One method of finding an intergroup consensus based on triangular fuzzy numbers

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Abstract: - This paper introduces one method of reaching consensus among several groups of experts. The method is based on the use of fuzzy numbers. It is meant that opinions of experts are expressed by fuzzy triangular numbers and, therefore, several finite collections of fuzzy triangular numbers are obtained. A method for aggregation of the obtained several finite collections of fuzzy sets into the resulting one is proposed. A new approach is introduced for determining degrees of experts' importance depending on the closeness of experts' estimates to the representative of a finite collection of all triangular fuzzy estimates. The specific fuzzy aggregation operator is offered. The proposed method is thoroughly discussed and its algorithm is presented.

Key-Words: - Consensus, Finite collection of fuzzy triangular numbers, Metric lattice, Regulation, Representative, Fuzzy aggregation operator, Algorithm

1 Introduction

An uncertainty assists at practically any situations in our life. One of the most effective ways to take account of this phenomenon is a using of group decision strategy. These processes by their nature represent a transformation from individual opinions of experts into the resulting one.

It is generally accepted that fuzzy numbers are a good tool to express uncertainty, limited knowledge (see e.g. [1,2,3]). For the solution of the considered problems we propose the use of triangular fuzzy numbers due to their computational efficiency [8]. In the present paper we suppose that there are a several groups of experts and each expert expresses his subjective estimate by a fuzzy triangular numbers that represents the rating to an alternative under a given criterion. As a result for each group of experts a finite collection of fuzzy sets is obtained. It is understood that experts are specialists of the same level, but this does not mean that their estimates may not be essentially different. The essence of our proposed approach consists in the following. A degree of agreement between experts' estimates is measured by the general metric defined by means of the isotone valuation introduced in the paper and embracing the whole class of distances between of fuzzy triangular numbers.

Further, the concepts of regulation and representative of a finite collection of fuzzy triangular numbers are introduced. The motivation of their introduction is explained in [6]. A representative is defined as a fuzzy triangular number such that the sum of distances between this triangular number and all other members of the considered finite collection of fuzzy triangular numbers is minimal. Speaking in general, a representative may take an infinite number of values, but it can be defined uniquely by using a special fuzzy aggregation operator. Our task is to aggregate the obtained finite collections of fuzzy triangular numbers into the resulting one in each group and to aggregate these representatives into the resulting one. Almost all the theoretical results represent, to one extent or the other, a modification of the results obtained in [5,6] including a terminology.

2 Essential Notions

We denote a *triangular fuzzy number* as $\tilde{R} = (a,b,c),\ a,b,c \in \mathbb{R},\ a \le b \le c$

$\Psi(X) = \left\{\tilde{R}_i = (a_i, b_i, c_i),\ a_i \le b_i \le c_i,\ i \in \mathbb{N}\right\}$ - the set of all triangular fuzzy numbers on the universe X.

$\tilde{R}_1 = \tilde{R}_2 \Leftrightarrow a_1 = a_2, b_1 = b_2,\ c_1 = c_2,\quad \tilde{R}_1, \tilde{R}_2 \in \Psi(X)$.

$\tilde{R}_1 \oplus \tilde{R}_2 = (a_1 + b_1, a_2 + b_2, c_1 + c_2), \quad \tilde{R}_1, \tilde{R}_2 \in \Psi(X)$.

$\alpha \odot \tilde{R} = (\alpha a, \alpha b, \alpha c) \ \ \alpha > 0, \ \tilde{R} \in \Psi(X)$.

