

The Graph of e-Abacus Diagram

AHMED MOHAMMED ALI^{1*}, AMMAR SEDDIQ MAHMOOD²

¹ Department of Mathematics, College of Computer science and Mathematics, University of Mosul, Mosul, IRAQ

² Department of Mathematics, College of Education for Pure science, University of Mosul, Mosul, IRAQ

Abstract: - Since James introduced the idea of an e-abacus diagram in 1978, this topic plays important roles with many topics, especially in algebra. In this paper, we will present new insights on this topic so that it appears to us as a graph theory; specifically a directed graph, according to certain conditions, to be a new start with another field. This proposed technique will be applied to the model that was approved by Mahmood and Mahmood in 2019 on English letters and to see how successful we are on Letter-Word-Sentence. It is completely natural that this method will have a new and very difficult method as an entrance to a new type of encryption process.

Key-Words: - directed graph, adjacent matrix, partition theory, e-abacus diagram.

Received: March 2, 2020. Revised: September 30, 2020. Accepted: October 13, 2020. Published: October 26, 2020.

1 Introduction

Let r be a nonnegative integer. The composition $\pi = (\pi_1, \pi_2, \dots, \pi_n) = (\theta_1^{\gamma_1}, \theta_2^{\gamma_2}, \dots, \theta_m^{\gamma_m})$ (where γ_z the number of times θ_z appeared, $z=1, 2, \dots, m$) of r is a sequence of non-negative integers such that $|\pi| = \sum_{j=1}^n \pi_j = \sum_{z=1}^m \theta_z^{\gamma_z} = r$. The composition is called a partition of r if $\pi_j \geq \pi_{j+1}, \forall j \geq 1$. Fix π is a partition of r and defining $\beta_i = \pi_i + b - i, 1 \leq i \leq b$. The set $\{\beta_1, \beta_2, \dots, \beta_b\}$ is said to be the set of β -number for π , see [10]. Let e be a positive integer number greater than or equal to 2, we can represent numbers by a diagram called e -abacus diagram, see[5], as shown:

Table 1.
e-Abacus Diagram

Runner 1	Runner 2	...	Runner e
0	1	...	e-1
e	e+1	...	2e-1
2e	2e+1	...	3e-1
⋮	⋮	...	⋮

where every β will be represented by a (●) and the rest of the sites by (-). In fact, the definition of e-Abacus diagram will leads us to the fact that the presence of an infinite number of diagrams are all suitable for any partition according to the value of e . Each partition will be represented first through by Young tableau $[\pi]$; see [4], and later by e-Abacus

diagram, for example, if $\pi = (3^3, 2, 1^2)$ and $e = 3$ or 4, there exist many of e -abacus diagrams:

Table 2.
Young Tableau and e-Abacus Diagram of
(3³, 2, 1²)

Young Tableau	e=3	e=3	e=4	e=4

Now, Many branches of mathematics, such as group theory, set theory, matrix theory and topology, have close connections with graph theory, therefore consider graph theory is one of the most important topics in applied sciences and engineering for because it is a mathematical model through which to analyse many concrete real-world problems successfully,[1]. There are two types of graphs: undirected graph is called graph and directed graph is called digraph. The graphs that we will be dealing with is directed graphs in this paper such that digraphs arise in a natural way in many applications of graph theory [3,6,11]. A graph G is an ordered pair of non-empty set $V(G)$ of elements called vertices and a family $E(G)$ (it may be empty) from unordered pairs of vertices, each element of $E(G)$ is called an edge. A digraph D as a set $V(D)$ that is non-empty of elements called vertices with a family $A(D)$ of ordered pairs of vertices is called each ordered pair of vertices (u, v) , where

$u, v \in V(D)$ a directed edge (directed edge from u to v). A directed graph is expressed as an ordered pair $(V(D), A(D))$. The number of vertex set $V(D)$ is the order digraph and the number of directed edges in $A(D)$ is the size digraph. A digraph D is called finite if both $V(D)$ and $A(D)$ are finite [2]. In this paper, we will connect between e-abacus diagram and directed graph by designing a specific matrix as a real input for this connection.

