

## Homomorphism Of Tripolar Fuzzy Soft $\Gamma$ –Semiring

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*Abstract:* Given the notion of tripolar fuzzy soft sets, the concepts of a tripolar fuzzy soft  $\Gamma$ –Semirings, a tripolar fuzzy soft  $\Gamma$ –Semiring homomorphism and a tripolar fuzzy soft ideal in  $\Gamma$ –Semirings are discussed, and related properties and corollaries are investigated. On the other hand, in this paper, we also define the image and pre-image of tripolar fuzzy soft  $\Gamma$ –Semirings. Some properties and results involving these concepts are stated and proved. .

*Key Words* Soft set, fuzzy Soft set, tripolar fuzzy soft set, tripolar fuzzy soft  $\Gamma$ –Semiring, tripolar fuzzy soft ideal, tripolar fuzzy soft  $\Gamma$ –Semiring homomorphism.

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### 1 Introduction

In 1934 Vandiver [1] introduced the concept of semiring, a semiring concept is the best algebraic structure because its common generalization of distributive lattices, rings and an universal algebra with two binary operations addition and multiplication such that one of them distributive over the other. Semiring used for solving problems in applied mathematics, information sciences and in the areas of theoretical computer science as well as in optimization theory, coding theory, graph theory and formal languages.

In 1964 Nobusawa [2] gave  $\Gamma$ –ring notation as a generalization of ring, after that Sen [3] discussed  $\Gamma$ –semigroup concept. While,  $\Gamma$ –semiring concept was given by Muirali Krishna [4] as a generalizes to the concept of  $\Gamma$ –ring and semiring. A fuzzy set theory was discussed and introduced by Zadeh [5] in 1965 as the most appropriate theory for dealing with uncertainty. Rosenfeld [6] in 1971 studied fuzzy subgroup concepts. The idea of fuzzy subgroup and its application on theory and properties studied by some researchers, see [7, 8, 9, 10, 11, 12]. In 1999, the concept of soft set theory was introduced by Molodtsov [13] as a new mathematical tool for dealing with uncertainties. Furthermore, in 2001 Maji et al [14] introduced fuzzy soft sets as extended of soft set theory. Ghosh et al and Murali [15, 16] introduced and studied fuzzy soft ring, fuzzy soft ideals and fuzzy soft k-ideals over a  $\Gamma$ –Semiring. öztürk et al [17] discussed soft  $\Gamma$ –rings and fuzzy subnear rings. In 1999, Atanassov [18] gave the idea of intuitionistic fuzzy set.

Massa'deh et al [19, 20, 21, 22, 23] extended the intuitionistic fuzzy set notation to  $\Gamma$ –Semiring and its ideals, Ku-ideals, subrings, M-subgroups and homomorphisms. On the other hand, Maji et al [24] studied and introduced the intuitionistic fuzzy soft set concept. Then Yaqoob et al [25] studied the concept on groups induced by (t, s)-norm.

In 1998, Zhang [26] introduced bipolar fuzzy sets concepts as a generalization of fuzzy sets. Lee [27] in 2000 used this concept and applied it to algebraic structures. Also Massa'deh [28, 29, 30] introduced the concepts of bipolar Q-fuzzy H-ideals over  $\Gamma$ –hemiring, anti bipolar Q–fuzzy normal semigroup and bipolar for any cosets, isomorphisms and  $\Gamma$ –hemiring. Bipolar fuzzy soft set concept introduced in 2013 by Akram [31] where he studied this concept on subalgebras.

The ideas of tripolar fuzzy set was introduced by Murali Krishna Rao [32] in 2018 where he discussed this concept on interior ideal of  $\Gamma$ –semigroup. Also, Murali et al discussed this concept on interior ideal of  $\Gamma$ –semiring and on soft interior ideal over semiring [33]. In this paper we introduced and discuss the concept of tripolar fuzzy soft  $\Gamma$ –semiring homomorphism and some of its theorems and properties of homomorphic image of tripolar fuzzy soft  $\Gamma$ –semiring.

### 2 Preliminaries

**Definition 1.** [4] If  $S$  is a set together with two associative operations called addition  $+$  and multiplica-

tion  $\cdot$  then will be called a semiring if the following conditions hold:

1.  $+$  is a commutative operation.
2.  $\exists 0 \in S$  such that  $s + 0 = s$  and  $s \cdot 0 = 0 \cdot s = 0 \forall s \in S$ .
3. Distribute law hold from left and right.

**Definition 2.** [4] If  $(S, +)$  and  $(\Gamma, +)$  are commutative semigroups. Then  $S$  is said to be  $\Gamma$ -semiring, if there exists a mapping  $S \times \Gamma \times S \rightarrow S$  written as  $(s_1, \alpha, s_2)$  as  $s_1\alpha s_2$  such that it satisfies the following conditions:

1.  $s_1\alpha(s_2 + s_3) = s_1\alpha s_2 + s_1\alpha s_3$
2.  $(s_1 + s_2)\alpha s_3 = s_1\alpha s_3 + s_2\alpha s_3$
3.  $s_1(\alpha + \beta)s_2 = s_1\alpha s_2 + s_1\beta s_2$ .
4.  $s_1\alpha(s_2\beta s_3) = (s_1\alpha s_2)\beta s_3, \forall s_1, s_2, s_3 \in S$  and  $\alpha, \beta \in \Gamma$ .

**Definition 3.** [4] If  $M$  is a  $\Gamma$ -semiring and  $I$  be a non empty subset of  $M$ . Then  $I$  is said to be a  $\Gamma$ -subsemiring of  $M$  if  $I$  is a sub-semigroup of  $(M, +)$  and  $I\Gamma I \subseteq I$ .

**Definition 4.** [4] If  $M$  is a  $\Gamma$ -semiring and  $I$  is a non empty subset of  $M$ . Then:

1.  $I$  is called a right ideal of  $M$  if:
  - (i)  $I$  is closed under addition.
  - (ii)  $I\Gamma M \subseteq I$ .
2.  $I$  is called a left ideal of  $M$  if:
  - (i)  $I$  is closed under addition.
  - (ii)  $M\Gamma I \subseteq I$ .
3.  $I$  is called an ideal of  $M$ , if it is both a right and left ideal.

