future claims cash flows and their variability. This work aims to illustrate the use of bootstrap methods in practice, taking as an example the claims development data of the personal accident portfolio from the largest insurance company in Albania, over a period of 10 years. It is not the objective of this work to provide a theoretical analysis of the bootstrap methods, rather, this work focuses on highlighting the benefits of using bootstrap methods to predict the distribution of future claims development, and estimate the standard error, for a better risk assessment of liabilities within insurance companies. This work is divided into two well-differentiated phases: the first is to select the theoretical probability distribution that best fits the available claims dataset. Comparison of distributions is facilitated by the possibilities offered by the R programming languages. Both, the maximum likelihood parameter estimation method and the chi-square goddess goodness of fit test, are used to specify the probability distribution that best fits the data, among a family of predefined distributions. The results show that the Gamma distribution better describes the claim development data. The next phase is to use bootstrap methods, based on the selected distribution, to estimate the ultimate value of claims, the claims reserve, and their standard error.

*Key-Words:* - Claim Reserving, Bootstrapping, R, Distribution Fitting

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## 1 Introduction

Claim modeling is a very important part in estimating liabilities of an insurance company. It will give the company an estimate of the capital needed to fulfill its obligations. Assessing better liabilities and assets of insurance companies will help them analyze their different insurance portfolios, project future new products, maintain an adequate solvency position, and estimate the need for additional reinsurance cover.

After a claim occurs, a period is needed to reach its final settlement. This period is known as the development period of claim [1]. It is the aim of claim reserving to estimate that period and the ultimate value of the claim at the settlement date. In addition to that, it is necessary to estimate the variability and the Value at Risk (VaR) of the estimated reserve.

When we analyze outstanding claims at different points in time, we divide them into two main groups: Claims that already have been incurred but aren't reported yet (IBNR), and claims that have been reported but aren't settled yet to the date of calculation (RBNS). For the second group, we have some information about the accident date and claim reporting dates, together with estimated values at different reporting dates. But the ultimate claim amount and the final settlement date are still unknown. For the first group, the only information we can use is the past data on the development of claims from the accident date through its reporting date until its settlement date. In both cases, it is necessary to use the available information on actual and past claim data to project future cashflows of these claims.

The most traditional method of reserving outstanding claims, especially in long-term business, is the Chain Ladder method [1] [2] [3]. It is a distribution-free method that gives a point estimator of reserves. This means that it doesn't give information on the risk that the estimated reserves will differ from the real reserves.

To analyze the variability of reserves, the Mack model [4] [5] is the most common. The method calculates the standard error and confidence intervals for reserves based on the estimated ones from the chain ladder results.

Meanwhile, bootstrap techniques [4] [6] [7] are a very good tool for predicting the distribution of claims and claim reserves. These techniques also estimate the standard error of these predictions. They are a very good alternative to the Mack model.

Both methods do not change the estimate of claim reserves, but variability and standard error are calculated with different assumptions. In the Mack model [5], the distribution of the underlying data is not predicted, but only the first two moments are specified. In the bootstrap model [6], we calculate the standard error as the standard deviation of the expected cash flows that would be obtained if we could repeat the experiment several times, from going backward in time and repeating the claim experience, each time evaluating the mean reserves. Differently from the Mack model, the bootstrap model predicts the fitting distribution of the data, and the difference between the fitting values and the real incurred data gives a signal of the deviation of the actual data from the model framework [4].

We will introduce these techniques using claims from personal accident claims in an insurance company in Albania. We will start by finding which distributions better fit these claims [4] [8] [9] [10]. Then we will estimate reserves and their variability using bootstrap models [4] [6] [11] as an alternative to the Mack Chain Ladder method [4] [5] [11] for the claim development triangles. We will highlight the differences between the estimation of reserves and their variability at different levels of confidence error on the models, so that we can find the best assessment for those claims

## 2 Materials and Methods

### 2.1 Claim Distribution

915 claims from the personal accident portfolio of an Albanian insurance company, incurred during the period 2005 – 2021, were taken into consideration. The volume of claims varies between Albanian lek (ALL) 5,000 to ALL 3,000,000. The average value of these claims was ALL 578,140. Fig. 1 shows the empirical density of these claims. There is a skewness (1.59) that results in a tail on the right in the empirical density of the claims.