2 e-Abacus Directed Graph

By looking at any e-Abacus diagram, we will always see the existence of (\bullet) and $(-)$, so if we assume that any (\bullet) corresponds to 1, any $(-)$ corresponds to 0 and that this diagram is a matrix of elements 1 and 0, thus becoming a matrix for this diagram, (this matrix we call it e-abacus diagram representative matrix). By above example:

Table 3
The Matrix of e-Abacus Diagram

e=3			e=3			e=4			e=4				
•	•	-	-	•	•	•	-	•	•	•	•	•	•
•	•	-	-	•	-	-	•	-	•	-	•	•	-
•	-	•	•	•	•	•	•	-	-	•	-	•	•
•	•	•	•	•	•	•	•	-	-	•	-	-	-
Diagram 1	↓	Diagram 2			Diagram 3	↓		Diagram 4					
1 1 0		0 1 1			1 0 1 1			1 1 1 1					
1 1 0		0 1 0			0 1 0 1			0 1 1 0					
1 0 1		1 1 1			1 1 0 0			1 0 1 1					
1 1 0								1 0 0 0					

Now, in graph theory, if $a = (u, v)$ is a directed edge of D , a is said to be incident out of u and incident into v , also u is said to be adjacent to v and v is adjacent from u . The number of directed edges incident out of a vertex v is the outdegree of v and it's denoted by $outdeg(v)$ and the number of directed edges incident into v is the indegree of v and it's denoted by $indeg(v)$. The sum outdegree and indegree of a vertex v is a degree of v , i.e., $deg(v) = outdeg(v) + indeg(v)$.

An directed edge having the same ends is called a loop of D [12]. We can represent the digraph D by matrix which is called adjacent matrix of directed graph and defined as:

$$m_{i,j} = \begin{cases} 1 & ; \text{if } (v_j, v_k) \text{ is an arc of } D, \\ 0 & ; \text{otherwise,} \end{cases}$$

we note that $m_{j,j} = 1$ if the arc (v_j, v_j) is a directed loop. Also, from adjacent matrix of digraph can be obtained the digraph. adjacent matrix of digraph is square matrix, 0 or 1 elements, unconditional symmetric and the sum elements of each row is representation the outdegree of corresponds vertex in row and the sum elements of each column is representation the indegree of corresponds vertex in

column. If the digraph without directed loop, then the elements of diagonal matrix are zero.

The following theorem gives the necessary and sufficient condition for an e-Abacus diagram to be an adjacent matrix corresponding to a digraph

Theorem: Every matrix $(0,1) \sim (-, \bullet)$ generated from an e-abacus diagram is square if and only if it's represents a matrix adjacent to a digraph.

Proof: An e-abacus diagram can be represented by a matrix by making each node or slash in the diagram an element with a value of one or zero respectively in the matrix M . Suppose that the matrix M of elements $(0,1)$ is a square matrix with size $n \times n$. In order to find an adjacent matrix of A for a digraph corresponding to the matrix resulting from an e-Abacus diagram, suppose ϕ bijective mapping between the row numbers and vertices labels to the indication digraph D of order n has vertex set $V(D) = \{v_1, v_2, \dots, v_n\}$, where $\phi: (Z_{n+1} - \{0\}) \rightarrow V(D)$ such that: $\phi(j) = v_j, j = 1, 2, \dots, n$.

