Option pricing under GARCH models applied to the SET50 index of Thailand

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Abstract: - Variance changes over time and depends on historical data and previous variances; as a result, it is useful to use a GARCH process to model it. In this paper, we use the notion of Conditional Esscher transform to GARCH models to find the GARCH, EGARCH and GJR risk-neutral models. Subsequently, we apply these three models to obtain option prices for the Stock Exchange of Thailand and compare to the well-known Black-Scholes model. Findings suggest that most of the pricing options under GARCH model are the nearest to the actual prices for SET50 option contracts with both times to maturity of 30 days and 60 days.

Key-Words: - Option pricing, GARCH model, Stochastic assets.
Received: November 26, 2020. Revised: March 11, 2021. Accepted: March 28, 2021. Published: April 14, 2021.

1 Introduction

The most important options in Thailand are the SET50 index options which are the contracts that provide the right to the buyer to buy or sell the SET50 index on or before an expired date at a specified contract price, depending on the form of the option. We consider, in our study, the European call option and therefore it can only be exercised at expiration. Investment in the security has a risk due to changes over time and the variance may aff ect the investment returns in an unexpected way. The most popular model which gives the option price is the Black and Scholes formula which assumes volatility to be constant. Several studies have shown that in the market, the volatility of the security depends on previous volatilities. Hence the GARCH model is suitable for modeling volatility and for forecasting asset prices since the GARCH is a model that applies the information of variance in the past to forecast variances in the future.

Many researchers were interested in GARCH option pricing models. In 2010, Costa, Veiga and Siu [1] valuated option prices by using some special GARCH models based on risk neutral assumption. In 1995, Duan [2], developed an option pricing model and its corresponding delta formula for GARCH models with by the generalization of the concept of risk neutral valuation and local risk neutral valuation relationship on assets. In 1994, Gerber and Shiu [3] proposed a method of Esscher Transforms for option pricing which was later be able to apply for GARCH models. In the later year of 1996, Schmitt [4] showed his empirical studies that the time-varying volatility of asset returns can be described by GARCH models. Siu, Tong and Yang in 2004 [5] developed an approach for derivative pricing by using the GARCH option pricing models under the dynamic environment based on the model of Gerber-Shui and utilized the concrete idea of the conditional Esscher transforms. In 2017, Huang, Su and Chen [6] explored the valuation performance of a special kind of GARCH model, Heston and Nandi GARCH model on the option pricing. The results showed another vision of the impact of liquidity on GARCH models and the pricing error during financial crisis. In 2017, Badescu, Cui and Ortega [7] investigated the pricing and weak convergence of an asymmetric non-affine, non-Gaussian GARCH model. The option data analysis illustrated the advantage of coupling the option pricing with non-Gaussian methods. In 2018, Hua and Jiang [8] explored option pricing based on the proposed hybrid GARCH models with improved ensemble empirical mode decomposition. The results indicated that the hybrid models of GARCH with the decomposition technique could reduce the option pricing error. In the recent years, many researchers have applied GARCH models for the new financial currency which was well known as Cryptocurrency. In 2020, Venter, Mare and Pindza [9] applied GARCH models for Bitcoin and Cryptocurrency index (CRIX). Later, in the same year of 2020, Venter and Mare [10] applied GARCH models to generate volatility indices of Bitcoin and Cryptocurrency index (CRIX). In 2021, Anel, Rastegari and Stentoft [11] proposed a new condition of GARCH models for option pricing including a dynamic variance dependent pricing kernel.

There are some researchers studied GARCH models applied to the environment of Thailand. In 2005, Khanthavit [12] considered the model for pricing and analyzed the behavior of options on the SET50. His results show that the GARCH (1,1) model could describe the random behavior of SET50 better than the other models. In 2014, Tanattrin [13] applied GARCH and GJRGARCH model for volatility analysis of international tourist arrival growth rate in Thailand.

In this paper, we study the GARCH, EGARCH and GJR-GARCH models to find the option prices. So we follow the Siu, Tong and Yang (2004) [5] method to find the risk neutral version of the GARCH, EGARCH and GJR-GARCH for pricing options on the SET50 index of Thailand.

2 Theoretical Background

In 1973, Black and Scholes [14] proposed a model for option pricing that has been frequently used in the financial researches. Nevertheless, the model that Black and Scholes proposed has some inapplicable assumptions of constant variance which is not practical. Hence, in 1982, Engle [15] presented the autoregressive conditional heteroskedasticity or ARCH model for modelling financial time series that present time-varying volatility clustering. In the ARCH model, a moving average of past error terms is utilized to forecast the variance:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \xi_{t-1}^2 + ... + \alpha_q \xi_{t-q}^2$$

and $\xi_t \mid \phi_{t-1} \in N(0, \sigma_t^2)$ where $\alpha_0 > 0$ and $\alpha_i > 0$, $i = 1, ..., q$. α_i must be estimated from the given data, ϕ_t is the information set of all information until time *t*. The stationarity condition of the ARCH model is that $\sum_{i=1}^{q} \alpha_i < 1$. The GARCH(*p*,*q*) model is well known as Generalized Autoregressive Conditionally Heteroskedastic with conditional variance with *p* GARCH coefficient terms β_j and *q* ARCH coefficient terms α_i. It practically determines a time series of return $y_t = \mu + \xi_t$ where μ is the expected return and ξ_t is a zero-mean white noise and

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \xi_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where $\xi_t = \sigma_t z_t$. The stationarity condition of the GARCH model is $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$.

