

# The Continuous Sampling Plan for Two Production Lines: The CSP-2L

PANNARAT GUAYJARERNPANISHK, TAWUN REMSUNGREN  
Faculty of Interdisciplinary Studies  
Nong Khai Campus, Khon Kaen University  
Tambon Nong Kom Koh, Muang District, Nong Khai, 43000  
THAILAND

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*Abstract:* This paper presents the CSP-2L continuous sampling plan, which is designed for product inspection on two independent production lines at the same time. The purpose of the CSP-2L is to improve the CSP-1-2L in order to reduce the number of defective products that have passed without being inspected during the temporary inspection stop, so the quality of the products is better. Therefore, if the manufacturer who uses the CSP-2L plan to inspect the production lines then resulting in higher quality of the products than the CSP-1-2L. This presentation includes the product inspection procedures and the formulas for performance measures such as average total fraction inspected (*ATFI*), average total outgoing quality (*ATOQ*), and average total outgoing quality limit (*ATOQL*) which are carried out using a Markov chain. The formulas for performance measures of the CSP-2L have been tested to be accurate. When defined, the probability of a unit produced by the process being nonconforming of line 1 and line 2 are equal ( $p_1 = p_2 = p = 0.005, 0.015$  and  $0.035$ ), the clearance number of line 1 and line 2 are equal ( $i_1 = i_2 = 10, 20, 30, 40$  and  $50$ ), the sampling fraction of line 1 and line 2 are equal ( $f_1 = f_2 = 1/2$  and  $1/3$ ) and the number of units to be found when inspection of line 1 and line 2 are in the phase of sampling inspection at the same time ( $m = i_1$  and  $2i_1$ ). Moreover, the *ATOQ* values from the CSP-2L and the CSP-1-2L plans were compared. The results showed that, the formulas for performance measures are accurate and in the case of  $p$ , the levels are low and moderate, the *ATOQ* of the CSP-2L are less than those of the CSP-1-2L in all cases. But in the case of  $p$  is at a high level, the *ATOQ* of the CSP-2L is less than those of the CSP-1-2L for some cases of  $i$ .

*Key-Words:* continuous sampling plan, Markov chain, average total fraction inspected, average total outgoing quality.

## 1 Introduction

An outgrowth of many years of research and consulting in the field of control system are developed to applied to real-world situations. The foundation of statistical tools and techniques of control system have been created by many researchers. Some relevant studies can be found in [1], [2] and [3]. A sampling plan is the one of important parts of control system. Sampling plans can adjust the efficiency of product inspection and reduce the manufacturer's workload whilst at the same time providing a rigorous statistical basis for the sampling and inspection process. The continuous sampling plans (CSPs) are the best sampling plan and used for inspecting each product unit on a production line. The result of the inspection is either conforming or non-conforming for a continuous process. The most advantages of CSPs is that the manufacturer can know the quality of the product after the inspection finished.

The CSPs have been designed and developed continuously from the past to the present. In order to check the quality of the product while it is being produced on the production belt and the characteristics of the product found on the production line. There are 2 types of continuous sampling plans, which are single-level continuous sampling plan and multi-level continuous sampling plan. The first continuous sampling plan, which was created as a single-level continuous sampling plan by Dodge in 1943, is called CSP-1 (Continuous Sampling Plan Type 1) [4]. The procedures of the CSP-1 are as follows. Start inspection with 100% inspection of units from the flow of production and continue the inspection until  $i$  successive conforming units are found, discontinue 100% inspection and start sampling inspection and inspect only a fraction  $f$  of the units, selecting a unit at random from each segment of  $1/f$  items. If a nonconforming unit is found then revert immediately to 100% inspection. When

nonconforming units are found, the manufacturer replaces or corrects all the nonconforming units found with conforming units. Therefore, the quality of the products will be better. After that, many researchers have designed and developed many sampling plans, that using CSP-1 as the basis, such as CSP-2, CSP-3, CSP-M, TCSP-1, MCSP-C, MCSP-2-C, CSP-F-L, and G-TF-CSP [5] - [11] and so on.

The CSPs have two product inspection phases: the inspection of every unit or 100% inspection phase, and the inspection of some units with sampling frequency or sampling inspection phase. While the production process is undergoing product inspection, if the product is found to be defective, that product unit will be withdrawn and replaced with a good product every time. Therefore, at the end of the inspection, the products that have passed inspection have better quality and continuous sampling plans are rectifying inspection plans. In each continuous sampling plan, in addition to designing a product inspection, formulas to measure performance were also created such as average fraction inspected (*AFI*), average outgoing quality (*AOQ*), and average outgoing quality limit (*AOQL*) of the plan. The value of the performance measure indicates the quality of the product after completion of the inspection or the number of products that are inspected when using this continuous sampling plan to inspect the production process. There are several text books concerning statistical quality control that can be found that offer reviews of CSPs [13] - [17].

The above CSPs are designed for product inspection under the condition that one inspector per production line is used. In 2005, Mayuresawan and Sudasna-Na-Ayudthaya proposed the CSP-1-2L continuous sampling plan to inspect the same type of product on two production lines at the same time but use only one inspector. The limitation of the CSP-1 -2 L is that there is only one inspector, therefore, during the 100% inspection phase, the inspector is unable to inspect two production lines at the same time. So the inspector has to temporarily stop inspecting one production line. The conditions regarding inspection sequences when inspected with the CSP-1-2L are as follows. First, start with 100% inspection at production line 1 and temporarily stop inspection at production line 2 and second, in case both production lines have to revert to 100% inspection at the same time, production line 2 will be reverted to 100% inspection and inspection of production line 1 will be temporarily stopped. There are four possible forms of the CSP-1-2L. Mayuresawan and Sudasna-Na-Ayudthaya presented the formulas for performance measures,

which consists of average total fraction inspected (*ATFI*), average total outgoing quality (*ATOQ*) and average total outgoing quality limit (*ATOQL*) [17].

With regard to the inspection procedure of the CSP-1-2L, the inspection process is complicated and there are many cases of inspection that can occur and this complexity may cause the inspector to confuse the inspection process and cause inspection errors. In addition, defective products may pass without inspection during the pause of the inspection, which causes the quality of the products to be low. Therefore, this paper has improved the inspection procedures of the CSP-1-2L in order to reduce complex inspection procedures. There is no case in which inspection is paused by replacing it with a sampling inspection to reduce the number of defective product units that have passed and expanding the sampling inspection procedures for both production lines when both production lines have a sampling inspection phase at the same time to extended restart 100% inspection. A new continuous sampling plan designated as the CSP-2L, plans to focus on the production line that is 100% inspection in progress more than the production line that is sampling inspection in progress and always beginning with the inspection on production line 1. the CSP-2L always defines that production line 1 is a production line that has a probability of a unit produced by the process being nonconforming ( $p_1$ ) more than or equal to production line 2 that has a probability of a unit produced by the process being nonconforming of production line 2 ( $p_2$ ) and still under conditions that, there is only one inspector to inspect two production lines at the same time and not 100% inspected at the same time on both production lines.

