Some Improved Generalized Ridge Estimators and their comparison

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Abstract: The problem of multicollinearity is often encountered in time series data since explanatory variables included in the model often share a common trend. Various methods exist in literatures to handle this problem. Among them is the most widely used ridge regression estimator which depends on the ridge parameter. This estimator can be subdivided into either generalized ridge or ordinary ridge estimators. Variance inflation factor is introduced to replace eigenvalue in the generalized ridge estimator proposed by Lawless and Wang (1976). Through this modification some new generalized ridge parameters are proposed and investigated via simulation study. The performances of these proposed estimators are compared with the existing ones using mean square error. Results show that the proposed estimators perform better than the existing ones. It is evident that increasing the level of multicollinearity and number of regressors has positive effect on the MSE. Also, the performance of the estimators depends on the level of error variances.

Keywords: Multicollinearity, Generalized ridge, Ordinary ridge, Simulation study

1.0 Introduction

In statistical modeling it's required that the explanatory variables should not be related. This problem is often encountered in time series data since explanatory variables included in the model often share a common trend, that is, they all increase or decrease over time. However, when this relationship exists, it is referred to as multicollinearity. The efficiency of Ordinary Least Squares (OLS) estimates is reduced when there is multicollinearity. Consequently, OLS estimator become unstable due to large standard error which in turns lead to poor prediction (Khalaf and Iguernane, 2016). Moreover, the regression coefficients might be statistically insignificant leading to wrong producing faulty statistical inference for practitioners (Dorugade, 2014). Various methods exist in literatures to handle the problem of multicollinearity. These include increasing the sample size, data disaggregation, dropping a variable, combining cross-sectional and time series data, reselecting variables, model reparametization, principal component regression estimator, Liu regression estimator, partial least squares estimator, stein estimator, ridge regression estimator and others. However, in this paper, the most widely used ridge regression estimator will be consider. This concept was first introduced by Hoerl and Kennard (1970) to handle multicollinearity problem in engineering data. They introduced a non-zero value of k (ridge parameter) to the $X'X$

matrix of OLS estimator which in turns make the mean squared error (MSE) for the ridge regression estimator smaller when compared with the MSE of OLS estimator. Different authors at different period of times have proposed different estimators for ridge parameter, k. To mention a few, Hoerl and Kennard (1970), Hoerl, Kennard, and Baldwin (1975), McDonald and Galarneau (1975), Lawless and Wang (1976), Dempster, Schatzoff, and Wermuth (1977), Nomura (1988), Troskie and Chalton (1996), Firinguetti (1999), Kibria (2003), Khalaf and Shukur (2005), Batah et al. (2008), Lukman and Ayinde (2016), Kibria and Banik (2016), Adnan et al. (2016) Lukman and Ayinde (2017), Lukman and Arowolo (2018), Lukman et al. (2018) and others. The aforementioned ridge regression estimators can be subdivided into two classes, namely, generalized ridge estimators and ordinary ridge estimators. The main aim of this study is to present a comprehensive review of some generalized ridge estimators and effects some modification. The results will be compare based on minimum AMSE criterion. Investigation conducted via Monte Carlo simulation by varying the following: variance of the random error, correlation among explanatory variables and the sample size. The paper is organized as follows. A comprehensive review of the available generalized ridge estimators for estimating k, followed by a Monte Carlo simulation study and finally, providing some concluding remarks.

2. Statistical Methodology

2.1 OLS and Ridge Regression Estimators

Consider a standard linear regression model

$$Y = X\beta + \varepsilon \tag{1}$$

where Y is an nx1vector of dependent variable, X is a design matrix of order $n \times p$,where p is the number of explanatory variables, β is a px1vector of coefficients and ε is the error vector of order nx1 distributed as $\mathrm{N}(0, \sigma^2 I_n)$. OLS estimator of β is given as:

$$\hat{\beta} = (X'X)^{-1} X'Y \tag{2}$$

In ridge regression, the $X'X$ predictor correlation matrix is modified by adding a small positive number $k\,(k > 0)$ called ridge parameter to the diagonal elements. Ridge estimator is defined as:

$$\hat{\beta}_{RR} = \left(X'X + kI_p\right)^{-1} X'Y,\ \ k > 0$$

(3)