In 1991, Nelson [16] presented the exponential GARCH or EGARCH model in which the logarithm of conditional variance is as follows.

$$\log \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left[\frac{|\xi_{t-i}|}{\sigma_{t-i}} - E\left(\frac{|\xi_{t-i}|}{\sigma_{t-i}} \right) \right] + \sum_{j=1}^{p} \beta_j \log \sigma_{t-j}^2 + \sum_{i=1}^{q} \gamma \left[\frac{\xi_{t-i}}{\sigma_{t-i}} \right]$$

where $\xi_t \in N(0, \sigma_t^2)$.

In the GJR model, $I[\xi_{t-i} < 0] = 1$ or $I[\xi_{t-i} \geq 0] = 0$ where $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and $\alpha_i + \xi_i \geq 0$.

The stationarity condition of the GJR model is:

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j + \frac{1}{2} \sum_{i=1}^{q} \gamma_i < 1.$$

In this paper, we consider the case of the GARCH, EGARCH and GJRGARCH option pricing models with the assumptions of the conditional distribution of the innovations ξ_t given that ϕ_{t-1} is normal.

2.1 GARCH Option Pricing Model

For GARCH option pricing model, it is provided that ε_t, given ϕ_{t-1}, is a noise with conditional mean zero and variance σ_t^2 under the statistical probability measure *P*. From the work of Duan in 1995 [2], we suppose that the process $\{\xi_t\}_{t \in T}$ can be modelled as a GARCH (*p*,*q*) with zero mean and variance σ_t^2 under the probability *P*.

Thus, we have that Y_t under the probability measure *P* is distributed under the normal distribution with the mean of $r + \lambda\sqrt{\sigma_t^2} - \sigma_t^2 / 2$ and the variance of σ_t^2. The conditional risk-neutralized Esscher parameter θ_t^q is given by

$$r = \ln\left\{ M_Y(t, \theta_t^q + 1) \right\}.$$

Using

$$\theta_t^q = \frac{r - \left(\mu + \left(\sigma_t^2 / 2 \right) \right)}{\sigma_t^2}$$

we have

$$M_Y^\theta\left(t, z + \theta_t^q\right) = \frac{M_Y\left(t, z + \theta_t^q\right)}{M_Y\left(t, \theta_t^q\right)} = e^{z\left(r - \sigma_t^2/2 + z^2\sigma_t^2/2\right)}$$

which is a normally distributed moment generating function with mean $r-\sigma_t^2/2$ and variance σ_t^2, with

$$Y_t^\theta \sim N\left(r-\sigma_t^2/2, \sigma_t^2/2\right).$$

Hence, the conditional distribution of ξ_t given ϕ_{t-1} under P where

$$Y_t=\mu+\varepsilon_t ; \xi_t=\varepsilon_t\sqrt{\sigma_t^2}$$

is

$$E_P\left[\xi_t \mid \phi_{t-1}\right]=E_P\left[Y_t \mid \phi_t\right]-\mu=r-\frac{\sigma_t^2}{2}-\mu$$

$$Var_P\left[\xi_t \mid \phi_{t-1}\right]=Var_P\left[Y_t\right]=\sigma_t^2 .$$

Under Q, nevertheless, the conditional distribution of ξ_t given ϕ_{t-1} is $N(0, \sigma_t^2)$,

$$\varepsilon_t=\xi_t-r+\mu+\sigma_t^2/2$$

and

$$Y_t=r-\sigma_t^2/2+\varepsilon_t .$$

It is noted that by applying the conditional mean, $\mu=r+\lambda\sqrt{\sigma_t^2}-\sigma_t^2/2$, the same way as in Duan's work [2], we obtain

$$\varepsilon_t=\xi_t-r+r+\lambda\sqrt{\sigma_t^2}-\sigma_t^2/2+\sigma_t^2/2 .$$

Thus,

$$\xi_t=\varepsilon_t-\lambda\sqrt{\sigma_t^2}$$

and therefore the dynamics of the variance is given by

$$\sigma_t^2=\alpha_0+\sum_{i=1}^P \alpha_i\left(\varepsilon_t-\lambda\sqrt{\sigma_{t-1}^2}\right)^2+\sum_{j=1}^q \beta_j \sigma_{t-j}^2 . \quad (1)$$

This result was obtained from Siu, Tong and Yang [5] and was in agreement with that obtained by Duan [2].

