A simplified fractional SEIR epidemic model and unique inversion of the fractional order

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Abstract: - A simplified linear time-fractional SEIR epidemic system is set forth, and an inverse problem of determining the fractional order is discussed by using the measurement at one given time. By the Laplace transform the solution to the forward problem is obtained, by which the inverse problem is transformed to a nonlinear algebraic equation. By choosing suitable model parameters and the measured time, the nonlinear equation has a unique solution by the monotonicity of the Mittag-Lellfer function. Theoretical testification is presented to demonstrate the unique solvability of the inverse problem.

Key-Words: - Fractional SEIR model; Laplace transform; Mittag-Lellfer function; inversion of fractional order; monotonicity; uniqueness

Received: May 22, 2021. Revised: January 25, 2022. Accepted: February 20, 2022. Published: March 23, 2022.

1 Introduction

Coronavirus disease 2019 (COVID-19), a pneumonia epidemic caused by a new type of coronavirus 2 (SARS-CoV-2), has become a global pandemic of the highest priority in the world [1, 2]. It is important to study its origin, its survival variation characteristics, the spread and infection laws, as well as the preventive vaccine. However, it is still of scientific significance for the COVID-19 to give mathematical modelling and perform optimal prevention and control strategies. There are quite a lot of mathematical models in describing the COVID-19 pandemic according to the spreading rules of human-to-human, most of them are based on the traditional models of SI, SIR and SEIR, etc. [3]. We refer to some recent literatures on mathematical modelling of the COVID-19, such as the integer-order models [4-6], and the fractional spreading models [7-10]. Actually, at the initial stage of the epidemic, the new coronavirus could survive in a special environment and it can spread by the infected goods during a relatively long time. If considering such transmission mode, and assuming there are only susceptible persons at the initial stage, such as the elder over 65 years old, and utilizing the fractional derivative to reveal the memory effect, a simplified linear SEIR model can be obtained following the ordinary SEIR model given as follows:

$$\begin{cases} \partial_t^\alpha S = -\mu S, \\ \partial_t^\alpha E = \mu S - \beta E, \\ \partial_t^\alpha I = \beta E - \gamma I, \\ \partial_t^\alpha R = \gamma I, \end{cases} \tag{1}$$

where $S = S(t)$, $E = E(t)$, $I = I(t)$ and $R = R(t)$ denote the number of the susceptible, the latent, the patient and the recovered people, $\partial_t^\alpha f$ denotes the α-order Caputo fractional derivatrive of $f(t)$ on $t > 0$, which defined by [11,12]

$$\partial_t^\alpha f = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\partial f(x,s)}{\partial s} \frac{ds}{(t-s)^\alpha}, \tag{2}$$

here $\alpha \in (0,1)$ is the fractional order, $\mu > 0$ is the latentive rate, $\beta > 0$ is the infected rate, $\gamma > 0$ is the recovered rate.

The system (1) is a linear fractional SEIR model. The linearity is its advantage as compared with the known SEIR models. The traditional SEIR models, including the interger-order and the fractional models, are nonlinear differential equations. In addition, the researches on the SEIR models are almost concentrated on the dynamical analysis with the spreading mechanism, and numerical simulations using numerical methods. Although the known SEIR models are studied under various conditions, there are no expressions of the solution in general due to the nonlinearity, and it is difficult to give deep researches in mathematics. On the contrary, we can get the expression of the solution of the simplified model (1) by the Laplace transform metod, and we can give more mathematical analysis on the model.

On the other hand, the fractional order in a fractional model is a key parameter to characterize the heavy-tail subdiffusion with memory effect. However, it is always unknown for real-life problems which leading to inverse fractional order problems. The inverse problems of identifying parameters including fractional orders have been studied during the last decade [13-24]. It is noted that in the existing work on inverse fractional order problems, most of them were studied by using subdomain measurements or one-point measurements at $t \in (0, T)$, or using subboundary data also at $t \in (0, T)$ for arbitrary given $T > 0$. An interesting problem is whether we can determine the fractional orders only using limited discretized data. Exactly speaking, there is one order $\alpha \in (0, 1)$ in model (1) which is unknown, can we determine it uniquely only using one measurement?