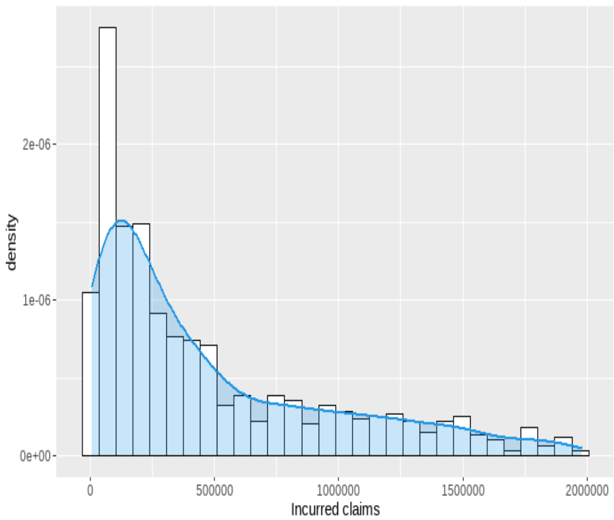


Fig. 1: The empirical density of claims incurred during years 2005 – 2021

Different theoretical distribution functions [4] [8] [9] [10] were used to find the best distributions that fit the amounts of claims. Fig. 2 and Fig. 3 below, show the first impression of which distribution fits best our data.