This paper has the following objectives: to design a new continuous sampling plan for two production lines, namely CSP-2L, to improve the formulas for performance measures, which are average total fraction inspected (*ATFI*), average total outgoing quality (*ATOQ*) and average total outgoing quality limit (*ATOQL*). This was carried out using a Markov chain, tested the accuracy of formulas to measure the performance of the CSP-2L and compared the *ATOQ* values from the CSP-2L and the CSP-1-2L plans.

## 2 Materials and Methods

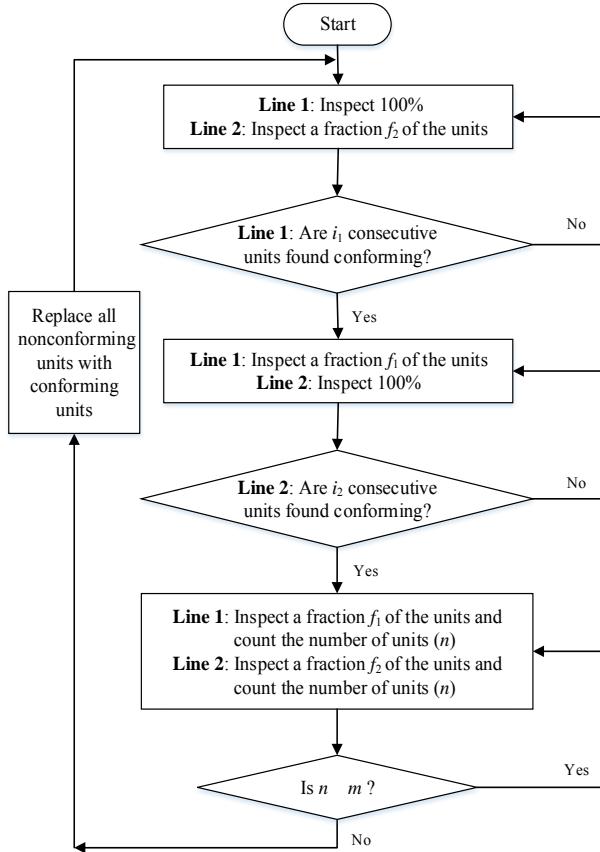
### 2.1 The Operating Procedure of the CSP -2L

In the process of the CSP-2L, the probability of a unit produced by the process being nonconforming of line 1 and line 2 are  $p_1$  and  $p_2$ , respectively. This

plan uses five parameters  $i_1, i_2, f_1, f_2$  and  $m$  which are defined by:

- $i_1$  = the clearance number of line 1,
- $i_2$  = the clearance number of line 2,
- $f_1$  = the sampling fraction of line 1,
- $f_2$  = the sampling fraction of line 2 and
- $m$  = the number of units to be found when inspection of line 1 and line 2 are in the phase of sampling inspection at the same time.

The inspection process of the CSP-2L is shown in Figure 1.



**Figure 1.** Flow diagram of the inspection process of the CSP-2L.

## 2.2 The CSP-2L Procedure as a Markov Chain

Stephens [18] used a Markov chain model to find certain measures of performance for CSP-1, CSP-2 and CSP-3 plans, assuming that the production process is in statistical control. In this paper the formulas of the CSP-2L procedure as a Markov chain is given. The CSP-2L computes three formulas, the average total fraction inspected (*ATFI*), the average total outgoing quality (*ATOQ*) and the average total outgoing quality limit (*ATOQL*). In addition, the assumption for derivation of these performance measures is both production lines of the CSP-2L are independent.

### Definition of symbols

- $S_n$  = the  $n^{\text{th}}$  state of the process,
- $P(S_n)$  = the steady-state probability for the state  $S_n$ ,
- $p_{in}$  = the probability that the process transits from state  $S_i$  to  $S_n$  in one step.

The conditions of the steady-state probabilities  $P(S_n)$  are as follows,

1.  $P(S_n) \geq 0$  for  $n = 1, 2, \dots, N$ ,
2.  $P(S_n) = \sum_{x=1}^N P(S_x) p_{xn}$  for  $n = 1, 2, \dots, N$ ,
3.  $\sum_{all n} P(S_n) = 1$ ,

where  $N$  is finite state [19].

Let  $[X_t]$  ( $t = 1, 2, \dots$ ) denote a discrete-parameter of a Markov chain with finite state space ( $S_n$ ).

The states of the process are defined, in a way similar to that of S. W. Roberts [20] and H. A. Lasater [21], as follows:

At line 1: the states of the process are defined,

$$S_j = A_{j-1} \quad (j = 1, 2, \dots, i_1+1)$$

= 100% inspection is being conducted and the last ( $j-1$ ) consecutive units inspected were conforming.

$$S_{3k+i+2} = f_1 N_k \quad (k = 0, 1, 2, \dots, i_2)$$

=  $k$  consecutive conforming units found during 100% inspection of line 2, while inspection of line 1 is in effect at sampling inspection phase and the last unit was not selected for inspection.

$$S_{3k+i+3} = f_1 I_n \quad (k = 0, 1, 2, \dots, i_2)$$

=  $k$  consecutive conforming units found during 100% inspection of line 2, while inspection of line 1 is in effect at sampling inspection phase and the last unit was selected which was found to be conforming.

$$S_{3k+i+4} = f_1 I_d \quad (k = 0, 1, 2, \dots, i_2)$$

=  $k$  consecutive conforming units found during 100% inspection of line 2, while inspection of line 1 is in effect at sampling inspection phase and the last unit was selected which was found to be nonconforming.

$$S_{3l+i+3i_2+2} = f_1 N^{l-1} \quad (l = 1, 2, \dots, m)$$

= the last unit was not inspected and the number of inspection units found during inspection is  $l-1$ , when the inspection of line 1 and line 2 are on the sampling inspection phase at the same time.

$$S_{3l+i_1+3i_2+3} = f_1 I_n^l (l = 1, 2, \dots, m)$$

= the last unit was found to be conforming and the number of inspection units found during inspection is  $l-1$ , when the inspection of line 1 and line 2 are on the sampling inspection phase at the same time.

$$S_{3l+i_1+3i_2+4} = f_1 I_d^l (l = 1, 2, \dots, m)$$

= the last unit was found to be nonconforming and the number of inspection units found during inspection is  $l-1$ , when the inspection of line 1 and line 2 are on the sampling inspection phase at the same time.

