Mathematical Uncertainty Cost Functions for Controllable Photo-Voltaic Generators considering Uniform Distributions

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Abstract: The economic dispatch of energy on power systems with high penetration of renewable generation is a mathematical problem of optimization. The solution techniques that have been used are programming techniques and heuristic approaches. In both cases, it is important to have a well-defined target function to be optimized. Nowadays, the power systems are more complex with the introduction of the renewable sources of energy with highly stochastic behavior. For this as follows it was pretended to obtain a model for the penalty costs in a photovoltaic generator. This paper shows a mathematical analysis with probabilistic methods contrasted with an analytic development for controllable renewable systems to be included in the target functions of economic dispatch problems. In order to validate the mathematical approach, Monte Carlo simulation was used to obtain the underestimation and overestimation penalty values of the scheduled power for the uncertainty cost of photo-voltaic (PV) generation in an instance of energy storage. Developed under a model with a uniform distribution of power, the document presents the validation for the uncertainty cost factor (UCF) comparing the Monte Carlo simulation with the analytic proposal where the low error in the results proved the advantages of using the analytic model due to its quadratic form and its coherence with the simulations that were performed.

Key–Words: Economic dispatch models, Mathematical modeling, Monte Carlo, Solar energy, Uncertainty cost.

1 Introduction

With the broad diffusion of alternative and renewable forms for energy generation in recent years, the necessity has raised to calculate the generation cost and to propose new ways to perform for this calculation [1], [2], [3]. To this end [1] shows the different components that affect the energy costs in an electric system e.g, the variation in demand. This case of variable in [2]. is gaining relevance, because it gives some uncertainty to the system and helps to make some important decisions in the economic dispatch.

The economic dispatch of energy on power systems with high penetration of renewable is a mathematical problem of optimization. The solution techniques that has been used are programming techniques and heuristic approaches [4]-[8]. These techniques can be linear or non-linear, however, great majority of these methods assume that the power system is deterministic, which is invalid in terms of clean energy [9]-[10]. It is for this reason that the heuristic approaches in this type of scheduling problems are used [12]-[14].

Nowadays, some loads for the electric systems are controllable, which means that the uncertainty of the system is growing with the use of solar photo-voltaic and wind generation, and with the implementation of electric vehicles. Thus, the need arises to describe probabilistically the behavior of loads and power generation. This mathematical concept was first studied with eolic generation in [15]. In the present document, the solar photo-voltaic generation is controlled with a battery bank.

In places where there is no historical data on demand the necessity to describe the system stochastically is showed in [15] and [16] , expanding the study for the uncertainty costs in different times of the day. The penalty cost for distributed generation connected with the electric network appears in [16]. This is possible using different probability distributions. In the case of the present article, the uniform distribution with uncertainty at twelve oclock is used in places where there is no certainty of the distribution. The most recent antecedent about uncertainty cost is in [17] on hydroelectric power plants. The table 1 shows the considerations of the previous work for economic dispatch with controllable renewable systems.

Table 1: Considerations of Scheduling Approaches

Considerations	References
Uncertainty Consideration	[1], [4], [5], [13]
Demand Response	[2]
Heuristic Optimization	[3], [7], [10], [13], [14], [15]
Stochastic optimization	[2], [4], [5], [17]
Energy Storage Systems	[5], [6]
Electric Vehicles	[9], [16]

2 Analytical method

The need to provide a model for the economic dispatch is evident. Thus, an analytic proposal will be developed and further proved with a Monte Carlo simulation. First, assume that the probability density function for the generated power $f[P]$ is defined with an uniform distribution:

$$f[P] = \begin{cases} \frac{1}{P_{max}-P_{min}} & \text{for } P_{min} \leq P \leq P_{max}, \\ 0 & \text{for } P < P_{min} \text{ or } P > P_{max} \end{cases}$$