It is difficult to give an answer in theory for the above question, but the situation could be changed if coping with numerical solutions. Here we are concerned with the inverse problem of determining the fractional order α in (1) using one measurement. By the Laplace transform method, the solution of the forward problem is expressed by the Mittag-Lellfer function, and the inverse problem is transformed to a nonlinear algebraic equation. By choosing the measured time, the nonlinear equation can be solved uniquely by the monotonicity of the Mittag-Lellfer function. The unique solvability of the inverse problem is testified by theoretical examples. The rest of the paper is organized as follows.

In Section 2, some preliminaries on the Mittag-Lellfer function, and the forward problem are introduced, and its solution is deduced by the Laplace transform. In Section 3, the inverse problem of identifying the fractional order using one measurement is given, which is transformed to solving of a nonlinear equation by the additional data, and the unique solvability of the inverse problem is proved by the monotonicity of the Mittag-Lellfer function, and numerical testification is presented. Conclusion is given in Section 4.

2 The forward problem and its solution

2.1 The forward problem and preliminaries

Assume the epidemic has occurred in a region, and the elder are the susceptible and high-risk groups of infectious diseases. At the initial stage, there are only the susceptible people, and some of them become latent persons, some of the latent persons become patients with the pandemic spreading. Therefor we give the initial condition for the model (1)

$$S(0) = S_0, E(0) = I(0) = R(0) = 0. \quad (3)$$

As a result the froward problem on the simplified SEIR model (1) is composed by the model (1), with the initial condition (3). When the parameters in the model are known in advance, the analytical solution of the above forward problem can be obtained by the Laplace transform with the aids of the Mittag-Lellfer function. Firstly we give some basic facts on the Mittag-Leffler function and its properties and the Laplace transform [25].

For real numbers $\alpha, \eta > 0$ and complex number $z \in \mathbb{C}$, the two-parametric Mittag-Leffler function is defined as:

$$E_{\alpha,\eta}(z) = \sum_{j=0}^{\infty} \frac{z^j}{\Gamma(\alpha j+\eta)}, \quad (4)$$

where $\Gamma(\cdot)$ denotes the Gamma function, and there is the so-called one-parametric Mittag-Leffler function $E_\alpha(z) := E_{\alpha,1}(z)$ as $\eta = 1$. On the Mittag-Leffler function, there holds the estimate in general

$$|E_{\alpha,\eta}(z)| \leq \frac{c}{1+|z|}, \ \alpha, \eta > 0, \quad (5)$$

where $c > 0$ is a constant.

For the real-valued Mittag-Leffler function, there holds the complete monotonicity [25].

Definition 1. A function $f: (0, \infty) \to \mathbf{R}$ is called *completely monotonic* if it posseses derivatives $f^{(n)}(t)$ of any order $n = 0,1,\cdots$, and the derivatives are alternative in sign, i.e.,

$$(-1)^n f^{(n)}(t) \geq 0, \forall t \in (0, \infty). \quad (6)$$

Lemma 1[25]. (i) The Mittag-Leffler function $E_\alpha(-t)$ is completely monotonic on $t \in (0, \infty)$ for all $0 \leq \alpha \leq 1$.

(ii) The two-parametric Mittag-Leffler function $E_{\alpha,\eta}(-t)$ is also completely monotonic on $t > 0$ for all $0 \leq \alpha \leq 1, \eta \geq \alpha$.

Corollary 1. For $0 \leq \alpha \leq 1$, the function $E_\alpha(-t)$ is strictly decreasing on $t > 0$, and the function $\frac{d}{dt}E_\alpha(-t)$ is strictly increasing on $t > 0$.

The function $\bar{f}(p)$of complex variable p defined by

$$\bar{f}(p) = \int_0^\infty e^{-pt} f(t)dt, \quad (7)$$

is called the Laplace transform of $f(t)$, where $f(t)$ satisfies the growth condition $|f(t)| \leq Me^{c_0 t}$as $t \to \infty$, and M, c_0 are positive constants.