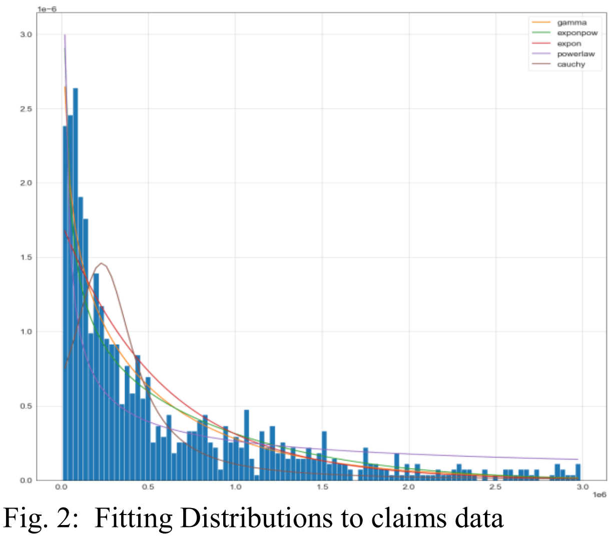


Fig. 2: Fitting Distributions to claims data

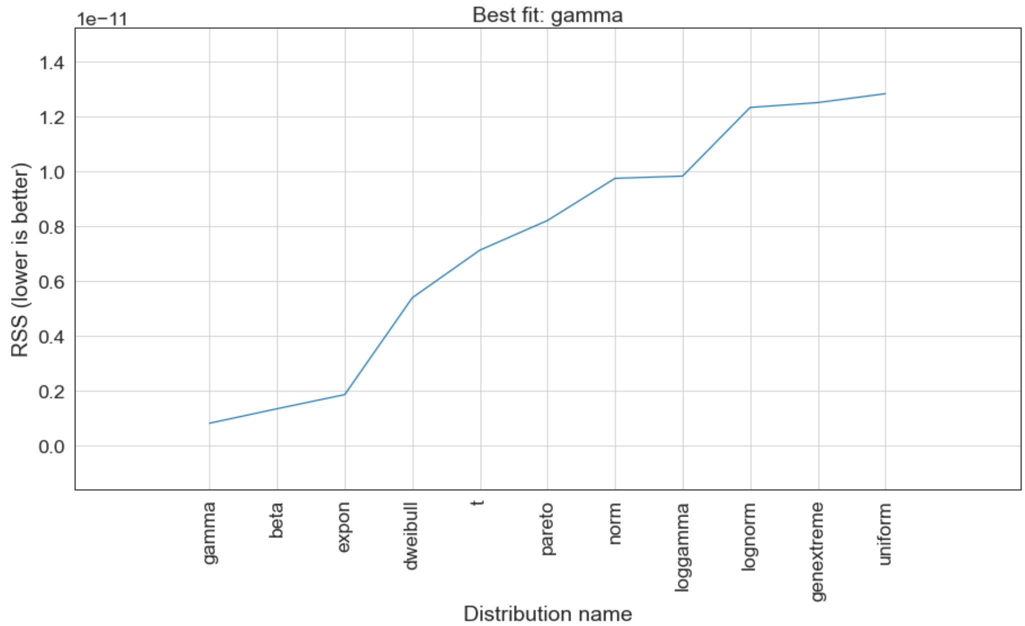


Fig. 3: Empirical density vs theoretical density functions

According to the results indicated in Fig. 3, the Gamma distribution is the closest distribution to the available data.

Moreover, if we go through the distribution parameters for each individual theoretical distribution as shown in Table 1, it results that Gamma distribution has one of the lowest Akaike Information Criterion (AIC), the one of the lowest square error, and the lowest Bayesian Information Criterion (BIC) coefficient among other distributions. Although the AIC for the gamma distribution is not the lowest, it is much lower than that for other distributions that have the closest BIC or sum-square error.

Table 1. Estimation results of statistics from different theoretical distributions for claim amounts

| Distribution | Sum square error | AIC | BIC |
|---|---|---|---|
| Gamma | $2.607959\ 10^{-12}$ | 3160.272483 | -30624.153540 |
| Burr | $3.221322\ 10^{-12}$ | 3219.839974 | -30424.064648 |
| Beta | $3.800095\ 10^{-12}$ | 3132.843308 | -30272.875312 |
| Normal | $2.055142\ 10^{-11}$ | 3230.413043 | -28742.081182 |
| Lognormal | $2.465234\ 10^{-11}$ | 3569.305206 | -28568.785330 |
| Exponential law | $3.642174\ 10^{-12}$ | 3144.688824 | -30318.531627 |
| Exponential | $4.606323\ 10^{-12}$ | 3171.578434 | -30110.463552 |
| Power law | $7.580380\ 10^{-12}$ | 3047.870805 | -29647.852603 |
| Cauchy | $1.136920\ 10^{-11}$ | 3308.384372 | -29283.781086 |

If we perform the Chi–squared test from R [12] as in Table 2, the closest distribution fitting our data is the Gamma distribution.

Table 2. Results of the Chi-squared test

| Goodness-of-fit statistics | Gamma | Weibull | Lognormal |
|---|---|---|---|
| Kolmogorov-Smirnov | 0.04643869 | 0.04765527 | 0.07975269 |
| Cramer-von Mises | 0.52057243 | 0.56061974 | 1.06145749 |
| Anderson-Darling | 3.61644563 | 3.81781521 | 8.55914819 |

From all graphical and statistical tests, the Gamma distribution results as the best fit for the distribution of claim data. We expect that this will lead to the same estimates as the standard distribution-free chain ladder method.

## 2.2 Claim Reserves

For the same set of data, we take into consideration the two models: the Mack model [5] and the bootstrap model [6].

After calculating the reserves with the chain ladder method, we will estimate and analyze the variability of those reserves with both models

### 2.2.1 Mack Model

The triangle of cumulative incurred claims and incremental claims in thousands of Albanian lek during the period 2012 - 2021 is shown in Table 3 and Table 4.