2.2 EGARCH Option Pricing Model

The EGARCH(1,1) model under the statistical probability measure P is given by:

$$\log\sigma_t^2=\alpha_0+\alpha\left[\frac{\left|\xi_{t-1}\right|}{\sigma_{t-1}}-\sqrt{\frac{2}{\pi}}\right]+\beta\log\sigma_{t-1}^2+\gamma\left[\frac{\xi_{t-1}}{\sigma_{t-1}}\right].$$

In a similar manner as in the previous case, we apply the method used by Sui, Tong and Yang in [5] to obtain the EGARCH under the Q measure:

$$\log\sigma_t^2=\alpha_0+\alpha\left[\frac{\left|\varepsilon_{t-1}-\lambda\sqrt{\sigma_{t-1}^2}\right|}{\sigma_{t-1}}-\sqrt{\frac{2}{\pi}}\right]$$

$$+\beta\log\sigma_{t-1}^2+\gamma\left[\frac{\varepsilon_{t-1}-\lambda\sqrt{\sigma_{t-1}^2}}{\sigma_{t-1}}\right].$$

2.3 GJR-GARCH Option Pricing Model

The GJR(1,1) model under the statistical probability measure P is given by :

$$\sigma_t^2=\alpha_0+\alpha\xi_{t-1}^2+\beta\sigma_{t-1}^2+\lambda I\left[\xi_{t-1}<0\right]\xi_{t-1}^2 .$$

In a similar manner as in the previous cases, we apply the method used by Sui, Tong and Yang in [5]. Now, we have the GJR-GARCH under Q measure as follows:

$$\sigma_t^2=\alpha_0+\alpha\left(\varepsilon_t-\lambda\sqrt{\sigma_t^2}\right)^2+\beta\sigma_{t-1}^2$$

$$+\gamma I\left[\varepsilon_t<\lambda\sqrt{\sigma_t^2}\right]\left(\varepsilon_t-\lambda\sqrt{\sigma_t^2}\right)^2 .$$

3 Methodology

In this work, we make some assumptions to construct a financial model for the discrete environment. We let T be the set $\{0,1,2,\ldots,T\}$ representing economic time under the probability space (Ω, F, P) where P is the probability measure with sample space Ω and event space F.

Let $\xi=\{\xi_t\}_{t\in T}$ be a stochastic process of underlying asset return and σ_t^2 denotes the stock variance. Next, we let λ be the risk premium and r be the risk free rate. Under P, we presume that the process $\{\beta_t\}_{t\in T}$, the bond price process, follows the relation $\beta_t=\beta_{t-1}e^r$, $\beta_0=1$ and the process of stock price $\{S_t\}_{t\in T}$ follows the dynamics,

$$Y_t=r+\lambda\sqrt{\sigma_t^2}-\frac{1}{2}\sigma_t^2+\varepsilon_t, \quad S_0=S,$$

$$\varepsilon_t \sim N(0,\sigma_t^2), \quad t\in T-\{0\}$$

where $Y_t=\ln\frac{S_t}{S_{t-1}}$ denotes the log-return of the stock S [2]. Subsequently, the method of conditional Esscher transform is applied for Y_t, the GARCH process with conditional Esscher parameter sequence of $\{\theta_t\}_{t\in T}$.

Under the probability measure P, we let $M_{Y_t|\phi_{t-1}}(z)$ be the moment generating function of Y_t given the information ϕ_{t-1} at time $t-1$ where $z\in\mathbb{R}$ and assume a stochastic process $\{\theta_t\}_{t\in T-\{0\}}$ given the information ϕ_{t-1}. We have

$$M_{Y_t}(t,z):=E_P(e^{zY_t}\mid\phi_{t-1})$$

$$=\int_{-\infty}^{\infty} e^{zY_t}dF\left(y\mid r+\lambda\sqrt{\sigma_t^2}-\frac{1}{2}\sigma_t^2+\varepsilon_t,\sigma_t^2\right) \quad (2)$$

where a conditional distribution function of Y_t is defined as

$$F\left(y \mid r+\lambda\sqrt{\sigma_t^2}-\frac{1}{2}\sigma_t^2+\varepsilon_t, \sigma_t^2\right)=P\left(Y_t \le \phi_{t-1}\right).$$

We assume that, for each $t \in T-\{0\}$ and $z \in \mathbb{R}$, $M_{Y_t}(t,z)$ exists if $E_P\left(e^{zY_t} \mid \phi_{t-1}\right)<\infty$ and $M_{Y_t}(t,\theta)$ exists and define $\Lambda_0=1$, where E is the expectation under P and

$$\Lambda_t=\prod_{k=1}^{t}\frac{e^{\theta_k Y_k}}{M_Y(k,\theta_k)}, \quad t \in T-\{0\}. \tag{3}$$