For a linear function for the penalty cost due to an underestimation $y = C_u[P] = C_u(P-P_s)$, it is possible to determine the corresponding expected penalty cost function as follows:

$$\begin{aligned} E[y] &= \int_{-\infty}^{\infty} y f(y) dy \\ &= \int_{0}^{C_u(P_{max}-P_s)} y \frac{1}{C_u(P_{max}-P_{min})} dy \\ &= \frac{1}{C_u(P_{max}-P_{min})} \left(\frac{y^2}{2}\right) \Big|_0^{C_u(P_{max}-P_s)} \end{aligned}$$

$$\begin{aligned} \rightarrow E[C_u(P)] =& \frac{C_u}{P_{max}-P_{min}} \left(\frac{P_s^2}{2} - \right. \\ & \left. P_s P_{max} + \frac{P_{max}^2}{2}\right) \end{aligned} \tag{1}$$

Similarly, the expected cost function for the overestimation with $z = C_o[P] = C_o(P_s - P)$ can be obtained:

$$\begin{aligned} E[z] &= \int_{-\infty}^{\infty} z f(z) dz \\ &= \int_{C_o(P_s-P_{min})}^{0} z \frac{-1}{C_o(P_{max}-P_{min})} dz \\ &= \frac{-1}{C_o(P_{max}-P_{min})} \left(\frac{z^2}{2}\right) \Big|_{C_o(P_s-P_{min})}^{0} \end{aligned}$$

$$\begin{aligned} \rightarrow E[C_o(P)] =& \frac{C_o}{P_{max}-P_{min}} \left(\frac{P_s^2}{2} - \right. \\ & \left. P_s P_{min} + \frac{P_{min}^2}{2}\right) \end{aligned} \tag{2}$$

The previous results make it possible to calculate the expected uncertainty cost function (UCF), which describes a remarkable quadratic pattern, something useful for conventional economic dispatch software.

$$\rightarrow E[UCF] = E[C_u(P)] + E[C_o(P)] \tag{3}$$

Furthermore, it is possible to define other helpful statistical variables that complement the model. For instance, a variance analysis can be performed with the following formulation:

$$Var[C_u(P)] = E[C_u(P)^2] - E[C_u(P)]^2 \tag{4}$$

$$Var[C_o(P)] = E[C_o(P)^2] - E[C_o(P)]^2 \tag{5}$$

Thus, for the underestimation case we have:

$$\begin{aligned} E[y^2] &= \int_{0}^{C_u(P_{max}-P_s)} y^2 \frac{1}{C_u(P_{max}-P_{min})} dy \\ &= \frac{1}{C_u(P_{max}-P_{min})} \left(\frac{y^3}{3}\right) \Big|_0^{C_u(P_{max}-P_s)} \end{aligned}$$

$$\rightarrow E[C_u(P)^2] = \frac{C_u^2}{P_{max}-P_{min}} \frac{(P_{max}-P_s)^3}{3} \tag{6}$$

$$\rightarrow Var[C_u(P)] = \frac{C_u^2(P_{max}-P_s)^3}{3(P_{max}-P_{min})} - E[C_u(P)]^2$$

and for the overestimation case we have:

$$\begin{aligned} E[z^2] &= \int_{C_o(P_s-P_{min})}^{0} z^2 \frac{-1}{C_o(P_{max}-P_{min})} dz \\ &= \frac{-1}{C_o(P_{max}-P_{min})} \left(\frac{z^3}{3}\right) \Big|_{C_o(P_s-P_{min})}^{0} \end{aligned}$$

$$\rightarrow E[C_o(P)^2] = \frac{C_o^2}{P_{max} - P_{min}} \frac{(P_s - P_{min})^3}{3} \quad (7)$$

$$\rightarrow Var[C_o(P)] = \frac{C_o^2 (P_s - P_{min})^3}{3(P_{max} - P_{min})} - E[C_o(P)]^2$$