**Definition 5.** [18] An intuitionistic fuzzy set of a non empty set  $A$  is an object of the form  $\delta = (\delta_\mu, \delta_\lambda) = \{(a, \delta_\mu(a), \delta_\lambda(a)); a \in A\}$ , such that  $\delta_\mu : A \rightarrow [0, 1], \delta_\lambda : A \rightarrow [0, 1]$  are membership functions,  $\delta_\mu, \delta_\lambda$  are respectively and  $0 \leq \delta_\mu(a) + \delta_\lambda(a) \leq 1, \forall a \in A$ .

**Definition 6.** [26] A bipolar fuzzy set  $\gamma$  of a non empty set  $A$  is an object of the form  $\gamma = \{(a, \gamma_\mu(a), \gamma_\lambda(a)); a \in A\}$  such that  $\gamma_\mu : A \rightarrow [0, 1]$  and  $\gamma_\lambda : A \rightarrow [-1, 0]$ .  $\gamma_\mu(a)$  represents satisfaction degree of  $a$  to the property corresponding to fuzzy set  $\gamma$  and  $\gamma_\lambda(a)$  represents satisfaction degree of  $a$  to the implicit counter property of fuzzy set  $\gamma$ .

**Definition 7.** [13] If  $U$  is an initial universe set,  $E$  is the set of parameters set,  $X \subseteq E$ . If  $P(U)$  represent to the power set of  $U$ . Then a pair  $(\phi, X)$  is said to be a soft set over  $U$  such that  $\phi$  is a map given by  $\phi : X \rightarrow P(U)$ .

**Definition 8.** [14] If  $U$  is an initial universe set,  $E$  is a parameters set and  $X \subseteq E$ . A pair  $(\phi, X)$  is said to be fuzzy soft over  $U$ , such that  $\phi$  is a map given by  $\phi : X \rightarrow I^U$  where  $I^U$  denotes the collection of all fuzzy subset of  $U$ .

**Definition 9.** [4] If  $R_1$  and  $R_2$  are two  $\Gamma$ -semirings, a function  $\Psi : R_1 \rightarrow R_2$  is called a homomorphism  $\Gamma$ -semiring if  $\Psi(x + y) = \Psi(x) + \Psi(y)$  and  $\Psi(x\alpha y) = \Psi(x)\alpha\Psi(y), \forall x, y \in R_1, \alpha \in \Gamma$ .

**Definition 10.** [4] If  $R_1$  and  $R_2$  are two sets and  $\Psi : R_1 \rightarrow R_2$  is any function. A bipolar fuzzy subset  $\delta$  of  $R_1$  is called a  $\Psi$ -invariant if  $\Psi(a) = \Psi(b) \Rightarrow \delta(a) = \delta(b)$ .

**Definition 11.** [30] If  $\psi : R_1 \rightarrow R_2$  is a map and  $\delta = (\delta^+, \delta^-)$  and  $\gamma = (\gamma^+, \gamma^-)$  are bipolar fuzzy subset in  $R_1$  and  $R_2$  respectively. Then the image  $\psi(\delta)$  of  $\delta$  is the bipolar fuzzy subset  $\psi(\delta) = ((\psi(\delta))^+, (\psi(\delta))^-)$  of  $R_2$  defined by:

$$(\psi(\delta))^+(a) = \begin{cases} \max\{\delta^+(a); a \in \psi^-(a); \text{if } \psi^-(a) \neq \emptyset\} \\ 0; \text{otherwise} \end{cases}$$

$$(\psi(\delta))^- (a) = \begin{cases} \max\{\delta^-(a); a \in \psi^-(a); \text{if } \psi^-(a) \neq \emptyset\} \\ 0; \text{otherwise} \end{cases}$$

and the pre-image  $\psi^{-1}(\gamma)$  of  $\gamma$  under  $\psi$  is the bipolar fuzzy subset of  $R_1$  defined by for  $a \in R_1, (\psi^{-1}(\gamma))^+(a) = \gamma^+(\psi(a))$  and  $(\psi^{-1}(\gamma))^- (a) = \gamma^-(\psi(a))$ .

**Definition 12.** [33] If  $Y$  is a universe set, a tripolar fuzzy set  $\gamma$  in  $Y$  is an object having the form  $\gamma = \{(a, \lambda_\gamma(a), \mu_\gamma(a), \delta_\gamma); a \in Y$  and  $0 \leq \lambda_\gamma(a) + \mu_\gamma(a) \leq 1\}$  such that,  $\lambda_\gamma : Y \rightarrow [0, 1], \mu_\gamma : Y \rightarrow [0, 1], \delta_\gamma : Y \rightarrow [-1, 0]; 0 \leq \lambda_\gamma(a) + \mu_\gamma(a) \leq 1$ . The degree of membership  $\lambda_\gamma(a)$  characterize the extent that a satisfies the property corresponding to tripolar fuzzy set  $\gamma$ ,  $\mu_\gamma(a)$  characterize the extent that a satisfies to the not property corresponding to tripolar fuzzy set  $\gamma$  and  $\delta_\gamma(a)$  characterize the extent that a satisfies to the implicit counter property of tripolar fuzzy set  $\gamma$ .

**Remark 13.**  $\gamma = (\lambda_\gamma, \mu_\gamma, \delta_\gamma)$  has been used for  $\gamma = \{(a, \lambda_\gamma(a), \mu_\gamma(a), \delta_\gamma); a \in Y$  and  $0 \leq \lambda_\gamma(a) + \mu_\gamma(a) \leq 1\}$ .

**Definition 14.** [16] Assume that  $R$  is a  $\Gamma$ -semiring,  $E$  is a set of parameter and  $X \subseteq E$ . If  $\phi$  is a mapping given by  $\phi : X \rightarrow \rho(R)$  such that  $\rho(R)$  is the power set of  $R$ . Then  $(\phi, X)$  is called a soft  $\Gamma$ -semiring over  $R$  if and only if for each  $x \in X, \phi(x)$  is  $\Gamma$ -subsemiring of  $R$ . This means that:

1.  $a, b \in R \Rightarrow a + b \in \phi(a)$ .

