k, t, d-Proximities in Rough Set

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Abstract: - The present paper is devoted to the study of k, t, d −proximity on rough sets from the relation point of view. The operator $O_0(\varphi, A)$ and $O(\varphi, A)$ is defined using the upper approximation and closure operators derived from the relation. The properties of induced operator and their connections are henceforth obtained. Moreover, our approach represents a new generalization of the operator using upper approximation only.

Key-Words: - Proximity spaces; Grill operator; Closure operators; Rough sets.

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1 Introduction

A classical rough theory set theory based on an equivalence relation was proposed by Pawlak in 1982 [17]. Rough set theory is a powerful mathematical tool to deal with vagueness. Recently, rough set has been combined with some mathematical theories such as algebra and topology.

Rough set theory has wide application in varied fields. Chen, et.al. [2] used decision theoretic rough sets in data mining. Wang [15] used rough set theory for scene image classification. Landowski and Landowska [4] presented the utilization of rough set hypothesis to get information from experimental data obtained from the examination of traffic intensity in chosen areas. In the present study, the concept rough sets are used to study k, t-proximities. The notion of basic proximity is due to Ĉech and the notion of Grill is due to Choquet [2]. The comprehensive study on the theory of proximity spaces was done by Naimpally and Warrack [8, 14]. The concept of proximities spaces has been generalized both in fuzzy setting [7]. Hosny [13] studied a type of proximity space based on Ideal I and three types of proximity δ. Peters, Tiwari and Singh [9] studied aassociated ε-approach merotopies. A proximity can be obtained from merotopy by restricting the cardinality.

2 Preliminaries

In this section, we collect the basic definitions of rough set, proximity spaces and other fundamental concepts which are used throughout this paper.

2.1 Definition

[17] Let X be a nonempty set and R be an equivalence relation on X. Then the pair *(*X, R*)* is called an **approximation space**.

2.2 Definition

[17] Let (X, R) be an approximation space and $[x]_R$ be the equivalence class of x under R. Then lower approximation and upper approximation of $A \subseteq X$ are, respectively, defined to be the sets;

$$\overline{A} = \{x \in X | [x]_R \cap A \neq \emptyset\},$$

$$\underline{A} = \{x \in X \mid [x]_R \subseteq A\}.$$

For an approximation space (X, R), $A \subseteq X$ is called a Definable set if it is a union of equivalence classes under R and a pair (L, U) of definable sets is called a rough set in (X, R) if $L \subseteq U$, also if equivalence class of x is a singleton set $\{x\}$ such that $\{x\} \in U$, then $\{x\} \in L$.

2.3 Definition

[17] Let $\emptyset$ be the empty set and A^c is the compliment of A in X, then we can get the following properties of the Pawalak's rough sets:

(i) $\underline{A} \subseteq A \subseteq \overline{A}$

(ii) $\overline{X} \subseteq X \subseteq X$

(iii) $\emptyset = \emptyset = \emptyset$

(iv) If $A \subseteq B$ then $\underline{A} \subseteq \underline{B}$ and $\overline{A} \subseteq \overline{B}$.
(v) $A = B$ iff $\underline{A} \subseteq \underline{B}$ and $\overline{A} = \overline{B}$.
(vi) $\underline{(A \cap B)} = (\underline{A}) \cap (\underline{B})$ and $\left(\overline{A \cup B}\right) = (\overline{A}) \cup (\overline{B})$.
(vii) $\left(\overline{A \cap B}\right) \subseteq (\overline{A}) \cap (\overline{B})$ and $\underline{(A \cup B)} \supseteq (\underline{A}) \cup (\underline{B})$.

2.4 Definition

[14] A binary relation δ on $P(X)$ is said to be a pre-basic proximity, if δ satisfies the following axioms:
(i) $A\delta B \implies B\delta A$.
(ii) $(A \cup B)\delta C \Leftrightarrow A\delta C$ or $B\delta C$.
(iii) $A\delta B \implies A \neq \emptyset$; and $\neq \emptyset$.
A pre basic proximity δ on $P(X)$ is said to be basic if it satisfies the following condition:
(iv) $A \cap B \neq \emptyset \implies A\delta B$.

