Generating Fuzzy Sets and Fuzzy Relations Based on Information

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Abstract: - Fuzzy set theory and fuzzy relation are important techniques in knowledge discovery in databases. In this work, we presented fuzzy sets and fuzzy relations according to some giving Information by using rough membership function as a new way to get fuzzy set and fuzzy relation to help the decision in any topic . Some properties have been studied. And application of my life on the fuzzy set was introduced.

Keywords : Fuzzy set, rough set, rough relation, fuzzy relation, equivalence relation.

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1 Introduction

Set theory is a basic branch of mathematics and it has a great effect in all branches of the natural sciences, especially mathematics. The usual definition of ordinary subset is not available in cases of collection with no sharp boundary for there are fuzzy problems in our life such as real numbers which are closely near to zero...etc. In 1965 L. Zadeh [7] introduced the fundamental concept of a fuzzy subset A of a given non-empty set X to be characterized by a membership function $\mu_A(x): X \to [0,1]$. Ordinary subsets of X are special case of fuzzy subsets. Usually a fuzzy subset A of X is defined with its membership function $\mu_A(x)$. Since L.Zadeh published his first research of fuzzy subsets, the scientists began to build up new branches of mathematics according to this theory, and so fuzzy mathematics grows up. Since 1971, many authors such as, Chakraborty and Das [9, 8], Highshi [2], Murali [13] and Seema [12] have applied the concept of fuzzy subset to the subject of binary relations and finding relationships from Fuzzy topological spaces. A membership Function is a tool of reduction for data. Pawlak in [16] expand the membership function into initial rough membership function. Also, El Atik in [1] used similarity as a membership function. The notions of relation play a fundamental role in applications of mathematics. They maybe generalized with respect to the notion of fuzzy subsets. One will then discover some new and very interesting properties. The concept of fuzzy relation is very important not only in theoretical studies but also, on a great wide, in practical applications. It contributed to the rapid development to computer and technology during the past two decades, from an industrial to an information society. It represents a key for bridging from real life data to mathematical models such as fuzzy topological structures, and other models that are concerned with neural networks...etc. It is as extension of ordinary relations, and their range of application is very wide. For example, they are frequently applied in clustering, pattern recognition [10], inference, system and control. They also have applications in the fields known as "soft sciences", such as psychology[7], economics and sociology[8,9], medical diagnosis[3], Multi-criteria Decision Making Method[4] and network controller design and analysis[5]. In this work, the second part we introduces preliminaries about fuzzy set theory and rough set theory. Third part we introduce a method which is used to create fuzzy set based on information by using rough membership function. Some basic properties of these sets are investigated.

The fourth part we introduce the new method to construct fuzzy relation based on information. Finally, last part we presents the conclusions of this paper. These new groups assist in the decision-making process based on the available information.

2 preliminaries

2.1 Definition [7]

Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A(x)$ from X to closed interval $I = [0, 1]$. The fuzzy set A is completely characterized by the set of pairs, $A = \{(x, \mu_A(x)): x \in X\}$ in particular, $\mu_A(x) = 1$, , $\mu\emptyset(x) = 0$ for all $x \in X$ are the universal fuzzy set and the empty fuzzy set, respectively. The family of all fuzzy sets in X is denoted by I^X.

2.2 Definition [11]

Let A and B be fuzzy sets in X, then.

a. $A \subseteq B \; iff \; \mu_A(x) \le \mu_B(x), \forall x \in X.$

b. $A = B \; iff \; \mu_A(x) = \mu_B(x), \; \forall x \in X.$

c. $\mu_{A^c}(x) = 1 - \mu_A(x), \; \forall x \in X$, where A^c is complement.

d. $\mu_{A\cup B}(x) = max\{\mu_A(x), \mu_B(x)\}$

e. $\mu_{A\cap B}(x) = min\{\mu_A(x), \mu_B(x)\}$

In the following we give an example to indicate some of the above notions.