Now, the distribution function for the probability of an underestimation or an overestimation is needed for the UCF. It can be expressed as follows:

$$\begin{aligned} P[P > P_s] &= \int_{P_s}^{P_{max}} f[P]dP \\ &= \int_{P_s}^{P_{max}} \frac{1}{P_{max} - P_{min}} dP \\ &= \frac{P}{P_{max} - P_{min}} \Big|_{P_s}^{P_{max}} \end{aligned}$$

$$\rightarrow P[P > P_s] = \frac{P_{max} - P_s}{P_{max} - P_{min}} \quad (8)$$

$$\begin{aligned} P[P < P_s] &= \int_{P_{min}}^{P_s} f[P]dP \\ &= \int_{P_{min}}^{P_s} \frac{1}{P_{max} - P_{min}} dP \\ &= \frac{P}{P_{max} - P_{min}} \Big|_{P_{min}}^{P_s} \end{aligned}$$

$$\rightarrow P[P < P_s] = \frac{P_s - P_{min}}{P_{max} - P_{min}} \quad (9)$$

It makes possible to calculate the probability for the UCF (only in the underestimation case) as follows:

$$1 = P[P < P_s] + \int_0^{C_u(P_{max} - P_s)} f[UCF_u]dP$$

$$1 = \frac{P_s - P_{min}}{P_{max} - P_{min}} + f[UCF_u]C_u(P_{max} - P_s)$$

This leads to a definition of the UCF_u probability density function:

$$\rightarrow f[UCF_u] = f_1 = \frac{1 - \frac{P_s - P_{min}}{P_{max} - P_{min}}}{C_u(P_{max} - P_s)} \quad (10)$$

Similarly, it is possible to calculate the probability for the UCF (only in the overestimation case) as follows:

$$1 = P[P > P_s] + \int_0^{C_o(P_s - P_{min})} f[UCF_o]dP$$

$$1 = \frac{P_{max} - P_s}{P_{max} - P_{min}} + f[UCF_o]C_o(P_s - P_{min})$$

This leads to a definition of the UCF_o probability density function:

$$\rightarrow f[UCF_o] = f_2 = \frac{1 - \frac{P_{max} - P_s}{P_{max} - P_{min}}}{C_o(P_s - P_{min})} \quad (11)$$

The definition for the variance associated with the UCF can be finally obtained with the equation:

$$Var[UCF] = E[UCF^2] - E[UCF]^2 \quad (12)$$

With $E[UCF]$ defined in Eq.(3), the remain term can be calculated as a piecewise function defined by $C_o(P_s - P_{min}) < C_u(P_{max} - P_s)$:

$$\begin{aligned} E[UCF^2] = &\int_0^{C_o(P_s - P_{min})} UCF^2(f_1 + f_2)dUCF \\ &+ \int_{C_o(P_s - P_{min})}^{C_u(P_{max} - P_s)} UCF^2 f_1 dUCF \\ &= (f_1 + f_2)\frac{(C_o(P_s - P_{min}))^3}{3} \\ &+ f_1 \frac{(C_u(P_{max} - P_s))^3 - (C_o(P_s - P_{min}))^3}{3} \end{aligned}$$

$$\rightarrow E[UCF^2] = \frac{f_2(C_o(P_s - P_{min}))^3}{3} + \frac{f_1(C_u(P_{max} - P_s))^3}{3} \quad (13)$$

And also defined by
$C_o(P_s - P_{min}) > C_u(P_{max} - P_s)$:

$$\begin{aligned} E[UCF^2] = &\int_0^{C_u(P_{max} - P_s)} UCF^2(f_1 + f_2)dUCF \\ &+ \int_{C_u(P_{max} - P_s)}^{C_o(P_s - P_{min})} UCF^2 f_2 dUCF \\ &= (f_1 + f_2)\frac{(C_u(P_{max} - P_s))^3}{3} \\ &+ f_2 \frac{(C_o(P_s - P_{min}))^3 - (C_u(P_{max} - P_s))^3}{3} \end{aligned}$$

$$\rightarrow E[UCF^2] = \frac{f_2(C_o(P_s - P_{min}))^3}{3} + \frac{f_1(C_u(P_{max} - P_s))^3}{3} \quad (14)$$

Here, it can be noticed that Eq. (13) and (14) are the same. Thus, we can conclude that Eq. (12) can be calculated only with Eq. (14) and (3).