3 Symmetric Relation of Rough Set

This section is devoted to study the symmetric relation on rough set determined by equivalence relation. Let X be a set and R be the equivalence relation on X. Let U_X^R denotes the approximation space.

3.1 Definition

A *rough-grill* $\mathcal{G}$ on U_X^R is a collection of upper approximations of rough sets defined on U_X^R, satisfying: $\phi \notin \mathcal{G}$; if $\bar{A} \in \mathcal{G}$ and $\bar{B} \supseteq \bar{A}$, then $\bar{B} \in \mathcal{G}$; $\overline{A \cup B} \in \mathcal{G}$ implies that $\bar{A} \in \mathcal{G}$ or $\bar{B} \in \mathcal{G}$. The family of all rough grills is denoted by $\Gamma(U_X^R)$.

3.2 Definition

Let U_X^R be an approximation space. Let A be the rough set in the form of $(\underline{A}, \overline{A})$. The family of all symmetric relation φ on U_X^R with the condition:

$$\varphi(A) = \{B \in U_X^R : (A, B) \in \varphi, A \neq \emptyset\} \in \Gamma(U_X^R)$$

is denoted by $\widetilde{U_X^R}$.

3.2.1 Remark

Every $\varphi \in U_X^R$ is the rough proximity on X if $\overline{A} \cap \overline{B} \neq \emptyset$ implies that $A \in \varphi(B)$.

3.3 Lemma

(i) For every $\varphi \in \widetilde{U_X^R}$ and $A \in U_X^R$ the operator given by, $cl_R A = A \cup \{x \in X : ([x]_R, A) \in \varphi\}$ is a (Rough) *Čech* closure operator on X.
(ii) For every A_1, A_2 $\in U_X^R$, $cl_R(X-(A_1 \cup A_2)) = X-(A_1 \cup A_2)$ if $cl_R(X-A_1) = X - A_1$ and $cl_R(X - A_2) = X - A_2$.

3.4 Definition

Let $A, E, F \in U_X^R$ and $\varphi \in \widetilde{U_X^R}$.
Define $\overline{O_0}(\varphi, A), \underline{O_0}(\varphi, A), \overline{O}(\varphi, A), and\ \underline{O}(\varphi, A)$ as follows:
(i) $\overline{O_0}(\varphi, A) = \{E : \overline{E} \supseteq \overline{A}$ and $cl_R(X - E) = X - E\}$
(ii) $\underline{O_0}(\varphi, A) = \{E : \underline{E} \supseteq \underline{A}$ and $cl_R(X - E) = X - E\}$
(iii) $\overline{O}(\varphi, A) = \{F : F \in \overline{O_0}(\varphi, A)$ and $(A, X - cl_R(F)) \notin \varphi\}$
(iv) $\underline{O}(\varphi, A) = \left\{F : F \in \underline{O_o}(\varphi, A)\right.$ and $\left.(A, X - cl_R(F)) \notin \varphi\right\}$

3.4.1 Remark

(i) For A, B $\in U_X^R$ with $B \subseteq A$,
a. $\overline{O_0}(\varphi, A) \subseteq \overline{O_0}(\varphi, B)$
b. $\underline{O_0}(\varphi, A) \subseteq \underline{O_0}(\varphi, B)$
c. $\overline{O}(\varphi, A) \subseteq \overline{O}(\varphi, B)$
d. $\underline{O}(\varphi, A) \subseteq \underline{O}(\varphi, B)$
(ii) Universal rough set contained in $\overline{O_0}(\varphi, A)$ for all $A \in U_X^R$.
(iii) For A$= \emptyset$, the operators defined in 3.4 are nonempty as it contains null set.

