Generalization Fixed Points of Multivalued α-Admissible Mappings in 2-Metric Spaces

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Abstract: - In this paper, we create some fixed-point results of multivalue α-admissible of 2-metric spaces. We introduce Hausdorff distance in 2-metric space, use it in our theorem. we investigated the existence of some fixed point results for new types of contraction. We study the stability of fixed point set.

Key-Words: - Metric spaces, 2-metric spaces, multivalued α −admissible mappings, fixed point, Hausdorff metric, α−ψ-contraction.

Received: March 25, 2021. Revised: December 17, 2021. Accepted: January 10, 2021. Published: February 4, 2022.

1 Introduction

One of the most valuable findings is the popular Banach contraction mapping principle [1] in nonlinear analysis. It was used in many different mathematical branches and in the general physical sciences. Metric fixed-point theory developed in various directions by mathematicians over the years. A comprehensive account is provided of that development Kirk and Sims in the Handbook [2]. Extended the contraction mapping theory of Banach by the usage of a legal Contractive Situation by Dass and Gupta [3]. Abu-Donia establishes some fixed point theorems in some types of metric spaces [cf.4-8]. Aubin and Cellina discuss some elements of this research in their book [9]. Nadler[10] expanded the Banach principle of contraction to set-valued mappings by the Hausdorff metric. Driven by Nadler's results, much research is done on fixed points multi-valued functions were conducted using this Hausdorff metric in different directions by multiple authors [11-15]. Stability is a approach associated with the limiting attitude of a system. It has been studied in various contexts of discrete and continuous dynamical systems [16,17]. Studies of the relation between the convergence of a mapping sequence and its fixed Points, known as stability of fixed points, were also widely studied in different settings [18-20]. the set of fixed points of multivalued mappings becomes bigger and hence more important for the study of stability. In [21] Samet et al. presented the definition of α-admissible mapping and a new group contractive mapping type known as α–ψ-contractive mapping type.[21] Expansion and generalization of current fixed-point literature results, in fact, the Banach's contraction principle. In addition, Karapinar and Samet [22] widespreaded the α -ψ -contractive type mappings and access assorted fixed point theorems for this generalized class of contractive mappings. Since then, fixed point results of α-admissible mappings have been established, such as [23,24]..

The concepts from setvalued analysis that we use in this paper are as follows. Let (X, d) be a metric space. Then

$$N(X) = \{\text{A: A is a non empty subset of X}\},$$

$$S(X) = \{\text{A: A is a non empty compact subset of X}\},$$

$$B(X) = \{\text{A: A is a non empty bounded subset of X}\} \text{ and}$$

$$SB(X) = \{\text{A: A is a non empty closed and bounded subset of X}\}.$$

For $x \in X$ and $B \in N(X)$, the function $D(x, B)$, and for $A, B \in SB(X)$, the function $H(A, B)$ are defined

as follows:

$$D(x, B) = \inf\{d(x, y): y \in B\}$$

and

$$H(A, B) = \max\{\sup_{x\in A} D(x, B), \sup_{y\in B} D(y, A)\}.$$

H is established the Hausdorff metric induced by the metric d on $SB(X)$[23]. encourage, if (X, d) is a complete then $SB(X), H$ is also complete.

The following lemma Nadler[18] generated by Nadler[18]

Lemma 1.1 [18] Let (X, d) be a metric space and $A, B \in SB(X)$. Let $q > 1$. next for every $x \in A$, there exists $y \in B$ so that $d(x, y) \leq qH(A, B)$.
In [18] Nadler certain that Lemma 1.1 is also accurate for $q \geq 1$, wherever $A, B \in S(X)$. Here we current the lemma

Lemma 1.2 [25] Let (X, d) be a metric space and $A, B \in SB(X)$. Let $q \geq 1$. next for every $x \in A$, there exists $y \in B$ so that $d(x, y) \leq qH(A, B)$.
The following is aftereffect of Lemma 1.2

Lemma 1.3 [25] Let two non-empty compact subsets of a metric space (X, d) are A and B and T is a multivalued mapping since $T: A \to S(B)$ Let $q \geq 1$. Next for $a, b \in A$ and $x \in Ta$, there endure $y \in Tb$ so that $d(x, y) \leq qH(Ta, Tb)$.