Then, we let $\{P_{t,\Lambda_t}\}_{t\in T-\{0\}}$ be a family of probabilities and define

$$P_{t,\Lambda_t}\left(\{Y_t \in X\} \mid \phi_{t-1}\right)$$
$$=E_{p_t}\left(I\left(Y_t \in X\right)\frac{e^{\theta_t Y_t}}{E_{P_t}\left(e^{\theta_t Y_t} \mid \phi_{t-1}\right)} \mid \phi_{t-1}\right) \tag{4}$$

where $I\{Y_t \in X\}$ is the indicator function of the event and X is an open set of the real line. Letting $M_Y^{\theta}\left(z,\theta_t\right)$ be the moment generating function of the return Y_t given the information ϕ_{t-1} with the conditional Esscher parameter ϕ_t, we consequently have

$$M_Y^{\theta}\left(z,\theta_t\right)=\frac{M_Y\left(t,z+\theta_t\right)}{M_Y\left(t,\theta_t\right)} \tag{5}$$

With a sequence of $\{\theta_t^q\}_{t\in T-\{0\}}$, we assume that log-return follows the following equation with the martingale condition.

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$$r=\ln\left\{M_Y\left(t,\theta_t^q+1\right)\right\}-\ln\left\{M_Y\left(t,\theta_t^q\right)\right\}, \quad t \in T-\{0\}$$

In 1994, Shiu et al [3] proposed the derivative pricing model V at time $t \in T$ as

$$V_t=e^{-r(T-t)}E_Q\left(V_T \mid \phi_t\right) \tag{6}$$

where Q is a conditional risk neutral Esscher pricing measure and P_{T,Λ_T^q} is a probability measure on $F=\phi_T$. The process $\{e^{-rt}S_t\}_{t\in T}$ is a martingale process under Q given the information ϕ.

Consequently, in this paper, the methodology is performed in the following steps.

1. We randomly choose the option contract on SET50 market of Thailand from years 2015 and 2016, since the data of these two years are publicly accessible with complete data.
2. We perform the residual test of ARCH effect from the chosen option data fitness by using Ljung-Box's $Q^2(n)$ test. The results of $Q(20)$ of 25.382 and $Q^2(20)$ of 52.320 show that SET50 option price samples have the ARCH effect.
3. In this paper, we use the lag order (1,1) for all testing models, since it was shown that models with a small lag are sufficient to cope with the changing variance [17].
4. We utilize the maximum likelihood estimation for parameter estimates of GARCH, EGARCH and GJR-GARCH.
5. From equation (6), the expected value of V_T is obtained by Monte Carlo simulation for a sample size of 10,000 with the variance reduction technique which is a control variate method.
6. The option pricing value from classical Black-Sholes model is obtained from a financial toolbox of Matlab.
7. The method of room mean square error is applied to evaluate the precision performance of the GARCH models.
8. We discuss the advantage and the limitation of the results for the investor in stock market of Thailand.

4 Numerical Results

In this section, we show some simulated option prices by using GARCH option pricing models (GARCH, EGARCH and GJR) and the classical Black-Scholes model (BS) under probability measure Q using the estimated values of SET50. The data have been obtained from the database www.set.or.th and the derivative security in which we are interested is the European Call option on the SET50 index.

We randomly pick the option contract on SET50 market from years 2015 and 2016. The contracts in year 2015 under study are S50Z15C950, S50Z15C925, S50Z15C900, S50Z15C875, S50Z15C850, S50Z15P950, S50Z15P925, S50Z15P900, S50Z15P875 and S50Z15P850. And in year 2016, the contracts under study are S50Z16C1000, S50Z16C975, S50Z16C950, S50Z16C925, S50Z16C900, S50Z16P1000, S50Z16P975, S50Z16P950, S50Z16P925 and S50Z16P900. We use daily closing prices in each index to estimate the parameters of the GARCH, EGARCH and GJR-GARCH models to be able to find the option price.

Table 1 The Estimated Parameters of the GARCH models for contracts year 2015

Contract	Models	Parameters			
		α_0	α	β	γ
S50Z15C950	GARCH	1.79e-5	0.92810	0.99183	-
	EGARCH	-6.29874	0.12032	-0.79121	-0.12890
	GJR	2.37e-5	0.89321	0.21083	0.13801
S50Z15C925	GARCH	2.28e-5	0.23018	0.64020	-
	EGARCH	-3.39023	-0.11930	0.20188	-0.13284
	GJR	2.11e-5	0.20912	0.80239	0.11121
S50Z15C900	GARCH	2.01e-5	0.39012	0.53082	-
	EGARCH	-1.39801	-0.00231	0.98210	-0.19021
	GJR	2.55e-5	0.32900	0.99821	0.11934
S50Z15C875	GARCH	2.83e-5	0.12903	0.10299	-
	EGARCH	-0.49781	-0.02190	-0.28301	-0.12190
	GJR	2.29e-5	0.23188	0.90128	0.13891
S50Z15C850	GARCH	1.88e-5	0.11408	0.22180	-
	EGARCH	-10.81006	0.93801	0.90721	-0.17781
	GJR	2.17e-5	0.15324	0.28102	0.11198
S50Z15P950	GARCH	3.74e-5	0.32301	0.92108	-
	EGARCH	-2.89102	-0.00513	-0.27912	-0.17782
	GJR	2.02e-5	0.34782	0.82109	0.11293
S50Z15P925	GARCH	3.25e-5	0.01812	0.80122	-
	EGARCH	-5.90123	-0.09913	0.21092	-0.18329
	GJR	2.65e-5	0.05635	0.99218	0.13245
S50Z15P900	GARCH	3.71e-5	0.52313	0.71221	-
	EGARCH	-0.44814	0.02921	0.82123	-0.19810
	GJR	2.62e-5	0.43872	0.39033	0.17812
S50Z15P875	GARCH	2.90e-5	0.62901	0.59023	-
	EGARCH	-8.11356	-0.13792	0.60298	-0.18921
	GJR	2.07e-5	0.58729	0.21203	0.12782
S50Z15P850	GARCH	1.66e-5	0.80001	0.12324	-
	EGARCH	-11.18182	-0.98921	-0.83232	-0.16692
	GJR	2.35e-5	0.78293	0.79231	0.19320