3.5 Lemma

For the sets $A_1, A_2 \in U_X^R$ and $\varphi \in \widetilde{U_X^R}$.
(i) $\overline{O_0}(\varphi, A_1 \cup A_2) = \overline{O_0}(\varphi, A_1) \cap \overline{O_0}(\varphi, A_2)$.
(ii) $\underline{O_0}(\varphi, A_1 \cup A_2) = \underline{O_0}(\varphi, A_1) \cap \underline{O_0}(\varphi, A_2)$.
(iii) $\overline{O}(\varphi, A_1 \cup A_2) = \overline{O}(\varphi, A_1) \cap \overline{O}(\varphi, A_2)$.
(iv) $\underline{O}(\varphi, A_1 \cup A_2) = \underline{O}(\varphi, A_1) \cap \underline{O}(\varphi, A_2)$.
(v) $\overline{O_0}(\varphi, A_1 \cup A_2)$ is contained in $\mathcal{G}$ iff $\overline{O_0}(\varphi, A_1)$ is contained in $\mathcal{G}$ or $\overline{O_0}(\varphi, A_2)$ is contained in $\mathcal{G}$.
(vi) $\underline{O_0}(\varphi, A_1 \cup A_2)$ is contained in $\mathcal{G}$ iff $\underline{O_0}(\varphi, A_1)$ is contained in $\mathcal{G}$ or $\underline{O_0}(\varphi, A_2)$ is contained in $\mathcal{G}$.
(vii) $\overline{O}(\varphi, A_1 \cup A_2)$ is contained in $\mathcal{G}$ iff $\overline{O}(\varphi, A_1)$ is contained in $\mathcal{G}$ or $\overline{O}(\varphi, A_2)$ is contained in $\mathcal{G}$.
(viii) $\underline{O}(\varphi, A_1 \cup A_2)$ is contained in $\mathcal{G}$ iff $\underline{O}(\varphi, A_1)$ is contained in $\mathcal{G}$ or $\underline{O}(\varphi, A_2)$ is contained in $\mathcal{G}$.

Proof. (i) Using remark 3.4.1, $\overline{O_0}(\varphi, A_1 \cup A_2) \subseteq \overline{O_0}(\varphi, A_1) \cap \overline{O_0}(\varphi, A_2)$.
For converse, it is sufficient to note that if $\overline{D} \in \overline{O_0}(\varphi, A_1) \cap \overline{O_0}(\varphi, A_2)$, then $\overline{D} \supseteq \overline{A_1 \cup A_2}$.
(ii) Analogous to (i).
(iii) Since $A_1 \subseteq A_1 \cup A_2$, we get $\overline{O_0}(\varphi, A_1 \cup A_2) \subseteq \overline{O_0}(\varphi, A_1) \cap \overline{O_0}(\varphi, A_2)$.
Let $D \in \overline{O_0}(\varphi, A_1) \cap \overline{O_0}(\varphi, A_2)$. Then using (i) D$\in \overline{O_0}(\varphi, A_1 \cup A_2)$. Since $\varphi(X - cl_R(D)$ is a grill and

$A_1, A_2 \notin \varphi(X - cl_R(D)$,we get $(A_1 \cup A_2, X - cl_R(D)) \notin \varphi$.

(iv) Analogous to (iii).

(v) Let $\overline{O_0}(\varphi, A_1) \subseteq \mathcal{G}$. Then $\overline{O_0}(\varphi, A_1 \cup A_2) \subseteq \mathcal{G}$, follows from remark 3.4.1. Conversely, if $D \in \overline{O_0}(\varphi, A_1 \cup A_2)$, then $\overline{D} \supseteq \overline{A_1} \cup \overline{A_2}$.Let $\overline{O_0}(\varphi, A_1) \nsubseteq \mathcal{G}$ and $\overline{O_0}(\varphi, A_2) \nsubseteq \mathcal{G}$. Then $D_1 \in \overline{O_0}(\varphi, A_1)$ and $D_2 \in \overline{O_0}(\varphi, A_2)$ such that $D_1, D_2 \notin \mathcal{G}$. This implies that $\overline{D_1} \cup \overline{D_2} \supseteq \overline{A_1 \cup A_2}$ but $D_1 \cup D_2 \notin \mathcal{G}$. Hence, $\overline{O_0}(\varphi, A_1 \cup A_2) \nsubseteq \mathcal{G}$.

(vi) Analogous to (v).