Hoerl and Kennard (1970) showed that the total variance decreases and the squared bias increases as k increases. The variance function is monotonically decreasing and the squared bias function is monotonically increasing. Thus, there is the probability that some k exists such that the MSE for $\hat{\beta}_{RR}$ is less than MSE for the usual $\hat{\beta}$ (Hoerl and Kennard, 1970). The general linear model (1) can be written in canonical form. Suppose there exist an orthogonal matrix T such that:

$$T'X'XT = \Lambda = diag\left(\lambda_1, \lambda_2, ..., \lambda_p\right) \tag{4}$$

where λ_i being the ith eigenvalue of $X'X$. Λ and T are the matrices of eigenvalues and eigenvectors of X' X respectively. Substituting $Z = XT$, $\alpha = T'\beta$ in model (1), then the equivalent model can be rewritten as:

$$Y = Z\alpha + \varepsilon$$

(5)

The OLS estimator of α is given by

$$\hat{\alpha}_{OLS} = (Z'Z)^{-1} Z'Y \qquad = \Lambda^{-1} Z'Y$$

(6)

Therefore, OLS estimator of β is given by

$$\hat{\beta}_{OLS} = T\hat{\alpha}_{OLS}$$

The generalized ridge estimator (GRR) estimator of α is defined as:

$$\hat{\alpha}_{GR} = \left(I - KA^{-1}\right)\hat{\alpha}_{OLS} \tag{7}$$

where $K = diag(k_1, k_2, ..., k_p), k_i \geq 0, i = 1,2,..., p$ be the different ridge parameters for different regressors and $A = \Lambda + K$. Hence, GRR estimator for β is $\hat{\beta}_{GR} = T\hat{\alpha}_{GR}$. The mean squared error $\hat{\alpha}_{GR}$ is given by:

$$MSE(\hat{\alpha}_{GR}) = Variance(\hat{\alpha}_{GR}) + [Bias(\hat{\alpha}_{GR})]^2$$
$$= \hat{\sigma}^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k_i)^2} + \sum_{i=1}^{p} \frac{k_i^2 \hat{\alpha}_i^2}{(\lambda_i + k_i)^2}$$
(8)

When k=0, the mean square error of OLS estimator is obtained

$$\text{MSE}(\hat{\alpha}_{\text{OLS}}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{1}{\lambda_i} \quad (9)$$

2.2. Review of Methods of Estimating Generalized Ridge Estimators

Hoerl and Kennard suggested the value of k_i as

$$k_{HK_i} = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}; \quad i = 1,2,..., p \quad (10)$$

Lawless and Wang (1976) proposed a ridge parameter by multiplying eigenvalues to the denominator of the ridge parameter proposed by Hoerl and Kennard (1970) given by

$$k_{LW_i} = \frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2} \quad i = 1, 2, ..., p$$
(11)

Nomura (1988) proposed a biasing parameter and it is given by

$$k_{N_i} = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} \left\{ 1 + \left[1 + \lambda_i \left(\frac{\hat{\alpha}_i^2}{\hat{\sigma}^2} \right)^{\frac{1}{2}} \right] \right\},$$
$i = 1, 2, ..., p$
(12)

Troskie and Chalton (1996) also proposed a generalized ridge parameter and it is given by

$$k_{TC_i} = \frac{\lambda_i \hat{\sigma}^2}{(\lambda_i \hat{\alpha}_i^2 + \hat{\sigma}^2)} \quad i = 1, 2, ..., p$$
(13)

Using a new approach, Firinguetti (1999) proposed the biasing parameter to be computed as

$$k_{F_i} = \frac{\lambda_i \hat{\sigma}^2}{[\lambda_i \hat{\alpha}_i^2 + (n-p)\hat{\sigma}^2]} \quad i = 1, 2, ..., p$$
(14)