Table 1 and 2 shows estimation results for randomized contracts from year 2015 and 2016 respectively, based on the same index. Several researchers assume the risk-free rate r to be 0 to make the interpretation of options easier such as Duan in 1995 [3] and Schmitt in 1996 [4]. Consequently, in this work, we also assume that the risk premium is zero.

Next, we use the estimated parameters from Table 1 and 2 in GARCH models under probability measure Q so that we find the option price by taking the conditional expectation of the terminal payoff under the pricing probability measure Q and discount at the risk-free interest rate as follows:

$$C_t = e^{-r(T-t)} E\left[\max\left(S_T - X, 0\right)\right],$$

and

$$P_t = e^{-r(T-t)} E\left[\max\left(0, X - S_T\right)\right]$$

where C_t and P_t are call option prices and put option prices respectively. Following the work of Duan's in 1995 [2], the terminal stock price at time T can be calculated as:

$$S_T = S_t \exp\left[(T-t)r - \frac{1}{2}\sum_{s=t+1}^{T} \sigma_s^2 + \sum_{s=t+1}^{T} \xi_s\right].$$

Tables 3 and 4 present the average simulated option prices C_t and P_t under GARCH, EGARCH and GJRGARCH models applied to the SET50 index option. This table uses the estimated parameters from Tables 1 and 2 to fit in the GARCH, EGARCH and GJRGARCH option pricing models. Then, we establish a set of parameters, the strike price (K) and stock price (S_0) at the start date of the year-2015 contracts and year-2016 contracts which are 19 August 2015 and 2 November 2016 for calculation,

Table 2 The Estimated Parameters of the GARCH models for contracts year 2016

Contract	Models	Parameters			
		α_0	α	β	γ
S50Z16C1000	GARCH	1.63e-5	0.42532	0.41512	-
	EGARCH	-6.34532	0.25104	-0.89412	-0.15343
	GJR	3.52e-5	0.23482	0.24255	0.13241
S50Z15C975	GARCH	1.58e-5	0.57923	0.84152	-
	EGARCH	-4.32144	0.94210	0.25143	-0.19423
	GJR	2.72e-5	0.45921	0.74231	0.18894
S50Z15C950	GARCH	2.32e-5	0.41325	0.42398	-
	EGARCH	-1.43253	-0.04221	0.74235	-0.11048
	GJR	4.23e-5	0.39802	0.74632	0.18432
S50Z15C925	GARCH	2.48e-5	0.24234	0.67333	-
	EGARCH	-0.23642	0.43152	-0.70244	-0.18923
	GJR	3.54e-5	0.28391	0.78234	0.12294
S50Z15C900	GARCH	2.09e-5	0.21425	0.23249	-
	EGARCH	-3.42220	0.43232	0.74125	-0.14214
	GJR	4.88e-5	0.23008	0.94325	0.10943
S50Z15P1000	GARCH	2.23e-5	0.24202	0.24244	-
	EGARCH	-5.23521	-0.09323	-0.94125	-0.10044
	GJR	2.56e-5	0.22329	0.73221	0.17935
S50Z15P975	GARCH	2.49e-5	0.04323	0.73242	-
	EGARCH	-1.94233	-0.03253	0.12532	-0.17354
	GJR	3.92e-5	0.03928	0.81435	0.19843
S50Z15P950	GARCH	2.47e-5	0.28423	0.54523	-
	EGARCH	-0.32424	-0.04238	0.82415	-0.19432
	GJR	1.34e-5	0.23654	0.43202	0.15324
S50Z15P925	GARCH	2.76e-5	0.69023	0.42150	-
	EGARCH	-5.10023	-0.13225	0.84215	-0.17842
	GJR	3.34e-5	0.69832	0.23114	0.11894
S50Z15P900	GARCH	1.91e-5	0.34221	0.74231	-
	EGARCH	-12.43253	-0.04247	-0.88349	-0.28340
	GJR	2.92e-5	0.42983	0.32566	0.29306

respectively. The values of parameters set in the models of each contract are clearly shown in Table 5. Thus, we apply the differences between the actual data of option prices and the simulated results in order to compare the performance of the models.