(vii) Let $\overline{O}(\varphi, A_1) \nsubseteq \mathcal{G}$ and $\overline{O}(\varphi, A_2) \nsubseteq \mathcal{G}$. Then $D_1 \in \overline{O}(\varphi, A_1)$ and $D_2 \in \overline{O}(\varphi, A_2)$ such that $D_1, D_2 \notin \mathcal{G}$. Therefore $D_1 \cup D_2 \in \overline{O}(\varphi, A_1 \cup A_2)$ and $D_1 \cup D_2 \notin \mathcal{G}$. Let $D_1 \cup D_2 \in \varphi(X - cl_R(A_1 \cup A_2))$. Then $A_1 \in \varphi(X - cl_R(D_1 \cup D_2))$ or $A_2 \in \varphi(X - cl_R(D_1 \cup D_2))$. $X - cl_R(D_1 \cup D_2) = [X - cl_R D_1\,] \cap [X - cl_R D_2] \in \varphi(A_1)$. i.e. $(A_1, X - cl_R D_1) \in \varphi$. Hence a contradiction. This gives that $(A_1 \cup A_2, X - cl_R(D_1 \cup D_2))) \notin \varphi$.

(viii) Analogous to (vii).

3.6 Definition

(i) The set of all functions $v: \widetilde{U_X^R} \times \Gamma(U_X^R) \to P(U_X^R)$, $v(\Pi, \mathcal{G}) \in U_X^R$ is denoted by $\Phi(X)$ where $\mathcal{G}$ and Π are rough grills and rough proximities on X.

(ii) For $\mathcal{G} \in \Gamma(U_X^R)$ and $\varphi \in \widetilde{U_X^R}$ we define:

$$t(\varphi, \mathcal{G}) = \{A \in U_X^R : \overline{O_o}(\varphi, A) \subseteq \mathcal{G}\}$$

$$k(\varphi, \mathcal{G}) = \{A \in U_X^R : \overline{O}(\varphi, A) \subseteq \mathcal{G}\}$$

$$d(\varphi, \mathrm{G}) = \{\mathrm{A} \in U_X^R : \exists \mathrm{D} \ni \varphi(\mathrm{D}) = \mathrm{G} \text{ and } \mathrm{E_A} \cap \mathrm{E_D} \neq \emptyset \text{ for all } \mathrm{E_A} \in \overline{\mathrm{O_o}}(\varphi, \mathrm{A}) \text{and all } \mathrm{E_D} \in \overline{\mathrm{O_o}}(\varphi, \mathrm{D})\} \cup \mathrm{G}.$$

3.7 Theorem

The functions $t, k, d \in \Phi(X)$.

3.8 Definition

A rough proximity Π on U_X^R is said to be λ– proximity on U_X^R iff for all $A \in U_X^R$, there exists a function $\lambda \in \Phi(X)$ satisfying $\lambda(\Pi, \Pi(A)) \subseteq \Pi(A)$. Further we denote the set of all λ–proximities on U_X^R by R_λ. A rough grill operator will be in class A_0 if $\lambda(\varphi, \mathcal{G}_1) \subseteq \lambda(\varphi, \mathcal{G}_2)$ where $\mathcal{G}_1$ and $\mathcal{G}_2$ are rough grills with $\mathcal{G}_1 \subseteq \mathcal{G}_2$; for all $\varphi \in (\widetilde{U_X^R})_\lambda$; λ will be in class A_1 if $\lambda(\varphi, \mathcal{G}_1 \cup \mathcal{G}_2) \subseteq \lambda(\varphi, \mathcal{G}_1) \cup \lambda(\varphi, \mathcal{G}_2)$, where $\mathcal{G}_1$ and $\mathcal{G}_2$ are rough grills.

3.9 Proposition

(i) $t, k \in A_0 \cap A_1$.

(ii) $d \in A_1$.

3.10 Proposition

Let Π be a rough proximity on U_X^R. Then,

(i) $\Pi \in \overline{R_k}$ iff it satisfies: $G \notin \Pi(F) \Leftrightarrow \exists \mathrm{H} \in \overline{O}(\Pi, G)$ and $F \notin \Pi(H)$.