3 Monte Carlo simulation

The Monte Carlo method has shown an extended approval for the validation of physical models including variables that have an associated probability density distribution (e.g. solar radiation). Therefore, a Monte Carlo simulation program was developed in order to study the behavior of overestimation and underestimation instances for a previously scheduled power value with a set of 100,000 uniformly distributed expected power values.

For this framework, test values were initially set to $P_s = 29$, $P_{min} = 26$, $P_{max} = 30$, $C_u = 300$, and $C_o = 700$. After an elapsed simulation time of around 1 second, multiple statistical parameters were obtained. It includes expected values and variances associated with the different cost functions that were modeled for the photovoltaic (PV) generation.

The resulting statistical values were: $E[C_u(P)] = 37.6$, $E[C_u(P)] = 787.2$, $E[UCF] = 824.8$, $Var[C_u(P)] = 6123.3$, $Var[C_u(P)] = 480921.6$ and $Var[UCF] = 427819.1$.

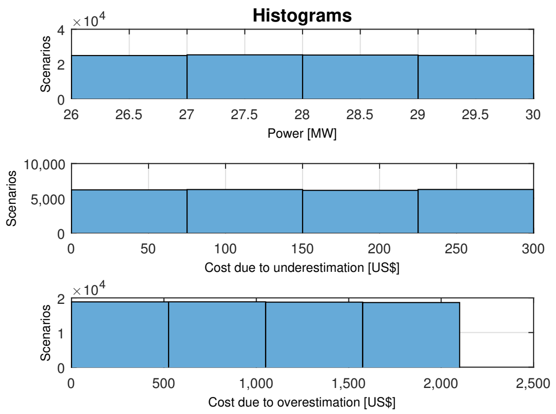

Figure 1: Histograms for the power and the different resulting costs.

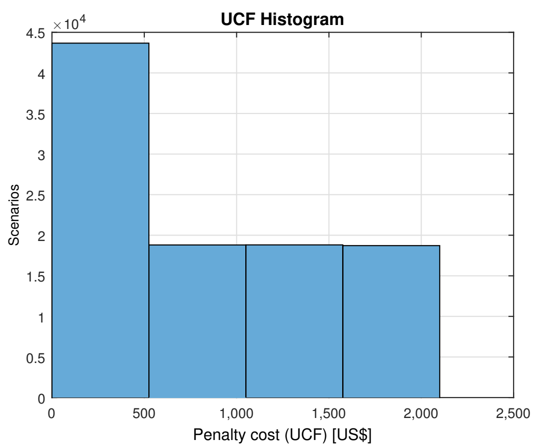

Figure 2: Histogram for the Uncertainty Cost Function.

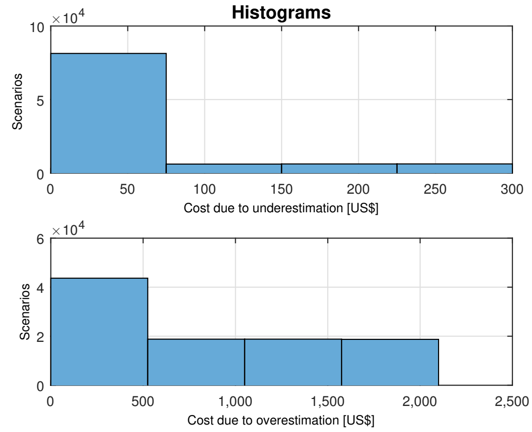

Figure 3: Histograms for underestimation only and overestimation only.

4 Validation

In order to validate the proposed analytic method, its equations were also calculated. It took around 500 milliseconds to obtain the resulting statistical values: $E[C_u(P)] = 37.5$, $E[C_u(P)] = 787.5$, $E[UCF] = 825$, $Var[C_u(P)] = 6093.8$, $Var[C_u(P)] = 482343.75$ and $Var[UCF] = 429375$. A simple comparison with the Monte Carlo results leads to errors below the $\epsilon = 1\%$ for all the variables under analysis.