2.2.1 Example

Let A denotes a fuzzy set of rivers that are long and B denote a fuzzy set of reveres that are navigable. Such that,

$\mathrm{A} = \{(\mathrm{Amazon}, 1), (\mathrm{Nile}, 0.9), (\mathrm{Yange} - \mathrm{Tse}, 0.8), (\mathrm{Danube}, 0.5), (\mathrm{Rhine}, 0.4)\}$ and $\mathrm{B} = \{(\mathrm{Amazon}, 0.8), (\mathrm{Nile}, 0.7), (\mathrm{Y\,ange} - \mathrm{T\,se}, 0.8), (\mathrm{Danube}, 0.6), (\mathrm{Rhine}, 0.3)\}$. We get $\mathrm{A} \cup \mathrm{B} = \{(\mathrm{Amazon}, 1), (\mathrm{Nile}, 0.9), (\mathrm{Y\,ange} - \mathrm{T\,se}, 0.8), (\mathrm{Danube}, 0.6), (\mathrm{Rhine}, 0.4)\}$ and

$\mathrm{A} \cap \mathrm{B} = \{(\mathrm{Amazon}, 0.8), (\mathrm{Nile}, 0.7), (\mathrm{Y\,ange} - \mathrm{T\,se}, 0.8), (\mathrm{Danube}, 0.5), (\mathrm{Rhine}, 0.4)\}.$

2.3 Definition [15]

Let X be a nonempty set called universe, and R be an equivalence relation on X. The pair (X, R) is called an approximation space. Let $[x]$ denote the equivalence class for an element $x \in X$ Let A be a subset of X, a rough set corresponding to A is the ordered pair $(\overline{A}, \underline{A})$, where $\overline{A}$ and $\underline{A}$ are defined as follow: $\underline{A} = \{[x] \subseteq A: x \in X\}$ (called lower approximation of A), $\overline{A} = \{[x] \cap A \neq \emptyset : x \in X\}$ (called upper approximation of A). Obviously $\underline{A} \subseteq A \subseteq \overline{A}$ The lower and upper approximation are used to divide the universe into three regions with respect to any subset $A \subseteq X$: $NEG(A) = X - \overline{A}$, , $POS(A) = \overline{A}$, $BND(A) = \overline{A} - \underline{A}$. An element of the negative region $NEG(A)$ definitely doesn't belong to A, an element of the positive region $POS(A)$ definitely belong to A and an element of the boundary region $BND(A)$only possibly to A .

In the following we give an example to indicate the above notion.

2.3.1 Example

Let $X = \{x_1, x_{2,} x_3, x_4, x_5, x_6\}$,

$X/R = \{\{x_5, x_6\}, \{x_1, x_3, x_4\}, \{x_2\}\}$ and $A = \{x_1, x_{5,} x_6\}$. We get $\underline{A} = \{x_{5,} x_6\}$ $\overline{A} = \{x_{1,}, x_{3,} x_{4,} x_{5,} x_6\}$.

Since fuzzy relations in general are fuzzy sets, we can define the cartesian product to be a relation between two or more fuzzy sets. Let A, B are a fuzzy sets on a universe X such that.

$$R \subseteq A \times B,$$

$$R : A \times B \to [0,1].$$

As shown in the following definition.

2.4 Definition [6]

The cartesian product of $A \in F(X)$ and $B \in F(X)$ is defined as $(A \times B)(x, y) = min\{\mu_A(x), \mu_B(y)\}\ \forall x \in X, y \in Y$ where $F(X)$, $F(Y)$ the family of fuzzy Sets. It is clear that the cartesian product of fuzzy sets is a fuzzy relation on X. We give in the following example to indicate the above notion.

2.4.1 Example

Let $A = \{(x_1, 0.4), (x_2, 0.8), (x_3, 0.0), (x_4, 0.7)\}$, $B = \{(y_1, 0.5), (y_2, 0.1), (y_3, 0.6), (y_4, 0.2)\}$ where $\mu_A : X \to [0,1]$ and $\mu_B : X \to [0,1]$ Then $\mu_{A\times B} : X \times Y \to [0,1]$ such that

$\mu_{A\times B} = min\{\mu_A(x), \mu_B(y)\}$. Can be expressed by the matrix

$$\begin{array}{c} A \times B \to \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{pmatrix} y_1 & y_2 & y_3 & y_4 \\ & & & \\ 0.4 & 0.1 & 0.4 & 0.2 \\ 0.5 & 0.1 & 0.6 & 0.2 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.1 & 0.6 & 0.2 \end{pmatrix}$$

2.5 Definition [11]

Fuzzy relation R from set X to set Y is a fuzzy set in the cartesian product $X \times Y = \{(x, y) : x \in X, y \in Y$ and is characterized by a membership function μ_R. Especially when $X = Y$, R is known as a fuzzy relation on X. The fuzzy binary relation can be expressed in the form: $R = \{((x, y), \mu_R)\}$ where μ_R is the degree of membership of the ordered pair (x, y) in R. We give the following example to indicate the above notion.