Definition 1.1 [21]. Let $T: X \to X$ and $\alpha: X \times X \to [0, \infty)$ be a function. We express that T is an α-admissible mapping if $x, y \in X$,

$$\alpha(x, y) \geq 1 \Longrightarrow \alpha(Tx, Ty) \geq 1.$$

In the following we characterize multivalued α-admissible mapping. $N(X)$ in the interpretation stand for the collection of all nonempty subsets of a nonempty set X.

Definition 1.2 [25]. Let $T: X \to N(X)$ a multivalued mapping since X is non-empty set and $\alpha: X \times X \to [0, \infty)$. For $x_0, y_0 \in X$ the mapping T called multivalued α-admissible if $\alpha(x, y) \geq 1 \Longrightarrow \alpha(x_1, y_1) \geq 1$ where $x_1 \in Tx_0$ and $y_1 \in Ty_0$.

Definition 1.3 [25]. Let $T: X \to SB(Y)$ be a multivalued mapping, since $(X, \sigma), (Y, d)$ are two metric spaces and H is the Hausdorff metric on $SB(Y)$. The mapping T is called continuous at $x \in X$ if for any sequence $\{x_n\}$ in X and $H(Tx, Tx_n) \to 0$ when $\sigma(x, x_n) \to 0$ as $n \to \infty$

Definition 1.4 [25] Let $p: X \to X$ a single-valued mapping, $T: X \to N(X)$ a multivalued mapping and X is a non-empty set. A point $x \in X$ is a fixed point of p (resp. T) iff $x = px$ (resp. $x \in Tx$).

2 Main Results

for $x \in X$ and $B, C \in N(X)$, for A, B and $C \in SB(X)$ the functions

$$H(A, B, C) = \max\{\sup_{x \in A} D(x, B, C), \sup_{y \in B} D(y, A, C), \sup_{z \in C} D(z, A, B)\},$$

where

$$D(x, B, C) = \inf\{d(x, y, z): y \in B, z \in C\}.$$

H is known as the Hausdorff metric induced by the 2-metric d on $SB(X)$.If (X, d) is complete then $(SB(X), H)$ is also complete.
We established the following lemma.

Lemma 2.1 Let (X, d) be a 2-metric space and A, B and $C \in SB(X)$. Let $q \geq 1$. next for every $x \in A$, there exists $y \in B$ and $z \in C$ so that $d(x, y, z) \leq qH(A, B, C)$.

proof. Let A, B and $C \in S(X)$. and $x \in A$. Since A, B and $C \in S(X)$, the resultbis true if $q > 1$. So, we shall prove the result for $q = 1$. Now, we know that

$$H(A, B, C) = \max\{\sup_{x \in A} D(x, B, C), \sup_{y \in B} D(y, A, C), \sup_{z \in C} D(z, A, B)\}.$$

From the definition,
$p = D(x, B, C) = \inf\{d(x, y, z): y \in B, z \in C\} \leq H(A, B, C)$. Then there exists a sequence $\{y_n\}$ in B such that $d(x, y_n, z) \to p$ as $n \to \infty$. Since B is compact, $\{y_n\}$ has a convergent subsequence $\{y_{n(k)}\}$. Hunce there exists $y \in X$ such that $y_{n(k)} \to y$ as $k \to \infty$. As B is compact, it is closed and $y \in B$. Now, $\lim_{n \to \infty} d(x, y_n, z) = p$
implies that $\lim_{k \to \infty} d(x, y_{n(k)}, z) = p$, that is, $d(x, y, z) = p = D(x, B, C) \leq H(A, B, C)$.
Hence the proof is completed.
the following is consequence of Lemma 2.1

Lemma 2.2 Let A, B and C are non-empty compact subsets of a 2-metric space (X, d), where $R = B \cup C$ and T is a multivalued mapping since $T: A \to S(R)$ Let $q \geq 1$. Next for a, b and $c \in A, x \in Ta$ there endure $y \in Tb$ and $z \in Tc$ so that $d(x, y, z) \leq qH(Ta, Tb, Tc)$.