Table 3. Incurred and reported claim amounts from the year of the accident to the reporting year (incremental data) in thousands of Albanian lek

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2012 | 35,132 | 2,437 | 691 | 1,512 | 128 | 150 | 84 | 13 | 3 | 1 |
| 2013 | 17,034 | 5,008 | 41 | 926 | 610 | 346 | 267 | 150 | 27 | |
| 2014 | 18,874 | 1,336 | 600 | 384 | 320 | 208 | 164 | 106 | | |
| 2015 | 26,956 | 3,211 | 876 | 691 | 140 | 175 | 146 | | | |
| 2016 | 50,844 | 10,915 | 552 | 389 | 300 | 129 | | | | |
| 2017 | 46,010 | 6,515 | 257 | 103 | 180 | | | | | |
| 2018 | 31,274 | 7,540 | 1,841 | 386 | | | | | | |
| 2019 | 47,524 | 8,915 | 1,206 | | | | | | | |
| 2020 | 56,289 | 8,455 | | | | | | | | |
| 2021 | 65,901 | | | | | | | | | |

Table 4. Incurred and reported claim amounts from the year of the accident to the reporting year (cumulative data) in thousands of Albanian lek

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2012 | 35,132 | 37,569 | 38,260 | 39,772 | 39,900 | 40,050 | 40,134 | 40,147 | 40,150 | 40,151 |
| 2013 | 17,034 | 22,042 | 22,083 | 23,009 | 23,619 | 23,965 | 24,232 | 24,382 | 24,409 | |
| 2014 | 18,874 | 20,210 | 20,810 | 21,194 | 21,514 | 21,722 | 21,886 | 21,992 | | |
| 2015 | 26,956 | 30,167 | 31,043 | 31,734 | 31,874 | 32,049 | 32,195 | | | |
| 2016 | 50,844 | 61,759 | 62,311 | 62,700 | 63,000 | 63,129 | | | | |
| 2017 | 46,010 | 52,525 | 52,782 | 52,885 | 53,065 | | | | | |
| 2018 | 31,274 | 38,814 | 40,655 | 41,041 | | | | | | |
| 2019 | 47,524 | 56,439 | 57,645 | | | | | | | |
| 2020 | 56,289 | 64,744 | | | | | | | | |
| 2021 | 65,901 | | | | | | | | | |

Table 5 gives the prediction of the claim development from the accident date to its final settlement. The predicted values in Table 5 are obtained from the incurred values as the average between every two subsequent periods [3]. For example, the value ALL 22,002 of the claim incurred in year 2014 predicted to accumulate in year 2022, is calculated as the ratio between the column 9 and column 8, then added to the claim incurred in year 2014 and accumulated to the year 2021

22,002=21,992*(1+(40,150+24,409)/(40,147+24,382))

The models used in Table 6 show the summarizing results for the ultimate value of the claims, the claim reserve, and the variability of reserves using the Mack model. The ultimate value of the claims and the claim reserve in Table 6 is obtained from Table 5. We calculate the value of the claim reserve for each accident year as the difference between the ultimate claim value with the reported value for that accident year in Table 5. For example, the claim reserve value of ALL 3,803,519 in Table 6 for the accident year 2020 is calculated as the difference (68,547,057 – 64,743,538) in Table 5. As a result, the estimated ultimate claims, using the Chain Ladder technique, is ALL 488,105,288, giving an estimate of the claim reserve ALL 23,832,820.