2.  $a, b \in R, \alpha \in \Gamma \Rightarrow a\alpha b \in \phi(a).$

**Definition 15.** If  $S$  is a  $\Gamma$ -semiring a tripolar fuzzy soft  $(\phi, X)$  over  $S$  is said to be tripolar fuzzy soft  $\Gamma$ -semiring over  $S$  if  $\phi(x) = \{\lambda_{\phi(x)}(s), \mu_{\phi(x)}(s), \delta_{\phi(x)}(s); s \in S, x \in X\}$  such that  $\lambda_{\phi(x)}(s) : S \rightarrow [0, 1], \mu_{\phi(x)}(s) : S \rightarrow [0, 1], \delta_{\phi(x)}(s) : S \rightarrow [-1, 0], 0 \leq \lambda_{\phi(x)}(s) + \mu_{\phi(x)}(s) \leq 1, \forall s \in S$  satisfying the following axioms:

1.  $\lambda_{\phi(x)}(s_1 + s_2) \geq \min\{\lambda_{\phi(x)}(s_1), \lambda_{\phi(x)}(s_2)\}$
2.  $\mu_{\phi(x)}(s_1 + s_2) \leq \max\{\mu_{\phi(x)}(s_1), \mu_{\phi(x)}(s_2)\}$
3.  $\delta_{\phi(x)}(s_1 + s_2) \leq \max\{\delta_{\phi(x)}(s_1), \delta_{\phi(x)}(s_2)\}$
4.  $\lambda_{\phi(x)}(s_1\alpha s_2) \geq \min\{\lambda_{\phi(x)}(s_1), \lambda_{\phi(x)}(s_2)\}$
5.  $\mu_{\phi(x)}(s_1\alpha s_2) \leq \max\{\mu_{\phi(x)}(s_1), \mu_{\phi(x)}(s_2)\}$
6.  $\delta_{\phi(x)}(s_1\alpha s_2) \leq \max\{\delta_{\phi(x)}(s_1), \delta_{\phi(x)}(s_2)\}, \forall s_1, s_2 \in S, x \in X$  and  $\alpha \in \Gamma.$

**Definition 16.** [4] If  $S$  is a  $\Gamma$ -semiring,  $E$  is a parameter set and  $X \subseteq E$ . If  $\phi$  is a mapping given by  $\phi : X \rightarrow \rho(S)$ . Then  $(\phi, X)$  is said to be a soft right (left) ideal over  $S$  if and only if for each  $x \in X, \phi(x)$  is a right (left) ideal of  $S$ . This means that:

1.  $s_1, s_2 \in \phi(X)$  then  $s_1 + s_2 \in \phi(X)$
2.  $s_1, s_2 \in \phi(X), \alpha \in \Gamma, s \in S$  then  $s_1\alpha s(s\alpha s_1) \in \phi(X).$

**Definition 17.** A tripolar fuzzy soft set  $(\phi, X)$  over  $\Gamma$ -semiring  $S$  is said to be a tripolar fuzzy soft right (left) ideal over  $S$  if

1.  $\lambda_{\phi(x)}(s_1 + s_2) \geq \min\{\lambda_{\phi(x)}(s_1), \lambda_{\phi(x)}(s_2)\}$
2.  $\mu_{\phi(x)}(s_1 + s_2) \leq \max\{\mu_{\phi(x)}(s_1), \mu_{\phi(x)}(s_2)\}$
3.  $\delta_{\phi(x)}(s_1 + s_2) \leq \max\{\delta_{\phi(x)}(s_1), \delta_{\phi(x)}(s_2)\}$
4.  $\lambda_{\phi(x)}(s_1\alpha s_2) \geq \lambda_{\phi(x)}(s_1)(\lambda_{\phi(x)}(s_2))$
5.  $\mu_{\phi(x)}(s_1\alpha s_2) \leq \mu_{\phi(x)}(s_1)(\mu_{\phi(x)}(s_2))$
6.  $\delta_{\phi(x)}(s_1\alpha s_2) \leq \delta_{\phi(x)}(s_1)(\delta_{\phi(x)}(s_2)), \forall s_1, s_2 \in S, x \in X$  and  $\alpha \in \Gamma.$

**Definition 18.** If  $S$  is a  $\Gamma$ -semiring,  $E$  is a parameter set and  $X \subseteq E$ . A tripolar fuzzy soft set  $(\phi, X)$  over  $S$  is said to be a tripolar fuzzy soft ideal if the following axioms are hold:

1.  $\lambda_{\phi(x)}(s_1 + s_2) \geq \min\{\lambda_{\phi(x)}(s_1), \lambda_{\phi(x)}(s_2)\}$
2.  $\mu_{\phi(x)}(s_1 + s_2) \leq \max\{\mu_{\phi(x)}(s_1), \mu_{\phi(x)}(s_2)\}$

3.  $\delta_{\phi(x)}(s_1 + s_2) \leq \max\{\delta_{\phi(x)}(s_1), \delta_{\phi(x)}(s_2)\}$
4.  $\lambda_{\phi(x)}(s_1\alpha s_2) \geq \max\{\lambda_{\phi(x)}(s_1), \lambda_{\phi(x)}(s_2)\}$
5.  $\mu_{\phi(x)}(s_1\alpha s_2) \leq \min\{\mu_{\phi(x)}(s_1), \mu_{\phi(x)}(s_2)\}$
6.  $\delta_{\phi(x)}(s_1\alpha s_2) \leq \min\{\delta_{\phi(x)}(s_1), \delta_{\phi(x)}(s_2)\}, \forall s_1, s_2 \in S, x \in X$  and  $\alpha \in \Gamma.$

### 3 Homomorphism in tripolar fuzzy soft $\Gamma$ -semiring

The homomorphism concept over tripolar fuzzy soft  $\Gamma$ -semiring is introduced and studied their properties in this section.

**Definition 19.** If  $(\phi_1, X)$  and  $(\phi_2, Y)$  are tripolar fuzzy soft set over  $\Gamma$ -semirings  $R_1$  and  $R_2$  respectively. Let  $\psi_1 : R_1 \rightarrow R_2$  and  $\psi_2 : X \rightarrow Y$  are two functions such that  $X$  and  $Y$  are parameter sets for the crisp sets  $R_1$  and  $R_2$  respectively. Then  $(\psi_1, \psi_2)$  is said to be a tripolar fuzzy soft function from  $R_1$  to  $R_2$ .