Definition 2.1 Let $T: X \to X$ and $\alpha: X \times X \times X \to [0,\infty)$ be a function. We say that T is an α-admissible mapping if $x, y, z \in X$,

$$\alpha(x,y,z) \geq 1 \Longrightarrow \alpha(Tx,Ty,Tz) \geq 1.$$

Definition 2.2 Let (X,d) be a 2-metric space and $T: X \to N(X)$ a multivalued mapping since $\alpha: X \times X \times X \to [0,\infty)$. For x_0, y_0 and $z_0 \in X$ the mapping T called multivalued α-admissible if

$$\alpha(x,y,z) \geq 1 \Longrightarrow \alpha(x_1,y_1,z_1) \geq 1$$

where $x_1 \in Tx_0,\ y_1 \in Ty_0$ and $z_1 \in Tz_0$

Definition 2.3 Let $(X,d),(Y,\sigma)$ are two 2-metric spaces, $T: X \to SB(Y)$ and H is the Hausdorff metric on $SB(Y)$. The mapping T is said to be continuous at $x \in X$ if for any sequence $\{x_n\}$ in X, $H(Tx,Tx_n,Tx_{n+1}) \to 0$ whenever $d(x,x_n,x_{n+1}) \to 0$ as $n \to \infty$.

Theorem 2.1 *Let (X,d) be a complete 2-metric space, $\alpha: X \times X \times X \to [0,\infty)$ and $T: X \to S(X)$ a multivalued mapping. Let T be multivalued α-admissible and continuous. Let $\psi: [0,\infty) \to [0,\infty)$ be a nondecreasing function and continuos with $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ and $\psi(t) < t$ for each $t > 0$. Suppose that for all $x, y, z \in X$,*

$$\alpha(x,y,z)H(Tx,Ty,Tz) \leq \psi(max\{d(x,y,z), D(x,Tx,T^2x), D(y,Ty,T^2y), \frac{D(x,y,Tz) - D(x,z,Tz)}{2}, \frac{D(y,Ty,T^2y)[1 + D(x,Tx,T^2x)]}{1 + d(x,y,z)}, \frac{D(z,Ty,T^2x)[1 + D(x,Ty,T^2z)]}{1 + d(x,y,z)}\}). \quad (1)$$

if there exist $x_0 \in X, x_1 \in Tx_0$ and $x_2 \in Tx_1$ such that $\alpha(x_0,x_1,x_2) \geq 1$, then T has a fixed point in X.

Proof From the condition, there exist $x_0 \in X, x_1 \in Tx_0$ and $x_2 \in Tx_1$ such that $\alpha(x_0,x_1,x_2) \geq 1$. By lemma 2.2, for $x_2 \in Tx_1$ there exists $x_3 \in Tx_2$ such that $d(x_1,x_2,x_3) \leq \alpha(x_0,x_1,x_2)H(Tx_0,Tx_1,Tx_2)$. Employ (1) and applying the monotone property of ψ, we have

$$\begin{aligned} d(x_1,x_2,x_3) &\leq \alpha(x_0,x_1,x_2)H(Tx_0,Tx_1,Tx_2) \\ &\leq \psi(\max\{d(x_0,x_1,x_2), D(x_0,Tx_0,T^2x_0), D(x_1,Tx_1,T^2x_1), \frac{D(x_0,x_1,Tx_2) - D(x_0,x_2,Tx_2)}{2}, \\ &\quad \frac{D(x_1,Tx_1,T^2x_1)[1 + D(x_0,Tx_0,T^2x_0)]}{1 + d(x_0,x_1,x_2)}, \frac{D(x_2,Tx_1,T^2x_0)[1 + D(x_0,Tx_1,T^2x_2)]}{1 + d(x_0,x_1,x_2)}\}) \\ &\leq \psi(\max\{d(x_0,x_1,x_2), d(x_0,x_1,x_2), d(x_1,x_2,x_3), \frac{d(x_0,x_1,x_3) - d(x_0,x_2,x_3)}{2}, \\ &\quad \frac{d(x_1,x_2,x_3)[1 + d(x_0,x_1,x_2)]}{1 + d(x_0,x_1,x_2)}, \frac{d(x_2,x_2,x_2)[1 + d(x_0,x_2,x_4)]}{1 + d(x_0,x_1,x_2)}\}) \\ &\leq \psi(\max\{d(x_0,x_1,x_2), d(x_1,x_2,x_3), d(x_0,x_1,x_2) + d(x_0,x_2,x_3) + \frac{d(x_1,x_2,x_3) - d(x_0,x_2,x_3)}{2}\}) \\ &\leq \psi(max\{d(x_0,x_1,x_2), d(x_1,x_2,x_3)\}). \end{aligned}$$