At line 2: the states of the process are defined,

$$S_{3j+i_1+3i_2+3m+5} = f_2 N_j (j = 0, 1, 2, \dots, i_1)$$

=  $j$  consecutive conforming units found during 100% inspection of line 1, while inspection of line 2 is in effect at sampling inspection phase and the last unit was not selected for inspection.

$$S_{3j+i_1+3i_2+3m+6} = f_2 I_n_j (j = 0, 1, 2, \dots, i_1)$$

=  $j$  consecutive conforming units found during 100% inspection of line 1, while inspection of line 2 is in effect at sampling inspection phase and the last unit was selected which was found to be conforming.

$$S_{3j+i_1+3i_2+3m+7} = f_2 I_d_j (j = 0, 1, 2, \dots, i_1)$$

=  $j$  consecutive conforming units found during 100% inspection of line 1, while inspection of line 2 is in effect at sampling inspection phase and the last unit was selected which was found to be nonconforming.

$$S_{k+4i_1+3i_2+3m+7} = B_{k-1} (k = 1, 2, \dots, i_2+1)$$

= 100% inspection is being conducted and the last  $(k-1)$  consecutive units inspected were conforming.

$$S_{3l+4i_1+4i_2+3m+6} = f_2 N_{l-1}^l (l = 1, 2, \dots, m)$$

= the last unit was not inspected and the number of inspection units found during inspection is  $l-1$ , when the inspection of line 1 and line 2 are on the sampling inspection phase at the same time.

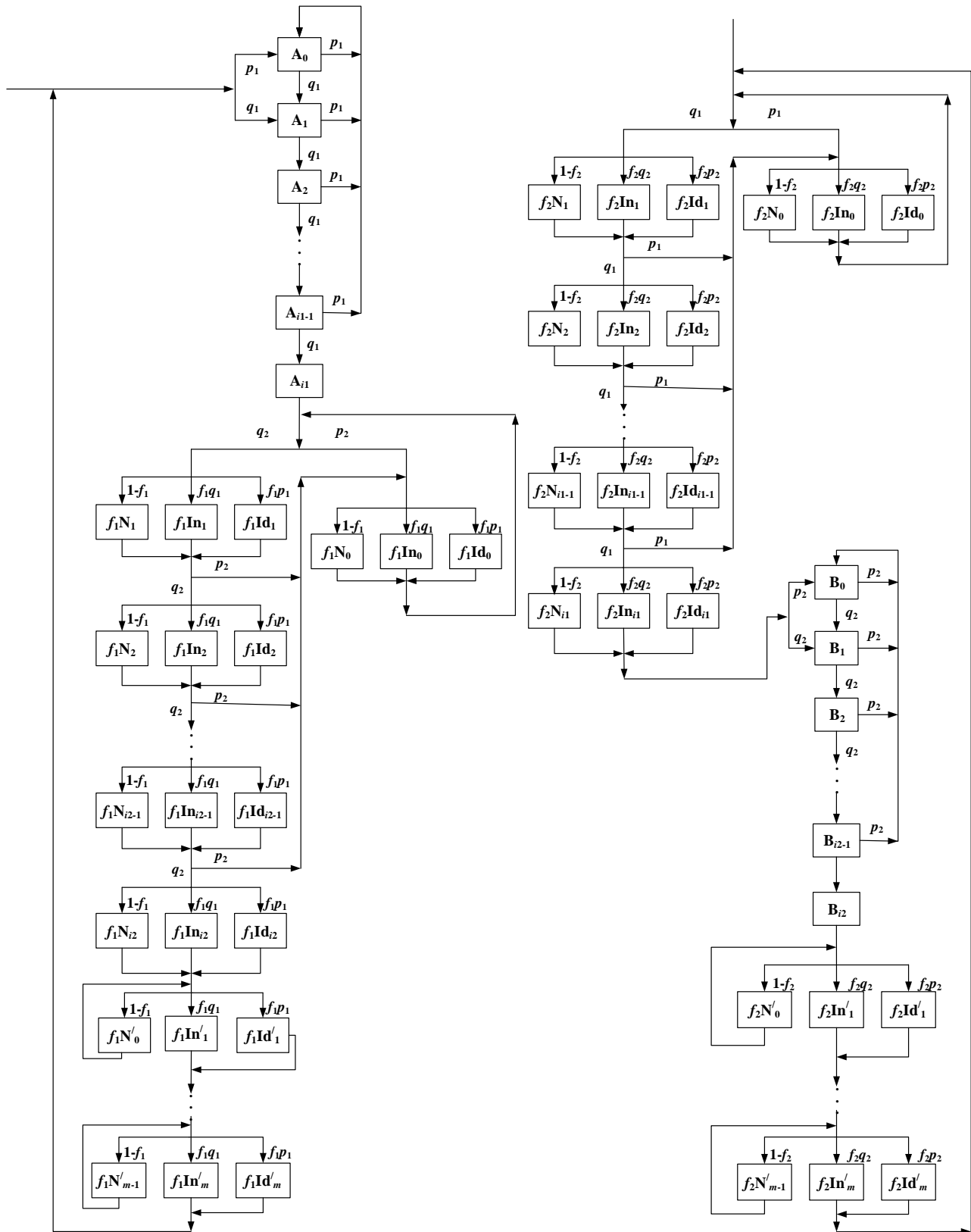
$$S_{3l+4i_1+4i_2+3m+7} = f_2 I_n^l (l = 1, 2, \dots, m)$$

= the last unit was found to be conforming and the number of inspection units found during inspection is  $l-1$ , when the inspection of line 1 and line 2 are on the sampling inspection phase at the same time.

$$S_{3l+4i_1+4i_2+3m+8} = f_2 I_d^l (l = 1, 2, \dots, m)$$

= the last unit was found to be nonconforming and the number of inspection units found during inspection is  $l-1$ , when the inspection of line 1 and line 2 are on the sampling inspection phase at the same time.

The set of  $N$  states (when  $N = 4i_1+4i_2+6m+8$ ) defined above completely describe the mutually exclusive phases of inspection for the CSP-2L procedure. A flow chart showing the description of the process by means of states and transitions between different states of the CSP-2L is shown in Figure 2.



a. Line 1  
b. Line 2  
**Figure 2.** Flow chart of states and transitions of the CSP-2L.

From Figure 2 and the definition of a Markov chain, the transition probabilities of a Markov chain and the transition matrix of a Markov chain, that are as follows [19],

**Definition 1:**

Let  $\{X_0, X_1, X_2, \dots\}$  be a sequence of discrete random variables. Then  $\{X_0, X_1, \dots\}$  is a Markov chain if it satisfies the Markov property:

$P(X_{t+1} = s | X_t = s_t, \dots, X_0 = s_0) = P(X_{t+1} = s | X_t = s_t)$ , for all  $t = 1, 2, \dots$  and for all states  $s_0, s_1, \dots, s_t, s$ .

**Definition 2:**

Let  $\{X_0, X_1, X_2, \dots\}$  be a Markov chain with state space  $S$ , where  $S$  has size  $n$  (possibly infinite). The transition probabilities of the Markov chain are

$$p_{ij} = P(X_{t+1} = j | X_t = i) \text{ for } i, j \in S, t = 0, 1, 2, \dots$$

**Definition 3:**

“The transition matrix of the Markov chain is  $P = (p_{ij})$ ”.

We get the transition matrix of line 1 and the transition matrix of line 2 of the CSP-2L shown in Table 1 and Table 2, respectively.