**Definition 20.** If  $(\phi_1, X)$  and  $(\phi_2, Y)$  are tripolar fuzzy soft set over  $\Gamma$ -semirings  $R_1$  and  $R_2$  respectively and  $(\psi_1, \psi_2)$  are tripolar fuzzy soft functions from  $R_1$  to  $R_2$ . Then  $(\psi_1, \psi_2)$  is called tripolar fuzzy soft  $\Gamma$ -semiring homomorphism if satisfying the following axioms:

1.  $\psi_1$  is a  $\Gamma$ -semiring homomorphism from  $R_1$  onto  $R_2$ .
2.  $\psi_2$  is a mapping from  $X$  onto  $Y$ .
3.  $\psi_1(\lambda_{\phi_1(x)}) = \phi_2\psi_2(x), \psi_1(\mu_{\phi_1(x)}) = \phi_2\psi_2(x)$  and  $\psi_1(\delta_{\phi_1(x)}) = \phi_2\psi_2(x), \forall x \in X.$

**Remark 21.** If there exist a tripolar fuzzy soft  $\Gamma$ -semiring homomorphism between  $(\phi_1, X)$  and  $(\phi_2, Y)$ . Then we say that  $(\phi_1, X)$  is soft homomorphic to  $(\phi_2, Y)$ .

**Definition 22.** If  $(\psi_1, \psi_2)$  is a tripolar fuzzy soft function from  $R_1$  to  $R_2$ . The pre-image of  $(\phi_2, Y)$  under the tripolar fuzzy soft function  $(\psi_1, \psi_2)$ , denoted by  $(\psi_1, \psi_2)^{-1}((\phi_2, Y))$  defined by  $(\psi_1, \psi_2)^{-1}((\phi_2, Y)) = (\psi_1^{-1}(\phi_2), \psi_2^{-1}(Y))$  is a tripolar fuzzy soft set.

**Theorem 23.** If  $(\phi, X)$  is a tripolar fuzzy soft  $\Gamma$ -semiring over  $R_2, \psi : R_1 \rightarrow R_2$  is monomorphism and for each  $x \in X$ , define  $(\psi\phi)_x(r) = \phi_x(\psi(r)), \forall r \in R$ , then  $(\psi\phi, X)$  is a tripolar fuzzy soft  $\Gamma$ -semiring over  $R_2$ .

*Proof.* Let  $r_1, r_2 \in R, x \in X$  and  $\alpha \in \Gamma$ . Then:

1.  $(\psi\phi)_x(r_1 + r_2) = \phi_x(\psi(r_1 + r_2))$   
 $= \lambda_{\phi(x)}(\psi(r_1) + \psi(r_2))$   
 $\geq \min\{\lambda_{\phi(x)}(\psi(r_1)), \lambda_{\phi(x)}(\psi(r_2))\}$   
 $= \min\{(\psi\phi)_x(r_1), (\psi\phi)_x(r_2)\}.$
2.  $(\psi\phi)_x(r_1 + r_2) = \phi_x(\psi(r_1 + r_2))$   
 $= \mu_{\phi(x)}(\psi(r_1) + \psi(r_2))$   
 $\leq \max\{\mu_{\phi(x)}(\psi(r_1)), \mu_{\phi(x)}(\psi(r_2))\}$   
 $= \max\{(\psi\phi)_x(r_1), (\psi\phi)_x(r_2)\}.$
3.  $(\psi\phi)_x(r_1 + r_2) = \phi_x(\psi(r_1 + r_2))$   
 $= \delta_{\phi(x)}(\psi(r_1) + \psi(r_2))$   
 $\leq \max\{\delta_{\phi(x)}(\psi(r_1)), \delta_{\phi(x)}(\psi(r_2))\}$   
 $= \max\{(\psi\phi)_x(r_1), (\psi\phi)_x(r_2)\}.$
4.  $(\psi\phi)_x(r_1\alpha r_2) = \phi_x(\psi(r_1\alpha r_2))$   
 $= \lambda_{\phi(x)}(\psi(r_1)\alpha\psi(r_2))$   
 $\geq \min\{\lambda_{\phi(x)}(\psi(r_1)), \lambda_{\phi(x)}(\psi(r_2))\}$   
 $= \min\{(\psi\phi)_x(r_1), (\psi\phi)_x(r_2)\}.$
5.  $(\psi\phi)_x(r_1\alpha r_2) = \phi_x(\psi(r_1\alpha r_2))$   
 $= \mu_{\phi(x)}(\psi(r_1)\alpha\psi(r_2))$   
 $\leq \max\{\mu_{\phi(x)}(\psi(r_1)), \mu_{\phi(x)}(\psi(r_2))\}$   
 $= \max\{(\psi\phi)_x(r_1), (\psi\phi)_x(r_2)\}.$
6.  $(\psi\phi)_x(r_1\alpha r_2) = \phi_x(\psi(r_1\alpha r_2))$   
 $= \delta_{\phi(x)}(\psi(r_1)\alpha\psi(r_2))$   
 $\leq \max\{\delta_{\phi(x)}(\psi(r_1)), \delta_{\phi(x)}(\psi(r_2))\}$   
 $= \max\{(\psi\phi)_x(r_1), (\psi\phi)_x(r_2)\}.$

Therefore  $(\psi\phi)_x$  is a tripolar fuzzy  $\Gamma$ -subsemiring of  $S$ . Thus  $(\psi f, X)$  is a tripolar fuzzy soft  $\Gamma$ -semiring over  $R_2$ .  $\square$

**Theorem 24.** *If  $(\gamma, X)$  is a tripolar fuzzy soft semiring over  $\Gamma$ -semiring  $R$ , if  $\psi$  is an endomorphism of  $R$  and defined  $(\gamma\psi)_x = \gamma_x\psi$  for each  $x \in X$ . Then  $(\gamma\psi, X)$  is a tripolar fuzzy soft  $\Gamma$ -semiring over  $R$ .*

*Proof.* Let  $r_1, r_2 \in R, x \in X$  and  $\alpha \in \Gamma$ . Then:

1.  $(\lambda\psi)_x(r_1 + r_2) = \lambda_{\gamma(x)}(\psi(r_1 + r_2))$   
 $= \lambda_{\gamma(x)}(\psi(r_1) + \psi(r_2))$   
 $\geq \min\{\lambda_{\gamma(x)}(\psi(r_1)), \lambda_{\gamma(x)}(\psi(r_2))\}$   
 $= \min\{(\lambda\psi)_x(r_1), (\lambda\psi)_x(r_2)\}.$
2.  $(\mu\psi)_x(r_1 + r_2) = \mu_{\gamma(x)}(\psi(r_1 + r_2))$   
 $= \mu_{\gamma(x)}(\psi(r_1) + \psi(r_2))$   
 $\leq \max\{\mu_{\gamma(x)}(\psi(r_1)), \mu_{\gamma(x)}(\psi(r_2))\}$   
 $= \max\{(\mu\psi)_x(r_1), (\mu\psi)_x(r_2)\}.$
3.  $(\delta\psi)_x(r_1 + r_2) = \delta_{\gamma(x)}(\psi(r_1 + r_2))$   
 $= \delta_{\gamma(x)}(\psi(r_1) + \psi(r_2))$   
 $\leq \max\{\delta_{\gamma(x)}(\psi(r_1)), \delta_{\gamma(x)}(\psi(r_2))\}$   
 $= \max\{(\delta\psi)_x(r_1), (\delta\psi)_x(r_2)\}.$