It follows that

$$d(x_1,x_2,x_3) \leq \psi(max\{d(x_0,x_1,x_2), d(x_1,x_2,x_3)\}). \quad (2)$$

Now,if $\max\{d(x_0,x_1,x_2), d(x_1,x_2,x_3)\} = d(x_1,x_2,x_3)$

Then from (2) and property of ψ
$d(x_1,x_2,x_3) \leq \psi(d(x_1,x_2,x_3) < d(x_1,x_2,x_3)$,

which is a contradiction. Hence $d(x_1,x_2,x_3) \leq d(x_0,x_1,x_2)$. Then from (2), we have

$$d(x_1,x_2,x_3) \leq \psi d(x_0,x_1,x_2). \quad (3)$$

Since $x_1 \in Tx_0, x_2 \in Tx_1$ and $x_3 \in Tx_2$ and $\alpha(x_0,x_1,x_2) \geq 1$, the α-admissibility of T implies that $\alpha(x_1,x_2,x_3) \geq 1$. By Lemma (2.2), for $x_3 \in Tx_2$ there exists $x_4 \in Tx_3$ such that

$$d(x_2,x_3,x_4) \leq \alpha(x_1,x_2,x_3)H(Tx_1,Tx_2,Tx_3).$$

employ (1) and using the monotone property of ψ, we have

$$d(x_2,x_3,x_4) \le \alpha(x_1,x_2,x_3)H(Tx_1,Tx_2,Tx_3)$$

$$\le \psi(\max\{d(x_1,x_2,x_3), D(x_1,Tx_1,T^2x_1), D(x_2,Tx_2,T^2x_2), \frac{D(x_1,x_2,Tx_3)-D(x_1,x_3,Tx_3)}{2}, \frac{D(x_2,Tx_2,T^2x_2)[1+D(x_1,Tx_1,T^2x_1)]}{1+d(x_1,x_2,x_3)}, \frac{D(x_3,Tx_2,T^2x_1)[1+D(x_1,Tx_2,T^2x_3)]}{1+d(x_1,x_2,x_3)}\})$$

$$\le \psi(\max\{d(x_1,x_2,x_3), d(x_1,x_2,x_3), d(x_2,x_3,x_4), \frac{d(x_1,x_2,x_4)-d(x_1,x_3,x_4)}{2}, \frac{d(x_2,x_3,x_4)[1+d(x_1,x_2,x_3)]}{1+d(x_1,x_2,x_3)}, \frac{d(x_3,x_3,x_3)[1+d(x_1,x_3,x_5)]}{1+d(x_1,x_2,x_3)}\})$$

$$\le \psi(max\{d(x_1,x_2,x_3), d(x_2,x_3,x_4), d(x_1,x_2,x_3)+d(x_1,x_3,x_4)+\frac{d(x_2,x_3,x_4)-d(x_1,x_3,x_4)}{2}\})$$

$$\le \psi(max\{d(x_1,x_2,x_3), d(x_2,x_3,x_4)\}).$$

Suppose that $d(x_1,x_2,x_3) < d(x_2,x_3,x_4)$. Then $d(x_2,x_3,x_4) \neq 0$ and it folows by (4) and property of ψ that