Definition 1. The triangular fuzzy number $\tilde{R}_1 = (a_i)$ is less than or equal to the triangular fuzzy number $\tilde{R}_2 = (b_i)$, $i = \overline{1,3}$, i.e. $\tilde{R}_1 \preceq \tilde{R}_2$ if and only if

$$a_1 \le b_1,\ a_2 \le b_2,\ a_3 \le b_3. \qquad (1)$$

It is known that [4]

$$\begin{cases} \widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\} = (\min\{a_1, b_1\},\ \min\{a_2, b_2\}, \min\{a_3, b_3\}), \\ \widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\} = (\max\{a_1, b_1\},\ \max\{a_2, b_2\}, \max\{a_3, b_3\}). \end{cases} \qquad (2)$$

Hence it follows that the above definition is equivalent to those given in the literature (see e.g. [1]):

$$\tilde{R}_1 \preceq \tilde{R}_2 \Leftrightarrow \begin{cases} \widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\} = \tilde{R}_1, \\ \widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\} = \tilde{R}_2 \end{cases}, \quad \tilde{R}_1, \tilde{R}_2 \in \Psi(X).$$

It is not difficult to verify that the distributivity of $\widetilde{\min}$ and $\widetilde{\max}$ holds in $\Psi(X)$:

$$\widetilde{\min}\{\tilde{R}_1, \widetilde{\max}\{\tilde{R}_2, \tilde{R}_3\}\} = \widetilde{\max}\{\widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\}, \widetilde{\min}\{\tilde{R}_1, \tilde{R}_3\}\},$$
$$\widetilde{\max}\{\tilde{R}_1, \widetilde{\min}\{\tilde{R}_2, \tilde{R}_3\}\} = \widetilde{\min}\{\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\}, \widetilde{\max}\{\tilde{R}_1, \tilde{R}_3\}\}. \qquad (3)$$

To determine distances between triangular fuzzy numbers we need to introduce the metric on $\Psi(X)$. Depending on the metric, distances between two objects can be measured in many ways. There exists a vast amount of literature on this issue (see e.g. [2]). Here we have chosen the following approach.

We say that the function $v(\tilde{R}) : \Psi(X) \to \Re^+$ is an *isotone valuation* on $\Psi(X)$ if

$$v(\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\}) + v(\widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\}) = v(\tilde{R}_1) + v(\tilde{R}_2)$$

and (4)

$$\tilde{R}_1 \preceq \tilde{R}_2 \Rightarrow v(\tilde{R}_1) \le v(\tilde{R}_2).$$

Consider the following equation:

$$\rho(\tilde{R}_1, \tilde{R}_2) = v(\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\}) - v(\widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\}) \qquad (5)$$

Proposition 1. *Eq.* (5) *represents the metric on* $\Psi(X)$, *i.e.* $\rho(\tilde{R}_1, \tilde{R}_2)$ *meets the following requirements*:

1) $\rho(\tilde{R}_1, \tilde{R}_2) = 0 \Leftrightarrow \tilde{R}_1 = \tilde{R}_2$;

2) $\rho(\tilde{R}_1, \tilde{R}_2) = \rho(\tilde{R}_2, \tilde{R}_1)$;

3) $\rho(\tilde{R}_1, \tilde{R}) + \rho(\tilde{R}, \tilde{R}_2) \ge \rho(\tilde{R}_1, \tilde{R}_2), \quad \forall \tilde{R} \in \Psi(X)$.

Proof. Let

$\rho(\tilde{R}_1, \tilde{R}_2) = 0 \Rightarrow v(\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\}) = v(\widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\})$.

By (2) and (4) we can conclude that

$\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\} = \widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\} \Rightarrow \tilde{R}_1 = \tilde{R}_2$. Now let

$\tilde{R}_1 = \tilde{R}_2 \Rightarrow \widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\} = \widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\} \Rightarrow v(\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\})$

$= v(\widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\}) \Rightarrow \rho(\tilde{R}_1, \tilde{R}_2) = 0$. So, 1) is true.

