Performance of a Robust Confidence Interval for the Process Capability Index C_p based on the Modified Trimmed Standard Deviation Under Non-Normal Distributions

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Abstract: In this paper, a robust interval estimator for the classical process capability index (C_p) based on the modified trimmed standard deviation ($\mathrm{MTSD} = S_T^*$) is considered under both normal and non-normal distributions. The performance of the newly proposed process capability index interval estimator over the existing method is compared using a simulation study. As a performance criterion, we consider both simulated coverage probability and average width. Simulation results evident that the proposed confidence interval based on the robust estimator performed well for most of cases. For illustration purposes, two real-life data from industry are analyzed which supported our simulation results to some extent. As a result, the proposed method can be recommend to be used by the practitioners in various fields of industry, engineering and physical sciences.

Key-Words: Confidence interval, process capability analysis, process capability index, robust statistics, modified trimmed standard deviation, coverage probability, average width, comparison, simulation study.

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1 Introduction

Process capability analysis (PCA) is a set of tools used to evaluate the capability of a manufacturing process. The outcome of this set of tools is known as the process capability index (PCI). The PCI was first developed by [1]; later, different PCIs found their origin when the underlying process distribution is normal. A process capability index (PCI) gives the indication on the quality process whether it is moving in line with the predefined standards or not. These standards can be determined by setting the lower specification limit (LSL) and upper specification limit (USL). One of the most commonly applied process capability indices used extensively in many fields is the process capability index (C_p). The C_p is a unit-free quantitative measure that compares the behavior of manufactured process characteristics [1, 2, 3].

Let us assume that the quality characteristic of interest has the double specification limits LSL and USL, and follows a normal distribution with mean (μ) and standard deviation (σ). A capable process is one where almost all the measurements fall inside the specification limits. Therefore, any product having the quality characteristic beyond the LSL and USL is considered to be a nonconforming product. The population process capability index (C_p) is defined as the ratio of specification width (USL – LSL) over the process spread (6σ), and given as follows:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} \quad (1)$$

The denominator in (1) gives the size of the range over which the process actually varies and the six-sigma coverage represents the spread of 99.73% of the data in normally distributed processes [4]. The process capability index (C_p) takes into account variability in process as it depends on spread for given specification limits. According to [5] the quality conditions and their corresponding C_p values are given in Table 1 as follows:

Table 1. The quality conditions for C_p values

C_p Value	Quality Condition
less than 0.67	Poor
between 0.67 and 1.00	Inadequate
between 1.00 and 1.33	Capable
between 1.33 and 1.67	Satisfactory
between 1.67 and 2.00	Excellent
greater than or equal to	Supper Excellent

The process capability index (C_p) is greatly depends on the assumption that the underlying quality characteristic measurements are independent and normally distributed. However, there are many situations in which the assumptions are not valid and therefore the exact confidence interval (CI) for the process capability index (C_p) is not accurate. These situations attributed to the use of another conventional approach for constructing CI. As an alternative, robust confidence intervals are put forward when the underlying normality assumption is not met. In this study, a robust confidence interval for the process capability index (C_p) by means of a robust scale estimator, namely the modified trimmed standard deviation ($MTSD = S_T^*$), is proposed. The proposed robust method seems to yield a better performance than the exact confidence interval (CI) method for non-normal distributions.

The rest of the paper is organized as follows: In Section 2, we presented the exact and suggested robust confidence intervals for the process capability index. A Monte Carlo simulation study has been conducted in Section 3. For illustration purposes, two real-life data examples are analyzed in Section 4. Finally, this paper ends up with some concluding remarks in Section 5.

2 Confidence Intervals

Since, in real life, most of data do not follow normality assumption, this paper considers the confidence interval estimation of the process capability index (C_p) when data is non-normal and there are some possible outliers, which may be an alert from an out-of-control manufacturing process.

2.1 The Exact Confidence Interval for C_p

Suppose that we have a quality characteristic under study, say X, and let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a a normal distribution with process mean μ and process standard deviation σ, that is $X_i \sim N(\mu, \sigma^2)$. If the process standard deviation σ is unknown, then it can be estimated from the random sample, and thus the process standard deviation σ can be replaced by the sample standard deviation (S). Therefore, the value of the point estimator of the process capability index (C_p) given in equation (1) can be calculated as follows:

