Fuzzy Soft Groups based on Fuzzy Space

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Abstract: -In this paper, a new theory of fuzzy soft groups (FSG) based on fuzzy spaces (FS) from Dib's point of view is found. The concept of FSG and fuzzy soft subgroup (FSSG) based on FS was elaborated. The relationship between FSG and FSG is also investigated.

Keywords: - FSG, FSSG, soft set (SS), fuzzy soft set (FSS).

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1 Introduction

Explored and discussed the topic of fuzzy sets (F-sets) [1]. The fuzzy group theory evolved in the following way: The concept of a fuzzy subgroup of a group was first described in [2]. The concept of FS was suggested by Dib [3]. In the ordinary case, this idea took the place of the universal set concept. Dib [3] defined fuzzy functions (FF), fuzzy binary operations (FBO), and fuzzy subspaces (FSB) in FS. The concepts of fuzzy group, fuzzy subgroup, and fuzzy group theory were constructed in the stated study after defining FS and FBO. Begins by introducing an SS [4]. It is a parameterized family of universal set subsets and also introduces and investigates FSS [5]. It is a more general idea that combines the F-set and the SS. The goal of this research was to combine two mathematical domains on the F-set, fuzzy algebra and SS theory, to create a new algebraic system called FSG based on FS. The theory of FSG is developed and FSSG is introduced. Based on the concept of FS, the theory of SS established by [4] has been applied to fuzzy subgroups to construct the notions of FSG and FSSG.

2 Preliminaries

Definition 2.1 ([3]) A FS $(Q,\ N=[0,1])$ is the set of all ordered pairs (q, N), $q \in Q$
$(Q,N)=\{(q,N): q\in Q\}$ where $(q,N)=\{(q,n): n\in N\}$.

The ordered pair $(q,\ N)$ is called a fuzzy element (FE) in the FS.

Definition 2.2 ([3]) A FSB U of the FS (Q, N) is the collection of all ordered pairs $(q,\ \alpha_q)$ where $q \in U_0$, $\exists U_0 \in Q$ and $\alpha_q \subseteq N$.

Definition 2.3 ([3]) Let $(Q,\ N)$ $(Y,\ N)$ and $(Z,\ N)$ be FS. The (FF) $\underline{D}$ from $(Q, N)\times(Y,\ N)=(Q\times Y,\ N\times N)$ into $(Z,\ N)$ is typified with the ordered pair $(D,\ J_{qy})$ where $D: Q\times Y \to Z$ a function and J_{qy} is a family function $J_{xy}: N\times N \to N$ fulfilling the requirements:

(i) J_{qy} is non-decreasing on $N\times N$,

(ii) $J_{qy}(0,0)=0$ and $J_{xy}(1,1)=1$.

Definition 2.4 ([3]) A FBO $\underline{D}=(D,\ J_{qy})$ on the FS $(Q, N=[0,1])$ is a FF from $(Q,N)\times(Q,N)\to(Q,N)$ where $D: Q\times Q\to Q$ and $J_{qy}: N\times N \to N$ are functions with satisfying

(i) $J_{qy}(n,m)=0$ if $n=0$ and $m=0$.
(ii) J_{qy} are onto.

3 FSG over FS

In this paper, we define (FSG) and (FSSG) and obtain some related results. Also, we will introduce FSSG induced by fuzzy soft subset.

Definition 3.1. An FSG typified by $((Q,\ N),\ \underline{D})$ is an FS iff for every, $((q, \alpha_{F(e)}(q)), N), ((y, \alpha_{F(e)}(y)), N)$ and $((z, \alpha_{F(e)}(z)), N)$ the following requirements are met:

(i) Associative:

$$((q,\alpha_{F(e)}(q)),N)\ \underline{D}\,((y,\alpha_{F(e)}(y)),N))\underline{D}((z,\alpha_{F(e)}(z)),N)$$

$$=((q,\alpha_{F(e)}(q)),N)\ \underline{D}\,((y,\alpha_{F(e)}(y)),N)\underline{F}\ ((z,\alpha_{F(e)}(z)),N)$$

It has an identity $((e,\alpha_{F(e)}(e)),N)$, for which

$$((q,\alpha_{F(e)}(q)),N)\ \underline{D}\,((e,\alpha_{F(e)}(e)),N)$$
$$=((\mathrm{e},\alpha_{F(e)}(x)),N)\ \underline{D}\,((q,\alpha_{F(e)}(q)),N)$$
$$=((q,\alpha_{F(e)}(q)),N),$$

(ii) Every fuzzy soft element (FSE) $((q,\alpha_{F(e)}(q)),N)$ has an inverse $((q,\alpha_{F(e)}(q)),N)^{-1}$ such that

$$((q,\alpha_{F(e)}(q)),N)\underline{D}((q,\alpha_{F(e)}(q)),N)^{-1}$$
$$=((q,\alpha_{F(e)}(q)),N)^{-1}\underline{D}((q,\alpha_{F(e)}(q)),N)$$
$$=((q,\alpha_{F(e)}(q)),N).$$

Denote $(q,\ N)^{-1}=(q,\ N)$, then

$$((qDy,\alpha_{F(e)}(qDy)),N)$$
$$=((yDq,\alpha_{F(e)}(q)),N)$$
$$=((e,\alpha_{F(e)}(e)),N)$$

It follows from (i), (ii), and (iii) which $(Q,\underline{D})$ is an FSG.