4.  $(\lambda\psi)_x(r_1\alpha r_2) = \lambda_{\gamma(x)}(\psi(r_1\alpha r_2))$   
 $= \lambda_{\gamma(x)}(\psi(r_1)\alpha\psi(r_2))$   
 $\geq \min\{\lambda_{\gamma(x)}(\psi(r_1)), \lambda_{\gamma(x)}(\psi(r_2))\}$   
 $= \min\{(\lambda\psi)_x(r_1), (\lambda\psi)_x(r_2)\}.$
5.  $(\mu\psi)_x(r_1\alpha r_2) = \mu_{\gamma(x)}(\psi(r_1\alpha r_2))$   
 $= \mu_{\gamma(x)}(\psi(r_1)\alpha\psi(r_2))$   
 $\leq \max\{\mu_{\gamma(x)}(\psi(r_1)), \mu_{\gamma(x)}(\psi(r_2))\}$   
 $= \max\{(\mu\psi)_x(r_1), (\mu\psi)_x(r_2)\}.$
6.  $(\delta\psi)_x(r_1\alpha r_2) = \delta_{\gamma(x)}(\psi(r_1\alpha r_2))$   
 $= \delta_{\gamma(x)}(\psi(r_1)\alpha\psi(r_2))$   
 $\leq \max\{\delta_{\gamma(x)}(\psi(r_1)), \delta_{\gamma(x)}(\psi(r_2))\}$   
 $= \max\{(\delta\psi)_x(r_1), (\delta\psi)_x(r_2)\}.$

Thus  $(\gamma\psi)_x$  is a tripolar fuzzy  $\Gamma$ -subsemiring of  $R$ . Then  $(\gamma\psi, X)$  is a tripolar fuzzy soft  $\Gamma$ -semiring over  $R$ .  $\square$

**Theorem 25.** *If  $\psi : R_1 \rightarrow R_2$  is an epimorphism of  $\Gamma$ -semiring and  $(\gamma, X)$  is a tripolar fuzzy soft right ideal over  $R_2$ . If for each  $x \in X, \zeta_x = \psi^{-1}(\gamma_x)$  then  $(\zeta, X)$  is a tripolar fuzzy soft right ideal over  $R_1$ .*

*Proof.* If  $x \in X$  and  $\alpha \in \Gamma$ . Then  $\gamma_x$  is a tripolar fuzzy soft right ideal over  $R_2$ . If  $r_1, r_2 \in R_1$  and  $\alpha \in \Gamma$ , then:

1.  $\psi^{-1}(\lambda_x)(r_1 + r_2) = \lambda_{\gamma(x)}(\psi(r_1 + r_2))$   
 $= \lambda_{\gamma(x)}(\psi(r_1) + \psi(r_2))$   
 $\geq \min\{\lambda_{\gamma(x)}(\psi(r_1)), \lambda_{\gamma(x)}(\psi(r_2))\}$   
 $= \min\{\psi^{-1}(\lambda_x)(r_1), \psi^{-1}(\lambda_x)(r_2)\}.$
2.  $\psi^{-1}(\mu_x)(r_1 + r_2) = \mu_{\gamma(x)}(\psi(r_1 + r_2))$   
 $= \mu_{\gamma(x)}(\psi(r_1) + \psi(r_2))$   
 $\leq \max\{\mu_{\gamma(x)}(\psi(r_1)), \mu_{\gamma(x)}(\psi(r_2))\}$   
 $= \max\{\psi^{-1}(\mu_x)(r_1), \psi^{-1}(\mu_x)(r_2)\}.$
3.  $\psi^{-1}(\delta_x)(r_1 + r_2) = \delta_{\gamma(x)}(\psi(r_1 + r_2))$   
 $= \delta_{\gamma(x)}(\psi(r_1) + \psi(r_2))$   
 $\leq \max\{\delta_{\gamma(x)}(\psi(r_1)), \delta_{\gamma(x)}(\psi(r_2))\}$   
 $= \max\{\psi^{-1}(\delta_x)(r_1), \psi^{-1}(\delta_x)(r_2)\}.$
4.  $\psi^{-1}(\lambda_x)(r_1\alpha r_2) = \lambda_x(\psi(r_1\alpha r_2))$   
 $= \lambda_x(\psi(r_1)\alpha\psi(r_2))$   
 $\geq \lambda_x(\psi(r_1))$   
 $= \psi^{-1}(\lambda_x)(r_1).$
5.  $\psi^{-1}(\mu_x)(r_1\alpha r_2) = \mu_x(\psi(r_1\alpha r_2))$   
 $= \mu_x(\psi(r_1)\alpha\psi(r_2))$   
 $\leq \mu_x(\psi(r_1))$   
 $= \psi^{-1}(\mu_x)(r_1).$
6.  $\psi^{-1}(\delta_x)(r_1\alpha r_2) = \delta_x(\psi(r_1\alpha r_2))$   
 $= \delta_x(\psi(r_1)\alpha\psi(r_2))$   
 $\leq \delta_x(\psi(r_1))$   
 $= \psi^{-1}(\delta_x)(r_1).$

Therefore  $\zeta_x = \psi^{-1}(\gamma_x)$  is a tripolar fuzzy right ideal of  $R_1$ . Thus  $(\zeta, X)$  is a tripolar fuzzy soft right ideal over  $R_1$ .  $\square$

Theorem 25 is true for tripolar fuzzy soft left ideal.