$$d(x_2,x_3,x_4) \le \psi\big(d(x_2,x_3,x_4)\big) < d(x_2,x_3,x_4),$$

which is a contradiction. Then from (4) we have

$$d(x_2,x_3,x_4) \le \psi(d(x_1,x_2,x_3)). \quad (5)$$

Since $x_2 \in Tx_1, x_3 \in Tx_2$ and $x_4 \in Tx_3$ and $\alpha(x_1,x_2,x_3) \ge 1$, the α-admissibility of T implies that $\alpha(x_2,x|_3,x_4) \ge 1$. Continuing this process, we build up a sequence $\{x_n\}$ such that for all $n \ge 0$

$$x_{n+1} \in Tx_n, \quad (6)$$

$$\alpha(x_n,x_{n+1},x_{n+2}) \ge 1, \quad (7)$$

and

$$d(x_{n+1},x_{n+2},x_{n+3}) \le \psi(d(x_n,x_{n+1},x_{n+2})). \quad (8)$$

By copied operation (8) and monotone property of ψ, we have

$$d(x_{n+1},x_{n+2},x_{n+3}) \le \psi\big(d(x_n,x_{n+1},x_{n+2})\big) \le \psi^2(d(x_{n-1},x_n,x_{n+1})) \le \ldots \le \psi^{n+1}(d(x_0,x_1,x_2)).$$

Then by a property of ψ, we have

$$\sum_n d(x_n,x_{n+1},x_{n+2}) \le \sum_n \psi^n(d(x_0,x_1,x_2)) < \infty.$$

This appearance that $\{x_n\}$ is a Cauchy sequence. From the completness of X, there exists $z \in X$ such that

$$x_n \to z \text{ as } n \to \infty. \quad (9)$$

Since $x_{n+1} \in Tx_n$, we have

$$D(x_{n+2},x_{n+1},Tz) \le H(Tx_{n+1},Tx_n,Tz).$$

Taking limit as $n \to \infty$ in the raised inequality, and accepting (9) and the continuity of T, we have

$$D(z,z,Tz) = \lim_{n\to\infty} D(x_{n+2},x_{n+1},Tz) \le \lim_{n\to\infty} H(Tx_{n+1},Tx_n,Tz) = 0,$$

that is, $D(z,z,Tz) = 0$.

Since $Tz \in S(x), Tz$ is compact and hence Tz is closed, that is, $Tz = \overline{Tz}$, where $\overline{Tz}$ denotes the closure of Tz. Now, $D(z,z,Tz) = 0$ implies that $z \in \overline{Tz} = Tz$, that is, z is a fixed point ot T.

Theorem 2.2 *Let (X,d) be a 2-metric space, $T_i: X \to S(X)$, $i = 1,2$ be two multivalued mapping and $\alpha: X \times X \times X \to [0,\infty)$. Let each $T_i, i = 1,2$ be continuous and multivalued α-admissible. Let $\psi: [0,\infty) \to [0,\infty)$ be a continuous and nondecreasing function with $\Phi(t) = \sum_{n=1}^{\infty} \psi^n(t) < \infty$, $\Phi(t) \to 0$ as $t \to 0$ and $\psi(t) < t$ for each $t > 0$. Suppose that*

(i) each $T_i, i = 1,2$ satisfing (1), that is, for all x, y and $z \in X$,

$$\alpha(x,y,z)H(T_ix,T_iy,T_iz) \le \psi(max\{d(x,y,z), D(x,T_ix,T_i^2x), D(y,T_iy,T_i^2y), \frac{D(x,y,T_iz)-D(x,z,T_iz)}{2},$$

$$\frac{D(y,T_iy,T_i^2y)[1+D(x,T_ix,T_i^2x)]}{1+d(x,y,z)}, \frac{D(z,T_iy,T_i^2x)[1+D(x,T_iy,T_i^2z)]}{1+d(x,y,z)}\});$$

(ii) for any $x \in F(T_1)$,and $y \in T_2x$, we have $\alpha(x,y,z) \geq 1$ whenever $z \in T_3x$; and for any $x \in F(T_2)$ and $y \in T_1x$, we have $\alpha(x,y,z) \geq 1$ whenever $z \in T_3x$.
Then $H(F(T_1),F(T_2),F(T_3)) \leq \Phi(w)$, where $w = \sup_{x\in X}H(T_1x,T_2x,T_3x)$.