Table 5. Ultimate claims in thousands of Albanian Lek

| **Year** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| 2012 | 35,132 | 37,569 | 38,260 | 39,772 | 39,900 | 40,050 | 40,134 | 40,147 | 40,150 | 40,151 |
| 2013 | 17,034 | 22,042 | 22,083 | 23,009 | 23,619 | 23,965 | 24,232 | 24,382 | 24,409 | 24,411 |
| 2014 | 18,874 | 20,210 | 20,810 | 21,194 | 21,514 | 21,722 | 21,886 | 21,992 | 22,002 | 22,004 |
| 2015 | 26,956 | 30,167 | 31,043 | 31,734 | 31,874 | 32,049 | 32,195 | 32,295 | 32310 | 32,313 |
| 2016 | 50,844 | 61,759 | 62,311 | 62,700 | 63,000 | 63,129 | 63,483 | 63,681 | 63,711 | 63,716 |
| 2017 | 46,010 | 52,525 | 52,782 | 52,885 | 53,065 | 53,362 | 53,662 | 53,829 | 53,854 | 53,858 |
| 2018 | 31,274 | 38,814 | 40,655 | 41,041 | 41,339 | 41,570 | 41,804 | 41,934 | 41,954 | 41,957 |
| 2019 | 47,524 | 56,439 | 57,645 | 58,590 | 59,015 | 59,345 | 59,678 | 59,865 | 59,892 | 59,897 |
| 2020 | 56,289 | 64,744 | 65,973 | 67,054 | 67,540 | 67,919 | 68,300 | 68,513 | 68,545 | 68,550 |
| 2021 | 65,901 | 76,753 | 78,210 | 79,491 | 80,068 | 80,517 | 80,969 | 81,221 | 81,259 | 81,265 |

Table 6. Process and parameter variables

| **Year** | Ultimate | Reserve | Process SD | CV | Parameter SD | CV | Total SD | CV |
|---|---|---|---|---|---|---|---|---|
| 2012 | 40,151,086 | 0 | | | | | | |
| 2013 | 24,410,145 | 958 | 4,908 | 513% | 3,827 | 400% | 6,224 | 650% |
| 2014 | 22,003,293 | 10,907 | 20,038 | 184% | 11,889 | 109% | 23,299 | 214% |
| 2015 | 32,311,438 | 116,832 | 101,619 | 87% | 62,763 | 54% | 119,439 | 102% |
| 2016 | 63,713,298 | 584,089 | 221,294 | 38% | 175,071 | 30% | 282,172 | 48% |
| 2017 | 53,856,734 | 791,433 | 291,280 | 37% | 186,322 | 24% | 345,775 | 44% |
| 2018 | 41,955,234 | 914,185 | 408,629 | 45% | 197,405 | 22% | 453,813 | 50% |
| 2019 | 59,894,530 | 2,249,596 | 891,103 | 40% | 446,058 | 20% | 996,510 | 44% |
| 2020 | 68,547,057 | 3,803,519 | 1,228,770 | 32% | 618,388 | 16% | 1,375,602 | 36% |
| 2021 | 81,262,472 | 15,361,301 | 3,583,797 | 23% | 1,656,889 | 11% | 3,948,276 | 26% |
| **Total** | **488,105,288** | **23,832,820** | **3,931,805** | **16%** | **2,537,855** | **11%** | **4,679,722** | **20%** |

From Table 6, we can also observe that we have a coefficient of variation (CV) of 20% and a standard deviation (SD) of 4,679,722, showing a considerable variation from the estimated reserve. The coefficient of variation is higher during the first calendar years due to the low value of reserves. The standard error increases during years 2019 – 2021, which is clearly observed during year 2021. These results can be summarized through graphs, as shown in Fig. 4 and Fig. 5.