**Proposition 26.** *If  $R_1$  and  $R_2$  are  $\Gamma$ -semirings,  $\psi : R_1 \rightarrow R_2$  is a  $\Gamma$ -semiring homomorphism and  $\phi$  is a  $\psi$ -invariant bipolar fuzzy subset of  $R_1$ , if  $b = \psi(a)$  then  $\psi(\phi)(b) = \phi(a)$ ;  $a \in R_1$ .*

*Proof.* straightforward.  $\square$

**Theorem 27.** *If  $(\gamma, X)$  is a tripolar fuzzy soft right ideal over  $\Gamma$ -semiring  $R_1$  and  $\psi$  is a homomorphism from  $R_1$  onto  $R_2$ . For each  $x \in X$ ,  $\gamma_x$  is a  $\psi$ -invariant bipolar fuzzy right ideal of  $R_1$ , if  $\zeta_x = \psi(\gamma_x)$  then  $(\zeta, X)$  is a tripolar fuzzy soft right ideal over  $R_2$ .*

*Proof.* Let  $r_1, r_2 \in R_2, x \in X$  and  $\alpha \in \Gamma$ . Then there exists  $r_3, r_4 \in R_1$  such that  $\psi(r_3) = r_1, \psi(r_4) = r_2, r_1 + r_2 = \psi(r_3 + r_4)$  and  $r_1\alpha r_2 = \psi(r_3\alpha r_4)$ .  $\gamma_x$  is  $\psi$ -invariant. Thus by proposition 26, we have:

$$\begin{aligned} 1. \lambda_{\zeta(x)}(r_1 + r_2) &= \psi(\lambda_{\gamma(x)}(r_1 + r_2)) \\ &= \lambda_{\gamma(x)}(r_3 + r_4) \\ &\geq \min\{\lambda_{\gamma(x)}(r_3), \lambda_{\gamma(x)}(r_4)\} \\ &= \min\{\psi(\lambda_{\gamma(x)}(r_3)), \psi(\lambda_{\gamma(x)}(r_4))\} \\ &= \min\{\lambda_{\zeta(x)}(r_1), \lambda_{\zeta(x)}(r_2)\} \\ 2. \mu_{\zeta(x)}(r_1 + r_2) &= \psi(\mu_{\gamma(x)}(r_1 + r_2)) \\ &= \mu_{\gamma(x)}(r_3 + r_4) \\ &\leq \max\{\mu_{\gamma(x)}(r_3), \mu_{\gamma(x)}(r_4)\} \\ &= \max\{\psi(\mu_{\gamma(x)}(r_3)), \psi(\mu_{\gamma(x)}(r_4))\} \\ &= \max\{\mu_{\zeta(x)}(r_1), \mu_{\zeta(x)}(r_2)\} \\ 3. \delta_{\zeta(x)}(r_1 + r_2) &= \psi(\delta_{\gamma(x)}(r_1 + r_2)) \\ &= \delta_{\gamma(x)}(r_3 + r_4) \\ &\leq \max\{\delta_{\gamma(x)}(r_3), \delta_{\gamma(x)}(r_4)\} \\ &= \max\{\psi(\delta_{\gamma(x)}(r_3)), \psi(\delta_{\gamma(x)}(r_4))\} \\ &= \max\{\delta_{\zeta(x)}(r_1), \delta_{\zeta(x)}(r_2)\} \\ 4. \lambda_{\zeta(x)}(r_1\alpha r_2) &= \psi(\lambda_{\gamma(x)}(r_1\alpha r_2)) \\ &= \lambda_{\gamma(x)}(\psi(r_3\alpha r_4)) \\ &= \lambda_{\gamma(x)}(\psi(r_3)\alpha\psi(r_4)) \\ &\geq \lambda_{\gamma(x)}(\psi(r_3)) \\ &= \psi(\lambda_{\gamma(x)}(r_3)) \\ &= \lambda_{\zeta(x)}(r_1) \\ 5. \mu_{\zeta(x)}(r_1\alpha r_2) &= \psi(\mu_{\gamma(x)}(r_1\alpha r_2)) \\ &= \mu_{\gamma(x)}(\psi(r_3\alpha r_4)) \\ &= \mu_{\gamma(x)}(\psi(r_3)\alpha\psi(r_4)) \\ &\leq \mu_{\gamma(x)}(\psi(r_3)) \\ &= \psi(\mu_{\gamma(x)}(r_3)) \\ &= \mu_{\zeta(x)}(r_1) \end{aligned}$$

$$\begin{aligned} 6. \delta_{\zeta(x)}(r_1\alpha r_2) &= \psi(\delta_{\gamma(x)}(r_1\alpha r_2)) \\ &= \delta_{\gamma(x)}(\psi(r_3\alpha r_4)) \\ &= \delta_{\gamma(x)}(\psi(r_3)\alpha\psi(r_4)) \\ &\leq \delta_{\gamma(x)}(\psi(r_3)) \\ &= \psi(\delta_{\gamma(x)}(r_3)) \\ &= \delta_{\zeta(x)}(r_1) \end{aligned}$$

then  $\zeta_x$  is a tripolar fuzzy ideal of  $R_2$ . Hence  $(\zeta, X)$  is a tripolar fuzzy soft right ideal over  $R_2$ .  $\square$

**Theorem 28.** *If  $(\gamma_1, X_1)$  and  $(\gamma_2, X_2)$  are two bipolar fuzzy soft  $\Gamma$ -semirings over  $R_1$  and  $R_2$  respectively, and  $(\phi, \psi)$  is a tripolar fuzzy soft  $\Gamma$ -semiring homomorphism from  $(\gamma_1, X_1)$  onto  $(\gamma_2, X_2)$ . Then  $(\phi(\gamma_1), X_2)$  is a tripolar fuzzy soft  $\Gamma$ -semiring over  $R_2$ .*