$\rho(\tilde{R}_1, \tilde{R}_2) = v(\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\}) - v(\widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\})$

$= v(\widetilde{\max}\{\tilde{R}_2, \tilde{R}_1\}) - v(\widetilde{\min}\{\tilde{R}_2, \tilde{R}_1\}) = \rho(\tilde{R}_2, \tilde{R}_1)$

2) is also true.

$$\begin{aligned}
\rho(\tilde{R}_1, \tilde{R}_2) &= v(\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\}) - v(\widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\}) \\
&\overset{(4)}{\le} v(\widetilde{\max}\{\widetilde{\max}\{\tilde{R}_1, \tilde{R}\}, \widetilde{\max}\{\tilde{R}, \tilde{R}_2\}\}) \\
&\quad - v(\widetilde{\min}\{\widetilde{\min}\{\tilde{R}_1, \tilde{R}\}, \widetilde{\min}\{\tilde{R}, \tilde{R}_2\}\}) \\
&\overset{(4)}{=} v(\widetilde{\max}\{\tilde{R}_1, \tilde{R}\}) + v(\widetilde{\max}\{\tilde{R}, \tilde{R}_2\}) \\
&\quad - v(\widetilde{\min}\{\widetilde{\max}\{\tilde{R}_1, \tilde{R}\}, \widetilde{\max}\{\tilde{R}, \tilde{R}_2\}\}) \\
&\quad + v(\widetilde{\max}\{\widetilde{\min}\{\tilde{R}_1, \tilde{R}\}, \widetilde{\min}\{\tilde{R}, \tilde{R}_2\}\}) \\
&\quad - v(\widetilde{\min}\{\tilde{R}_1, \tilde{R}\}) - v(\widetilde{\min}\{\tilde{R}, \tilde{R}_2\}) \\
&\overset{(3),(4)}{=} \rho(\tilde{R}_1, \tilde{R}) + \rho(\tilde{R}, \tilde{R}_2) + v(\widetilde{\min}\{\tilde{R}, \widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\}\}) \\
&\quad - v(\widetilde{\max}\{\tilde{R}, \widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\}\}) \overset{(4)}{\le} \rho(\tilde{R}_1, \tilde{R}) + \rho(\tilde{R}, \tilde{R}_2)
\end{aligned}$$

3) is proved and the proof is completed. □

$\Psi(X)$ with isotone valuation v and metric (5) is called the *metric space* of triangular fuzzy numbers.

Definition 2. In the metric space the triangular fuzzy number $\tilde{R}^*$ is *the representative* of the finite collection of triangular fuzzy numbers $\{\tilde{R}_j\}$, $j = \overline{1,m}$, $m = 2,3,\ldots$, if

$$\sum_{j=1}^{m}\rho\left(\tilde{R}^{*},\tilde{R}_{j}\right)\leq\sum_{j=1}^{m}\rho\left(\tilde{S},\tilde{R}_{j}\right),\ \forall\tilde{S}\in\Psi\left(X\right) \qquad (6)$$

To simplify further theoretical constructions we need to introduce the concept of *regulation* of a finite collection of triangular fuzzy numbers. We begin with an example.

Suppose we have the finite collection of triangular fuzzy numbers:

	a_j	b_j	c_j
$\tilde{R}_1$	7	7.5	8
$\tilde{R}_2$	6	6.1	7.7
$\tilde{R}_3$	1	3	5
$\tilde{R}_4$	7.6	7.9	8.1

Compare with it the following finite collection of triangular fuzzy numbers:

	a'_j	b'_j	c'_j
$\tilde{R}'_1$	1	3	5
$\tilde{R}'_2$	6	6.1	7.7
$\tilde{R}'_3$	7	7.5	8
$\tilde{R}'_4$	7.6	7.9	8.1

We see that the columns contain the same values but they are increasingly ordered in the second table By the regulation of the finite collection of triangular fuzzy numbers $\left\{\tilde{R}_{1,}\tilde{R}_{2,}\tilde{R}_{3,}\tilde{R}_{4}\right\}$ we will mean the finite collection of triangular fuzzy numbers $\left\{\tilde{R}'_{1,}\tilde{R}'_{2,}\tilde{R}'_{3,}\tilde{R}'_{4}\right\}$. The strict definition of regulation of the set of trapezoidal fuzzy numbers will be given below, whereas now we have to answer one question, namely: whether the members of this regulation are also triangular fuzzy numbers. The question can be rephrased as follows: do the inequalities $a_j' \leq b_j' \leq c_j' \leq d_j'$ ($j=\overline{1,m},\ m=2,3,...$) hold? The next proposition provides a positive answer.