Also, in the same method with respect to the column numbers and vertices labels, that is:

$\phi(k) = v_k, k = 1, 2, \dots, n$.
 Then $\hat{\phi}(M(j, k)) = A(v_j, v_k), j, k = 1, 2, \dots, n$,
 where $\hat{\phi}$ is a bijective mapping of the elements of the matrix M to the elements of the adjacent matrix A of a digraph, i.e., $\hat{\phi}: M(j, k) \rightarrow A(v_j, v_k), j, k = 1, 2, \dots, n$. $A(v_j, v_k) = 1$ when $M(j, k) = 1$, this means that the vertex v_j is adjacent to the vertex v_k , and $A(v_j, v_j) = 1$ when $M(j, j) = 1$, this means that the vertex is v_j incident on a directed loop, as well as $A(v_j, v_k) = 0$ when $M(j, k) = 0$, this means that the vertex v_j is not adjacent to the vertex v_k . So the square matrix M generated from an e-Abacus diagram is adjacent matrix of digraph. Likewise, we can get an e-Abacus diagram from the adjacent matrix of an digraph, this complete the proof. ■

The matrix $A(3 - \text{abacus diagram}) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

in Diagram 2 is a square matrix, therefore it's represent the adjacent matrix and which corresponding digraph below in Figure 1.

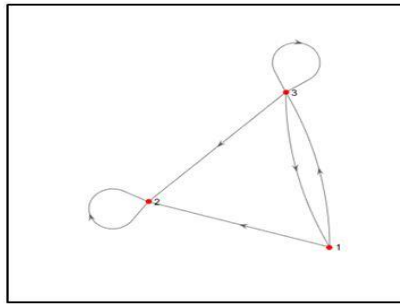


Fig. 1.
Directed Graph of 3 – Abacus Diagram in
Diagram 2.

But the e-abacus diagram representative matrix when $e = 3$ in Diagram 1, we can't representation by directed graph because it's not square matrix. If the number of rows in e- Abacus diagram is less than from the number of columns, then we can be obtain on square matrix by addition new rows have elements zeros, this operation does not effected on partition; see Diagram 3, but if the number of columns in e- abacus diagram is less than from the number of rows, then we can be obtain on square matrix by addition new columns have elements zeros, this operation is effected of value of partition, this means choosing e different from the original model, then this case is not taken.

A digraph is symmetric if $(u, v) \in E(D)$, then $(v, u) \in E(D)$. The complete symmetric digraph K_n^* of order n has both arcs (u, v) and (v, u) for any distinct vertices u and v . A digraph is called an oriented graph if $(u, v) \in E(D)$ but $(v, u) \notin E(D)$. Thus, an oriented graph D can be obtained from a graph G by assigning a direction to each edge of G and transforming G itself into an oriented graph. The digraph D is also called an orientation of G . An orientation of a complete graph is called a tournament [2].

From the relationship between the of e – abacus diagram and digraphs, we can get many properties, such as:

1. If the e-abacus diagram representative matrix is identity matrix, then digraph has only directed loop on every vertex.
2. If the e-abacus diagram representative matrix has zero diagonal, then digraph is a simple [2].

3. If the elements of the e-abacus diagram representative matrix $m_{j,k} = 1$, for all $j \neq k$, $j, k = 1, 2, \dots, n$, then digraph is complete.
4. If the e-abacus diagram representative matrix is upper triangular (or lower triangular) matrix, then digraph is not strong [2].
5. If the e-abacus diagram representative matrix is symmetric matrix, then digraph is symmetric.
6. If $m_{j,j+1} = 1$ for $j = 1, 2, \dots, n - 1$, (n is the number of rows(columns)), then digraph is path digraph [2].
7. If $m_{j,j+1} = 1$ for $j = 1, 2, \dots, n - 1$, (n is the number rows (columns)) and $m_{1,n} = 1$, then digraph is cycle digraph also it is strong digraph [2].
8. If $m_{j,k} = 1$ and $m_{k,j} = 0$ for all $j \neq k$ and $j, k = 1, 2, \dots, n$, (n is the number rows (columns)), then digraph is tournament [1].

Finally, there are another properties can be resulting from the e-Abacus diagram representative matrix.