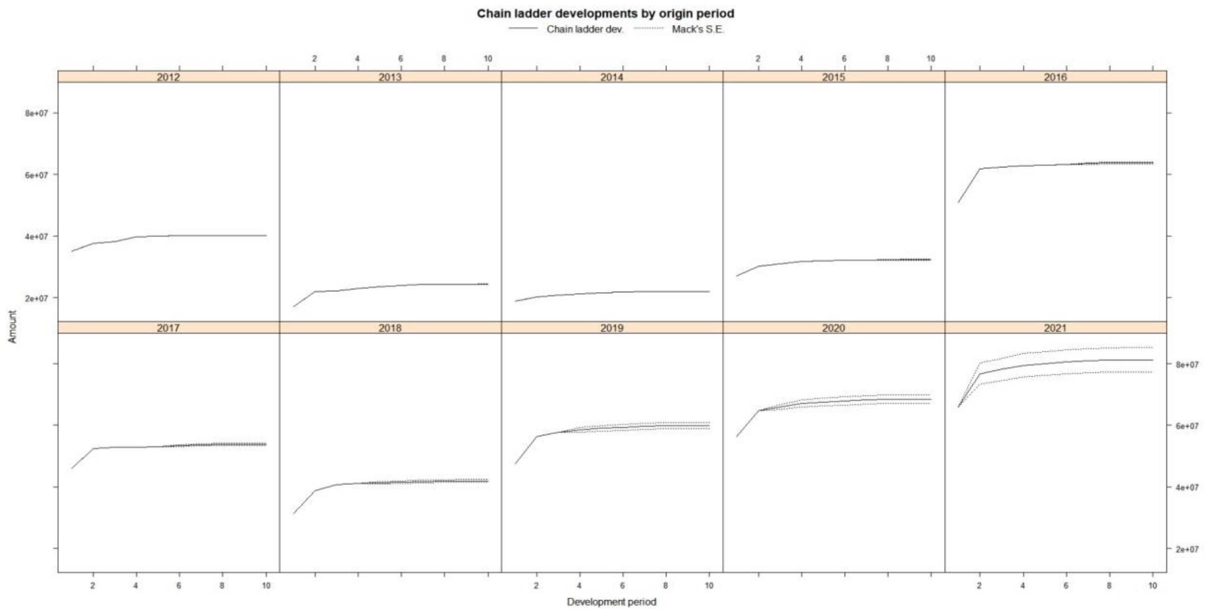


Fig. 4: Chain ladder developments by origin period

 

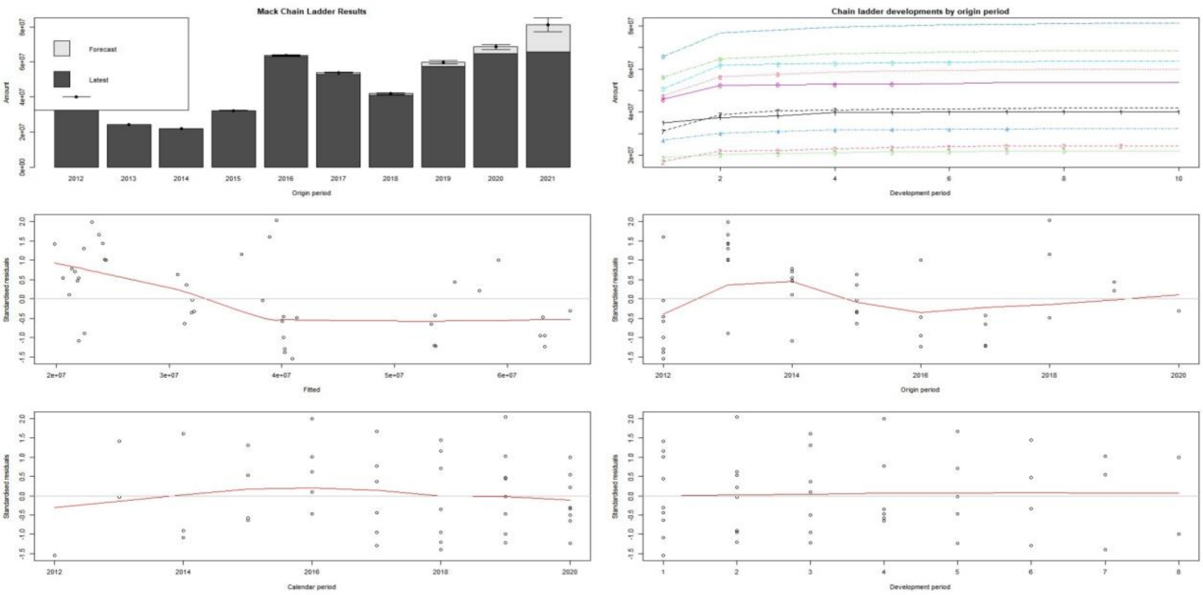


Fig. 5: Development of reserve and its standard error for the Mack model

From the graphs in Fig. 4 and Fig. 5, there are no trends in the residuals of the Mack model. As it is shown in Table 6 and Fig. 4, the standard error (SE) is slightly visible during period 2018 – 2020, and more noticeable during the last year (2021). The confidence intervals for the estimated reserve at different levels, result as in Table 7:

Table 7. Confidence intervals for claim reserves at different confidence levels

| Confidence level | Confidence interval |
|---|---|
| 0.75 | (18,451,140: 29,214,501) |
| 0.95 | (14,660,565: 33,005,075) |
| 0.99 | (11,782,536: 35,883,105) |
| 0.995 | (10,682,801: 36,982,839) |

### 2.2.2 Bootstrapping

The bootstrap technique with 999 simulations is used to the incremental claim data incurred during the period 2012 – 2021 from Table 3 and Table 4. Gamma as the best fitting distribution to the dataset was used for process distribution.

After bootstrapping, the results compared with Mack model are shown in Table 8. The reserves calculated with bootstrapping (ALL 24,802,264) are slightly higher than the chain ladder reserve (24,802,264 − 23,832,820)/23,832,820 = 0.0406 = 4%. The standard error and variation from bootstrapping are much higher than Mack during period 2012 – 2018, but it is lower during year 2021.

Table 7. Data Summary for Mack and Bootstrap

| Year | Mean IBNR (Mack) | Mean IBNR (bootstrap) | Mack SE | Bootstrap SE | CV (Mack) | CV (Bootstrap) |
|---|---|---|---|---|---|---|
| 2012 | - | - | | | | |
| 2013 | 958 | 2,864 | 6,224 | 57,658 | 650% | 2013% |
| 2014 | 10,907 | 16,692 | 23,299 | 152,148 | 214% | 912% |
| 2015 | 116,832 | 131,812 | 119,439 | 315,712 | 102% | 240% |
| 2016 | 584,089 | 632,954 | 282,172 | 722,245 | 48% | 114% |
| 2017 | 791,433 | 869,364 | 345,775 | 774,275 | 44% | 89% |
| 2018 | 914,185 | 947,746 | 453,813 | 767,830 | 50% | 81% |
| 2019 | 2,249,596 | 2,429,705 | 996,510 | 1,271,573 | 44% | 52% |
| 2020 | 3,803,519 | 4,000,632 | 1,375,602 | 1,503,605 | 36% | 38% |
| 2021 | 15,361,301 | 15,770,495 | 3,948,276 | 3,272,987 | 26% | 21% |
| **Total** | **23,832,820** | **24,802,264** | **4,679,722** | **4,983,026** | **20%** | **20%** |

Graphically, the result of the bootstrapping is shown in Fig. 6. The best fitting distribution result is the gamma distribution with parameters α=25.36386 and β=1.015098 10-03 Apart from the year 2021, there is not much difference between the real and simulated values.