(ii) $\Pi \in \overline{R_t}$ iff it satisfies: $G \notin \Pi(F) \Leftrightarrow \exists \mathrm{H_F} \in \overline{O_o}(\Pi, F)$ and $\mathrm{H_G} \in \overline{O_o}(\Pi, G)$ such that $(\mathrm{H_G}, \mathrm{H_F}) \notin \Pi$.

(iii) $\Pi \in \overline{R_d}$ iff it satisfies $B \notin \Pi(\mathrm{C}) \Leftrightarrow$ there exists $\mathrm{H_B} \in \overline{\mathrm{O_o}}(\varphi, \mathrm{C})$ such that $\mathrm{H_B} \cap \mathrm{H_C} = \emptyset$.

3.11 Theorem

$\overline{R_k} \subseteq \overline{R_t} \subseteq \overline{R_d}$.

Proof. Let $\Pi \in \overline{R_k}$. Then $G \notin \Pi(F)$ iff $\exists \mathrm{H_1} \in \overline{O}(\Pi, G)$ and $\mathrm{F} \notin \Pi(H_1)$ i.e iff $\mathrm{H_1} \in \overline{O}(\Pi, G)$ and $\mathrm{H_2} \in \overline{O}(\Pi, F)$ such that $\mathrm{H_2} \notin \Pi(H_1)$. Hence $\Pi \in \overline{R_t}$. So $\overline{R_k} \subseteq \overline{R_t}$.

It is sufficient to show that $\overline{R_t} \subseteq \overline{R_d}$. Let $\Pi \in \overline{R_t}$ then $D_1 \notin \Pi(D_2)$ iff there exist $H_{D_1} \in \overline{\mathrm{O_o}}(\varphi, \mathrm{D_1})$ and $H_{D_2} \in \overline{\mathrm{O_o}}(\varphi, \mathrm{D_2})$ such that $H_{D_1} \notin \Pi(H_{D_2})$ and so $H_{D_1} \cap H_{D_2} = \emptyset$. This gives that $\Pi \in \overline{R_d}$.

3.11.1 Remark

It is to be noted that $\underline{R_k} \subseteq \underline{R_t} \subseteq \underline{R_d}$.

3.12 Definition

For a rough closure space (U_X^R, cl_R) and the operator $\overline{O_o}(cl_R, F)$, the rough sets $\mathrm{H_1}$ and $\mathrm{H_2}$are said to be separated with respect to cl_R iff $\exists G_i \in \overline{O_o}(cl_R, \mathrm{H_i}), i = 1,2,3$ .... such that G_1 and G_2 are disjoint rough sets.

3.13 Proposition

For a rough closure space (U_X^R, cl_R),

$$\Pi_{Cl_R}^{+} = \{(F, G): \text{F and G are not seperated with respect to } cl_R\}$$

is a rough proximity on U_X^R.

Proof. Let$(F, G) \notin \Pi_{Cl_R}^{+}$. Then there exist $\mathrm{H_1} \in \overline{O_o}(cl_R, F)$ and $\mathrm{H_2} \in \overline{O_o}(cl_R, G)$ such that $\mathrm{H_1} \cap \mathrm{H_2} = \emptyset$. Since $\mathrm{H_1} \supseteq \mathrm{F}$ and $\mathrm{H_2} \supseteq \mathrm{G}$, we get $\mathrm{F} \cap \mathrm{G} = \emptyset$. Since $\emptyset \in \overline{O_o}(cl_R, \emptyset)$, $\emptyset$ and F are separated for all F. Hence $\emptyset \notin \Pi_{Cl_R}^{+}(F)$. Let $G \in \Pi_{Cl_R}^{+}(F)$ and $\supseteq$ G. Then for all $\mathrm{H_1} \in \overline{O_o}(cl_R, F)$ and all $\mathrm{H_2} \in \overline{O_o}(cl_R, G)$, $\mathrm{H_1} \cap \mathrm{H_2} \neq \emptyset$. Since for $\mathrm{H} \in \Pi_{Cl_R}^{+}(F)$, $\overline{O_o}(cl_R, F) \subseteq \overline{O_o}(cl_R, G)$. Let $F, G \notin \Pi_{Cl_R}^{+}(H)$. Then there exists $\mathrm{S_1} \in \overline{O_o}(cl_R, F)$ and $\mathrm{H_1} \in \overline{O_o}(cl_R, H)$