Batah et al. (2008) proposed a ridge parameter given by

$$k_{B_i} = \hat{\sigma}^2 \frac{\left\{ \left[\left(\frac{\hat{\alpha}_i^4 \lambda_i^2}{4\hat{\sigma}^2} \right) + \left(\frac{6\hat{\alpha}_i^4 \lambda_i}{\hat{\sigma}^2} \right) \right]^{\frac{1}{2}} - \left(\frac{\hat{\alpha}_i^2 \lambda_i}{2\hat{\sigma}^2} \right) \right\}}{\hat{\alpha}_i^2}$$
$$i = 1, 2, ..., p \quad (15)$$

where α_i is the *ith* element of $\hat{\alpha}_{OLS}$, $i = 1, 2, ..., p$ and $\hat{\sigma}^2$ is the OLS estimator of σ^2, i.e. $\hat{\sigma}^2 = \frac{Y'Y - \hat{\alpha}'Z'Y}{n - p - 1}$

Dorugade (2014) modified Lawless and Wang (1976) by multiplying $\frac{\lambda_{\max}}{2}$ to the denominator of Hoerl and Kennard (1970a). The estimator is as:

$$k_{AD_i} = \frac{2\hat{\sigma}^2}{\lambda_{\max} \hat{\alpha}_i^2} \quad i = 1, 2, ..., p \quad (16)$$

where $\lambda_{\max}$ is the largest eigenvalue of $X'X$.

Adnan et al. (2016) modified Dorugade (2014) by multiplying $\frac{\lambda_{\max}}{\sqrt{5}}$ to the denominator of Hoerl and Kennard (1970). The estimator is as:

$$k_{AD_i} = \frac{\sqrt{5}\hat{\sigma}^2}{\lambda_{\max} \hat{\alpha}_i^2} \quad i = 1, 2, ..., p \quad (17)$$

2.3 Proposed Estimator

Lawless and Wang (1976) proposed a ridge parameter by multiplying eigenvalues to the denominator of the ridge parameter proposed by Hoerl and Kennard (1970a). Dorugade (2014) suggested an estimator that significantly improves the mean square error of the regression parameter by modifying the proposed estimator by Lawless and Wang (1976). It is observed that eigenvalue introduced by Lawless and Wang (1976) are often used to detect the presence of multicollinearity in a model. This necessitated the modification in this study with the use of variance inflation factor. The following generalized ridge estimators are proposed in this study.

$$k_{P1_i} = \frac{p\hat{\sigma}^2}{VIF_{AM}^{1/P}\alpha_i^2} \qquad i = 1, 2, ..., p \qquad (18)$$

where VIF_{AM} is the arithmetic mean of variance inflation factor

$$k_{P2_i} = \frac{n\hat{\sigma}^2}{VIF_{\min}^{1/P}\alpha_i^2} \qquad i = 1, 2, ..., p \qquad (19)$$

where VIF_{min} is the minimum value of variance inflation factor

$$k_{P3_i} = \frac{n\hat{\sigma}^2}{VIF_{\min}^{1/n}\alpha_i^2} \qquad i = 1, 2, ..., p \qquad (20)$$

where VIF_{min} is the minimum value of variance inflation factor

3. Monte-Carlo Simulation

Most of the works that have been done under ridge estimators have been conducted using Monte Carlo Simulation since a theoretical comparison is not possible. Most researchers have generated data from a normal population, different number of regressors has been used and mostly MSE was used as a performance criterion. These can be seen in the works of McDonald and Galarneau (1975), Kibria (2003), Dorugade (2014), and Kibria and Banik (2016). The simulation study was conducted in this paper using TSP 5.0. Following Kibria (2003), the explanatory variables are generated as follows:

$$X_{ti} = (1 - \rho^2)^{\frac{1}{2}} Z_{ti} + \rho Z_{tp} \qquad (21)$$

t=1, 2, 3,…, n. i=1, 2,…p.

where Z_{ti} is independent standard normal distribution with mean zero and unit variance, ρ is the correlation between any two explanatory variables and p is the number of explanatory variables. The value of ρ were taken as 0.95, 0.99 and 0.999 respectively. The number of explanatory variable (p) was taken to be three (3) and seven (7) respectively. The true values of the model regression coefficient are taken as in the study of Lukman and Ayinde (2017): β_0 was taken to be identically zero. When the number of explanatory variable is three (3): β_1=0.8, β_2=0.1, β_3=0.6. When the number of explanatory variable is seven (7): β_1=0.4, β_2=0.1, β_3=0.6, β_4=0.2, β_5=0.25, β_6=0.3, β_7=0.53. The parameter values were chosen such that $\beta'\beta$=1 which is a common restriction in simulation studies of this type (Muniz and Kibria, 2009). To examine the impact of sample size on the performance of the estimators, the following sample sizes were considered: n= 30, 50 and 70. For the purpose of evaluating the influence of the variance error on the proposed estimators, σ was taken to be 0.5, 5 and 10. This experiment was replicated 5000 times.