**Table 1.** One-step transition probability matrix of line 1 of the CSP-2L.

	$A_s$	$A_i$	...	$A_{i-1}$	$f_i N_0$	$f_i In_0$	$f_i Id_0$	$f_i N_1$	$f_i In_1$	$f_i Id_1$	...	$f_i N_2$	$f_i In_2$	$f_i Id_2$	$f_i N'_0$	$f_i In'_1$	$f_i Id'_1$	$f_i N'_1$	$f_i In'_2$	$f_i Id'_2$	...	$f_i N'_{m-1}$	$f_i In'_m$	$f_i Id'_m$
$A_s$	$p_s$	$q_s$	...								...										...			
$A_i$	$p_i$		...								...										...			
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
$A_{i-1}$	$p_i$		...	$q_i$							...										...			
$A_{i-1}$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$	$q_2(1-f_i)$	$q_2 f_i q_1$	$q_2 f_i p_1$	...										...			
$f_i N_0$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$	$q_2(1-f_i)$	$q_2 f_i q_1$	$q_2 f_i p_1$	...										...			
$f_i In_0$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$	$q_2(1-f_i)$	$q_2 f_i q_1$	$q_2 f_i p_1$	...										...			
$f_i Id_0$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$	$q_2(1-f_i)$	$q_2 f_i q_1$	$q_2 f_i p_1$	...										...			
$f_i N_1$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$				...										...			
$f_i In_1$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$				...										...			
$f_i Id_1$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$				...										...			
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
$f_i N_{2-1}$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$				...	$q_2(1-f_i)$	$q_2 f_i q_1$	$q_2 f_i p_1$							...			
$f_i In_{2-1}$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$				...	$q_2(1-f_i)$	$q_2 f_i q_1$	$q_2 f_i p_1$							...			
$f_i Id_{2-1}$			...		$p_2(1-f_i)$	$p_2 f_i q_1$	$p_2 f_i p_1$				...	$q_2(1-f_i)$	$q_2 f_i q_1$	$q_2 f_i p_1$							...			
$f_i N_2$			...								...				$1-f_i$	$f_i q_1$	$f_i p_1$				...			
$f_i In_2$			...								...				$1-f_i$	$f_i q_1$	$f_i p_1$				...			
$f_i Id_2$			...								...				$1-f_i$	$f_i q_1$	$f_i p_1$				...			
$f_i N'_0$			...								...				$1-f_i$	$f_i q_1$	$f_i p_1$				...			
$f_i In'_1$			...								...							$1-f_i$	$f_i q_1$	$f_i p_1$	...			
$f_i Id'_1$			...								...							$1-f_i$	$f_i q_1$	$f_i p_1$	...			
$f_i N'_1$			...								...							$1-f_i$	$f_i q_1$	$f_i p_1$	...			
$f_i In'_2$			...								...										...			
$f_i Id'_2$			...								...										...			
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
$f_i N'_{m-1}$			...								...										...	$1-f_i$	$f_i q_1$	$f_i p_1$
$f_i In'_m$	$p_s$	$q_s$	...								...										...			
$f_i Id'_m$	$p_s$	$q_s$	...								...										...			



### 2.3 Test of the Accuracy of Performance Measures for the CSP-2L

For testing the accuracy of the formulas to measure the performance that are defined for the CSP-2L, the results from the formulas were compared with the values obtained from extensive simulations and the simulation was repeated 500 times which more than [17] who repeated 60 times.

Two different lines were examined for the probability of a unit produced by the process being nonconforming of line 1 and line 2 are  $p_1$  and  $p_2$ , respectively. When  $p_1 = p_2 = p = 0.005, 0.015$  and  $0.035$ . For each  $p$ , values of  $i_1 = i_2 = 10, 20, 30, 40$  and  $50$ , values of  $f_1 = f_2 = 1/2$  and  $1/3$ , and values of  $m = i_1$  and  $2i_1$ .

The average total fraction inspected (*ATFI*) and the average total outgoing quality (*ATOQ*) were calculated, and then compared the values *ATFI* and *ATOQ* from the simulations with the values of *ATFI* and *ATOQ* computed from the formulas.

The percentage difference of the *ATFI* values (%dif\_*ATFI*) is the ratio of the difference between the *ATFI* values from formula and simulations over the *ATFI* values from simulations shown in equation (1) and the percentage difference of the *ATOQ* values (%dif\_*ATOQ*) is the ratio of the difference between the *ATOQ* values from formula and simulations over the *ATOQ* values from simulations shown in equation (2). The *ATFI* and *ATOQ* formulas are accepted as the accurate formulas if %dif\_*ATFI* and %dif\_*ATOQ* are less than or equal to 2% [11].

$$\%dif\_ATFI = \left| \frac{ATFI\_F - ATFI\_S}{ATFI\_S} \right| \times 100 \quad (1)$$

$$\%dif\_ATOQ = \left| \frac{ATOQ\_F - ATOQ\_S}{ATOQ\_S} \right| \times 100 \quad (2)$$

Where

*ATFI\_F* was the *ATFI* value from the formula of the CSP-2L,

*ATFI\_S* was the *ATFI* value from the simulations of the CSP-2L,

*ATOQ\_F* was the *ATOQ* value from the formula of the CSP-2L,

*ATOQ\_S* was the *ATOQ* value from the simulations of the CSP-2L.

## 3 Results

### 3.1 The Formulas to Measure the Performance of the CSP-2L

Let  $p_1$  and  $p_2$  be the probability of a unit produced by the process being nonconforming of line 1 and line 2, when  $q_1 = 1 - p_1$  and  $q_2 = 1 - p_2$ . The following

formulas to measure the performance may be obtained:

The average number of units inspected under the 100% inspection of line 1,  $u_1$ :

$$u_1 = \frac{1 - q_1^{i_1}}{p_1 q_1^{i_1}}. \quad (3)$$

The average number of units inspected under the 100% inspection of line 2,  $u_2$ :

$$u_2 = \frac{1 - q_2^{i_2}}{p_2 q_2^{i_2}}. \quad (4)$$

The average number of units passed under the sampling inspection of line 1,  $v_1$ :

$$v_1 = \frac{f_1(1 - q_2^{i_2}) + m(q_2^{i_2} - q_2^{i_2+1})}{f_1 p_2 q_2^{i_2}}. \quad (5)$$

The average number of units passed under the sampling inspection of line 2,  $v_2$ :

$$v_2 = \frac{f_2(1 - q_1^{i_1}) + m(q_1^{i_1} - q_1^{i_1+1})}{f_2 p_1 q_1^{i_1}}. \quad (6)$$

The average total fraction inspected, *ATFI*:

$$ATFI = \frac{u_1 + f_1 v_1 + u_2 + f_2 v_2}{u_1 + v_1 + u_2 + v_2}. \quad (7)$$

The average total outgoing quality, *ATOQ*:

$$ATOQ = \frac{p_1 + p_2}{2} [1 - ATFI]. \quad (8)$$

The average total outgoing quality limit, *ATOQL*:

$$ATOQL = \max_{\text{all } p_1, p_2} ATOQ. \quad (9)$$

Details of the steps for finding the performance measure formulas are presented in the Appendix.