Tables 6 and 7 show the Root Mean Square Error (or RMSE), calculated by the square root of average square difference between the simulated option price and the option price in the market. The formula of RMSE is as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\hat{y}_i - y_i\right)^2}$$

where $\hat{y}_i$ are predicted prices, and y_i market prices. We simulated the prices of option under the GARCH, EGARCH and GJRGARCH models by using MATLAB. The contracts under study from year 2015 are S50Z15C950, S50Z15C925, S50Z15C900, S50Z15C875, S50Z15C850, S50Z15P950, S50Z15P925, S50Z15P900, S50Z15P875 and S50Z15P850.

The results show that, at the maturity time of 30 days, the GARCH model's value is the nearest to the actual option price in the market in all of our sample contracts except that the option prices for the contract S50Z15C850 under the EGARCH model are slightly nearer to the actual option prices as it can be seen that RMSE of contract S50Z15C850 under GARCH model is 0.17189 and the RMSE of EGARCH model is 0.16727.

At maturity time of 60 days, the option prices under the GARCH model are the nearest to the actual option prices in the market for all of the contracts as well, except that the for the contract of S50Z15C950, the EGARCH model shows a slightly better performance as indicated by the results that the RMSE of contract S50Z15C950 with GARCH model is 0.01332, while it is 0.01325 with the EGARCH.

Table 3 The average simulated option prices under GARCH and BS models for contracts year 2015

Contract	GARCH	EGARCH	GJR	BS
Call option		$T = 30$		
S50Z15C950	2.5823	3.2409	11.6788	9.7283
S50Z15C925	6.3679	7.9823	12.8396	11.2452
S50Z15C900	11.5321	14.0032	16.8921	17.3340
S50Z15C875	24.0002	25.3245	29.9892	32.2391
S50Z15C850	34.8772	34.7458	35.1242	40.9811
Put option		$T = 30$		
S50Z15P950	63.5023	63.4903	68.5322	65.0927
S50Z15P925	46.1098	46.5436	58.3423	56.3902
S50Z15P900	25.7320	27.3432	28.0321	35.0924
S50Z15P875	14.2232	16.9823	19.9302	25.2501
S50Z15P850	8.20102	11.2422	17.4325	20.2239
Call option		$T = 60$		
S50Z15C950	3.4245	3.3129	17.3274	15.3022
S50Z15C925	7.5241	7.6432	35.4325	26.3247
S50Z15C900	12.353	12.453	50.4522	49.2932
S50Z15C875	26.432	29.432	70.0432	72.4320
S50Z15C850	36.534	40.024	80.3245	93.7882
Put option		$T = 60$		
S50Z15P950	66.2932	66.4565	140.0623	80.3202
S50Z15P925	48.4324	49.5343	75.9023	69.3936
S50Z15P900	27.0801	28.4326	47.0343	49.9457
S50Z15P875	17.0023	20.0342	36.0325	38.9021
S50Z15P850	11.8425	15.0425	24.3883	29.0287

Table 4 The average simulated option prices under GARCH and BS models for contracts year 2016

Contract	GARCH	EGARCH	GJR	BS
Call option		$T = 30$		
S50Z16C1000	7.24254	7.13254	9.42423	10.1324
S50Z16C975	15.42344	15.04353	18.23494	19.9932
S50Z16C950	26.42435	26.14252	31.32544	35.3211
S50Z16C925	41.12325	41.04659	42.33256	53.2038
S50Z16C900	55.62234	58.12524	61.04235	73.9217
Put option		$T = 30$		
S50Z16P1000	67.83224	67.18942	75.45336	78.3231
S50Z16P975	47.32532	47.10425	49.23343	52.3292
S50Z16P950	32.32556	31.54342	31.24797	43.4893
S50Z16P925	22.22145	24.52345	30.09842	35.4581
S50Z16P900	14.23223	16.52533	29.04253	29.9210
Call option		$T = 60$		
S50Z16C1000	9.45233	10.42352	11.42026	12.9332
S50Z16C975	18.23123	18.34235	23.89421	19.3292
S50Z16C950	28.24252	30.24235	37.42363	35.9844
S50Z16C925	43.43235	45.25324	46.42546	56.2001
S50Z16C900	58.42224	61.02644	59.42084	65.7862
Put option		$T = 60$		
S50Z16P1000	69.42352	69.94326	75.42634	78.4522
S50Z16P975	50.23425	51.14253	53.24336	65.0901
S50Z16P950	34.53426	36.00234	43.23509	54.2995
S50Z16P925	23.32098	24.14253	30.42540	41.7890
S50Z16P900	15.94204	17.45530	25.42543	30.0087