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\ S} \quad (2)$$

where $$S = \sqrt{(n-1)^{-1} \sum_{i=1}^{n} (X_i - \bar{X})^2} \quad (3)$$

Now, the exact $(1-\alpha)100\%$ confidence interval (CI) for the process capability index (C_p) can be obtained as follows:

$$\text{LCL} = \frac{USL - LSL}{6s} \sqrt{\frac{\chi^2_{\left(\frac{\alpha}{2}, n-1\right)}}{(n-1)}} \quad (4)$$

$$\text{UCL} = \frac{USL - LSL}{6s} \sqrt{\frac{\chi^2_{\left(1-\frac{\alpha}{2}, n-1\right)}}{(n-1)}} \quad (5)$$

where $\chi^2_{\left(\frac{\alpha}{2}, n-1\right)}$ and $\chi^2_{\left(1-\frac{\alpha}{2}, n-1\right)}$ are the α/2 and the 1−α/2 quantiles of the chi-squared distribution respectively with $n-1$ degrees of freedom.

2.2 Proposed Robust Confidence Interval for C_p

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a normal distribution with process mean μ and process standard deviation σ, that is $X_i \sim N(\mu, \sigma^2)$. If $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ denoted the corresponding order statistics of this random sample. The r-times symmetrically trimmed random sample is obtained by dropping both lowest and highest r values, then the trimmed mean ($\bar{X}_T$) can be calculated as follows:

$$\bar{X}_T = \frac{1}{n-2r}\sum_{i=r+1}^{n-r} X_{(i)} \tag{6}$$

where $r = [\alpha n]$ represents the greatest integer and trimming is done for $\alpha\%$; $0 \leq \alpha \leq 0.5$ of the sample size n. The sample standard deviation of trimmed observation is denoted by (S_T) and can be calculated as follows:

$$S_T = \sqrt{\frac{1}{n-2r-1}\sum_{i=r+1}^{n-r}(X_{(i)} - \bar{X}_T)^2} \tag{7}$$

Following [6], the modified trimmed standard deviation can be calculated as follows:

$$\hat{\sigma}_T = MTSD = s_T^* = 1.4826\, s_T \tag{8}$$

The constant multiplier 1.4826 is used to control on loss due to trimming. Now, the proposed $(1-\alpha)100\%$ confidence interval (CI) for the process capability index can be obtained with the following steps:

Step 1: Select a random sample of size (n), $X_1, X_2, \ldots, X_n$, from the production process and measure the quality variable X.

Step 2: Calculate the trimmed mean ($\bar{X}_T$), trimmed standard deviation (S_T) and the modified trimmed standard deviation (S_T^*) from equations (6), (7) and (8) respectively.

Step 3: The process capability index (C_p) based on the modified trimmed standard deviation (S_T^*) is obtained using the following formula:

$$\hat{C}_p^* = \frac{\text{USL} - \text{LSL}}{6\, S_T^*} \tag{9}$$

Step 4: The lower and upper confidence limit for process capability index, $\hat{C}_p^*$, are obtained as follows:

$$LCL = \hat{C}_p^* \sqrt{\frac{\chi^2_{\left(\frac{\alpha}{2},\ n-1\right)}}{n-1}} \tag{10}$$

$$UCL = \hat{C}_p^* \sqrt{\frac{\chi^2_{\left(1-\frac{\alpha}{2},\ n-1\right)}}{n-1}} \tag{11}$$

where $\chi^2_{\left(\frac{\alpha}{2},n-1\right)}$ and $\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$ are the α/2 and the 1−α/2 quantiles of the central chi-squared distribution respectively with $n-1$ degrees of freedom.

3 Simulation Study

Using statistical package R 3.5.2, a simulation study has been conducted to compare and evaluate the performance of the exact and proposed robust confidence intervals in this section.

3.1 Performance Evaluation

The coverage probability (CP) and the average width (AW) were considered as performance criterion. The estimated coverage probability ($\widehat{\text{CP}}$) and the estimated average width ($\widehat{\text{AW}}$) based on M replicates are given by:

$$\widehat{\text{CP}} = \frac{\#(\text{LCL} \leq \text{C}_\text{p} \leq \text{UCL})}{\text{M}} \tag{12}$$

$$\widehat{AW} = \frac{\sum_{i=1}^{M}(\text{UCL}_i - \text{LCL}_i)}{\text{M}} \tag{13}$$

where $\#(\text{LCL} \leq \text{C}_\text{p} \leq \text{UCL})$ denotes the number of simulation runs for which the true value of C_p falls within the limits of the confidence interval. Our objective is to find a good confidence interval for non-normal cases, therefore, we consider the following probability distributions: normal, Student's t, Exponnetial, Chi-Square, Gamma, Beta and Lognormal probability with specified parameters, which cover a range of normal and non-normal distributions. The data were generated from these different distributions and they are given in Table 2. We consider $C_p = 1.00$ as the true value of the process capability index. The true values for the lower specification limit (LSL) and the upper specification limit (USL) are calculated and given in the Table 3.