### 3.2 The Accuracy of Formulas to Measure the Performance for the CSP-2L

#### 3.2.1 The *ATFI* Formula

The *ATFI* values from the formula (*ATFI\_F*), the *ATFI* values from the simulations (*ATFI\_S*) and the percentage difference of the *ATFI* values (%dif\_*ATFI*) for each set of  $p_1, p_2, i_1, i_2, f_1$  and  $f_2$  values are shown in Table 3 and Table 4 for  $m = i_1$  and  $2i_1$  values, respectively. It was also found that, at all levels of  $p_1$  and  $p_2$ , %dif\_*ATFI* was less than 2 % for all sets of  $i_1, i_2, f_1, f_2$  and  $m$  values. Therefore, the simulations signified that the *ATFI* formula is accurate.

#### 3.2.2 The *ATOQ* and the *ATOQL* Formulas

In Table 5 and Table 6, the *ATOQ* values from the formula (*ATOQ\_F*), the *ATOQ* values from the simulations (*ATOQ\_S*) and the percentage



difference of the *ATOQ* values (%dif\_*ATOQ*) for each set of  $p_1, p_2, i_1, i_2, f_1$  and  $f_2$  values are shown for  $m = i_1$  and  $2i_1$  values, respectively. It was found that, at all levels of  $p_1$  and  $p_2$ , the %dif\_*ATOQ* was less than 2% for all sets of  $i_1, i_2, f_1, f_2$  and  $m$  values. So

the simulations implied that the *ATOQ* formula is accurate. In addition, from the relationship between the *ATOQL* and the *ATOQ*, the *ATOQL* formula is also accurate.

**Table 3.** The *ATFI\_F*, *ATFI\_S* and %dif\_*ATFI* values of the CSP-2L when  $m = i_1$

$p_1 = p_2$	$i_1 = i_2$	$f_1 = f_2 = 1/2$			$f_1 = f_2 = 1/3$		
		<i>ATFI_F</i>	<i>ATFI_S</i>	%dif_ <i>ATFI</i>	<i>ATFI_F</i>	<i>ATFI_S</i>	%dif_ <i>ATFI</i>
0.005	10	0.62673	0.62651	0.04	0.46889	0.46219	1.45
	20	0.62831	0.62849	0.03	0.47093	0.46488	1.30
	30	0.62991	0.62970	0.03	0.47300	0.47282	0.04
	40	0.63152	0.63177	0.04	0.47510	0.47396	0.24
	50	0.63314	0.63397	0.13	0.47722	0.47586	0.29
0.015	10	0.63025	0.63025	0.00	0.47345	0.46709	1.36
	20	0.63513	0.63490	0.04	0.47985	0.47353	1.33
	30	0.64010	0.63946	0.10	0.48647	0.48585	0.13
	40	0.64514	0.64367	0.23	0.49330	0.49012	0.65
	50	0.65022	0.64859	0.25	0.50030	0.49605	0.86
0.035	10	0.63753	0.63783	0.05	0.48304	0.47748	1.1
	20	0.64938	0.64680	0.40	0.49914	0.49034	1.79
	30	0.66138	0.65707	0.65	0.51610	0.51079	1.04
	40	0.67322	0.66682	0.95	0.53354	0.52399	1.82
	50	0.68458	0.68071	0.57	0.55097	0.54473	1.15

**Table 4.** The *ATFI\_F*, *ATFI\_S* and %dif\_*ATFI* values of the CSP-2L when  $m = 2i_1$

$p_1 = p_2$	$i_1 = i_2$	$f_1 = f_2 = 1/2$			$f_1 = f_2 = 1/3$		
		<i>ATFI_F</i>	<i>ATFI_S</i>	%dif_ <i>ATFI</i>	<i>ATFI_F</i>	<i>ATFI_S</i>	%dif_ <i>ATFI</i>
0.005	10	0.58488	0.58479	0.02	0.41841	0.41426	1.00
	20	0.58631	0.58658	0.05	0.42003	0.41635	0.88
	30	0.58776	0.58766	0.02	0.42168	0.42160	0.02
	40	0.58923	0.58955	0.05	0.42336	0.42278	0.14
	50	0.59073	0.59159	0.15	0.42508	0.42448	0.14
0.015	10	0.58807	0.58831	0.04	0.42203	0.41822	0.91
	20	0.59259	0.59264	0.01	0.42723	0.42336	0.91
	30	0.59732	0.59697	0.06	0.43274	0.43242	0.07
	40	0.60225	0.60109	0.19	0.43857	0.43636	0.51
	50	0.60736	0.60604	0.22	0.44471	0.44171	0.68
0.035	10	0.59486	0.59575	0.15	0.42986	0.42691	0.69
	20	0.60651	0.60463	0.31	0.44368	0.43779	1.35
	30	0.61914	0.61543	0.60	0.45924	0.45500	0.93
	40	0.63252	0.62627	1.00	0.47640	0.46806	1.78
	50	0.64630	0.64291	0.53	0.49489	0.48988	1.02

### 3.3 The Comparison of the *ATOQ* values between the CSP-1-2L and the CSP-2L plans.

To say which production line is better quality, it can be considered from the *ATOQ* value. If the *ATOQ* value of any plan is lower, that plan will have better quality production lines.

Therefore, the values of *ATOQ* for the CSP-2L were compared with the values of *ATOQ* for the

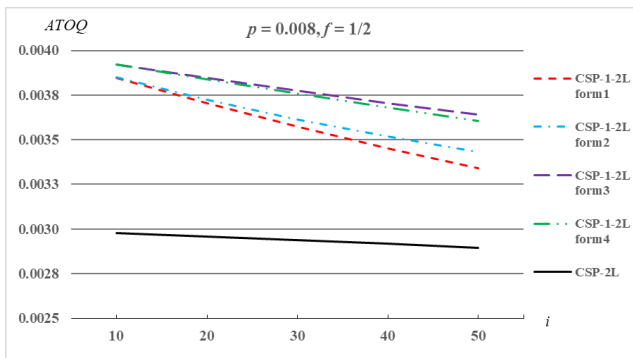
CSP-1-2L. When defined, the values of the parameters are  $p_1 = p_2 = p = 0.008, 0.01$  and  $0.03$ . For each  $p$ , values of  $i_1 = i_2 = i = 10, 20, 30, 40$  and  $50$ , values of  $f_1 = f_2 = f = 1/2$  and  $1/3$ , and the value of  $m = i$ . The *ATOQ* values of the CSP-2L and the CSP-1-2L plans are shown in Figure 3 to Figure 5.