On performing Laplace transform for a fractional derivative function, some regularity is needed for the performed function, see [26] for detailed analysis. In this work we mainly focus on the inverse fractional order problem, and we assume that the solution to the forward problem has the regularity such that the Laplace transform on the Caputo fractional derivative of the solution can be performed. For the fractional derivative $\partial_t^\alpha f(0 < \alpha < 1)$, there holds:

$$\overline{\partial_t^\alpha f}(p) = p^\alpha \bar{f}(p) - p^{\alpha-1} f(0), \quad (8)$$

where $\bar{f}(p)$ denotes the Laplace transform of $f(t)$. In addition, the following formula plays a key role in solving fractional differential equations by the Laplace transform:

$$\int_0^\infty e^{-pt}\, t^{\eta-1} E_{\alpha,\eta}(-c\, t^\alpha) dt = \frac{p^{\alpha-\eta}}{p^\alpha+c}, \quad (9)$$

where $\alpha, \eta \in (0, 1)$, and $c > 0$ be constant.

2.2 The solution of the forward problem

By performing the Laplace transform on the first equation in model (1), we have

$$p^\alpha \bar{S}(p) - p^{\alpha-1} S(0) = -\mu \bar{S}(p).$$

Noting $S(0) = S_0$, there holds

$$\bar{S}(p) = \frac{p^{\alpha-1}}{p^\alpha+\mu} S_0.$$

By utilizing the formula (9), we get the expression of $S(t)$:

$$S(t) = S_0 E_\alpha(-\mu t^\alpha). \quad (10)$$

Next, by performing the Laplace transform on the second equation in (1), there holds

$$p^\alpha \bar{E}(p) - p^{\alpha-1} E(0) = \mu \bar{S}(p) - \beta \bar{E}(p),$$

and thanks to $E(0) = 0$, we have

$$\bar{E}(p) = \frac{p^{\alpha-1}}{(p^\alpha+\mu)((p^\alpha+\beta)} \mu S_0.$$

Let $\mu \neq \beta$. Then by using (9) again we get:

$$E(t) = \frac{\mu S_0}{\mu-\beta}[E_\alpha(-\beta t^\alpha) - E_\alpha(-\mu t^\alpha)]. \quad (11)$$

Similarly, if $\mu \neq \beta \neq \gamma$, then by the Laplace transform method, we can obtain the expressions of $I(t)$ and $R(t)$ given as follows:

$$I(t) = \frac{\mu\beta S_0}{(\mu-\beta)(\mu-\gamma)(\beta-\gamma)}[(\beta-\gamma)E_\alpha(-\mu t^\alpha) - (\mu-\gamma)E_\alpha(-\beta t^\alpha) + (\mu-\beta)E_\alpha(-\gamma t^\alpha)]. \quad (12)$$

$$R(t) = \frac{\mu\beta\gamma S_0 t^\alpha}{(\mu-\beta)(\mu-\gamma)(\beta-\gamma)}[(\beta-\gamma)E_{\alpha,1+\alpha}(-\mu t^\alpha) - (\mu-\gamma)E_{\alpha,1+\alpha}(-\beta t^\alpha) + (\mu-\beta)E_{\alpha,1+\alpha}(-\gamma t^\alpha)]. \quad (13)$$

Theorem 1. Assume the parameters μ, β and γ are positive constants and satisfy the condition: $\gamma > \mu > \beta$ or $\beta > \mu > \gamma$. Then for any given $S_0 > 0$, the forward problem (1), (3) has a unique, nonnegative solution $(S(t), E(t), I(t), R(t))$ expressed by (10)-(13), and the solution has the asymptotic behaviors $S(t), E(t), I(t) \to 0$, $R(t) \to$ M as $t \to \infty$, where $M > 0$ is a constant.