The average mean square error of the estimators over 5000 replication was computed using the following equation:

$$AMSE(\hat{\beta}) = \frac{1}{5000}\sum_{i=1}^{p}\sum_{j=1}^{5000}\left(\hat{\beta}_{ij} - \beta_i\right)^2 \qquad (22)$$

where $\hat{\beta}_{ij}$ is i[th] element of the estimator β in the j[th] replication which gives the estimate of β_i . β_i are the true value of the parameter previously mentioned. Estimators with the minimum AMSE are considered best.

4. Results

The AMSE of the OLS estimator, existing generalized ridge estimators and the proposed are provided in Table 1. From Table 1, it was observed that the proposed estimators perform better than OLS. Also, the proposed estimators (especially KP2 and

KP3) perform better than all the reviewed ridge estimators at the different correlation levels, sample sizes and variance of the error terms used in this simulation study. It is evident that increasing the level of multicollinearity and number of regressors has positive effect on the MSE. Also, the performance of the estimators depends on the level of error variances.

5. Conclusion

Variance inflation factor is introduced in this study to improve the performance of the estimator proposed by Lawless and Wang (1976). The proposed estimator performance was evaluated via simulation study at different level of correlation between the explanatory variable, sample size, error variance and number of explanatory variables using average mean square error (AMSE). Results show that the proposed estimators perform better than all the existing ones reviewed in this study.

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Table 1: AMSE of OLSE and some generalized ridge estimators