Table 2. Probability distributions and coefficient of skewness

Probability Distribution	Skewness Coefficient
Normal, N(50, 1)	0.000
Student-t, t(5)	0.000
Exponential, Expo(2)	2.000
Chi-Square, df = 1	2.828
Gamma, G(6, 1)	0.816
Beta, Beta (3, 3)	0.000
Lognormal, LN(0, 1)	6.185

Table 3. True values of LSL and USL used for the simulation study

Probability Distribution	LSL	USL
Normal, N(50, 1)	47	53
Student-t, t(5)	-3.873	3.873
Exponential, Expo(2)	-1	2
Chi-Square, df = 1	-3.243	5.243
Gamma, G(6, 1)	-1.348	13.384
Beta, Beta (3, 3)	-0.067	1.067
Lognormal, LN(0, 1)	-4.835	8.132

The algorithm steps for the simulation study are given as follows:

Step 1: Generate random samples of size (n) equals to 5, 10, 25, 50 and 100 from each one of the above mentioned probability distributions.

Step 2: The number of replications used for each simulation is $M = 50{,}000$.

Step 3: The nominal confidence level was fixed at 0.95, a widely used confidence level.

Step 4: Several trimming levels (5%, 10%, 20% 25% & 30%) are used.

Step 5: Compute $\hat{C}_p$ from equation (2) and $\hat{C}_p^*$ from equation (9) for each random sample. Then compute LCL and UCL for the $M = 50{,}000$ random samples.

Step 6: The estimated coverage probability ($\widehat{CP}$) and average width ($\widehat{AW}$) are calculated for the true value $C_p = 1.00$ by using equations (12) and (13), respectively, and $M = 50{,}000$ for each one of the probability distributions described above and all cases of sample size (n).

Table 4. Estimated coverage probability of 95% CI

PDF	n	Confidence Interval Method					
		Exact CI	Proposed Robust CI				
			5%	10%	20%	25%	30%
N(50,1)	5	0.9976	---	---	0.7870	---	---
	10	0.9449	---	0.8561	0.7605	---	---
	25	0.9490	0.5612	0.8262	0.4975	0.2536	0.1000
	50	1.0000	0.3886	0.8846	0.1966	0.0378	0.0009
	100	0.9495	0.3280	0.8833	0.0233	0.0002	0.0000
t(5)	5	0.9954	---	---	0.7620	---	---
	10	0.7557	---	0.7260	0.7756	---	---
	25	0.5919	0.2587	0.5905	0.6157	0.3646	0.1654
	50	1.0000	0.0982	0.5929	0.3637	0.1016	0.0040
	100	0.1621	0.0381	0.6380	0.1078	0.0024	0.0000
$\chi^2_{(1)}$	5	0.7025	---	---	0.5501	---	---
	10	0.6394	---	0.6115	0.3844	0.1663	---
	25	0.5953	0.6182	0.5686	0.1112	0.0465	0.0158
	50	0.5775	0.6099	0.3233	0.0115	0.0017	0.0000
	100	0.5625	0.5900	0.1370	0.0001	0.0000	0.0000
Expo (2)	5	0.7523	---	---	0.5743	---	---
	10	0.2737	---	0.5981	0.2908	---	---
	25	0.3719	0.6151	0.3646	0.0206	0.0051	0.0008
	50	0.1928	0.4463	0.0552	0.0003	0.0000	---
	100	0.0001	0.1101	0.0013	0.0000	---	---
Gamma (6,1)	5	0.9285	---	---	0.7724	---	---
	10	0.9178	---	0.8468	0.7289	---	---
	25	0.9013	0.6025	0.8316	0.4383	0.2205	0.0852
	50	0.8995	0.4425	0.8592	0.1634	0.0303	0.0009
	100	0.8963	0.3453	0.8477	0.0001	---	---
LN(0,1)	5	0.6036	---	---	0.4548	---	---
	10	0.4648	---	0.4271	0.1856	---	---
	25	0.3698	0.4486	0.2449	0.0097	0.0027	0.0005
	50	0.3178	0.3355	0.0315	0.0000	0.0000	0.0000
	100	0.2768	0.0925	0.0012	0.0000	---	---
Beta (3,3)	5	0.9722	---	---	0.7938	---	---
	10	0.9771	---	0.8312	0.8083	---	---
	25	0.9810	0.4422	0.7417	0.6520	0.3988	0.1891
	50	0.9829	0.2020	0.8051	0.4165	0.1313	0.0068
	100	0.9837	0.1067	0.7754	0.1372	0.0033	0.0000

The results of simulated coverage probability and average width of the confidence intervals are respectively presented in Tables 4 and 5.