3 Application

Since the e-abacus diagram representative matrix for English letters is square 5×5 , which were presented by Mahmood and Mahmood in [8,9], it can be taken as an application between digraph and partition theory, as the digraph in the encoding formula can be abbreviated as follows: To coding the digraph by 10 –tuple out degrees and indegrees of vertices in D .

$$\text{Code}(\text{any letter}) = (\text{outdeg}v_1, \dots, \text{outdeg}v_5; \text{indeg}v_1, \dots, \text{indeg}v_5),$$

where $0 \leq \text{outdeg}v_j \leq 5$ and $0 \leq \text{indeg}v_j \leq 5$, for all $1 \leq j \leq 5$. See the appendix 1.

Perhaps the next question is most important in this area, what will we gain after we have done this step for each letter? The answer will be, if we have any word, how will we be able to convert it according to what has been proposed here!! It is quite clear that we have a square matrix with 5 for the first letter and the same for the second letter and so on for the last letter, let it be ϑ letters from the chosen word. This means that we now have a new matrix with a capacity of 5 rows and 5ϑ of columns. Now, added $5(\vartheta - 1)$ rows in down the original

Acknowledgements:

The authors sincerely thank the University of Mosul/College of Education for Pure Sciences and the College of Computer Science and Mathematics for moral support in order to complete this research.

References:

- [1] R. Balakrishnan and K. Ranganathan, *A Textbook of Graph Theory*, Springer Science, New York 2012.
- [2] G. Chartrand and L. Lesniak, *Graphs and Digraphs*, 6th ed., Wadsworth and Brooks / Cole. California, 2016.
- [3] J.C. Fournier, *Graph Theory and Applications*, Library of Congress Cataloging-in-Publication Data, 2009.
- [4] W. Fulton, *Young Tableaux, with Applications to Representation Theory and Geometry*, Cambridge University Press, 1997.
- [5] G. D. James, Some combinatorial results involving Young diagrams, *Mathematical Proceeding of the Cambridge Philosophical Society*, Vol.83,1978,pp. 1-10.
- [6] L. G. Jonathan, Y. Jay and A. Mark, *Graph Theory and Its Applications*, 3rd edition. LLC CRC Press is an imprint of Taylor & Francis Group, 2019.
- [7] A. S. Mahmood, Replace the content in e-abacus diagram. *Open Access Library J.*, Vol.7,2020, pp.1-6.
- [8] A. B. Mahmood and A. S. Mahmood, Secret-word by e-abacus diagram I. *Iraqi J. of Science*, Vol.60, No.3, 2019, pp.638-646.
- [9] A. B. Mahmood and A. S. Mahmood, Secret-text by e-abacus diagram II. *Iraqi J. of Science*, Vol.60, No.4,2019, pp.840--846.
- [10] A. Mathas, Iwahori-Hecke Algebras and Schur Algebras of the Symmetric Group, *University Lecture Series*, Vol.15, 1999.
- [11] D. Matthias and E. Frank, Quantitative Graph Theory Mathematical Foundations and Applications, *Chapman and Hall/CRC*. 2019.
- [12] J. W. Robin, *Introduction to Graph Theory*, 4th edition, Library of Congress Cataloging in Publication Data, 1996.

Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

Ahmed Mohammed Ali was study the directed graph, some properties and sketch the digraphs. Ammar Seddik Mahmmod give idea partition theory with connection to digraph.

Sources of funding for research presented in a scientific article or scientific article itself

There are no sources of funding

Creative Commons Attribution License 4.0 (Attribution 4.0 International , CC BY 4.0)