such that $S_1 \cap H_1 = \emptyset$. Also there exist $S_2 \in \overline{O_o}(cl_R, G)$ and $H_2 \in \overline{O_o}(cl_R, H)$ such that $S_2 \cap H_2 = \emptyset$. Since $S_1 \cup S_2 \in \overline{O_o}(cl_R, F \cup G), H_1 \cap H_2 \in \overline{O_o}(cl_R, H)$ and $(S_1 \cup S_2) \cap (H_1 \cap H_2) = (S_1 \cap (H_1 \cap H_2)) \cup (S_2 \cap (H_1 \cap H_2)) = \emptyset$. We have $F \cup G \notin \Pi^+_{cl_R}(H)$.

3.14 Theorem

If $\Pi \in R_i$, where $i = k, t$ or d, then (X, C_Π) is a regular rough topological space satisfying property **RT**, where

RT: For any closed set E and any rough point x with $x \notin F$, there exist rough sets C and D such that $x \in C$ and $F \subseteq D$ such that $C \cap D = \emptyset$.

Proof. It is sufficient to show for d. Let $x \notin C_\Pi(B)$,
$\Rightarrow \exists H_x \in \overline{O_o}(\varphi, \{x\})$ and $H_B \in \overline{O_o}(\varphi, B)$ such that $H_x \cap H_B = \emptyset$.
$\Rightarrow B \subset H_B \subset X - H_x$ and $C_\Pi(X - H_x) = X - H_x$
$\Rightarrow C_\Pi(B) \subset X - H_x \Rightarrow C_\Pi(C_\Pi(B)) \subset X - H_x \Rightarrow$
$x \notin C_\Pi(C_\Pi(B)) \Rightarrow C_\Pi(C_\Pi(B)) \subset C_\Pi(B)$.
Hence (X, C_Π) is a topological space. Now to show regularity:
Let $x \notin F$. Then $\{x\} \notin \Pi(F) \Rightarrow E_x \supset \{x\}$ and $E_F \supset F$ such that $E_x \cap E_F = \emptyset$.
This proves that (X, C_Π) is regular.

3.15 Theorem

Let (X, C) be a rough topological space satisfying property **RT.** Then $\Pi^+_C \in M_i(X, C_\Pi), i = k, t, d$.

Proof. Since (X, C) is rough topological space satisfying property RT, and by [16] a rough topological space induced by similarity relation is regular, the result follows for $d -$ proximity. Hence the result follows for k and t proximities as well.

3.16 Preposition

Let $i = k, t$ and d. Then $M_i(X, C_\Pi) \neq \emptyset$ iff (X, C_Π) is a rough topological space satisfying **RT.** Moreover Π^+_C is the smallest $i -$proximity in each case.

3.17 Lemma

Let (X, C) is a regular topological space. Then $\Pi^+_C \in M_i(X, C_\Pi), i = k, t, d$.
Proof. First, we have to show that $C = C_{\Pi^+_C}$ (for $k -$proximity). Let $x \notin C_{\Pi^+_C}(A)$.
$\Rightarrow ([x], A) \notin \Pi^+_C$
$\Rightarrow N_x \cap N_A = \emptyset$ for some $N_x \in O_o(C_\Pi, [x])$ and $N_A \in (C_\Pi, A)$.
$\Rightarrow x \in N_x$ and $A \subseteq N_A$ such that $N_x \cap N_A = \emptyset \Rightarrow x \notin C(A)$. [Because this is the property of regular topological space and C is regular topological space]. Since $\overline{R_k} \subseteq \overline{R_t} \subseteq \overline{R_d}$. We have to show only that $\Pi^+_C \in M_k(X, C_\Pi)$.
Let $G \notin \Pi^+_C(F)$. Then F and G are separated. This implies that $H_F \in \overline{O_o}(Cl_R, F)$ and $H_G \in \overline{O_o}(Cl_R, G)$ such that $H_F \cap H_G = \emptyset$. This implies $F \subseteq H_F$ and $G \subseteq H_G$ and $H_F \cap H_G = \emptyset$.
This implies that $X - Cl_R H_G$ and G are disjoint sets.
$\Rightarrow G \notin \Pi^+_C(X - Cl_R H_G) \Rightarrow H_G \notin \Pi^+_C(F)$ [Because $F \subseteq H_F$ and $H_F \cap H_G = \emptyset$. So $F \cap H_G = \emptyset$].