*Proof.* By definition 20,  $\phi$  is a  $\Gamma$ -semiring homomorphism from  $R_1$  onto  $R_2$  and  $\psi$  is a mapping from  $X_1$  onto  $X_2$  for each  $y \in X_2$  there exist  $x \in X_1$  such that  $\psi(x) = y$ . Define  $(\phi(\gamma_1))_y = \phi(\gamma_{1x})$ . If  $r_1, r_2 \in R_2$  and  $\alpha \in \Gamma$ , then there exist  $r_3, r_4 \in R_1$  such that  $\phi(r_3) = r_1, \phi(r_4) = r_2$  and  $\phi(r_3 + r_4) = r_1 + r_2$  and  $\phi(r_3\alpha r_4) = r_1\alpha r_2$ . Thus we have:

$$\begin{aligned} 1. (\phi(\lambda_{\gamma_1})_{\psi(x)}(r_1 + r_2) &= \phi(\lambda_{\gamma_1(x)}(r_1 + r_2)) \\ &= \lambda_{\gamma_1(x)}(r_3 + r_4) \\ &\geq \min\{\lambda_{\gamma_1(x)}(r_3), \lambda_{\gamma_1(x)}(r_4)\} \\ &= \min\{\phi(\lambda_{\gamma_1(x)}(r_3)), \phi(\lambda_{\gamma_1(x)}(r_4))\} \\ &= \min\{\phi(\lambda_{\gamma_1})_{\psi(x)}(r_1), \phi(\lambda_{\gamma_1})_{\psi(x)}(r_2)\} \\ 2. (\phi(\mu_{\gamma_1})_{\psi(x)}(r_1 + r_2) &= \phi(\mu_{\gamma_1(x)}(r_1 + r_2)) \\ &= \mu_{\gamma_1(x)}(r_3 + r_4) \\ &\leq \max\{\mu_{\gamma_1(x)}(r_3), \mu_{\gamma_1(x)}(r_4)\} \\ &= \max\{\phi(\mu_{\gamma_1(x)}(r_3)), \phi(\mu_{\gamma_1(x)}(r_4))\} \\ &= \max\{\phi(\mu_{\gamma_1})_{\psi(x)}(r_1), \phi(\mu_{\gamma_1})_{\psi(x)}(r_2)\} \\ 3. (\phi(\delta_{\gamma_1})_{\psi(x)}(r_1 + r_2) &= \phi(\delta_{\gamma_1(x)}(r_1 + r_2)) \\ &= \delta_{\gamma_1(x)}(r_3 + r_4) \\ &\leq \max\{\delta_{\gamma_1(x)}(r_3), \delta_{\gamma_1(x)}(r_4)\} \\ &= \max\{\phi(\delta_{\gamma_1(x)}(r_3)), \phi(\delta_{\gamma_1(x)}(r_4))\} \\ &= \max\{\phi(\delta_{\gamma_1})_{\psi(x)}(r_1), \phi(\delta_{\gamma_1})_{\psi(x)}(r_2)\} \\ 4. (\phi(\lambda_{\gamma_1})_{\psi(x)}(r_1\alpha r_2) &= \phi(\lambda_{\gamma_1(x)}(r_1\alpha r_2)) \\ &= \lambda_{\gamma_1(x)}(r_3\alpha r_4) \\ &\geq \min\{\lambda_{\gamma_1(x)}(r_3), \lambda_{\gamma_1(x)}(r_4)\} \\ &= \min\{\phi(\lambda_{\gamma_1(x)}(r_3)), \phi(\lambda_{\gamma_1(x)}(r_4))\} \\ &= \min\{\phi(\lambda_{\gamma_1})_{\psi(x)}(r_1), \phi(\lambda_{\gamma_1})_{\psi(x)}(r_2)\} \\ 5. (\phi(\mu_{\gamma_1})_{\psi(x)}(r_1\alpha r_2) &= \phi(\mu_{\gamma_1(x)}(r_1\alpha r_2)) \\ &= \mu_{\gamma_1(x)}(r_3\alpha r_4) \\ &\leq \max\{\mu_{\gamma_1(x)}(r_3), \mu_{\gamma_1(x)}(r_4)\} \\ &= \max\{\phi(\mu_{\gamma_1(x)}(r_3)), \phi(\mu_{\gamma_1(x)}(r_4))\} \\ &= \max\{\phi(\mu_{\gamma_1})_{\psi(x)}(r_1), \phi(\mu_{\gamma_1})_{\psi(x)}(r_2)\} \end{aligned}$$

$$\begin{aligned} 6. & (\phi(\delta_{\gamma_1}))_{\psi(x)}(r_1\alpha r_2) = \phi(\delta_{\gamma_1(x)})(r_1\alpha r_2) \\ & = \delta_{\gamma_1(x)}(r_3\alpha r_4) \\ & \leq \max\{\delta_{\gamma_1(x)}(r_3), \delta_{\gamma_1(x)}(r_4)\} \\ & = \max\{\phi(\delta_{\gamma_1(x)})(r_1), \phi(\delta_{\gamma_1(x)})(r_2)\} \\ & = \max\{\phi(\delta_{\gamma_1})_{\psi(x)}(r_1), \phi(\delta_{\gamma_1})_{\psi(x)}(r_2)\}. \end{aligned}$$

Then  $\phi(\gamma_1)_y$  is a tripolar fuzzy  $\Gamma$ -subsemiring of  $R_2$ . Hence  $(\phi(\gamma_1), X_2)$  is a tripolar fuzzy soft  $\Gamma$ -semiring over  $R_2$ .  $\square$

**Theorem 29.** *If  $R_1, R_2$  are two  $\Gamma$ -semirings,  $\phi : R_1 \rightarrow R_2$  is a  $\Gamma$ -semiring homomorphism,  $(\gamma_1, X_1), (\gamma_2, X_2)$  are tripolar fuzzy soft  $\Gamma$ -semirings over  $R_1$  and  $(\gamma_1, X_1)$  is a tripolar fuzzy soft  $\Gamma$ -subsemiring of  $(\gamma_2, X_2)$ . Then  $(\phi(\gamma_1), X_1)$  and  $(\phi(\gamma_2), X_2)$  are tripolar fuzzy soft  $\Gamma$ -subsemirings over  $R_2$  and  $(\phi(\gamma_1), X_1)$  is a tripolar fuzzy soft  $\Gamma$ -subsemiring of  $(\phi(\gamma_2), X_2)$ .*

*Proof.* Since  $(\phi(\gamma_1))_x = \phi(\gamma_{1(x)})$  is a tripolar fuzzy  $\Gamma$ -subsemiring of  $R_2$  for all  $x \in X_1$  and  $(\phi(\gamma_2))_y = \phi(\gamma_{2(y)})$  is a tripolar fuzzy  $\Gamma$ -subsemiring of  $R_2$  for all  $y \in X_2$ . Hence  $(\phi(\gamma_1), X_1)$  and  $(\phi(\gamma_2), X_2)$  are tripolar fuzzy soft  $\Gamma$ -semiring over  $R_2$ . Since  $(\gamma_1, X_1)$  is a tripolar fuzzy soft  $\Gamma$ -subsemiring of  $(\gamma_2, X_2)$ . And  $\gamma_{1(x)}$  is a tripolar fuzzy subsemiring of  $\gamma_{2(x)}$ . Hence  $\phi(\gamma_{1(x)})$  is a tripolar fuzzy  $\Gamma$ -subsemiring of  $\phi(\gamma_{2(x)})$  for all  $x \in X_1$ . Therefore  $(\phi(\gamma_1), X_1)$  is a tripolar fuzzy soft  $\Gamma$ -subsemiring of  $(\phi(\gamma_2), X_2)$ .  $\square$