Proposition 2. Consider the finite collections of triangular fuzzy numbers $\left\{\tilde{R}_j\right\}=\left\{\left(a_j,b_j,c_j\right)\right\}$ and $\left\{\tilde{R}'_j\right\}=\left\{\left(a'_j,b'_j,c'_j\right)\right\}$, where the sets $\{a_j\}$ and $\{a_j'\}$, $\{b_j\}$ and $\{b_j'\}$, $\{c_j\}$ and $\{c_j'\}$ are pairwise equal and $a_1' \leq a_2' \leq \ldots \leq a_m'$, $b_1' \leq b_2' \leq \ldots \leq b_m'$, $c_1' \leq c_2' \leq \ldots \leq c_m'$, $j=\overline{1,m},\ m=2,3,...$. Then $a_j' \leq b_j' \leq c_j'$, i.e. $\left\{\tilde{R}'_j\right\}\in\Psi\left(X\right)$.

Proof. We prove the proposition for the sets $\{a_j\}$, $\{a_j'\}$, and $\{b_j\}$, $\{b_j'\}$ $j=\overline{1,m},\ m=2,3,...$. The proof for the other sets of vertices of triangular fuzzy numbers is analogous. We proceed by induction.

(i) $m = 2$. We have $\{a_1, b_1\}$, $\{a_2, b_2\}$, where $a_1 \leq b_1$, $a_2 \leq b_2$; $\{a_j'\} = \{a_j\}$, $\{b_j'\} = \{b_j\}$, $j \in \{1,2\}$, where $a_1' \leq a_2', b_1' \leq b_2' \Rightarrow a_1' = \min\{a_1,a_2\} \leq \min\{b_1,b_2\} = b_1'$ and $a_2' = \max\{a_1,a_2\} \leq \max\{b_1,b_2\} = b_2'$;

(ii) assume that $a_j' \leq b_j'$ for all $j \in \{3,4,\ldots,m-1\}$, $m = 4,5,\ldots$;

(iii) consider a_m' and b_m'. In fact that $a_m' = \max\{a_j', a_m\}$ and $b_m' = \max\{b_j', b_m\}$. This and (ii) imply $a_m' \leq b_m'$. □

Now we are able to introduce a strict definition of the regulation.

Definition 3. The finite collection of triangular fuzzy numbers $\left\{\tilde{R}'_j\right\}$ is the regulation of the finite collection of triangular fuzzy numbers $\left\{\tilde{R}_j\right\}$ if the finite sets $\left\{a_j\right\}$ and $\left\{a'_j\right\}$, $\left\{b_j\right\}$ and $\left\{b'_j\right\}$, $\left\{c_j\right\}$ and $\left\{c'_j\right\}$ are pairwise equal and $a'_1 \leq a'_2 \leq ... \leq a'_m$, $b'_1 \leq b'_2 \leq ... \leq b'_m$, $c'_1 \leq c'_2 \leq ... \leq c'_m$, $j=\overline{1,m},\ m=2,3,...$.

By this definition and (5) it is obvious that the equality

$$\sum_{j=1}^{m}\rho\left(\tilde{S},\tilde{R}_{j}\right)=\sum_{j=1}^{m}\rho\left(\tilde{S},\tilde{R}'_{j}\right) \qquad (7)$$

holds in the metric space for any $\tilde{S}\in\Psi\left(X\right)$ and the finite collection of triangular fuzzy numbers $\left\{\tilde{R}_j\right\}$, $j=\overline{1,m},\ m=2,3,...$. From (7) it follows that the *representatives of the finite collection of triangular fuzzy numbers and its regulation coincide.*

By Definition 3 and (1), the following proposition is true.