**Table 5.** The  $ATOQ$  F,  $ATOQ$  S and %dif  $ATOQ$  values of the CSP-2L when  $m = i_1$

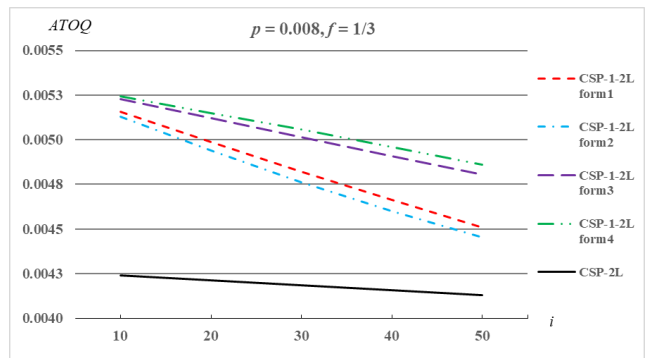
$p_1 = p_2$	$i_1 = i_2$	$f_1 = f_2 = 1/2$			$f_1 = f_2 = 1/3$		
		$ATOQ\_F$	$ATOQ\_S$	%dif $ATOQ$	$ATOQ\_F$	$ATOQ\_S$	%dif $ATOQ$
0.005	10	0.00187	0.00185	0.85	0.00266	0.00263	0.89
	20	0.00186	0.00183	1.33	0.00265	0.00266	0.49
	30	0.00185	0.00183	1.17	0.00263	0.00267	1.42
	40	0.00184	0.00186	0.74	0.00262	0.00259	1.45
	50	0.00183	0.00181	1.20	0.00261	0.00257	1.64
0.015	10	0.00555	0.00564	1.61	0.00790	0.00794	0.57
	20	0.00547	0.00537	1.85	0.00780	0.00767	1.71
	30	0.00540	0.00535	0.91	0.00770	0.00760	1.41
	40	0.00532	0.00525	1.37	0.00760	0.00746	1.84
	50	0.00525	0.00515	1.80	0.00750	0.00744	0.78
0.035	10	0.01269	0.01263	0.41	0.01809	0.01785	1.39
	20	0.01227	0.01213	1.19	0.01753	0.01732	1.20
	30	0.01185	0.01172	1.19	0.01694	0.01708	0.84
	40	0.01144	0.01146	0.22	0.01633	0.01612	1.29
	50	0.01104	0.01093	1.01	0.01572	0.01550	1.37

**Table 6.** The  $ATOQ$  F,  $ATOQ$  S and %dif  $ATOQ$  values of the CSP-2L when  $m = 2i_1$

$p_1 = p_2$	$i_1 = i_2$	$f_1 = f_2 = 1/2$			$f_1 = f_2 = 1/3$		
		$ATOQ\_F$	$ATOQ\_S$	%dif $ATOQ$	$ATOQ\_F$	$ATOQ\_S$	%dif $ATOQ$
0.005	10	0.00208	0.00204	1.57	0.00291	0.00288	1.02
	20	0.00207	0.00204	1.42	0.00290	0.00290	0.11
	30	0.00206	0.00208	0.90	0.00289	0.00289	0.17
	40	0.00205	0.00204	0.84	0.00288	0.00284	1.45
	50	0.00205	0.00202	1.37	0.00287	0.00286	0.51
0.015	10	0.00618	0.00612	0.99	0.00867	0.00856	1.28
	20	0.00611	0.00601	1.63	0.00859	0.00852	0.80
	30	0.00604	0.00597	1.22	0.00851	0.00842	1.04
	40	0.00597	0.00590	1.19	0.00842	0.00833	1.07
	50	0.00589	0.00583	1.00	0.00833	0.00825	0.98
0.035	10	0.01418	0.01397	1.50	0.01995	0.01964	1.59
	20	0.01377	0.01356	1.58	0.01947	0.01920	1.42
	30	0.01333	0.01312	1.56	0.01893	0.01866	1.41
	40	0.01286	0.01293	0.53	0.01833	0.01803	1.66
	50	0.01238	0.01241	0.28	0.01768	0.01740	1.61

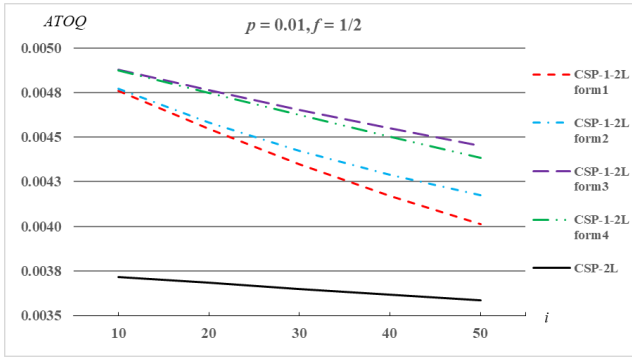


(c)

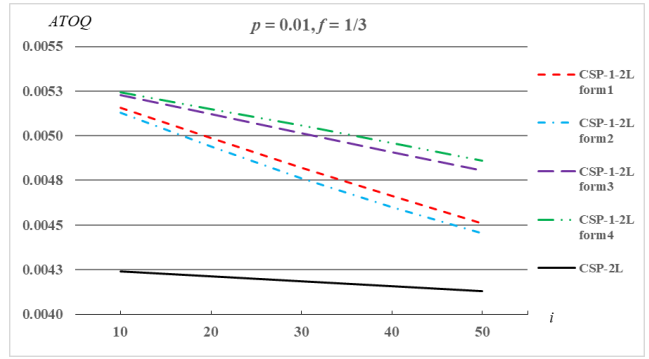


(d)

**Figure 3.** The  $ATOQ$  values of the CSP-1-2L and the CSP-2L plans when  $p = 0.008$ .

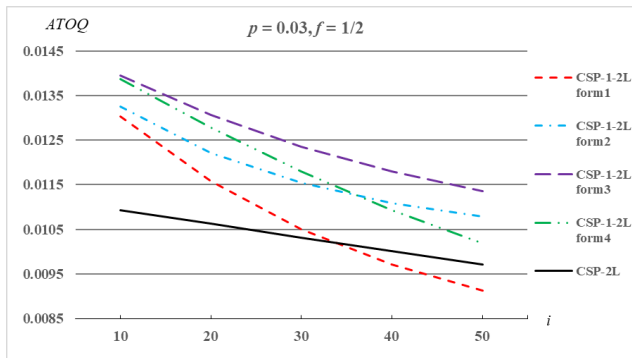


(e)

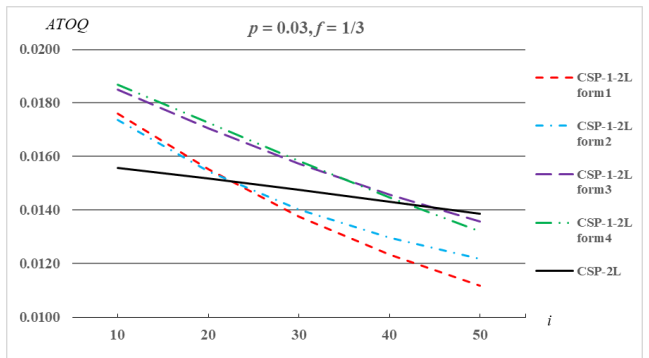


(f)

**Figure 4.** The  $ATOQ$  values of the CSP-1-2L and the CSP-2L plans when  $p = 0.01$ .



(g)



(h)

**Figure 5.** The  $ATOQ$  values of the CSP-1-2L and the CSP-2L plans when  $p = 0.03$ .

It can be seen from Figure 3 to Figure 5 that, in the case of  $p$ , the levels are low (0.008, c, d) and moderate (0.01, e, f), the  $ATOQ$  of the CSP-2L are less than those of the CSP-1-2L in all cases. In the case of  $p$  is at a high level (0.03, g, h), the  $ATOQ$  of the CSP-2L is less than those of the CSP-1-2L only in cases of  $i = 10, 20$  and  $30$  and  $i = 10$  and  $20$  for  $r = 2$  and  $3$ , respectively.