We noticed from Tables 4 and 5 that we didn't calculate values for all cells because the symmetric trimming for any sample size meaningful only if the trimming percentage is less than 50%. If we review Tables (4 - 5), the following conclusions can be made:

Table 5. Estimated average width of the 95% CI

PDF	n	Confidence Interval Method					
		Exact CI	Proposed Robust CI				
			5%	10%	20%	25%	30%
N(50,1)	5	1.5224	---	---	2.6647	---	---
	10	0.9645	---	0.9302	1.3668	1.3668	---
	25	0.5736	0.4597	0.5298	0.8142	0.9626	1.1599
	50	0.3989	0.3227	0.3998	0.5725	0.6739	0.8961
	100	0.2795	0.2369	0.2827	0.4039	0.4960	0.6305
t(5)	5	1.8451	---	---	0.7147	---	---
	10	1.0687	---	0.4900	0.4900	---	---
	25	0.6049	0.3963	0.4719	0.7534	0.8947	1.0848
	50	0.4109	0.2802	0.3457	0.5303	0.6281	0.8416
	100	0.2844	0.2077	0.2550	0.3751	0.4641	0.5943
$\chi^2_{(1)}$	5	3.0238	---	---	6.6301	---	---
	10	1.3671	---	1.5225	2.4934	4.7708	---
	25	0.6696	0.5986	0.7444	1.3111	1.5968	1.9786
	50	0.7647	0.4064	0.5618	0.8912	1.0794	1.4857
	100	0.5468	0.3019	0.3920	0.6193	1.0259	4.5610
Expo (2)	5	2.1595	---	---	3.8173	---	---
	10	1.1598	---	1.1690	1.7726	---	---
	25	0.6234	0.5251	0.6271	1.0120	1.2049	1.4671
	50	0.4171	0.3616	0.4694	0.6992	0.8294	---
	100	0.2865	0.2678	0.3315	0.4931	---	---
Gamma (6,1)	5	4.2190	---	---	7.0099	---	---
	10	2.4923	---	2.3578	3.4785	---	---
	25	1.4376	1.1505	1.3342	2.0651	2.4363	---
	50	0.9878	0.8021	1.0016	1.4374	1.6944	2.2609
	100	0.6888	0.5810	0.7041	1.2442	---	---
LN (0,1)	5	4.4217	---	---	7.1773	---	---
	10	2.1457	---	2.0147	3.8383	---	---
	25	0.8591	0.8138	1.0276	1.8087	2.1875	2.6970
	50	0.6448	0.5508	0.7803	1.2408	1.5024	2.0604
	100	0.4196	0.4943	0.6561	1.0437	---	---
Beta (3,3)	5	1.5891	---	---	2.4755	---	---
	10	0.9677	---	0.8691	1.2504	---	---
	25	0.5740	0.4381	0.4965	0.7419	0.8722	1.0498
	50	0.3986	0.3056	0.3690	0.5169	0.6042	0.8000
	100	0.2795	0.2235	0.3661	0.4468	0.4468	0.5657

1. In general, for 10% trimmed data, the coverage probability (CP) is better for both normal and non-normal distributions.

2. For 5% trimmed data, the coverage probability (CP) is better for chi-square, exponential and lognormal distributions.

3. To obtain large coverage probability (CP) for a small sample size, say $n = 5$, the 20% trimming value is needed.

4. For a smaller sample size, the large trimming value is not practically effective.

5. For large sample sizes, such as $n = 50, 100$, for all levels of trimming considered, the improvement in coverage probability (CP) is marginal.

6. The trimming and usage of modified trimmed standard deviation ($MTSD = S_T^*$) has advantage over the sample standard deviation (S) in improving in coverage probability (CP) of C_p is effective for sample sizes $n = 10$ and 25 for small levels of trimming.

7. The average width (AW) of the proposed robust confidence interval for C_p is better than that for the exact confidence interval where we found that small samples with large trimming values or moderate sizes of samples with 5 to 10 percentages are advisable.

8. The simulation study shows that an estimate of C_p based on the modified trimmed standard deviation ($MTSD = S_T^*$) has large coverage probability (CP) for small levels of trimming in moderate sample size. In addition, the 5% to 10% trimming are advisable for getting a better coverage probability (CP) in compare with C_p based on the sample standard deviation (S). The same trend is observed for the average width (AW) for the normal and non-normal distributions considered in this simulation study.