Proposition 3. *The regulation is a finite collection of nested triangular fuzzy numbers*:

$$\tilde{R}_1^{'} \preceq \tilde{R}_2^{'} \preceq ... \preceq \tilde{R}_m^{'},\ m = 2,3,...\ .$$

It is easy to see that

$$\tilde{R}_1 \preceq \tilde{R}_2 \Rightarrow \rho\left(\tilde{R}_1, \tilde{R}_2\right) = v\left(\tilde{R}_2\right) - v\left(\tilde{R}_1\right). \qquad (8)$$

The proof of Theorem 1 is analogous of the proof of Theorem 3.1 in [6]

Theorem 1. *In the metric space of triangular fuzzy numbers the representative* $\tilde{R}^*$ *of the finite collection of triangular fuzzy numbers* $\left\{\tilde{R}_j\right\}$, $j = \overline{1, m}$, m=2,3,… *is determined as follows:*

$$\tilde{R}_{m/2}^{'} \preceq \tilde{R}^* \preceq \tilde{R}_{m/2+1}^{'} \quad \textit{if m is even;}$$
$$\tilde{R}^* = \tilde{R}_{(m+1)/2}^{'} \quad \textit{if m is odd.}$$

To realize our method, we need a special aggregation operator that satisfies certain requirements. Today, in fuzzy sets theory there are a lot of well known fuzzy aggregation operators such as triangular norms, conorms, averaging, with weight coefficients and so on (see e.g. [7]).

In the metric space of triangular fuzzy numbers the representative of a finite collection of triangular fuzzy numbers, as it is stated in Definition 2, is a new kind of fuzzy aggregation operator. By Theorem 1 the representative of the finite collection of triangular fuzzy numbers $\left\{\tilde{R}_j\right\}$, $j = \overline{1, m}$, $m = 2,3,...$ yields $\tilde{R}_{m/2}^{'} \preceq \tilde{R}^* \preceq \tilde{R}_{m/2+1}^{'}$ if m is even, and $\tilde{R}^* = \tilde{R}_{(m+1)/2}^{'}$ - if m is odd. We introduce a new fuzzy aggregation operator that always meets these requirements using the same arguments as in the entry of similar operator for finite collection of fuzzy sets in [4]. Here and in the sequel the symbol [ ] denotes the integer part of a number.

$$\tilde{R}^* = \begin{cases} \left(a_{[m/2],i}^{'} + a_{[(m+3)/2],i}^{'}\right)/2 & \text{if } \sum 1 = \sum 2 \\ a_{[m/2],i}^{'} + \left(\dfrac{\sum 1}{\sum 1 + \sum 2}\left(a_{[(m+3)/2],i}^{'} - a_{[m/2],i}^{'}\right)\right) & \text{otherwise} \end{cases} \quad i = \overline{1,3}, \qquad (9)$$

where $\sum 1 = \sum_{j=1}^{[(m+1)/2]} \rho\left(\tilde{R}_j^{'}, \tilde{R}_{[m/2]}^{'}\right)$,

$$\sum 2 = \sum_{j=[m/2]+1}^{m} \rho\left(\tilde{R}_j^{'}, \tilde{R}_{[(m+3)/2]}^{'}\right)$$

Remark 1. It is easy to show that the representative determined by (9) is a triangular fuzzy number

3 The Method of Finding an Intergroup Consensus

Let p groups of experts be attracted to consider of a certain project and each of these groups consists of m persons. Each expert expresses his subjective estimate by a triangular fuzzy number that represents the rating to an alternative under a given criterion. As a result for each group of experts a finite collection of triangular fuzzy numberს is obtained. So, we have p finite collections of m triangular fuzzy numbers.

We propose a new approach for achieving an intergroup consensus. At the first stage, we aggregate the expert estimates in each group. In constructing any kind of aggregation method under group decision-making, the key task is to determine the well-justified weights of importance for each expert. Let us consider the finite collection of triangular fuzzy numbers obtained by conversion of experts' quantitative estimates. To our mind, the representative of this collection, i.e. a triangular fuzzy number such that the sum of distances between it and all other members of the given finite collection is minimal, is of certain particular interest. The representative can be regarded as a kind of group consensus, but in that case the degrees of experts' importance are neglected. The representative is something like a standard for the members of the considered collection. As the weights of physical bodies are measured by comparing them with the Paris standard kilogram, it seems natural for us to determine experts' weights of importance depending on how close experts' estimates are to the representative [6].