Table 5 Parameters set in the models for contracts

Contract	K	S_0	Variance	Actual option price
Call option		$T = 30$		
S50Z15C950	950	859.51	0.234523	2.90
S50Z15C925	925	859.51	0.134425	6.00
S50Z15C900	900	859.51	0.135234	11.80
S50Z15C875	875	859.51	0.342543	24.00
S50Z15C850	850	859.51	0.109452	34.50
Put option		$T = 30$		
S50Z15P950	950	859.51	0.123253	63.80
S50Z15P925	925	859.51	0.143567	46.00
S50Z15P900	900	859.51	0.193235	25.90
S50Z15P875	875	859.51	0.289223	14.30
S50Z15P850	850	859.51	0.242674	8.80
Call option		$T = 30$		
S50Z16C1000	1000	942.56	0.192523	7.90
S50Z16C975	975	942.56	0.145437	15.70
S50Z16C950	950	942.56	0.183257	26.60
S50Z16C925	925	942.56	0.235367	40.40
S50Z16C900	900	942.56	0.323426	56.00
Put option		$T = 30$		
S50Z16P1000	1000	942.56	0.132567	67.90
S50Z16P975	975	942.56	0.282363	48.00
S50Z16P950	950	942.56	0.427336	32.30
S50Z16P925	925	942.56	0.291325	21.30
S50Z16P900	900	942.56	0.225623	13.90
Call option		$T = 60$		
S50Z15C950	950	859.51	0.242536	3.60
S50Z15C925	925	859.51	0.292677	7.00
S50Z15C900	900	859.51	0.192364	12.60
S50Z15C875	875	859.51	0.242673	25.20
S50Z15C850	850	859.51	0.109452	36.00
Put option		$T = 60$		
S50Z15P950	950	859.51	0.224256	64.60
S50Z15P925	925	859.51	0.142267	48.00
S50Z15P900	900	859.51	0.122566	27.90
S50Z15P875	875	859.51	0.133554	15.30
S50Z15P850	850	859.51	0.272342	10.80
Call option		$T = 60$		
S50Z16C1000	1000	942.56	0.152623	9.90
S50Z16C975	975	942.56	0.183265	16.50
S50Z16C950	950	942.56	0.223563	28.20
S50Z16C925	925	942.56	0.243003	42.50
S50Z16C900	900	942.56	0.132578	58.00
Put option		$T = 60$		
S50Z16P1000	1000	942.56	0.225367	69.50
S50Z16P975	975	942.56	0.242774	49.00
S50Z16P950	950	942.56	0.243268	34.00
S50Z16P925	925	942.56	0.112532	23.20
S50Z16P900	900	942.56	0.173564	15.10

Table 6 Model comparison for 2015 contracts.

Contract	GARCH	EGARCH	GJR	BS
Call option	$T = 30$			
S50Z15C950	0.1982	1.4823	3.5802	3.4293
S50Z15C925	0.2452	1.4562	2.2063	2.1293
S50Z15C900	0.2639	1.7892	2.2340	2.5324
S50Z15C875	0.1455	0.0945	1.0178	2.2088
S50Z15C850	0.1718	0.1672	0.8753	1.2398
Put option	$T = 30$			
S50Z15P950	0.2103	0.2242	2.0329	1.9923
S50Z15P925	0.3801	0.3901	3.2238	3.1208
S50Z15P900	0.8321	1.9323	1.3223	2.9023
S50Z15P875	0.3012	1.3223	2.4387	3.8922
S50Z15P850	0.6231	1.1021	4.2981	5.3892
Call option	$T = 60$			
S50Z15C950	0.0133	0.0132	4.0820	4.0037
S50Z15C925	0.0013	0.3254	5.3244	5.1220
S50Z15C900	0.0023	0.6543	7.1021	8.3212
S50Z15C875	0.0021	1.1435	6.1213	7.0092
S50Z15C850	0.0032	1.1354	3.2103	3.9326
Put option	$T = 60$			
S50Z15P950	0.1423	0.5424	8.0329	7.8023
S50Z15P925	0.1124	0.9043	5.2923	4.2922
S50Z15P900	0.0942	0.4214	4.3232	5.9201
S50Z15P875	0.0032	1.9323	4.9324	6.2873
S50Z15P850	0.0012	1.5932	3.3200	4.6700

From year 2016, the contracts under study are S50Z16C1000, S50Z16C975, S50Z16C950, S50Z16C925, S50Z16C900, S50Z16P1000, S50Z16P975, S50Z16P950, S50Z16P925 and S50Z16P900. Table 7 shows that the option prices from all of our randomized contracts with both times to maturity of 30 days and 60 days under GARCH model are the nearest to the option prices in the market, except only for the contract S50Z16C925 under the EGARCH model with time to maturity of 30 days that has slightly less RMSE, 0.6389, compared to the RMSE of the contract under the GARCH model, 0.6434.

From the results it is obvious that BS model performs the highest RMSE in most of the contracts compared to GARCH, EGARCH and GJRGARCH. It is noted that, among GARCH models, all contracts under the GJRGARCH model have higher RMSE than the other contracts under the other models but lower RMSE than BS model. Also, clearly, most contracts under the GARCH model have the lowest RMSE. Although, there are few contracts that show a slightly better performance with the EGARCH model, the differences in RMSE are insignificant when compared to that of the GARCH model. As a result, it could be concluded that the GARCH model may be a more suitable model for option pricing for SET50 in Thailand, based on our sample of data.