3.2 Contamination Effect on Exact and Proposed Robust Confidence Intervals

This section deals with the performance of the exact and proposed interval estimators for the process capability index (C_p) using several trimming levels (5%, 10%, 20% 25% & 30%) when we have contaminated data. For the said purpose, we have generated 100 random observations form a contaminated environment, where 80 observations come from the standard normal distribution while the remaining 20 observations are taken from the standard exponential distribution. The resulting data set is given in Table 6.

Table 6. A Contaminated data set simulated from a contaminated process

-0.74	1.52	0.18	1.73	-0.71	-1.98	-0.84	-0.65	8.11	5.28
-0.90	1.23	-0.19	-0.11	0.30	1.17	0.10	-0.39	7.20	1.51
0.25	-0.34	-0.05	-0.48	0.66	2.16	0.77	-0.98	9.04	7.73
0.62	0.35	0.68	1.17	-1.10	-1.05	0.75	-0.86	7.41	3.39
1.35	0.10	1.72	0.88	0.64	0.29	0.59	-0.72	9.86	8.37
1.60	0.35	1.16	-1.10	0.52	-0.24	1.29	-1.71	7.15	11.8
0.44	0.18	-0.02	-0.94	0.41	-0.60	1.07	0.47	5.72	8.74
0.21	1.00	0.31	-0.01	-0.57	0.01	2.97	0.79	3.07	11.6
-0.35	-1.65	0.02	0.04	1.33	-0.17	0.01	-0.77	9.64	6.01
1.53	-1.44	-0.35	0.19	-0.57	0.11	-0.71	-1.90	2.38	15.1

We have created different useful plots for this data set and presented them in Figure 1 and obvious outliers are evident. To analyze the data, we used the Minitab® Release 18 (Minitab Inc., 2017).

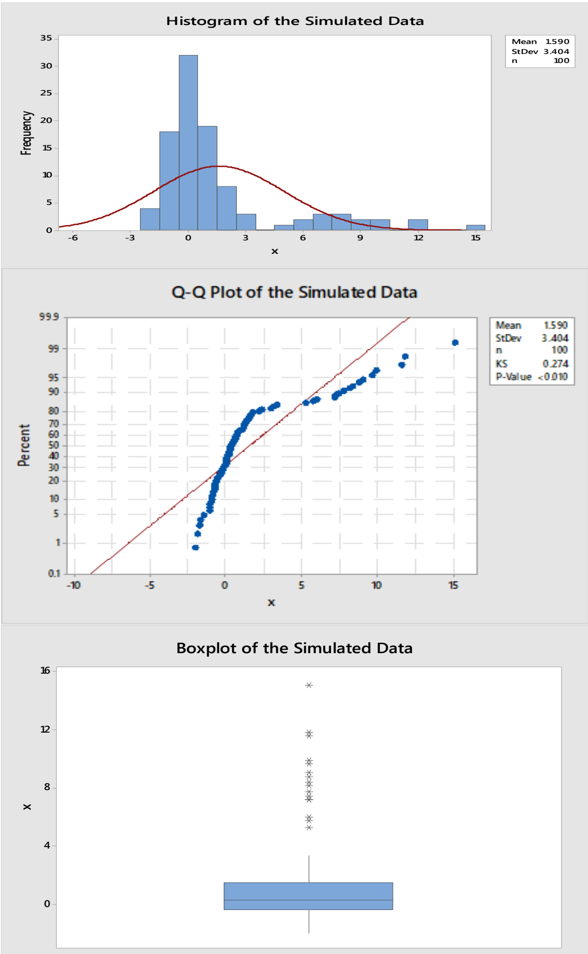

Figure 1. Graphical displays for the contaminated data

The sample standard deviation (S) and the modified trimmed standard deviation ($MTSD = S_T^*$), are calculated and given in Table 7.

Table 7. Summary of scale statistics

Statistics	Value
Sample Standard Deviation (S)	3.404
MTSD (5%)	3.724
MTSD (10%)	2.740
MTSD (20%)	0.983
MTSD (25%)	0.766
MTSD (30%)	0.582

Suppose that for this distribution we have the specified lower limit LSL = -8.622 and the specified upper limit USL = 11.802, then we will calculate the estimated $\hat{C}_p$ from equation (2) and $\hat{C}_p^*$ from equation (9), and constructed the 95% CI and provided them in Table 8.