This article is published under the terms of the Creative Commons Attribution License 4.0

https://creativecommons.org/licenses/by/4.0/deed.en_US

See the appendix

The digraphs of English letters

	e-Abacus Diagram	Adjacent Matrix of Digraphs	Directed Graphs (Digraphs)																																				
A		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <th>4</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p>Code A = (3,2,5,2,2; 4,2,2,2,4)</p>		1	2	3	4	5	1	0	1	1	1	0	2	1	0	0	0	1	3	1	1	1	1	1	4	1	0	0	0	1	5	1	0	0	0	1	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	1	0	0	0	1																																		
3	1	1	1	1	1																																		
4	1	0	0	0	1																																		
5	1	0	0	0	1																																		
B		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table> <p>Code B = (3,2,3,2,3; 0,5,3,3,2)</p>		1	2	3	4	5	1	0	1	1	1	0	2	0	1	0	0	1	3	0	1	1	1	0	4	0	1	0	0	1	5	0	1	1	1	0	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	0	1	0	0	1																																		
3	0	1	1	1	0																																		
4	0	1	0	0	1																																		
5	0	1	1	1	0																																		
C		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>4</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table> <p>Code C = (3,2,1,2,3; 3,2,2,2,2)</p>		1	2	3	4	5	1	0	1	1	1	0	2	1	0	0	0	1	3	1	0	0	0	0	4	1	0	0	0	1	5	0	1	1	1	0	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	1	0	0	0	1																																		
3	1	0	0	0	0																																		
4	1	0	0	0	1																																		
5	0	1	1	1	0																																		
D		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table> <p>Code D = (3,2,2,2,3; 0,5,2,2,3)</p>		1	2	3	4	5	1	0	1	1	1	0	2	0	1	0	0	1	3	0	1	0	0	1	4	0	1	0	0	1	5	0	1	1	1	0	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	0	1	0	0	1																																		
3	0	1	0	0	1																																		
4	0	1	0	0	1																																		
5	0	1	1	1	0																																		

E		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table> <p>Code E = (4,1,3,1,4; 0,5,3,3,2)</p>		1	2	3	4	5	1	0	1	1	1	1	2	0	1	0	0	0	3	0	1	1	1	0	4	0	1	0	0	0	5	0	1	1	1	1	
	1	2	3	4	5																																		
1	0	1	1	1	1																																		
2	0	1	0	0	0																																		
3	0	1	1	1	0																																		
4	0	1	0	0	0																																		
5	0	1	1	1	1																																		
F		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table> <p>Code F = (4,1,3,1,1; 0,5,2,2,1)</p>		1	2	3	4	5	1	0	1	1	1	1	2	0	1	0	0	0	3	0	1	1	1	0	4	0	1	0	0	0	5	0	1	0	0	0	
	1	2	3	4	5																																		
1	0	1	1	1	1																																		
2	0	1	0	0	0																																		
3	0	1	1	1	0																																		
4	0	1	0	0	0																																		
5	0	1	0	0	0																																		
G		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>3</th> <td>1</td> <td>0</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <th>4</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table> <p>Code G = (3,1,4,2,3; 3,2,3,3,2)</p>		1	2	3	4	5	1	0	1	1	1	0	2	1	0	0	0	0	3	1	0	1	1	1	4	1	0	0	0	1	5	0	1	1	1	0	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	1	0	0	0	0																																		
3	1	0	1	1	1																																		
4	1	0	0	0	1																																		
5	0	1	1	1	0																																		
H		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p>Code H = (2,2,4,2,2; 0,5,1,1,5)</p>		1	2	3	4	5	1	0	1	0	0	1	2	0	1	0	0	1	3	0	1	1	1	1	4	0	1	0	0	1	5	0	1	0	0	1	
	1	2	3	4	5																																		
1	0	1	0	0	1																																		
2	0	1	0	0	1																																		
3	0	1	1	1	1																																		
4	0	1	0	0	1																																		
5	0	1	0	0	1																																		