Theorem 3.1 *Associated to each* FSG $((Q,\ N),\ \underline{D})$ *there is a fuzzy group* $((Q,\ N),\ \underline{D})$ *and they are isomorphic to each other by the correspondence*

$$((q,\ \alpha_{F(e)}(q)),\ N)\leftrightarrow(q,N).$$

Definition 3.2 A FSG $((Q,N),\ \underline{D})$ is said to be a *commutative* FSG if

$$((q,\ \alpha_{F(e)}(q)),\ N)\underline{D}\ ((y,\ \alpha_{F(e)}(y)),\ N)$$
$$=((y,\ \alpha_{F(e)}(y)),\ N)\ \underline{D}\ ((q,\ \alpha_{F(e)}(q)),\ N)$$

for all FSE $((q, \alpha_{F(e)}(q)), N)$ and $((y,\ \alpha_{F(e)}(y)),\ N)$ of the FS $(Q,\ N)$.

Example 3.2 Consider the set $C=\mathbb{Z}_4=\{0,1,2,3\}$, $E=\{0,1,2\}$. Define FBO $\underline{D}=(D,\ J_{qy})$ over the FS $(\mathbb{Z}_4,\ N=[0,1])$ set as follow:

$D(q,y)=q+_4 y$, where $+_4$ refers to addition modulo 4,

$J_{xy}(n,m)=n\wedge m$ and

$\tilde{F}:\mathbb{Z}_4\rightarrow N^A,\ \tilde{\mathrm{F}}(\mathrm{e}):C\rightarrow N=[0,1]\cdot$

$$(\tilde{F},C)=\{\alpha_{F(e)}(0)=\{\tfrac{0}{1},\tfrac{1}{1},\tfrac{2}{1},\tfrac{3}{1}\},$$
$$\alpha_{F(e)}(1)=\{\tfrac{0}{0.5},\tfrac{1}{0},\tfrac{2}{0},\tfrac{3}{0}\},$$
$$\alpha_{F(e)}(2)=\{\tfrac{0}{0.8},\tfrac{1}{0.8},\tfrac{2}{0.8},\tfrac{3}{0.8}\},$$
$$\alpha_{F(e)}(3)=\{\tfrac{0}{0.2},\tfrac{1}{0},\tfrac{2}{0.2},\tfrac{3}{0}\}\}.$$

The element of FSS $\alpha_{F(e)}(0)=\{\tfrac{0}{1},\tfrac{1}{1},\tfrac{2}{1},\tfrac{3}{1}\}$. Thus $(\mathbb{Z}_4,\ N),\ \underline{D})$ is an FSG.

Theorem 3.2 The statements are correct for any FSG:

(1) For fuzzy soft identity, the element is unique.

(2) The inverse of each FSE $((q,\ \alpha_{F(e)}(q)),\ N)\in((Q,N),\ \underline{D})$ is unique.

$$(((q^{-1})^{-1},\ \alpha_{qD(e)}(q^{-1})^{-1}),\ N)$$
$$=((q,\ \alpha_{qD(e)}(q)),\ N).$$

Definition 3.3. Let $((Q, N = [0,1]), \underline{D})$ be an FSG and let $U = \{(q, \alpha_q) : q \in U_0\}$ be an FSB of (Q, N). $(U, \underline{D})$ is called an FSSG of the FSG $((Q, N), \underline{D})$ if:

(i) $\underline{D}$ is closed on the *FSB U*, i.e.,

$$(q, \alpha_q)\, \underline{D}\, (y, \alpha_y) = (qDy,\ \alpha_{qDy}) = (qDy,\ (\alpha_q J_{qy} \alpha_y))$$

(ii) $(U,\ \underline{D})$ meets the requirements of an FSG

Theorem 3.3 $(U,\ \underline{D})$ *is an* FSSG *of the* FSG $((Q, N = [0,1]), \underline{D})$ *if and only if:*

(1) (U_0, D) *is a subgroup of* $(Q,\ D)$,

(2) $J_{qy}(\alpha_q,\ \alpha_y) = \alpha_{xDy} = (\alpha_q J_{qy} \alpha_y)$.