Thus, the main idea of the proposed aggregation reduces to the following. The weight of importance for each expert is determined by a function inversely proportional to the distance between his/her transformed estimate and the representative of the finite collection of all experts' transformed estimates, i.e. the smaller the distance between an expert's estimate and the representative, the larger the weight of his importance.

Let us give a formal description of the method. Let $\tilde{R}_j$, $j \in \{1,2,...,m\}$, $m = 2,3,...$ be a triangular fuzzy number representing the jth expert's subjective estimate of the rating to an alternative under a given criterion in certain group. The transformed estimates of all experts in this group form the finite collection of triangular fuzzy numbers $\{\tilde{R}_j\}$. By Definition 2 and formula (9) we find the regulation $\{\tilde{R}'_j\}$ and the representative $\tilde{R}^*$ of this collection. Denote the *j*th expert's aggregation weight (weight of importance) and the final result of aggregation by ω_j and $\tilde{R}$ respectively.

By the above reasoning, the weights and the final result of aggregation can be defined in the form

$$\omega_j = \frac{\left(\rho\left(\tilde{R}^*, \tilde{R}_j\right)\right)^{-1}}{\sum_{j=1}^{m}\left(\rho\left(\tilde{R}^*, \tilde{R}_j\right)\right)^{-1}} \tag{10}$$

and

$$\tilde{R} = \sum\nolimits_{j=1}^{m}(\omega_j \odot \tilde{R}_j) \tag{11}$$

Here $\odot$ is the fuzzy multiplication operator [3]. It is obvious that $\sum\nolimits_{j=1}^{m}\omega_j = 1$.

This approach looks plausible, but if at least one member of the finite collection of expert estimates coincides with the representative, then the function ω_j discontinues. It can be shown that the following proposition is true (see Proposition 3.1 in [6]).

Proposition 3. *For any finite collection of triangular fuzzy numbers* $\{\tilde{R}_j\}$, $j = \overline{1,m}$, $m = 2,3,...$ *the following holds:*

(a) $\tilde{R} = \sum\nolimits_{j=1}^{m}(\omega_j \odot \tilde{R}_j)$ *is always continuous* (*here* ω_j *is given by* (10));

(b) *If there exists at least one j such that* $\rho\left(\tilde{R}_j, \tilde{R}^*\right) = 0$ *then* $\tilde{R} = \tilde{R}^*$.

Corollary 1. *If for all t, j* ∈ {1,2,…, *m*} $\tilde{R}_t = \tilde{R}_j \Rightarrow \tilde{R} = \tilde{R}^*$

Corollary 2. *If all estimates are identical then* $\omega_j = 1/m$.

Proof. This assertion immediately follows from (11). □

4 The Algorithm

To realize our method we will arrive as follows. First of all, using formulas (10) and (11), we found a well-argument way of finding consensus in each group of experts. As a result the finite collection of consensus $\{\tilde{R}_k\}$, $k = \overline{1,p}$, $p = 2,3,...$ is obtained. Finally, we define the intergroup consensus as the representative of the resulting finite collection $\{\tilde{R}_k\}$.

Here we modifye the algorithm developed in [6].

Step 0: *Initialization: There are p finite collections of triangular fuzzy numbers* $\{\tilde{R}_{kj}\}$ *and their regulations* $\{\tilde{R}'_{kj}\}$, *denote the aggregation weight of the j-th expert of the k-th group by* ω_{kj}, *the consensus in each group by* $\tilde{R}_k$, $k = \overline{1,p}$, $j = \overline{1,m}$, $m, p = 2,3,...$ *and the intergroup consensus by* $\tilde{R}^{**}$.

Step 1: *Do* Step 2 *for* $k = \overline{1,p}$.

Step 2: *Compute the representatives* $\tilde{R}^*_k$ *of* $\{\tilde{R}'_{kj}\}$, $j = \overline{1,m}$, $m = 2,3,...$ *by* (9).