I	<p style="text-align: center;">I</p>	<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table> <p style="text-align: center;"><i>Code I = (3,1,1,1,3; 0,2,5,2,0)</i></p>		1	2	3	4	5	1	0	1	1	1	0	2	0	0	1	0	0	3	0	0	1	0	0	4	0	0	1	0	0	5	0	1	1	1	0	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	0	0	1	0	0																																		
3	0	0	1	0	0																																		
4	0	0	1	0	0																																		
5	0	1	1	1	0																																		
J	<p style="text-align: center;">J</p>	<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>4</th> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table> <p style="text-align: center;"><i>Code J = (3,1,1,2,1; 1,2,4,1,0)</i></p>		1	2	3	4	5	1	0	1	1	1	0	2	0	0	1	0	0	3	0	0	1	0	0	4	1	0	1	0	0	5	0	1	0	0	0	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	0	0	1	0	0																																		
3	0	0	1	0	0																																		
4	1	0	1	0	0																																		
5	0	1	0	0	0																																		
K	<p style="text-align: center;">K</p>	<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p style="text-align: center;"><i>Code K = (2,2,2,2,2; 0,5,1,2,2)</i></p>		1	2	3	4	5	1	0	1	0	0	1	2	0	1	0	1	0	3	0	1	1	0	0	4	0	1	0	1	0	5	0	1	0	0	1	
	1	2	3	4	5																																		
1	0	1	0	0	1																																		
2	0	1	0	1	0																																		
3	0	1	1	0	0																																		
4	0	1	0	1	0																																		
5	0	1	0	0	1																																		
L	<p style="text-align: center;">L</p>	<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table> <p style="text-align: center;"><i>Code L = (1,1,1,1,4; 0,5,1,1,1)</i></p>		1	2	3	4	5	1	0	1	0	0	0	2	0	1	0	0	0	3	0	1	0	0	0	4	0	1	0	0	0	5	0	1	1	1	1	
	1	2	3	4	5																																		
1	0	1	0	0	0																																		
2	0	1	0	0	0																																		
3	0	1	0	0	0																																		
4	0	1	0	0	0																																		
5	0	1	1	1	1																																		

M	<p style="text-align: center;">M</p>	<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>4</th> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	0	1	0	2	1	0	1	0	1	3	1	0	1	0	1	4	1	0	1	0	1	5	1	0	0	0	1	
	1	2	3	4	5																																		
1	0	1	0	1	0																																		
2	1	0	1	0	1																																		
3	1	0	1	0	1																																		
4	1	0	1	0	1																																		
5	1	0	0	0	1																																		
N	<p style="text-align: center;">N</p>	<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	0	0	1	2	0	1	1	0	1	3	0	1	1	1	1	4	0	1	0	1	1	5	0	1	0	0	1	
	1	2	3	4	5																																		
1	0	1	0	0	1																																		
2	0	1	1	0	1																																		
3	0	1	1	1	1																																		
4	0	1	0	1	1																																		
5	0	1	0	0	1																																		
O	<p style="text-align: center;">O</p>	<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>4</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	1	1	0	2	1	0	0	0	1	3	1	0	0	0	1	4	1	0	0	0	1	5	0	1	1	1	0	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	1	0	0	0	1																																		
3	1	0	0	0	1																																		
4	1	0	0	0	1																																		
5	0	1	1	1	0																																		
P	<p style="text-align: center;">P</p>	<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>5</th> <td>1</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	1	1	0	2	0	1	0	0	1	3	0	1	1	1	0	4	0	1	0	0	0	5	1	1	1	0	0	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	0	1	0	0	1																																		
3	0	1	1	1	0																																		
4	0	1	0	0	0																																		
5	1	1	1	0	0																																		