Table 8. The 95% CIs for the simulated data

Confidence Interval Method	PCI		Confidence Interval		
			LCL	UCL	Widt
CI-Exact	$\hat{C}_p$	1.000	0.861	1.139	0.278
CI-MTSD (5%)	$\hat{C}_p^*$	0.914	0.780	1.048	0.268
CI-MTSD (10%)		1.242	1.049	1.435	0.386
CI-MTSD (20%)		3.463	2.839	4.085	1.246
CI-MTSD (25%)		4.444	3.566	5.320	1.754
CI-MTSD (30%)		5.849	4.555	7.140	2.585

Table 8 gives the expected results as data is non-normal with outliers. The width of the robust confidence interval is smaller than that of the exact confidence interval for 5% trimming and is larger at 10% trimming onwards. According to the quality conditions for C_p value given in Table 1, we observe from Table 8 that when the sample standard deviation (S) is used to estimate σ, then the process is capable, approximately inadequate, of meeting the given specifications, but when the modified trimmed standard deviation ($MTSD = S_T^*$) is used to estimate σ, then the process is inadequate, approximately capable, at 5% level of trimming, capable for 10% level of trimming and at 20% level of trimming onwards, the process is supper excellent capable of meeting the given specifications. Therefore, the proposed robust process capability index ($\hat{C}_p^*$) performed comparatively better than the exact process capability index ($\hat{C}_p$). Therefore, the exact process capability index ($\hat{C}_p$) is not adequate to assess a non-normal data with outliers.

4 Real Data Applications

In this section, the application of the exact and proposed robust confidence intervals for the process capability index (C_p) in reality is studied here by using two real-life industrial data sets for illustration purposes.

4.1 Weight of the Rubber Edge

The data for this example, which is presented in Table 9 was analyzed by [7, 8] among others. The study involved a manufacturer and supplier of audio-speaker components. Data represents the weight (*in grams*) of rubber edge. This weight of the rubber edge, is one of the key components that reflect the sound quality of drive unit.

Table 9. Weight of rubber edge data

8.63	8.65	8.57	8.57	8.54	8.69	8.63	8.64	8.59	8.61
8.60	8.66	8.65	8.50	8.61	8.61	8.63	8.67	8.54	8.62
8.65	8.58	8.65	8.67	8.67	8.65	8.69	8.66	8.62	8.63
8.59	8.65	8.64	8.64	8.52	8.69	8.66	8.66	8.61	8.55
8.57	8.64	8.63	8.57	8.61	8.59	8.56	8.71	8.53	8.51
8.72	8.58	8.64	8.69	8.64	8.75	8.59	8.61	8.58	8.65
8.73	8.70	8.65	8.56	8.66	8.65	8.66	8.68	8.62	8.54
8.67	8.62	8.54	8.62	8.66	8.56	8.60	8.62	8.61	8.66

From past experiences, the company decided that the process for the weight of the rubber edge will have upper and lower specifications limits respectively LSL = 8.46 gram and USL = 8.94 gram. Any measurement of the data falls outside the given specification limits will be considered as unacceptable. Some useful plots (box, histogram and QQ) are presented in Figure 2.

Plots in Figure 2 and the KS goodness-of-fit test for normality has a p-value = 0.314 greater than $\alpha = 0.05$, which indicate that the weight of the rubber edge data follow a normal distribution. Moreover, no obvious outlier values present in the data set. For the above analysis, we used Minitab® Release 18 (Minitab Inc., 2017).

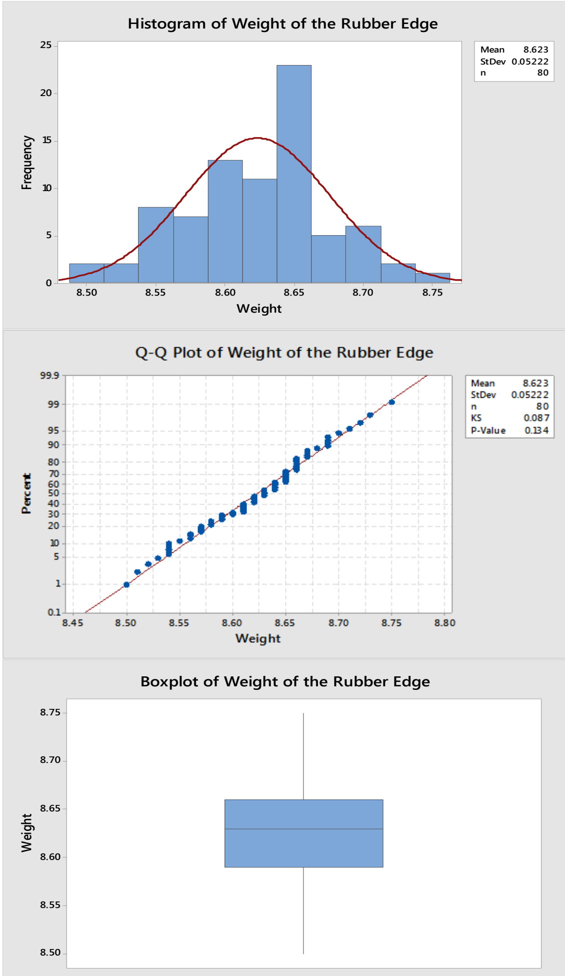

Figure 2. Graphical displays of the rubber edge data

A summary of the scale statistics values, the sample standard deviation (S) and the modified trimmed standard deviation ($MTSD = S_T^*$), are calculated and given in Table 10.