Also, an stronger validation was obtained due to a more general simulation with P_s, P_{min} and P_{max} randomly chosen between a valid range of values that lead to UCF errors below $\epsilon = 0.7\%$.

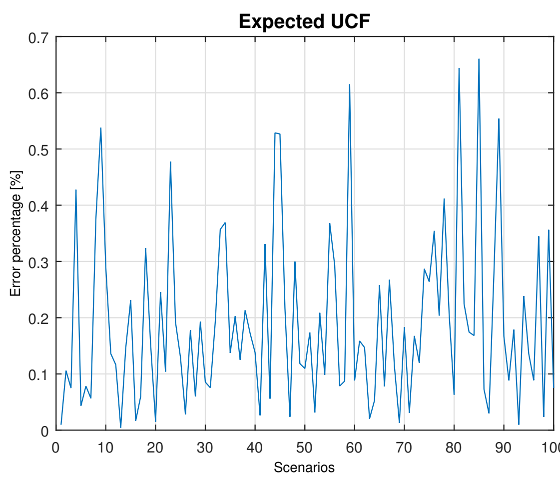

Figure 4: Error plot for different UCF calculations.

Finally, the importance of the resulting quadratic formula for UCF motivated a simulation were P_s was evaluated for several values between 100 and 200 with fixed limits $P_{min} = 50$ and $P_{max} = 250$.

The results between Monte Carlo simulations and the analytic method validated the proposal, both of them showing a quadratic trend in accordance to the expected analytic equation (15) and the curve fitting results (16) that confirm the same for the Monte Carlo data with a coefficient of determination $R^2 = 0.9999$.

$$\rightarrow E[UCF]_{analytic} = 2.5P_s^2 - 550P_s + 51250 \quad (15)$$

$$\rightarrow E[UCF]_{fit} = 2.48P_s^2 - 546.78P_s + 51044.24 \quad (16)$$

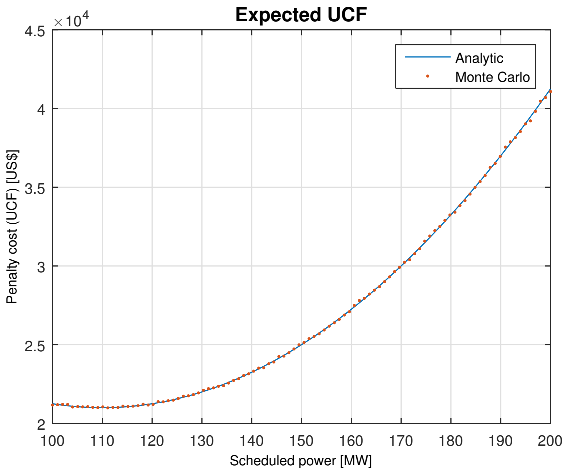

Figure 5: Comparison between Analytic and Monte Carlo results for UCF.

5 Conclusions and future work

A simple mathematical model in eq.(1) and eq.(2) exhibits a quadratic structure for the calculation of expected penalty cost values associated with the overestimation and underestimation of scheduled power in a PV generator working under uniform distribution for its available power.

The proposed model has been verified under Monte Carlo simulations with a remarkably low error associated with the expected UCF values Fig.(4). Thus, a simple method could be used to improve penalty cost estimations for PV generators lacking information for its available power probability distribution. This way, only with a uniform distribution model and a classic economic dispatch software handling eq.(1) and eq.(2) an optimized scheduled power could be easily obtained.

Research related to a deeper understanding of the practical implications in the results that were obtained is important. The analysis of the limitations using this model with specialized software for classic economic dispatch, experimental testing of the analitical methods that were exposed, analysis under other probability distributions and applications to different generation technologies are some important topics that can be reviwed in the future.

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