This means that for all sets of  $i$  and  $r$  values when the probability of a unit produced by the process being nonconforming of both lines are at low or moderate levels, and for all sets of  $r$  and  $i$  are at low values when the probability of a unit produced by the process being nonconforming of both lines is at a high level, the CSP-2L yields better product quality than those of the CSP-1-2L.

## 4 Conclusion

In this paper, the new continuous sampling plan was designed for the inspection of two production lines, which are independent at the same time, namely CSP-2L. The CSP-2L has given priority to the production line that is inspection at 100% inspection phase more than the production line that is inspection at sampling inspection phase and defines the probability of a unit produced by the process

being nonconforming of the first production line is not less than the second production line. Moreover, the formulas to measure the performance of the CSP-2L such as the average total fraction inspected ( $ATFI$ ) and the average total outgoing quality ( $ATOQ$ ) are derived using a Markov chain, assuming that the production process is under statistical control.

In testing the accuracy of the formulas to measure the performance of the CSP-2L uses the percentage difference of the  $ATFI$  values ( $\%dif\_ATFI$ ) and the percentage difference of the  $ATOQ$  values ( $\%dif\_ATOQ$ ). If  $\%dif\_ATFI$  and  $\%dif\_ATOQ$  are less than or equal to 2%, then the formulas will be accepted to be accurate [11]. The test results showed that,  $\%dif\_ATFI$  and  $\%dif\_ATOQ$  were found to agree within 2% in all cases. Therefore, the formulas to measure the performance of the CSP-2L are accurate.

The comparison results of the  $ATOQ$  values between the CSP-1-2L and the CSP-2L plans showed that, for all sets of  $i$  and  $r$  values when  $p$  are at low or moderate levels, and for all sets of  $r$  and  $i$  are at low values when  $p$  is at a high level, the CSP-2L yields better product quality than those of the CSP-1-2L.

## 5 Discussion

The advantage of the CSP-2L plan is that it has reduced the number of defective product units that have passed, resulting in higher quality of the products when compare with the CSP-1-2L plan when the probability of a unit produced by the process being nonconforming of both lines are low or moderate. However, it will also increase the number of inspection units. As a manufacturer that considers operating costs as the primary factor, if the number of inspection units increases then the operating costs will be higher. So, the selection of plan for inspection has to be based on the aims of the manufacturer. In the case of the manufacturer who need the high quality of production and low operating costs, this is possible under the CSP-2L conditions. At present, there is nothing designed to solve this objective. Therefore, the aforementioned objective will be a way to design a new plan in the future. So, in the future research, the authors are interested in sampling plan that can reduce operating costs and make high quality of production at the same time.

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## Appendix

### Derivation of Performance Measures of the CSP-2L

At line 1: from the first and second conditions of the steady-state probabilities  $P(S_n)$  and relations from Table 1, we get

$$P(A_0) = p_1 \sum_{j=0}^{i_1} P(A_j), \quad (10)$$

$$P(A_j) = \frac{q_1^j}{1-q_1^i} P(A_0); j = 1, 2, \dots, i_1, \quad (11)$$

$$\begin{aligned} &P(f_1N_0) + P(f_1In_0) + P(f_1Id_0) \\ &= \frac{1-q_2^{i_2}}{q_2^{i_2}} \frac{q_1^{i_1}}{1-q_1^i} P(A_0), \end{aligned} \quad (12)$$

$$\begin{aligned} &P(f_1N_k) + P(f_1In_k) + P(f_1Id_k) \\ &= \frac{q_2^k}{q_2^{i_2}} \frac{q_1^{i_1}}{1-q_1^i} P(A_0); k = 1, 2, \dots, i_2, \end{aligned} \quad (13)$$

$$\begin{aligned} &P(f_1N'_{l-1}) + P(f_1In'_l) + P(f_1Id'_l) \\ &= \frac{1}{f_1} \frac{q_1^{i_1}}{1-q_1^i} P(A_0); l = 1, 2, \dots, m. \end{aligned} \quad (14)$$

From the third condition, we get

$$\begin{aligned} &P(A_0) + \sum_{j=1}^{i_1} P(A_j) + P(f_1N_0) + P(f_1In_0) + P(f_1Id_0) \\ &+ \sum_{k=1}^{i_2} P(f_1N_k) + P(f_1In_k) + P(f_1Id_k) \\ &+ \sum_{l=1}^m P(f_1N'_{l-1}) + P(f_1In'_l) + P(f_1Id'_l) = 1. \end{aligned}$$

From equations (11) to (14), it can be written (10) as

$$\begin{aligned} &P(A_0) \\ &= \frac{f_1 p_1 p_2 q_2^{i_2} (1-q_1^i)}{f_1 p_2 q_2^{i_2} (1-q_1^i) + f_1 p_1 q_1^i (1-q_2^{i_2}) + m p_1 p_2 q_1^i q_2^{i_2}}. \end{aligned} \quad (15)$$

When replacing equation (11) to (14) with equation (15), we get

$$\begin{aligned} &P(A_j) \\ &= \frac{f_1 p_1 p_2 q_1^j q_2^{i_2}}{f_1 p_2 q_2^{i_2} (1-q_1^i) + f_1 p_1 q_1^i (1-q_2^{i_2}) + m p_1 p_2 q_1^i q_2^{i_2}}; \\ &j = 1, 2, \dots, i_1, \end{aligned} \quad (16)$$

$$\begin{aligned} &P(f_1N_0) + P(f_1In_0) + P(f_1Id_0) \\ &= \frac{f_1 p_1 p_2 q_1^{i_1} (1-q_2^{i_2})}{f_1 p_2 q_2^{i_2} (1-q_1^i) + f_1 p_1 q_1^i (1-q_2^{i_2}) + m p_1 p_2 q_1^i q_2^{i_2}}, \end{aligned} \quad (17)$$

$$\begin{aligned} &P(f_1N_k) + P(f_1In_k) + P(f_1Id_k) \\ &= \frac{f_1 p_1 p_2 q_1^k q_2^{i_2}}{f_1 p_2 q_2^{i_2} (1-q_1^i) + f_1 p_1 q_1^i (1-q_2^{i_2}) + m p_1 p_2 q_1^i q_2^{i_2}}; \\ &k = 1, 2, \dots, i_2, \end{aligned} \quad (18)$$

$$\begin{aligned} &P(f_1N'_{l-1}) + P(f_1In'_l) + P(f_1Id'_l) \\ &= \frac{p_1 p_2 q_1^i q_2^{i_2}}{f_1 p_2 q_2^{i_2} (1-q_1^i) + f_1 p_1 q_1^i (1-q_2^{i_2}) + m p_1 p_2 q_1^i q_2^{i_2}}; \\ &l = 1, 2, \dots, m. \end{aligned} \quad (19)$$