**Theorem 30.** *If  $(\gamma_1, X)$  and  $(\gamma_2, Y)$  are tripolar fuzzy soft  $\Gamma$ -semirings over  $R_1$  and  $R_2$  respectively and  $(\phi, \psi)$  is a tripolar fuzzy soft homomorphism from  $(\gamma_1, X)$  onto  $(\gamma_2, Y)$  then the pre-image of  $(\gamma_2, Y)$  under tripolar fuzzy soft  $\Gamma$ -semiring homomorphism is a tripolar fuzzy soft  $\Gamma$ -subsemiring of  $(\gamma_1, X)$  over  $R_1$ .*

*Proof.* By Definition 22  $(\phi, \psi)^{-1}(\gamma_2, Y) = (\phi^{-1}(\gamma_2), \psi^{-1}(Y))$ . Define  $(\phi^{-1}(\gamma_2))_x(r_1) = \gamma_{2\psi(x)}(\phi(r_1))$  for all  $r_1 \in R_1$  and  $x \in \psi^{-1}(Y)$ . Take  $v, w \in R_1$  and  $\alpha \in \Gamma$ . Then

$$\begin{aligned} 1. & (\phi^{-1}(\lambda_{\gamma_2}))_x(v+w) = \lambda_{\gamma_{2\psi(x)}}(\phi(v+w)) \\ & = \lambda_{\gamma_{2\psi(x)}}(\phi(v) + \phi(w)) \\ & \geq \min\{\lambda_{\gamma_{2\psi(x)}}(\phi(v)), \lambda_{\gamma_{2\psi(x)}}(\phi(w))\} \\ & = \min\{\phi^{-1}(\lambda_{\gamma_2})_x(v), \phi^{-1}(\lambda_{\gamma_2})_x(w)\} \\ 2. & (\phi^{-1}(\mu_{\gamma_2}))_x(v+w) = \mu_{\gamma_{2\psi(x)}}(\phi(v+w)) \\ & = \mu_{\gamma_{2\psi(x)}}(\phi(v) + \phi(w)) \\ & \leq \max\{\mu_{\gamma_{2\psi(x)}}(\phi(v)), \mu_{\gamma_{2\psi(x)}}(\phi(w))\} \\ & = \max\{\phi^{-1}(\mu_{\gamma_2})_x(v), \phi^{-1}(\mu_{\gamma_2})_x(w)\} \\ 3. & (\phi^{-1}(\delta_{\gamma_2}))_x(v+w) = \delta_{\gamma_{2\psi(x)}}(\phi(v+w)) \\ & = \delta_{\gamma_{2\psi(x)}}(\phi(v) + \phi(w)) \\ & \leq \max\{\delta_{\gamma_{2\psi(x)}}(\phi(v)), \delta_{\gamma_{2\psi(x)}}(\phi(w))\} \\ & = \max\{\phi^{-1}(\delta_{\gamma_2})_x(v), \phi^{-1}(\delta_{\gamma_2})_x(w)\} \end{aligned}$$

$$\begin{aligned} 4. & (\phi^{-1}(\lambda_{\gamma_2}))_x(v\alpha w) = \lambda_{\gamma_{2\psi(x)}}(\phi(v\alpha w)) \\ & = \lambda_{\gamma_{2\psi(x)}}(\phi(v)\alpha\phi(w)) \\ & \geq \min\{\lambda_{\gamma_{2\psi(x)}}(\phi(v)), \lambda_{\gamma_{2\psi(x)}}(\phi(w))\} \\ & = \min\{\phi^{-1}(\lambda_{\gamma_2})_x(v), \phi^{-1}(\lambda_{\gamma_2})_x(w)\} \\ 5. & (\phi^{-1}(\mu_{\gamma_2}))_x(v\alpha w) = \mu_{\gamma_{2\psi(x)}}(\phi(v\alpha w)) \\ & = \mu_{\gamma_{2\psi(x)}}(\phi(v)\alpha\phi(w)) \\ & \leq \max\{\mu_{\gamma_{2\psi(x)}}(\phi(v)), \mu_{\gamma_{2\psi(x)}}(\phi(w))\} \\ & = \max\{\phi^{-1}(\mu_{\gamma_2})_x(v), \phi^{-1}(\mu_{\gamma_2})_x(w)\} \\ 6. & (\phi^{-1}(\delta_{\gamma_2}))_x(v\alpha w) = \delta_{\gamma_{2\psi(x)}}(\phi(v\alpha w)) \\ & = \delta_{\gamma_{2\psi(x)}}(\phi(v)\alpha\phi(w)) \\ & \leq \max\{\delta_{\gamma_{2\psi(x)}}(\phi(v)), \delta_{\gamma_{2\psi(x)}}(\phi(w))\} \\ & = \max\{\phi^{-1}(\delta_{\gamma_2})_x(v), \phi^{-1}(\delta_{\gamma_2})_x(w)\} \end{aligned}$$

Thus  $(\phi^{-1}(\gamma_2))_x$  is a tripolar fuzzy  $\Gamma$ -subsemiring of  $R_1$  for all  $x \in \psi^{-1}(Y)$ . Therefore  $((\phi^{-1}(\gamma_2)), (\psi^{-1}(Y)))$  is a tripolar fuzzy soft  $\Gamma$ -subsemiring of  $(\gamma_1, X)$  over  $R_1$ .  $\square$

## 4 Conclusion

In this paper, we studied the concept of tripolar fuzzy soft  $\Gamma$ - semiring homomorphism and discussed some properties of homomorphic image and pre image of tripolar fuzzy soft  $\Gamma$ -semiring. These concepts are basic supporting structures for development the theory of soft set. This work can be extended to the properties of different notions of kernel of tripolar fuzzy soft  $\Gamma$ - semiring homomorphism, tripolar fuzzy soft filters over  $\Gamma$ - semirings and tripolar fuzzy soft prime and maximal ideals.

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