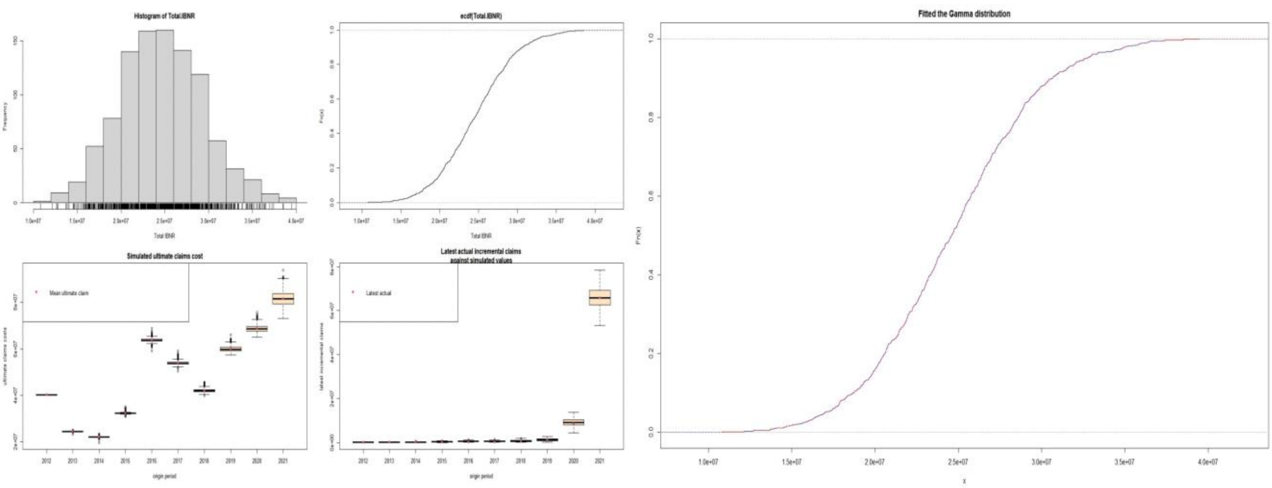

Fig. 6: Summarizing Results for the Bootstrap model

To calculate the Value at Risk (VaR), we use the bootstrap IBNR quantiles at 75%, 90%, 95%, and 99.5% as in Table 9. The highest VaR is recorded in the last year (2021).

Table 8. Value at Risk (VaR) at different confidence intervals

| Year | IBNR75% | IBNR95% | IBNR99% | IBNR99.5% |
|---|---|---|---|---|
| 2012 | - | - | - | - |
| 2013 | 0 | 7,376 | 152,917 | 275,180 |
| 2014 | 530 | 128,366 | 493,993 | 636,302 |
| 2015 | 184,111 | 793,624 | 1,297,922 | 1,471,080 |
| 2016 | 998,648 | 1,990,579 | 2,993,631 | 3,348,337 |
| 2017 | 1,267,387 | 2,259,353 | 3,057,382 | 3,455,701 |
| 2018 | 1,383,022 | 2,448,951 | 3,683,693 | 4,028,342 |
| 2019 | 3,113,885 | 4,728,319 | 5,780,345 | 6,505,809 |
| 2020 | 4,973,068 | 6,814,957 | 8,390,076 | 8,860,049 |
| 2021 | 17,754,400 | 21,063,575 | 24,601,077 | 25,908,415 |
| **Total** | **29,675,051** | **40,235,101** | **50,451,035** | **54,489,215** |

## 4 Conclusion

Claims distribution and claims reserving play a very important role in the solvency and operations of an insurance company. Actuaries must be able to assess risk reserve to the level of prudency required from the legal framework and from their own risk assessment criteria within the insurance company.

The aim of this study is divided into two parts. The first one is to analyze the empirical data and to find the best distribution. When fitting techniques with several theoretical distributions and using different diagnostic techniques with the help of the statistical language it resulted that the gamma distribution was the best fitting distribution to the claim data.

The second is to analyze and compare results of claim reserves and their variability, to the Mack chain ladder model, before and after introducing bootstrap techniques, with real data from an Albanian insurance company. With the Mack model, the value of claim reserves is the same as in the chain ladder method. The coefficient of variability is 20% which shows a considerable variability of the estimated claim reserves. When applying the bootstrap technique with 999 iterations, the gamma distribution was used to fit the distribution of claim reserves, as it resulted in the best fitting among other theoretical distributions. With the bootstrapping model, it resulted that the estimation of reserves differs only by 4% from the claim reserve calculated with the chain ladder model, but the prediction errors were much higher. This is mainly because the Mack model is a distribution-free technique, whereas with the bootstrap model the Gamma distribution was used with the predictions as it resulted as the best distribution fitting our dataset, giving a more accurate estimate of the variability of the claim reserves. Value at Risk was estimated at different levels of confidence, showing higher values, especially for the year 2021 which has to develop in the succeeding years.

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**Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)**

Oriana Zaçaj collected the statistics, analysed the statistical data and the outputs from the statistical tests working and comparing with similar papers.
Endri Raço executed the algorithms [9] [11] [12] to the statistical data, finding their best distributional features.
Kleida Haxhi, Etleva Llagami, Kostaq Hila reviewed the results and statistical analyses