Step 3: *Do* Step 4 *for* $k = \overline{1,p}$.

Step 4: *Compute* $\Delta_{kj} = \rho\left(\tilde{R}^*_k, \tilde{R}_{kj}\right)$, $j = \overline{1,m}$, $m = 2,3,...$:

- *If at least one* $\Delta_{kj} = 0$ *then* $\tilde{R}_{kj} = \tilde{R}^*_k$;
- *If* $\Delta_{kj} > 0$ *for all j then compute* ω_{kj} *by* (10) *and obtain the* $\{\tilde{R}_k\}$ *by* (11), *taking into account the fact that in both formulas m is replaced by p*.

Step 5: *Determine the regulation* $\{\tilde{R}'_k\}$ *and its representative* $\{\tilde{R}^*\}$ *by* (9), *taking into account the fact that here m is replaced by p*.

Step 6: *Do* Step 4 *for* $k = \overline{1,p}$.

Step 7: *Compute* $\Delta_k = \rho\left(\tilde{R}^*, \tilde{R}_k\right)$:

- *If at least one* $\Delta_k = 0$ *then* $\tilde{R}_k = \tilde{R}^{**}$;
- *If* $\Delta_k > 0$ *for all j then compute* ω_k *by* (10) *and obtain the* $\tilde{R}^{**}$ *by* (11), *taking into account the fact that in both formulas m is replaced by p*.

Step 8: *The representation is* $\mu = \left\{\mu(x_1), \mu(x_2), ..., \mu(x_p)\right\}$

5 Conclusion

We consider the approach to making decisions for different problems in nonstandard situations with a lack of the previous experience and incomplete knowledge of the considered problem. In such cases we usually cannot do without expert evaluations which lead to the process of group decision-making, and it becomes necessary to solve a problem of alternatives aggregation. It is proposed to solve such problems by means of fuzzy numbers. It is meant that there are several groups of experts. An approach is proposed for the processing of quantitative expert evaluations given as triangular fuzzy numbers.

Here we summarize the essence of the proposed method. We present the method based on the general metric covering the whole class of metrics on the set of triangular fuzzy numbers. A new approach is proposed for determining degrees of experts' importance depending on the closeness of experts' estimates to the representative of a finite collection of all triangular fuzzy estimates. A special fuzzy aggregation operator is offered for the realization of the proposed approach and its algorithm is given.

References:

[1] D. Dubois, H. Prade, Systems on linear fuzzy constraints, *Fuzzy Sets and Systems,* Vol.3, No.1, 1980, pp. 37-48.

[2] Debashree Guha, Debjani Chakraborty, A new approach to fuzzy distance measure and similarity measure between two generalized fuzzy numbers, *Applied Soft Computing,* Vol.10, Issue 1, 2010, pp. 90-99

[3] A. Kauffman and M.M. Gupta, *Introduction to Fuzzy Arithmetic: Theory and Applications* [Van Nostrand Reinhold, New York, 1985].

[4] T. Tsabadze, A method for fuzzy aggregation based on grouped expert evaluations, *Fuzzy Sets and Systems* 157 (2006) pp. 1346-1361.

[5] T. Tsabadze**,** An Approach for Aggregation of Experts' Qualitative Evaluations by Means of Fuzzy Sets. *2013 IFSA World Congress NAFIPS Annual Meeting*, Edmonton, Canada, 2013.

[6] T. Tsabadze, A method for aggregation of trapezoidal fuzzy estimates under group decision-making, *Int. J. Fuzzy Sets and Systems*, 266(2015), pp. 114-130.

{7] J. Vaníček, I. Vrana and S. Aly, *Fuzzy aggregation and averaging for group decision making: A generalization and survey. Knowledge-Based Systems* 22 (2009) pp. 79-84.

[8] L. A. Zadeh, The concept of linguistic variable and its application to approximate reasoning – Part 1, *Information Sciences*, 8, 1975 , pp. 199-249.