Proof. If (1) and (2) are satisfied, then

(a) $\underline{D}$ is closed on the *FSB* U. Let $(q, \alpha_q),\ (y,\ \alpha_y) \in U$. Then

$$(q, \alpha_q)\, \underline{D}\, (y, \alpha_y) = (qDy,\ J_{qy}(\alpha_q,\ \alpha_y)) = (qDy, \alpha_{qDy}) \in U.$$

(b) $(U,\ \underline{D})$ satisfies the FSG. Let $(q,\ \alpha_q)$, $(y, \alpha_y), (z, \alpha_z) \in U$. Then

$(b1)$
$$((q, \alpha_q)\underline{D}(y, \alpha_y)(\underline{D}(z, \alpha_z) = (qDy,\ \alpha_{qDy})\underline{D}(z,\ \alpha_z) = ((qDy)Dz,\ \alpha_{(qDy)Dz}) = (qD(yDz), \alpha_{qD(yDz)}) = (q,\ \alpha_q)\underline{D}\,(yDz,\ \alpha_{yDz}) = (q,\ \alpha_q)\,\underline{D}\,((y, \alpha_y)\,\underline{D}\,(z, \alpha_z).$$

(b2)
$$(q,\ \alpha_q)\,\underline{D}\,(e, \alpha_e) = (qDe,\ \alpha_{qDe}) = (q,\ \alpha_q) = (eDq,\ \alpha_{eDq}) = (e, \alpha_e)\,\underline{D}\,(q, \alpha_e).$$

(b3) Each (q, α_q) has an inverse $(q^{-1}, \alpha_{q^{-1}})$, since

$$(q, \alpha_q)\,\underline{D}\,(q^{-1},\ \alpha_{q^{-1}}) = (qDq^{-1},\ \alpha_{qDq^{-1}}) = (q^{-1}Dq,\ \alpha_{q^{-1}Dq}) = (e\ ,\ \alpha_e) = (q^{-1},\ \alpha_{q^{-1}})\,\underline{D}(q, \alpha_q).$$

We can deduce from (a) and (b) that $(U, \underline{D})$ is an FSSG of FSG $((Q, N), \underline{D})$. Conversely if $(U, \underline{D})$ is an FSSG of FSG $((Q, N), \underline{D})$ Then (i) is restricted by associativity. The following is also valid:

$$(\alpha_q J_{qy} \alpha_y) = J_{qy}(\alpha_q \times \alpha_y) = \alpha_{qDy}.$$

Theorem 3.4 $(U; \underline{D})$ *is an FSSG of the* FSG $((Q, N = [0,1]), \underline{D})$ *iff:*

(i) $qDy^{-1} \in U_0$, *for every* $q,\ y \in\ U_0$;

(ii) $J_{qy}(\alpha_q,\ \alpha_y) = \alpha_{qDy} = (\alpha_q J_{qy} \alpha_y)$.

Theorem 3.5 let $(H_0(C),\ \underline{D})$, $(\underline{H}(C),\ \underline{D})$ *and* $(\overline{H}(C),\ \underline{D})$ *are* FSSG of *the* FSG $\left((Q,\ N), \underline{D}\right)$ *if;*

(i) $(C_0, D\ )$ *is a subgroup of* (Q, D) *where* $C_0 = \{q \in Q : C(q) \neq 0\}$;

(ii) $J_{xy}\left(\underline{C}(q), \underline{C}(y)\right) = C(qd\,y)\ , \forall q, y \in C_0$.

Proof. We prove the result for $(H_0(C)\ ,\ \underline{D})$ and the rest would be done in the same way.

Let $(q, \alpha_q) = H_0(\alpha_q)$ and let be $(H_0(\alpha_q), \underline{D})$ an *FSSG* of the FSG $\left((Q, N),\ \underline{D}\right)$. Then (i) holds by Again by Theorem 3.2 we have:

$$
\begin{aligned}
J_{qy}(\alpha_q,\ \alpha_y) &= J_{qy}\left((0,\ \alpha_q),\ (0, \alpha_y)\right) \\
&= \alpha_{qDy} \\
&= (0,\ \alpha_{qFy})
\end{aligned}
$$

That is, $J_{qy}\left((0,\ \alpha_q),\ (0,\ \alpha_y)\right) = J_{qy}\left((0,\ \alpha_q),\ (0,\ \alpha_y)\right)$
$= u_{qDy}$.

Conversely, assume that conditions (i) and (ii) hold, then:

$$
\begin{aligned}
J_{qy}\left((0,\ \alpha_q),(0,\ \alpha_y)\right) &= J_{qy}\left((0,\ \alpha_q),(0,\ \alpha_y)\right) \\
&= J_{qy}\left((0,\ 0),\ J_{qy}(0,\alpha_q),\ (0,\ \alpha_y)\right) \\
&= \alpha_{qDy}.
\end{aligned}
$$

Similarly, we can prove that $J_{qy}\left((0,\ \alpha_q),\ (0,\ \alpha_y)\right) = \alpha_{qDy}$ which proves by Theorem 3.2 that $(H_0(C),\ \underline{D})$ is fuzzy subgroups of $((Q,\ N),\ \underline{D})$.

4 Conclusion

In this paper, we have applied the fuzzy group study started by Dib (1994) to the context of FSG.

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