2.5.1 Example

Let S be a set of symptoms and I be a set of illness where,

$S = \{\text{Cough}, \text{Fever}, \text{Shortnness of breath}\}$ $I = \{Asthma, Bronchitis, Influenza\}$. Where $R : S \times I \times\to [0,1]$.

$$\begin{array}{c} S\ R\ I \to \\ Cough \\ Fever \\ Shortness \\ \text{of breath} \end{array} \begin{pmatrix} Asthma & Bronchitis & Influenze \\ & & \\ 0.72 & 0.98 & 0.62 \\ 0.34 & 0.81 & 0.94 \\ 0.98 & 0.67 & 0.13 \end{pmatrix}$$

2.6 Definition [15]

Let $\omega^* = (U^n, R)$ be a product approximation space. For any $Q \subseteq U^n$, the relation Q is called rough in ω^* iff $\underline{R}(Q) \neq \overline{R}(Q)$, otherwise Q is an exact relation in ω^*.

2.7 Definition [15]

Let $\omega^* = (U^n, R)$ be a product approximation space. For any $Q \subseteq U^n$, the difference $\underline{R}(Q) - \overline{R}(Q)$ is called the boundary of Q in ω^* and denoted by $BNDR(Q)$. The relation Q is rough in ω^* iff $BNDR(Q) \neq \emptyset$.

The following example indicates the above definition.

2.7.1 Example

If X be a universe and $X/R_1, X/R_2$ are two partition on X where $X = \{a, b, c\}$. $X/R_1 = \{\{a\}, \{b, c\}\}$, $X/R_2 = \{\{b\}, \{\{a, c\}\}$,

$R_r = \{(a, a), (c, b), (a, b)\}$ such that $R_r \subseteq X \times X, (X/R_1, X/R_2) =$
$\{\{(a, b)\}, \{(a, a), (a, c)\}, \{(b, b), (b, c)\},$

$\{(b, a), (b, c), (c, a), (c, c)\}\}$.

We get

$\underline{R_r} = \{(a, b)\}$, $\overline{R_r} =$
$\{(a, b), (b, b), (c, b), (a, a), (a, c)\}$

$since\ \overline{R_r} - \underline{R_r} \neq \emptyset$. This implies that R_r is a rough relation on $X \times X$.

3 Fuzzy set based on information

In this section, we introduce a new method to construct fuzzy set from data by using the equivalence class used by **Pawlak** in the rough membership function for an for an equivalence relation R[11].

3.1 Definition

Let X be a universal set and R be an equivalence relation on X , A is a subset of X we get the membership function for using the equation.

$$\mu_A^R(x) = \frac{|[x]_R \cap A|}{|[x]_R|}, \ \forall\, x \in X$$

Such that $[x]_R$ is an equivalence class and $x \in X$. In the following figure shows that the degree of membership function of elements based on information. The degree of element in interior set equal to 1, the degree of element in exterior set equal to 0, the degree of element in boundary set take between value 0 and 1.