ρ	Estimator	n=30					
		p=3			**p=7**		
		σ^2=0.25	25	100	σ^2=0.25	25	100
	OLS	0.00002826	0.0113030	0.045212	0.00009735	0.0097355	0.038942
	KHK	0.00000094	0.0036297	0.014566	0.00004877	0.0041010	0.016238
	KAD	0.00002735	0.0109860	0.043935	0.00009510	0.0094450	0.037783
	KAYA	0.00002725	0.0109500	0.043788	0.00009496	0.0094166	0.037669
0.95	KLW	**0.00000078**	0.0026730	0.010646	0.000070013	0.0063308	0.025249
	KTC	0.00001476	0.0054852	0.021980	0.00005861	0.0053381	0.021234
	KHMO	0.00001602	0.0012317	0.004501	0.00004051	0.0010837	0.039104
	KFM	0.00002647	0.0105440	0.042177	0.00008972	0.0089999	0.035999
	KP1	**0.00000055**	**0.0002994**	**0.000791**	**0.00001818**	**0.0002613**	**0.000576**
	KP2	**0.00000073**	**0.0002560**	**0.000581**	**0.00001911**	**0.0002441**	**0.000495**
	KP3	0.00001587	**0.0002055**	**0.000302**	**0.00002128**	**0.0002190**	**0.000370**
	OLS	0.000644	0.064395	0.257580	0.00048100	0.048100	0.19240
	KHK	0.000205	0.020759	0.083185	0.00020744	0.019578	0.078132
	KAD	0.000625	0.062488	0.249930	0.00046250	0.046527	0.18614
	KAYA	0.000622	0.062269	0.249050	0.00046102	0.046373	0.18553
0.99	KLW	**0.000022**	**0.001313**	**0.005164**	0.00013838	0.012427	0.049661
	KTC	0.000315	0.031368	0.125570	0.00026875	0.025912	0.10347
	KHMO	0.000119	0.008356	0.032959	0.00010324	0.007250	0.028622
	KFM	0.000602	0.060087	0.240330	0.00044433	0.044458	0.17783
	KP1	0.000048	0.002317	0.008822	**0.00002809**	**0.000881**	**0.0034427**
	KP2	**0.000045**	**0.001579**	**0.005825**	**0.00002779**	**0.000735**	**0.0024901**
	KP3	**0.000046**	**0.000363**	**0.000869**	**0.00002851**	**0.000424**	**0.0011996**
0.999	OLS	0.007075	0.707460	2.829840	0.0048164	0.481640	1.926570
	KHK	0.002264	0.228370	**0.914080**	0.0019455	0.193090	0.772150
	KAD	0.006855	0.685430	2.741620	0.0046371	0.464630	1.858700
	KAYA	0.006830	0.682910	2.731540	0.0046204	0.462980	1.852110
	KLW	**0.000045**	**0.000208**	**0.000681**	**0.0000714**	**0.005230**	**0.021013**
	KTC	0.003442	0.344950	1.380130	0.0025939	0.257280	1.028720
	KHMO	0.001096	0.105100	0.419870	0.0009813	0.095504	0.381790
	KFM	0.006605	0.660090	2.640240	0.0044512	0.445130	1.780500
	KP1	0.000775	0.074115	0.295950	0.0001435	0.017401	0.078362
	KP2	**0.000560**	**0.051617**	0.205760	**0.0001195**	**0.010124**	**0.040159**
	KP3	**0.000081**	**0.002257**	**0.008207**	**0.0000563**	**0.003061**	**0.011788**
	n=50						
ρ	**P=3**	**P=3**			**P=7**		
	OLS	0.000100	0.010055	0.040219	0.00005774	0.0057740	0.023096
	KHK	0.000068	0.006729	0.026947	**0.00001656**	0.0007837	0.003214
	KAD	0.000100	0.010004	0.040009	0.00005768	0.0056598	0.022632
	KAYA	0.000100	0.009998	0.039985	0.0005763	0.0056465	0.022578
0.95	KLW	0.000084	0.008284	0.033079	**0.00002412**	0.0018588	0.007461
	KTC	0.000072	0.007204	0.028867	**0.00002489**	0.0021337	0.008633
	KHMO	0.000047	0.002406	0.009267	0.00003826	0.0002745	**0.000702**
	KFM	0.000096	0.009673	0.038700	0.00005476	0.0055142	0.022065
	KP1	**0.000022**	**0.000545**	**0.001783**	0.00003107	**0.0001774**	0.002392
	KP2	**0.000023**	**0.000497**	**0.001575**	0.00002930	**0.0001486**	**0.000199**
	KP3	**0.000034**	**0.000262**	**0.000543**	0.00003034	**0.0001582**	**0.000198**
0.99	**OLS**	0.000554	0.055454	0.221810	0.00033699	0.033699	0.13479
	KHK	0.000367	0.036912	0.147790	0.000046341	0.0046774	0.018922
	KAD	0.000552	0.055145	0.220560	0.00033110	0.033058	0.13221
	KAYA	0.000551	0.055108	0.220410	0.00033042	0.032983	0.13191
	KLW	0.000224	0.022523	0.090212	**0.000027465**	0.0026059	0.010610
	KTC	0.000393	0.039634	0.158720	0.00012040	0.012597	0.050695
	KHMO	0.000211	0.018895	0.075284	0.000041519	**0.0013597**	**0.0051611**
	KFM	0.000533	0.053363	0.213470	0.00032042	0.032189	0.12879
	KP1	**0.000072**	**0.004924**	**0.019316**	**0.000032432**	0.0032697	0.015615
	KP2	**0.000068**	**0.004338**	**0.016951**	**0.000031166**	**0.0001508**	**0.0002504**
	KP3	**0.000050**	**0.000693**	**0.002209**	0.000033046	**0.0001606**	**0.0002167**
	OLS	0.005968	0.596790	2.387170	0.0037742	0.377420	1.509680
	KHK	0.003937	0.395780	1.583750	0.0005147	0.053925	0.216450
	KAD	0.005934	0.593200	2.372720	0.0037058	0.370260	1.480940