Table 10. Scale statistics of rubber edge data

Statistics	Value
Sample Standard Deviation (S)	0.0522
MTSD (5%)	0.0619
MTSD (10%)	0.0517
MTSD (20%)	0.0363
MTSD (25%)	0.0299
MTSD (30%)	0.0233

The estimated $\hat{C}_p$ from equation (2) and $\hat{C}_p^*$ from equation (9) for C_p, the 95% CI and the corresponding widths are calculated and the results present in Table 11.

Table 11. The 95% CIs for C_p for the weight of rubber edge data

Confidence Interval Method	PCI		Confidence Interval Limits		
			LCL	UCL	Width
CI-Exact	$\hat{C}_p$	1.53	1.294	1.771	0.477
CI-MTSD (5%)	$\hat{C}_p^*$	1.29	1.080	1.504	0.424
CI-MTSD (10%)		1.54	1.277	1.816	0.539
CI-MTSD (20%)		2.20	1.760	2.648	0.888
CI-MTSD (25%)		2.27	2.084	3.267	1.183
CI-MTSD (30%)		3.43	2.582	4.282	1.700

Table 11 gives the expected results, as data is normal. The width of the proposed robust confidence interval for C_p is shorter than that of the exact confidence interval for 5% trimming level, while larger at 10% trimming level and onwards. In addition, according to the quality conditions (C_p given in Table 1), we observe from Table 11 is that when the sample standard deviation (S) is used to estimate σ, then the process is satisfactory capable, but when the modified trimmed standard deviation $(MTSD = S_T^*)$ is used to estimate σ, then the process is capable at 5% level of trimming, satisfactory for 10% level of trimming and at 20% level of trimming onwards. The results of this example supported the simulation study results to some extent. It is noted that for large values of trimming levels, MTSD is small and hence process capability level improves, for normally distributed process. These results indicated that the process is being capable and normal distribution is adequate for modelling this data.

4.2 Weight Measurements for Major League Baseball's

In this example, we will use data set consists of weight measurements (*in ounces*) for a random sample of size $n = 60$ major league baseballs (see [1], page 169, problem 24). The same data set was used by [3]. The observations for this data are presented in Table 12.

Table 12. The weight measurements for 60 major league baseball's data

5.09	5.08	5.21	5.17	5.07	5.24	5.12	5.16	5.18	5.19
5.26	5.10	5.28	5.29	5.27	5.09	5.24	5.26	5.17	5.13
5.27	5.26	5.17	5.19	5.28	5.28	5.18	5.27	5.25	5.26
5.26	5.18	5.13	5.08	5.25	5.17	5.09	5.16	5.24	5.23
5.28	5.24	5.23	5.23	5.27	5.22	5.26	5.27	5.24	5.27
5.25	5.28	5.24	5.26	5.24	5.24	5.27	5.26	5.22	5.09

The process mean of this product is known as 5.25 ounce and the specified lower limit and upper limit are LSL = 4.85 and USL = 5.45 (It's reasonable that, for this product, we allow more deviation from the lower side of the process mean). Any weight of the major league baseball falls outside the given specification limits will be considered as unacceptable, and the major league baseball should be sent to repair, which depends on whether the weight is inside of [LSL, USL] or not.

Some useful plots for the major league baseball's data are presented in Figure 3, which obviously indicated no outlier in the data. The KS goodness-of-fit test for normality has a p-value less than α = 0.01, which indicates that the data do not follow a normal distribution. Thus, it may be concluded that the weight measurements for major league baseball's data can be regarded as taken from a non-normal process. For the above analysis, we used Minitab® Release 18 (Minitab Inc., 2017).

A summary of the scale statistics values, the sample standard deviation (S) and the modified trimmed standard deviation $(MTSD = S_T^*)$, are calculated and given in Table 13.