4 Conclusion

This paper investigates k, t and $d -$proximities on rough sets based on approximation operations. The method basically deals with general symmetric relation and approximation operation. The present study deals with proximities taking upper approximations only; however, the results hold good for lower approximations also. The non-trivial semi-proximity are in correspondence with the digital image used in computer graphics and can be seen in Latecki and Prokop [3]. The significance of this generalization is that we can generate new method to get rough proximities spaces with respect to each similarity relation and corresponding closure operator induced from relation. In view of this the present work will help to investigate or generalized the concept of proximities on rough set theory and generalized rough set theory.

References:

[1] H.Chen, T. Li, C.Luo, S.J.Horng, G.Wang, A Decision-Theoretic Rough Set Approach for Dynamic Data Mining, *IEEE Transactions on Fuzzy Systems*, 23(2015), 1958–1970.
[2] G.Choquet, Sur Les Notions De Filter at De Grille, *Compt. Rendurs Acad., Sci.Paris,* 224(1947), 171-173.
[3] L.Latecki, F.Prokop, Semi-proximity continuous in digital images, *Pattern Recognit Lett*, 16(1995), 1175-1187.
[4] M. Landowski, A. Landowska, Usage of the rough set theory for generating decision rules of number of traffic vehicles, *Transportation Research Procedia*, 39(2019) 260–269.
[5] M.Khare, R.Singh, Complete *ξ*-grills and *(L,n)*-Merotopies, *Fuzzy Set and Systems*, 159(2008), 620-628.

[6] M.Khare, R.Singh, *L*-Contiguities and their ordered structures, *Fuzzy Set and Systems*, 158(2007), 399-408.
[7] M.Khare, R.Singh, *L*-guild and Binary *L*-Merotopies, *Novi Sad J.Maths*, 36(2006), 57-64.
[8] M.S. Gagrat and S.A. Naimpally, Proximity approach to extension problems, *Fund. Math*., (1971), 63-76.
[9] J.F. Peters, S.Tiwari, R.Singh, Approach Merotopies and Associated Near Sets, *Theory and Applications of Mathematics & Computer Science*, 3(1) (2013) 1–12
[10] R.Singh, On symmetric relation on I^X, *AIP conference Proceeding*, (2017), 1897, 020038.
[11] R.Singh, A.K.Umrao, A Study on Soft $d-$Proximity, *Journal of Advanced Research in Dynamical and Control Systems*., 05- Sp, 2018, 1911- 1914.
[12] R.Singh, A.K.Umrao, On Finite Order Nearness in Soft Set Theory, *WSEAS TRANSACTIONS on MATHEMATICS*, Vol. 18, July 2019, 118-122.
[13] R.A.Hosny, Relations and Applications on Proximity Structures, *Gen. Math. Notes*, Vol. 11, No. 1(2012), 24-40.
[14] S.A.Naimpally, B.D.Warrack, Proximity spaces, *Cambridge Univ.Press,New York*, (1970).
[15] X. Wang, N. Liu and K. Me, Application of Rough Set Theory on scene image classification,*Chinese Control and Decision Conference*, (2008), 2338-2342.
[16] Z. Li, Topological properties of generalized rough sets, in: 2010 Seventh International Conference on Fuzzy Systems and Knowledge Discovery, vol. 5, 2010, pp. 2067–2070.
[17] Z. Pawlak, Rough sets, *Int. J. Inform. Comput.Sci.*,11(1982), 341–356.