Proof From Theorem 2.1, the set of fixed point of $T_i(i = 1,2)$ are non-empty, that is, $F(T_i) \neq \emptyset$, for $i = 1,2$. Let $y_0 \in F(T_1)$, that is, $y_0 \in T_1y_0$. Then by Lemma 2.1, there exists $y_1 \in T_2y_0$ and $y_2 \in T_3y_0$ such that

$$d(y_0,y_1,y_2) \leq H(T_1y_0,T_2y_0,T_3y_0). \qquad (10)$$

Since $y_0 \in F(T_1)$, $y_1 \in T_2y_0$ and $y_2 \in T_3y_0$, by condition (ii) ,we have $\alpha(y_0,y_1,y_2) \geq 1$. By lemma 2.2, for $y_1 \in T_2y_0, y_2 \in T_2y_1$ there exists $y_3 \in T_2y_2$ such that

$$d(y_1,y_2,y_3) \leq \alpha(y_0,y_1,y_2)H(T_2y_0,T_2y_1,T_2y_2).$$

Then contend similarly as in the proof of Theorem 2.1, we construct a sequence y_n such that for all $n \geq 0$

$$y_{n+1} \in T_2y_n, \qquad (11)$$

$$\alpha(y_n,y_{n+1}y_{n+2}) \geq 1, \qquad (12)$$

$$d(y_{n+1},y_{n+2},y_{n+3}) \leq \psi\big(d(y_n,y_{n+1},y_{n+}2)\big) \qquad (13)$$

and

$$d(y_{n+1},y_{n+2},y_{n+3}) \leq \psi(d(y_n,y_{n+1},y_{n+2})) \leq \psi^2(d(y_{n-1},y_n,y_{n+1})) \leq \ldots \leq \psi^{n+1}(d(y_0,y_1,y_2)). \qquad (14)$$

Contend similarly as in the proof of Theorem 2.1, we prove that $\{y_n\}$ is a Cauchy sequence X and there exists $v \in X$ such that

$$y_n \to v \text{ as } n \to \infty, \qquad (15)$$

further v is a fixed point of T_2, that is, $v \in T_2v$.

Now, from (10) and the definition of w, we have

$$d(y_0,y_1,y_2) \leq H(T_1y_0,T_2y_0,T_3y_0) \leq w = \sup_{x\in X}H(T_1x,T_2x,T_3x). \qquad (16)$$

Repeatedly, by the triangle inequality and using (14), we have

$$d(y_0,y_1,v) \leq \sum_{i=0}^{n}(d(y_i,y_{i+1},y_{i+2})) + d(y_n,y_{n+2},v) + d(y_{n+1},y_{n+2},v) \leq \sum_{i=0}^{n}\psi^i\big(d(y_0,y_1,y_2)\big) + d(y_n,y_{n+2},v + d(y_{n+1},y_{n+2},v).$$

Taking limit $n \to \infty$ in the above inequality, using (15),(16) and the propertyies of ψ, we have
$d(y_0,y_1,v) \leq \sum_{i=0}^{\infty}\psi^i(d(y_0,y_1,y_2)) \leq \sum_{i-0}^{\infty}\psi^i(w) = \Phi(w)$.

Thus, given arbitrary $y_0 \in F(T_1)$, we can find $v \in F(T_2)$ for which

$$d(y_0,y_1,v) \leq \Phi(w).$$

Similarly, we can prove that for arbitrary $c_0 \in F(T_2)$, there exists a $p \in F(T_1)$ such that $d(c_0,c_1,p) \leq \Phi(w)$. Hence, we conclude that $H(F(T_1),F(T_2,F(T_3))) \leq \Phi(w)$.

3 Conclusion

In this paper we established the existence of fixed points of multivalued α-admissible mappings in 2-metric spaces. and we investigated the stability of fixed point, also we introduced and studied the notion of multivalued α-admissible in 2-metric spaces

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