Q		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>4</th> <td>1</td> <td>0</td> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	1	1	0	2	1	0	0	0	1	3	1	0	0	0	1	4	1	0	0	1	1	5	0	1	1	1	1	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	1	0	0	0	1																																		
3	1	0	0	0	1																																		
4	1	0	0	1	1																																		
5	0	1	1	1	1																																		
R		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	1	1	0	2	0	1	0	0	1	3	0	1	1	1	0	4	0	1	0	0	1	5	0	1	0	0	1	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	0	1	0	0	1																																		
3	0	1	1	1	0																																		
4	0	1	0	0	1																																		
5	0	1	0	0	1																																		
S		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <th>5</th> <td>1</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	1	1	0	2	1	0	0	0	0	3	0	1	1	0	0	4	0	0	0	1	0	5	1	1	1	0	0	
	1	2	3	4	5																																		
1	0	1	1	1	0																																		
2	1	0	0	0	0																																		
3	0	1	1	0	0																																		
4	0	0	0	1	0																																		
5	1	1	1	0	0																																		
T		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>5</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	1	1	1	2	1	0	1	0	0	3	0	0	1	0	0	4	0	0	1	0	0	5	0	0	1	0	0	
	1	2	3	4	5																																		
1	0	1	1	1	1																																		
2	1	0	1	0	0																																		
3	0	0	1	0	0																																		
4	0	0	1	0	0																																		
5	0	0	1	0	0																																		

U		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	0	0	1	2	0	1	0	0	1	3	0	1	0	0	1	4	0	1	0	0	1	5	0	0	1	1	0	
	1	2	3	4	5																																		
1	0	1	0	0	1																																		
2	0	1	0	0	1																																		
3	0	1	0	0	1																																		
4	0	1	0	0	1																																		
5	0	0	1	1	0																																		
<p>Code U = (2,2,2,2,2; 0,4,1,1,4)</p>																																							
V		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <th>5</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	0	0	0	0	2	1	0	0	0	1	3	1	0	0	0	1	4	0	1	0	1	0	5	0	0	1	0	0	
	1	2	3	4	5																																		
1	0	0	0	0	0																																		
2	1	0	0	0	1																																		
3	1	0	0	0	1																																		
4	0	1	0	1	0																																		
5	0	0	1	0	0																																		
<p>Code V = (0,2,2,2,1; 2,1,1,1,2)</p>																																							
W		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>4</th> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	0	0	0	0	2	1	0	0	0	1	3	1	0	1	0	1	4	1	0	1	0	1	5	0	1	0	1	0	
	1	2	3	4	5																																		
1	0	0	0	0	0																																		
2	1	0	0	0	1																																		
3	1	0	1	0	1																																		
4	1	0	1	0	1																																		
5	0	1	0	1	0																																		
<p>Code W = (0,2,3,3,2; 3,1,2,1,3)</p>																																							
X		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <th>2</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <th>4</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>5</th> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table>		1	2	3	4	5	1	0	1	0	1	0	2	0	0	1	0	0	3	0	1	0	1	0	4	1	0	0	0	1	5	0	0	0	0	0	
	1	2	3	4	5																																		
1	0	1	0	1	0																																		
2	0	0	1	0	0																																		
3	0	1	0	1	0																																		
4	1	0	0	0	1																																		
5	0	0	0	0	0																																		
<p>Code X = (2,1,2,2,0; 1,2,1,2,1)</p>																																							

<p>Y</p>		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>2</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>5</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> </tbody> </table> <p>Code Y = (0,2,3,1,1; 1,1,3,1,1)</p>		1	2	3	4	5	1	0	0	0	0	0	2	1	0	0	0	1	3	0	1	1	1	0	4	0	0	1	0	0	5	0	0	1	0	0	
	1	2	3	4	5																																		
1	0	0	0	0	0																																		
2	1	0	0	0	1																																		
3	0	1	1	1	0																																		
4	0	0	1	0	0																																		
5	0	0	1	0	0																																		
<p>Z</p>		<table border="1"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> <tr> <th>2</th> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <th>3</th> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <th>5</th> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table> <p>Code Z = (4,1,1,1,5; 1,3,3,3,2)</p>		1	2	3	4	5	1	0	1	1	1	1	2	0	0	0	1	0	3	0	0	1	0	0	4	0	1	0	0	0	5	1	1	1	1	1	
	1	2	3	4	5																																		
1	0	1	1	1	1																																		
2	0	0	0	1	0																																		
3	0	0	1	0	0																																		
4	0	1	0	0	0																																		
5	1	1	1	1	1																																		