	KAYA	0.005930	0.592780	2.371040	0.0036979	0.369430	1.477610
	KLW	**0.000157**	**0.011511**	**0.045757**	0.0000424	**0.001044**	**0.003998**
0.999	KTC	0.004231	0.425690	1.703510	0.0013841	0.142780	0.572270
	KHMO	0.002564	0.255800	1.023370	0.0001933	0.018829	0.075544
	KFM	0.005742	0.574320	2.297300	0.0036003	0.360630	1.442640
	KP1	0.001305	0.129100	0.516330	**0.0000261**	0.039090	0.156410
	KP2	**0.001177**	**0.115960**	**0.463700**	**0.0000322**	**0.000454**	**0.001565**
	KP3	**0.000114**	**0.005866**	**0.022705**	**0.0000346**	**0.000226**	**0.000503**
	n=70						
ρ		**P=3**			P=7		
	OLS	0.000028931	0.0028931	0.0115730	0.0001512	0.015120	0.060479
	KHK	**0.000015174**	0.0012310	0.0047919	0.0001115	0.0091095	0.036304
	KAD	0.000028715	0.0028490	0.0114250	0.0001512	0.015072	0.060297
	KAYA	0.000028689	0.0028449	0.011408	0.0001512	0.015067	0.060277
0.95	KLW	**0.000021072**	0.0020141	0.0079586	0.0001251	0.011375	0.045430
	KTC	**0.000018204**	0.0016063	0.0063223	0.0001130	0.010143	0.040460
	KHMO	0.000043013	0.0004191	0.0011927	0.0000671	0.0028180	0.010890
	KFM	0.000028279	0.0028132	0.0112480	0.0001464	0.014674	0.058700
	KP1	0.000037146	**0.0001700**	**0.0002334**	**0.0000275**	**0.0002035**	**0.000487**
	KP2	0.000038354	**0.0001714**	**0.0002271**	**0.0000274**	**0.0001941**	**0.000431**
	KP3	0.000045940	**0.0001833**	**0.0002056**	**0.0000284**	**0.0001783**	**0.000318**
	OLS	0.00016891	0.0168910	0.0675650	0.0009137	0.091366	0.36547
	KHK	0.000095620	0.0071635	0.0284920	0.00059461	0.056374	0.22515
	KAD	0.00016774	0.016679	0.0667570	0.00091119	0.091132	0.36453
	KAYA	0.00016760	0.0166560	0.066665	0.00091092	0.091107	0.36443
0.99	KLW	0.000057610	0.0039641	0.015719	0.00032325	0.030331	0.12123
	KTC	0.00010608	0.0093151	0.037094	0.00064118	0.062141	0.24830
	KHMO	0.000079573	0.0026645	0.010210	0.00030817	0.026734	0.10650
	KFM	0.00016425	0.0164130	0.065648	0.00088563	0.088680	0.35475
	KP1	**0.000048064**	**0.0002950**	**0.0008361**	**0.00002835**	**0.0009001**	**0.003482**
	KP2	**0.000048255**	**0.0002695**	**0.0007158**	**0.00002777**	**0.0007612**	**0.002897**
	KP3	**0.000051413**	**0.0001884**	**0.0002389**	**0.00002799**	**0.0003707**	**0.001203**
	OLS	0.0018864	0.0168910	0.754570	0.0103920	1.039160	4.156650
	KHK	0.00086431	0.0071635	0.323030	0.0065629	0.649450	2.596560
	KAD	0.0018754	0.0166790	0.745590	0.0103690	1.036710	4.146830
	KAYA	0.0018735	0.016656	0.744560	0.0103670	1.036430	4.145730
0.999	KLW	**0.0000561**	0.003964	**0.004842**	0.0003524	0.034825	0.139720
	KTC	0.0010751	0.0093151	0.416730	0.0071705	0.712340	2.848480
	KHMO	0.00045475	0.0026645	0.155810	0.0040745	0.400260	1.599840
	KFM	0.0018327	0.016413	0.733130	0.0100830	1.008750	4.035100
	KP1	0.00011215	**0.0002951**	0.028970	**0.0001662**	**0.018172**	**0.073387**
	KP2	**0.0000998**	**0.0002695**	**0.023927**	**0.0001383**	**0.015170**	**0.061323**
	KP3	**0.00005125**	**0.0001884**	**0.000784**	**0.0000409**	**0.003524**	**0.014263**