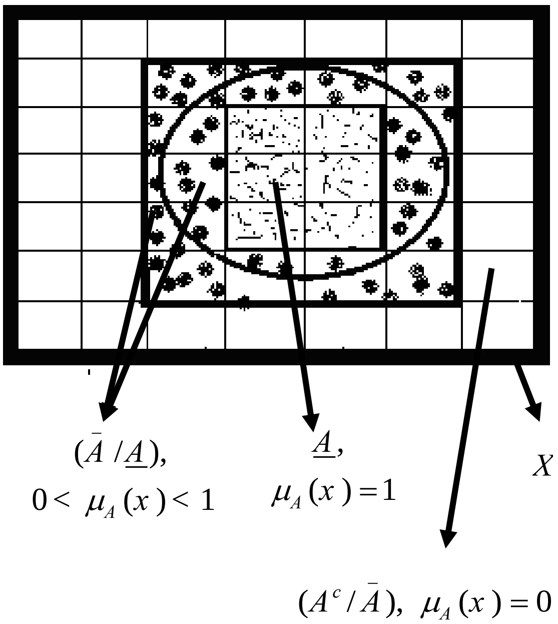

Fig.1 (Membership functions)

The following example indicate definition 3.1

3.1.1 Example

Let $X = \{x_1, x_2, x_3, x_4,\}$, $A = \{x_2, x_3\}$,

$$R = \{(x_1,x_1),(x_2,x_2),(x_3,x_3),(x_4,x_4),(x_3,x_4),(x_4,x_3)\}$$

and $[x_1]_R = \{x_1\}$, $[x_2]_R = \{x_2\}$, $[x_3]_R = \{x_3, x_4,\}$, $[x_4]_R = \{x_3, x_4\}$
we use the definition 3.1

$$\mu_A^R(x_1) = \frac{|[x_1]_R \cap A|}{|[x_1]_R|} = \frac{0}{1} = 0 \ ,$$

$$\mu_A^R(x_2) = \frac{|[x_2]_R \cap A|}{|[x_2]_R|} = \frac{1}{1} = 1 \ ,$$

$$\mu_A^R(x_3) = \frac{|[x_3]_R \cap A|}{|[x_3]_R|} = \frac{1}{2} = 0.5,$$

$$\mu_A^R(x_4) = \frac{|[x_4]_R \cap A|}{|[x_4]_R|} = \frac{1}{2} = 0.5$$

We get the fuzzy set

$$A = \{(x_1, 0), (x_2, 1), (x_3, 0.5), (x_4, 0.5)\}$$

Now we give the theorem to obtain the degree of all element that take value between 0 and 1 .

3.1 Theorem

If X is a nonempty set, $R = X \times X$ and A is a subset of X, then the fuzzy set defined by $A = \left\{\left(a_i, \frac{m}{n}\right) : m = |A|,\ n = |X|, i = 1, 2, 3, \dots, n\right\}$.

Proof: Since $R = X \times X$, $A \subseteq X$, $x_i \in A$, $i = 1, 2, 3, \dots m$, $m \le n$. From definition 3.1 . We get

$$\mu_A^R(x) = \frac{|[x]_R \cap A|}{|[x]_R|} = \frac{m}{n}, \ \ \forall\, i.$$

The following example illustrate a method of determining the membership function.

3.1.2 Example

Let $X = \{a, b, c\}$, $A = \{a, b\}$ and

$$R = \left\{\begin{matrix}(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),\\ (c,a),(c,b),(c,c)\end{matrix}\right\}$$

. From definition 3.1, we have $[a]_R = \{b\} = [c]_R = \{a, b, c\}$, we get

$$\mu_A^R(a) = \mu_A^R(b) = \mu_A^R(c) = \frac{2}{3} = 0.67$$

$A = \{(a, 0.67), (b, 0.67), (c, 0.67)\}$.

Now we notice that we can obtain the crisp set from the fuzzy set if the relation is reflexive as following.

3.2 Theorem

If X is a nonempty set , $R = \{(x, y) : x = y,\ \forall\, x, y \in X\}$, A is a subset of X, then $\mu_A(x) = \begin{cases} 1 & if \quad x \in A, \\ 0 & if \quad x \notin A. \end{cases}$

Proof: Since $R = \{(x, y): x = y, \forall x, y \in X\}$ i.e. R is reflexive relation and X is a nonempty set , $A \subseteq X$, hence
$[x_i]_R = \{x_i\}$, $\forall i = 1, 2, 3, \ldots, n$ if $x_i \in A$. Then
$$\mu_A^R(x_i) = \frac{|[x_i]_R \cap A|}{|[x_i]_R|} = \frac{1}{1} = 1, \ \forall x_i \in X, \ x_i \in A.$$
Because R is reflexive, then
$$\mu_A^R(x_i) = |[x_i]_R \cap A| = |[x_i]_R| .$$
Or if $x_i \notin A$, Then
$\mu_A^R(x_i) = \frac{|[x_i]_R \cap A|}{|[x_i]_R|} = \frac{0}{1} = 0 \ \forall x_i \in X, \ x_i \notin A.$
Because $[x_i]_R \cap A = \emptyset$ and $|\emptyset| = 0.$
This theorem indicates as for crisp case.
As shown in the following example.