Proof. Obviously the solution $S(t)$ is positive, and strictly decreasing on $t > 0$ due to the monotonicity of the Mittag-Lellfer function given in Corollary 1.

Next, if $\mu \neq \beta$, by the monocinity of the Mittag-Lellfer function, the solution $E(t)$ is positive too, whether $\mu > \beta$ or $\mu < \beta$, and there holds $E(t) \to 0$ by the estimate of $|E_\alpha(-ct^\alpha)| \leq \frac{C_0}{1+|ct^\alpha|}$ as $t \to \infty$.

Thirdly we are to prove the nonnegativity and monotonicity of $I(t)$ and $R(t)$. Rewrite $I(t)$ as

$$I(t) = \mu\beta S_0\left[\frac{E_\alpha(-\gamma t^\alpha) - E_\alpha(-\mu t^\alpha)}{(\mu-\gamma)(\beta-\gamma)} + \frac{E_\alpha(-\mu t^\alpha) - E_\alpha(-\beta t^\alpha)}{(\mu-\beta)(\beta-\gamma)}\right], \quad (14)$$

For given $t > 0$ and $\alpha \in (0,1)$, define a function

$$g(x) = E_\alpha(-x\, t^\alpha),\ x > 0. \quad (15)$$

By Corollary 1 there holds

$$g'(x) \leq 0 \text{ and } g''(x) \geq 0, x > 0. \quad (16)$$

Then by the meanvalue theorem we get

$$I(t) = \mu\beta S_0 t^\alpha g''(\xi_3)\frac{\xi_2-\xi_1}{\beta-\gamma}, \quad (17)$$

where ξ_1 is between μ and γ, ξ_2 is between μ and β, and ξ_3 is between ξ_1 and ξ_2. According to the conditions of the parameters, we deduce that $\xi_2 - \xi_1$ has the same sign with $\beta - \gamma$, and thus there must have $I(t) \geq 0$ by (16). Noting the expression of $R(t)$ is of a similar form as $I(t)$, and the Mittag-Lellfer function $E_{\alpha,1+\alpha}(-xt^\alpha)$ has the same properties as $E_\alpha(-xt^\alpha)$, we can get its nonnegative as done in the above, as well as its asymptotic behavior based on (5). The proof is over.

Based on the above solutions, the trends of the pandemic with the time are plotted in Fig.1, where the fractional order is chosen as $\alpha = 0.75$, and other model parameters are given as $\mu = 0.25$, $\beta = 0.1$, $\gamma = 0.5$ and $S_0 = 1000$. From Fig.1 it can be seen that, the number of the susceptible is strictly decreasing, the number of the latent is going up firstly and then decreasing as well as the patient, and the number of the recovered is increasing, which basically coincide with the spreading trends of the pandemic at its initial stage.

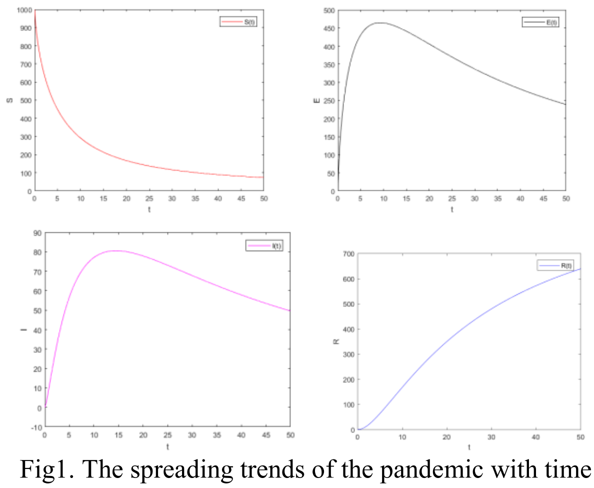

Fig1. The spreading trends of the pandemic with time

3 Inverse fractional order problem

3.1 The inverse problem

Although we get the solution to the forward problem given by (10)-(13), it cannot be put into practice if there are unknown parameters in the model. Actually, the fractional order in (1) is an important index characterizing the slower spreading with memory effect in time, but it is always unknown which leads to inverse fractional order problems.