Table 7 Model comparison for 2016 contracts.

Contract	GARCH	EGARCH	GJR	BS
Call option	$T = 30$			
S50Z16C1000	0.1424	0.1644	1.0432	1.3020
S50Z16C975	0.1242	0.1453	1.3252	1.5234
S50Z16C950	0.4522	0.5543	2.4325	3.2992
S50Z16C925	0.6434	0.6389	1.0093	1.5236
S50Z16C900	0.8422	1.3042	2.9432	3.42321
Put option	$T = 30$			
S50Z16P1000	0.4233	1.2325	3.4252	4.5823
S50Z16P975	0.2394	0.5352	0.4256	1.2832
S50Z16P950	0.3827	0.5623	3.4256	4.2324
S50Z16P925	0.3235	1.0992	3.9342	5.2312
S50Z16P900	0.1125	0.3453	4.4256	5.9921
Call option	$T = 60$			
S50Z16C1000	0.0024	0.1235	1.2425	1.5422
S50Z16C975	0.0425	0.2352	2.2452	3.9812
S50Z16C950	0.1842	1.2423	3.2425	2.9902
S50Z16C925	0.0242	1.3523	1.2014	1.1021
S50Z16C900	0.0083	1.5042	0.3256	1.6202
Put option	$T = 60$			
S50Z16P1000	0.0923	0.3425	2.5326	3.2201
S50Z16P975	0.0031	0.0212	1.4256	2.3324
S50Z16P950	0.0892	0.0993	3.2362	4.9821
S50Z16P925	0.0523	0.5242	2.2567	3.0583
S50Z16P900	0.1100	1.2154	3.42264	4.9122

5 Conclusion

In this paper, we studied the GARCH, EGARCH and GJRGARCH models in finding the option price for the SET50 index of Thailand. We followed the method of Sui, Tong and Yang's [5] which uses the conditional Esscher transform to find the risk neutral version of the GARCH, EGARCH and GJRGARCH model which is required for finding option prices.

We carried out the exercise of simulating option prices using the GARCH, EGARCH and GJRGARCH models in the risk neutral measure Q. Our computations of the risk neutral version of the models are in agreement with those of Duan's [2] and Schmitt's [4].

The contracts that we studied were randomized from two years, 2015 and 2016. The option contracts from year 2015 are S50Z15C950, S50Z15C925, S50Z15C900, S50Z15C875, S50Z15C850, S50Z15P950, S50Z15P925, S50Z15P900, S50Z15P875 and S50Z15P850. The option contracts from year 2016 are S50Z16C1000, S50Z16C975, S50Z16C950, S50Z16C925, S50Z16C900, S50Z16P1000, S50Z16P975, S50Z16P950, S50Z16P925 and S50Z16P900. All of our sample option contracts are traded in the Thailand Futures Exchange (TFEX). In most of the contracts, we observed that the option prices under the GARCH model is the closest to the actual option prices in the market. Especially, when the GARCH model is compared to the well-known Black-Sholes (BS) model, GARCH model can significantly outperform BS model. Only three out of twenty contracts in this study, which are S50Z15C850 with time to maturity of 30 days, S50Z15C950 S50Z15C850 with time to maturity of 60 days and S50Z16C925 S50Z15C850 with time to maturity of 30 days, show that the option prices under the EGARCH model is the closest to the actual prices in the market. However, the results of these three contracts under the EGARCH model only slightly outperform the results of the GARCH model. It can be concluded that the GARCH model might be a good candidate for these three contracts as well. This implies that the GARCH option pricing model may be the most suitable tool, compared to the EGARCH and GJRGARCH models, including the well-known benchmark of BS model, for the investors to valuate the options in Thailand.

In conclusion, the advantages of this study is to indicate that option prices in Thailand under SET50 have the ARCH effect based on Ljung-Box's $Q^2(n)$ test. As a result, it is suggested to use GARCH models for option pricing. Our study indicates that GARCH model obviously outperform the other pricing models, including the well-known Black-Schole model because of the least RMSE in our option price samples. This illustrates the main benefit for the investor to analyze the option pricing in Thailand. The GARCH option pricing model is a good candidate as an attractive tool for model pricing in SET50 options of Thailand. The investor can confidently use GARCH model as the most important tool to valuate option prices of SET50 and to determine the status of being overpriced and underpriced of the underlying assets. Moreover, it is suggested that GARCH model can be applied to predict the option price in the future as well based on our study of data samples of SET50 in Thailand.

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Acknowledgment
The authors would like to greatly thank the Centre of Excellence in Mathematics, CHE, Thailand, for all supports under Contract RG-01-63-01-1.

Contribution of individual authors to the creation of a scientific article (ghostwriting policy)
Both authors contributed equally to this work.

Sources of funding for research presented in a scientific article or scientific article itself
This work was financially supported by the Centre of Excellence in Mathematics, CHE, Thailand, under Contract RG-01-63-01-1.