3.2.1 Example

Let $X = \{a, b, c\}$, $A = \{a, c\}$ and
$R = \{(a, a), (b, b), (c, c)\}$. Then we have $[a]_R = \{a\}$, $[b]_R = \{b\}$, $[c]_R = \{c\}$. . From definition 3.1, $\mu_A^R(a) = 1$, $\mu_A^R(b) = 0$, $\mu_A^R(ac) = 1$ and

$$A = \{(a, 1), (b, 0), (c, 1)\}$$

3.3 Theorem

If A, B are two subset of X and R is an equivalence relation on X and $A \subseteq B$. Then $\mu_A^R(x) \le \mu_B^R(x)$. Where $\mu_A^R(x)$ and $\mu_B^R(x)$ are fuzzy set, $x \in X$..
Proof: Let $x \in A$ this implies that $x = (a, \mu_A^R(a))$, $\mu_A^R(a) = \frac{|[a]_R \cap A|}{|[a]_R|}$. Since $A \subseteq B$ this implies that $\mu_A^R(a) \le \frac{|[a]_R \cap B|}{|[a]_R|}$ for $|[a]_R \cap A| \le |[a]_R \cap B|$ this leads to $\mu_A^R(a) \le \mu_B^R(a)$ therefore $\mu_A^R(x) \le \mu_A^R(a)$ $\forall x \in X$.
The following example indicate the above theorem.

3.3.1 Example

Let $X = \{a, b, c, d, e\}$, $A = \{b, d\}$, $B = \{a, b, d\}$ and
$$R = \begin{Bmatrix} (a, a), (a, e), (a, d), (b, b), (d, d), (d, a), \\ (c, c), (d, e), (e, a), (e, d), (e, e) \end{Bmatrix}$$

Since $[a]_R = \{a, e, d\}$, $[b]_R = \{b\}$, $[c]_R = \{c\}$, $[d]_R = \{a, e, d\}$, $[e]_R = \{a, e, d\}$.
We use the definition 3.1, in state set A, we get $\mu_A^R(a) = 0.33$, $\mu_A^R(b) = 1$, $\mu_A^R(c) = 0$, $\mu_A^R(d) = 0.33$, $\mu_A^R(e) = 0.33$

$$A = \{(a, 0.33), (b, 1), (c, 0), (d, 0.33), (e, 0.33)\}$$

We use the definition 3.1 in state set B, we get $\mu_A^R(a) = 0.33$, $\mu_A^R(b) = 1$, $\mu_A^R(c) = 0$, $\mu_A^R(d) = 0.33$, $\mu_A^R(e) = 0.33$

From set A and set B , we get $A \subseteq B$, i.e. $\mu_A^R(x) \le \mu_B^R(x), \forall x \in X$.

Now we obtain some results which show the relation between fuzzy subset.

3.1 Remark

If X is a universal set , A, B, C are subsets on X , $\mu_A(x) \le \mu_B(x)$ and $\mu_B(x) \le \mu_C(x)$. Then $\mu_A(x) \le \mu_C(x)$.

3.2 Remark

If A, B are two subset of X, $A \ne B$. Then $\mu_A(x) \ne \mu_B(x)$.

The following example indicate the next remark.