Noting that the recovered persons in the pandemic are known every day, so we can get some additional data on $R(t)$ at given time t_k:

$$R(t_k),\ k = 1, 2, \cdots, K, \quad (18)$$

where $K \geq 1$. The inverse fractional order problem is to identify the order $\alpha \in (0,1)$ by using the data (18) based on the forward problem, where the parameters μ, β, γ are known and different each other.

An interesting problem for the above inverse problem is:

Can we determine $\alpha \in (0,1)$ uniquely only using one measurement of $R(t_1)$ at given $t_1 > 0$?

Generally speaking, it is very difficult to answer the above question in theory. Nevertheless, we can deal with such problem in some special cases. Denote $d = R(t_1)$ at $t_1 > 0$ as the data. Then noting the solution's expression (13), we get a nonlinear algebraic equation on $\alpha \in (0,1)$:

$$F(\alpha) := \frac{\mu\beta\gamma S_0 t_1^{\alpha}}{(\mu-\beta)(\mu-\gamma)(\beta-\gamma)}[(\beta-\gamma)E_{\alpha,1+\alpha}(-\mu t_1^{\alpha}) - (\mu-\gamma)E_{\alpha,1+\alpha}(-\beta t_1^{\alpha}) + (\mu-\beta)E_{\alpha,1+\alpha}(-\gamma t_1^{\alpha})] = d, \quad (19)$$

As a result the inverse fractional order problem is transformed to solving of the nonlinear equation (19).

3.2 Unique solvability

As said in the above, it is very difficult to prove the uniqueness of a nonlinear algebra equation in theory. However, it is possible and feasible to get a unique solution of the nonlinear equation (19) under suitable conditions.

Theorem 2. Assume the parameters μ, β and γ are positive constants ranged in $(0,1)$, and satisfy the condition of Theorem 1. Then the nonlinear equation (19) has a unique solution for suitable $t_1 > 1$.

Proof. We are to prove the function $F(\alpha)$ defined by (19) is monotonic increasing on the fractional order, i.e., $F'(\alpha) > 0$ on $\alpha \in (0,1)$. As done in the proof of Theorem 1, define three functions on $\alpha \in (0,1)$ by

$$\begin{cases} h_1(\alpha) = E_{\alpha,1+\alpha}(-\gamma t_1^{\alpha}), \\ h_2(\alpha) = E_{\alpha,1+\alpha}(-\mu t_1^{\alpha}), \\ h_3(\alpha) = E_{\alpha,1+\alpha}(-\beta t_1^{\alpha}), \end{cases} \quad (20)$$

and rewrite $F(\alpha)$ as

$$F(\alpha) = \mu\beta\gamma S_0 t_1^{\alpha} J(\alpha), \quad (21)$$

where $J(\alpha) = \frac{h_1(\alpha)-h_2(\alpha)}{(\mu-\gamma)(\beta-\gamma)} + \frac{h_2(\alpha)-h_3(\alpha)}{(\mu-\beta)(\beta-\gamma)}$.

Then we have

$$F'(\alpha) = \mu\beta\gamma S_0 t_1^{\alpha}[\ln(t_1)J(\alpha) + J'(\alpha)]. \quad (22)$$

With a simlar method as usd in the proof of Theorem 1, we can deduce that $J(\alpha) > 0$ in the case of $\gamma > \mu > \beta$ or $\beta > \mu > \gamma$, as well as $J'(\alpha) > 0$. So by suitably choosing the measured time $t_1 > 1$, there holds

$$F'(\alpha) > 0,$$

which implies that the equation (19) has a unique solution. The proof is completed.

3.2.1 Example 1

Let the exact fractional order in model (1) be $\alpha = 0.8$, and other parameters be $\mu = 0.25, \beta = 0.1$ and $\gamma = 0.5, S_0 = 1000$, and the measured time be $t_1 = 50$. By (13) we get the additional data $d = R(t_1) \approx 724$. By solving the nonlinear equation (13), the fractional order can be reconstructed. In order to see the unique solvability of the equation, we plot the functions $y = F(\alpha)$, and $y = d$ on $\alpha \in (0,1)$ in Fig.2.