Let

$$\begin{aligned} &P(100_1) = \sum_{j=0}^{i_1} P(A_j) \\ &= \frac{f_1 p_2 q_2^{i_2} (1-q_1^i)}{f_1 p_2 q_2^{i_2} (1-q_1^i) + f_1 p_1 q_1^i (1-q_2^{i_2}) + m p_1 p_2 q_1^i q_2^{i_2}}. \end{aligned} \quad (20)$$

$$\begin{aligned} &P(S_1) = P(f_1N_0) + P(f_1In_0) + P(f_1Id_0) \\ &+ \sum_{k=1}^{i_2} P(f_1N_k) + P(f_1In_k) + P(f_1Id_k) \\ &+ \sum_{l=1}^m P(f_1N'_{l-1}) + P(f_1In'_l) + P(f_1Id'_l) \\ &= \frac{p_1 q_1^i [f_1 (1-q_2^{i_2}) + m p_2 q_2^{i_2}]}{f_1 p_2 q_2^{i_2} (1-q_1^i) + f_1 p_1 q_1^i (1-q_2^{i_2}) + m p_1 p_2 q_1^i q_2^{i_2}}. \end{aligned} \quad (21)$$

From equations (20) and (21), we can find the average number of units inspected under the 100% inspection of line 1 or  $u_1$  and the average number of units passed under the sampling inspection of line 1 or  $v_1$  from

$$\begin{aligned} &u_1 = \frac{P(100_1)}{P(f_1In'_m) + P(f_1Id'_m)} \\ &= \frac{1-q_1^i}{p_1 q_1^i}. \end{aligned} \quad (22)$$

$$\begin{aligned} &v_1 = \frac{P(S_1)}{P(f_1In'_m) + P(f_1Id'_m)} \\ &= \frac{f_1 (1-q_2^{i_2}) + m (q_2^{i_2} - q_2^{i_2+1})}{f_1 p_2 q_2^{i_2}}. \end{aligned} \quad (23)$$

At line 2: in a similar way to line 1 and relations from Table 2, we get

$$\begin{aligned} &P(B_0) \\ &= \frac{f_2 p_1 p_2 q_1^i (1-q_2^{i_2})}{f_2 p_1 q_1^i (1-q_2^{i_2}) + f_2 p_2 q_2^{i_2} (1-q_1^i) + m p_1 p_2 q_1^i q_2^{i_2}}, \end{aligned}$$

$$\begin{aligned} &P(B_k) \\ &= \frac{f_2 p_1 p_2 q_1^k q_2^{i_2}}{f_2 p_1 q_1^i (1-q_2^{i_2}) + f_2 p_2 q_2^{i_2} (1-q_1^i) + m p_1 p_2 q_1^i q_2^{i_2}}; \\ &k = 1, 2, \dots, i_2, \end{aligned}$$

$$\begin{aligned} &P(f_2N_0) + P(f_2In_0) + P(f_2Id_0) \\ &= \frac{f_2 p_1 p_2 q_2^{i_2} (1-q_1^i)}{f_2 p_1 q_1^i (1-q_2^{i_2}) + f_2 p_2 q_2^{i_2} (1-q_1^i) + m p_1 p_2 q_1^i q_2^{i_2}}, \end{aligned}$$

$$\begin{aligned} & P(f_2 N_j) + P(f_2 I_n_j) + P(f_2 I_d_j) \\ &= \frac{f_2 p_1 p_2 q_1^j q_2^{i_2}}{f_2 p_1 q_1^i (1 - q_2^{i_2}) + f_2 p_2 q_2^{i_2} (1 - q_1^i) + m p_1 p_2 q_1^i q_2^{i_2}} \\ & ; j = 1, 2, \dots, i_1, \end{aligned}$$

$$\begin{aligned} & P(f_2 N'_{l-1}) + P(f_2 I_n'_l) + P(f_2 I_d'_l) \\ &= \frac{p_1 p_2 q_1^i q_2^{i_2}}{f_2 p_1 q_1^i (1 - q_2^{i_2}) + f_2 p_2 q_2^{i_2} (1 - q_1^i) + m p_1 p_2 q_1^i q_2^{i_2}} \\ & ; l = 1, 2, \dots, m. \end{aligned}$$

Let

$$\begin{aligned} P(100_2) &= \sum_{k=0}^{i_2} P(B_k) \\ &= \frac{f_2 p_1 q_1^i (1 - q_2^{i_2})}{f_2 p_1 q_1^i (1 - q_2^{i_2}) + f_2 p_2 q_2^{i_2} (1 - q_1^i) + m p_1 p_2 q_1^i q_2^{i_2}}. \end{aligned} \quad (24)$$

$$\begin{aligned} P(S_2) &= P(f_2 N_0) + P(f_2 I_n_0) + P(f_2 I_d_0) \\ &+ \sum_{j=1}^{i_1} P(f_2 N_j) + P(f_2 I_n_j) + P(f_2 I_d_j) \\ &+ \sum_{l=1}^m P(f_2 N'_{l-1}) + P(f_2 I_n'_l) + P(f_2 I_d'_l) \\ &= \frac{p_2 q_2^{i_2} [f_2 (1 - q_1^i) + m p_1 q_1^i]}{f_2 p_1 q_1^i (1 - q_2^{i_2}) + f_2 p_2 q_2^{i_2} (1 - q_1^i) + m p_1 p_2 q_1^i q_2^{i_2}}. \end{aligned} \quad (25)$$

From equations (24) and (25), we can find the average number of units inspected under the 100% inspection of line 2 or  $u_2$  and the average number of units passed under the sampling inspection of line 2 or  $v_2$  from

$$\begin{aligned} u_2 &= \frac{P(100_2)}{P(f_2 I_n'_m) + P(f_2 I_d'_m)} \\ &= \frac{1 - q_2^{i_2}}{p_2 q_2^{i_2}} \end{aligned} \quad (26)$$

$$\begin{aligned} v_2 &= \frac{P(S_2)}{P(f_2 I_n'_m) + P(f_2 I_d'_m)} \\ &= \frac{f_2 (1 - q_1^i) + m (q_1^i - q_1^{i+1})}{f_2 p_1 q_1^i} \end{aligned} \quad (27)$$

Then we can get the performance measures of the CSP-2L which are given from equations (22), (23), (26) and (27).