3.3.2 Example

Let $X = \{a, b, c, d, e\}$, $A = \{a, d\}$, $B = \{b, e\}$ and
$$R = \begin{Bmatrix} (a, a), (b, b), (c, c), (d, d), (e, e), (a, d), (d, a), \\ (c, b), (b, c), (a, b), (b, a) \end{Bmatrix}$$

Since $[a]_R = \{a, b, d\}$, $[b]_R = \{b, c, a\}$, $[c]_R = \{c, b\}$, $[d]_R = \{a, d\}$, $[e]_R = \{e\}$.
We use the definition 3.1, in state A, we get $\mu_A^R(a) = 0.66$, $\mu_A^R(b) = 0.33$, $\mu_A^R(c) = 0$, $\mu_A^R(d) = 1$, $\mu_A^R(e) = 0$

$$A = \{(a, 0.66), (b, 0.33), (c, 0), (d, 1), (e, 0)\}$$

And in state , we get $\mu_B^R(a) = 0.33$, $\mu_B^R(b) = 0.33$, $\mu_B^R(c) = 0.33$, $\mu_B^R(d) = 0$, $\mu_B^R(e) = 1$

$$B = \{(a, 0.33), (b, 0.33), (c, 0.33), (d, 0), (e, 1)\}$$

From set A and set B , we get $\mu_A(x) \ne \mu_B(x)$ $\forall x \in X$.

From this third part, we extract a fuzzy set of information that we have to help the decision maker, as we see in the next application.

Application

The application to the problem of chikungunya, which is a disease transmitted to humans by a virus that carries aedes mosquitoes. Associated symptoms are fever and severe joint pain, other symptom in clued muscle pain, headache and nausea. Initial symptoms are similar to dengue fever. Joint pain can last for a long time and full recovery may take months. In recent decades the disease has spread in Africa and Asia. The following table shows information on eight patients.

Table 1(Giving data about eight patients)

Symptoms	Joint pain	Headache	Nausea	Temperature	Chikungunya
P_1	Very high	Very high	Very high	high	Yes
P_2	Very high	Normal	Normal	high	No
P_3	Very high	Normal	Normal	high	Yes
P_4	Normal	high	Normal	Very high	No
P_5	Normal	Very high	high	high	No
P_6	Very high	Very high	Normal	Very high	Yes
P_7	high	high	Normal	Normal	No
P_8	Very high	Very high	Normal	Very high	Yes

The columns of the table represent the symptoms

for Chikungunya(Joint pain, Headache, Nausea, Temperature) and the rows represent the patients(P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8) from the table, we find that those who have the disease are P1, P3, P6, P8. Now we are generating a fuzzy set from the information available in the table 1. As an application to what was mentioned in definition 3.1 to assist the decision maker. From the following table take $X = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$

patients. $A = \{P_1, P_3, P_6, P_8\}$ they have the disease , and the equivalence relation is $R = \left\{\begin{matrix} (P_1, P_1), (P_2, P_2), (P_2, P_3), (P_3, P_2), (P_3, P_3) \\ (P_4, P_4), (P_5, P_5), (P_6, P_6), (P_6, P_8), (P_7, P_7), \\ (P_8, P_8), (P_8, P_6) \end{matrix}\right\}$

We have $[P_1] = \{P_1\}$, $[P_2] = [P_3] = \{P_2, P_3\}$, $[P_4] = \{P_4\}$, $[P_5] = \{P_5\}$, $[P_6] = [P_8] = \{P_6, P_8\}$, $[P_7] = \{P_7\}$. We use the definition 3.1 $\mu_A^R(P_1) = 1$, $\mu_A^R(P_2) = 0.5$, $\mu_A^R(P_3) = 0.5$, $\mu_A^R(P_4) = 0$, $\mu_A^R(P_5) = 0$, $\mu_A^R(P_6) = 1$, $\mu_A^R(P_7) = 0$, $\mu_A^R(P_8) = 1$. We get the fuzzy set is $\left\{\begin{matrix} (P_1, 1), (P_2, 0.5), (P_3, 0.5), (P_4, 0), (P_5, 0), (P_6, 1) \\ (P_7, 0)(P_8, 1) \end{matrix}\right\}$

From obtaining the fuzzy set, we find it helps in medical diagnosis. We deduce from the fuzzy set that we got patients P_1, P_6, P_8 100% injury, while patients P_2, P_3 is 50% sick, and this is new on the patients data , so the importance of this research is to convert the descriptive information into digital data that helps decision-makers in obtaining an accurate decision.

4 Fuzzy relation based on information

In this part, we introduce a new method for constructing a fuzzy relation from data by using a rough relation as the following.