It can be seen clearly that the function $F(\alpha)$ is strictly monotonic on $\alpha \in (0,1)$, and there exists unique solution to the equation $F(\alpha) = d$, which demonstrates the uniqueness of the inverse problem.

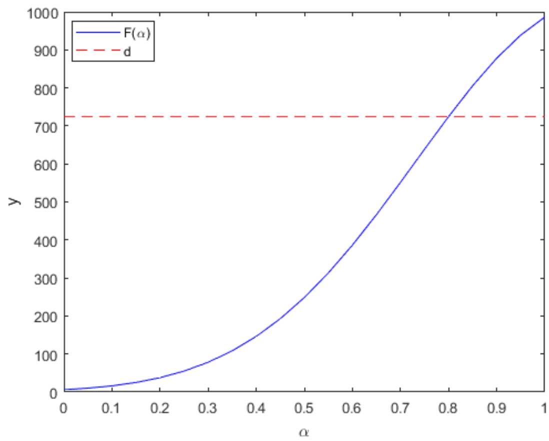

Fig.2 The pictures of $y = F(\alpha)$ and $y = d$ in Ex.1

3.2.2 Example 2

Let the exact fractional order be $\alpha = 0.5$ in this example, and other parameters be $\mu = 0.2, \beta = 0.5$ and $\gamma = 0.1, S_0 = 1000$, and the measured time be $t_1 = 10$. Also by (13) the additional data is given as $d \approx 62$. As done in Example 1, the functions $y = F(\alpha)$ and $y = d$ on $\alpha \in (0,1)$ are plotted in Fig.3.

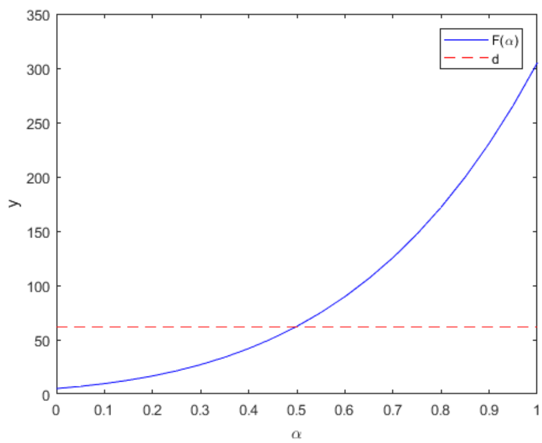

Fig.3 The pictures of $y = F(\alpha)$ and $y = d$ in Ex.2

From Fig.3 it can be seen again that the function $F(\alpha)$ is also strictly monotonic on $\alpha \in (0,1)$, and the inverse problem is of uniqueness too.

4 Conclusion

A linear time-fractional SEIR epidemic model and the inverse fractional order problem using one measurement are investigated in mathematics. Based on the expression of the solution to the forward problem, the inverse problem is reduced to a nonlinear algebraic equation on the fractional order, and the unique solvability can be obtained by the complete monotonicity of the Mittag-Lellfer function of real variable under suitable order conditions for the parameters. Theoretical examples are presented to illustrate the uniqueness of the inverse problem. It is noted that the derivative of the function $F(\alpha)$ on $\alpha \in (0,1)$ can be computed by (22), and some gradient-type iterative algorithms can be applied to solve the nonlinear equation for which we will give details in the near future.

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Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

Yi Zhang, deduced the solution of the model, and carried out the numerical algorithm and computations of Figures 1-3.
Gongsheng Li, as the corresponding author, put forward the fractional SEIR model, and finished the proofs of Theorems 1-2.

Sources of funding for research presented in a scientific article or scientific article itself

This work is supported by the National Natural Science Foundation of China (no. 11871313), and the Natural Science Foundation of Shandong Province, China (no. ZR2019MA021).

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