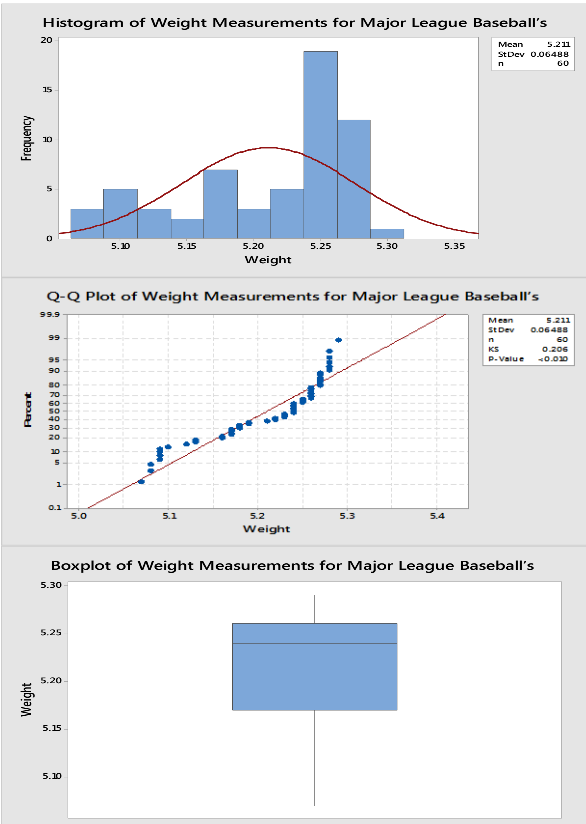

Figure 3. Graphical displays for baseballs data

Table 13. Scale statistics for weight measurements of league baseball's data

Statistics	Value
Sample Standard Deviation (S)	0.0649
MTSD (5%)	0.0859
MTSD (10%)	0.0742
MTSD (20%)	0.0506
MTSD (25%)	0.0442
MTSD (30%)	0.0359

Table 14. The 95% CIs for the weight measurements of league baseball's data

Confidence Interval Method	PCI		Confidence Interval Limits		
			LCL	UCL	Width
CI-Exact	$\hat{C}_p$	1.541	1.263	1.818	0.555
CI-MTSD (5%)	$\hat{C}_p^*$	1.164	0.943	1.385	0.442
CI-MTSD (10%)		1.348	1.076	1.619	0.543
CI-MTSD (20%)		1.976	1.515	2.436	0.921
CI-MTSD (25%)		2.262	1.683	2.840	1.157
CI-MTSD (30%)		2.786	1.986	3.585	1.599

The estimated $\hat{C}_p$ from equation (2) and $\hat{C}_p^*$ from equation (9), the 95% CI and the corresponding confidence interval width for all confidence intervals are calculated and the results present in Table 14. Table 14 gives the expected results as data is non-normal (left-skewed). The width of the proposed interval is shorter than that of the exact interval for 5% and 10% trimming levels, and is larger at 20% trimming onwards. Furthermore, according to the quality conditions (C_p in Table 1), we observe from Table 14 that when the sample standard deviation (S) is used to estimate σ, the process is satisfactory capable of meeting the given specifications, but when the modified trimmed standard deviation ($MTSD = S_T^*$) is used to estimate σ, then the process is capable at 5% level of trimming, satisfactory for 10% level of trimming and at 20% level of trimming onwards, the process is supper excellent capable. The results of this example supported simulation study results to some extent. Therefore, the proposed robust process capability index performed comparatively better than exact process capability index in case of non-normal data without outliers. The exact C_p is not adequate to assess a non-normal data without outliers.

5 Conclusion

In this paper, a robust confidence interval for the process capability index (C_p) based on the modified trimmed standard deviation ($MTSD = S_T^*$) is proposed. Since a theoretical comparison among the interval estimators is difficult, a simulation study has been conducted to compare the performance of the exact and proposed confidence intervals for the process capability index (C_p) in terms of coverage probability (CP) and average width (AW). To pursue the simulation study both normal and a variety of non-normal distributions are considered. It appears from simulation study is that the proposed robust confidence interval is a better index for both normal and non-normal data. In general, the coverage probability (CP) of robust C_p is better with smaller width in confidence interval for 5% and 10% trimmed data compared with exact C_p. Though, for small samples with large trimming shows better results, it may not be practically advisable. Samples with moderate size and a symmetric trimming of 5% and 10% is advisable for measuring process capability using this robust C_p. The methodology was illustrated using two real-life data sets. The practitioners can decide whether or not the level of trimming value is enough to accept the capability of the process and how changing its value affects the value of the process capability index (C_p), and then choose which case is more suitable for their process. For further research, there are some other robust scale estimators available that might have interesting properties in application to capability indices. Furthermore, the evaluation of different non-normal PCIs, under different distributions, is an open problem.

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