4.1 Definition

The relation R_r is called rough relation on the partition $(X \times X)/R$ if R_r is not equal to the union of some equivalence classes.

4.2 Definition

If X is a universal set $(X \times X)/R$ is a partition on X , R is a binary relation, then we get a fuzzy relation by the use of.

$$\mu_R(x, y) = \frac{|R_r \cap [(x, y)]|}{|[(x, y)]|}$$

Where R_r is called rough relation and $|[(x, y)]|$ is the equivalence class of

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(x, y) . As shown in the following example.

4.2.1 Example

Let $X = \{a, b, c, d\}$,

$$X \times X = \left\{ \begin{array}{c} (a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c), \\ (b,d),(c,a),(c,b),(c,c),(c,d),(d,a), \\ (d,b),(d,c),(d,d) \\ , \end{array} \right\},$$

$$(X \times X)/R = \left\{ \begin{array}{c} \{(a,b),(a,d),(b,c),(d,a)\}, \\ \{(a,c),(b,b),(c,a),(c,c),(d,d)\}, \\ \{(d,b),(c,b),(b,d),(c,d)\}, \\ \{(a,a),(b,a),(d,c)\} \end{array} \right\}$$

And
$R_r = \{(a,a),(a,b),(a,c),(d,a),(c,b),(b,b)\}$.

We use the definition 4.2 $\mu_R(a,a) = \mu_R(b,a) = \mu_R(d,c) = 0.33$, $\mu_R(a,b) = \mu_R(a,d) = \mu_R(b,c) = \mu_R(d,a) = 0.5$, $\mu_R(a,c) = \mu_R(b,b) = \mu_R(c,a) = \mu_R(c,c) = \mu_R(d,d) = 0.4$,

$$\mu_R(b,d) = \mu_R(c,b) = \mu_R(c,d) = \mu_R(d,b) = 0.25$$

The fuzzy relation can be written by the matrix

$$\begin{array}{c} R_F \\ a \\ b \\ c \\ d \end{array} \begin{pmatrix} a & b & c & d \\ & & & \\ 0.33 & 0.5 & 0.4 & 0.5 \\ 0.33 & 0.4 & 0.5 & 0.25 \\ 0.4 & 0.25 & 0.4 & 0.25 \\ 0.5 & 0.25 & 0.33 & 0.4 \end{pmatrix}$$

4.3 Definition

If X is a universal set X/R is a partition on X , $A \subseteq X$ and $((X/R) \times (X/R))$ product a partition on X . Then we get a fuzzy relation by the use of .

$$\mu_{A\times A}^{((X/R)\times(X/R))}(x,y) = \frac{\big||A \times A| \cap [(x,y)]\big|}{|[(x,y)]|}$$

As shown in the following example.

4.3.1 Example

Let $X = \{a, b, c, d\}$, $A = \{a, b\}$, $X/R = \{\{a\},\{b,d\},\{c\}\}$,
$A \times A = \{(a,a),(a,b),(b,a)(b,b)\}$

and

$$((X/R) \times (X/R)) = \left\{ \begin{array}{c} \{(a,a)\},\{(a,b),(a,d)\},\{(b,a),(d,a)\},\{(a,c)\}, \\ \{(b,b),(b,d),(d,b),(d,d)\},\{(b,c),(d,c)\}, \\ \{(c,a)\},\{(c,b),(c,d)\},\{(c,c)\} \end{array} \right\},$$

From definition 4.3 , we get the following fuzzy relation

$$\begin{array}{c} R_F \\ a \\ b \\ c \\ d \end{array} \begin{pmatrix} a & b & c & d \\ & & & \\ 1 & 0.5 & 0.0 & 0.5 \\ 0.5 & 0.25 & 0.0 & 0.25 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.25 & 0.0 & 0.25 \end{pmatrix}$$

5 Conclusion

Our result in this paper. The new concept of fuzzy set method helps in honest and accurate expression of things for decision maker. Also the new fuzzy relation helps to express ambiguous relationships by representing it on computer systems. Based on these results, It becomes important in generating fuzzy set and the formation of fuzzy relationships based on information is useful in solving many fuzzy life problems.

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