Global Existence and Exponential Decay of Solutions for a Class of Nonlinear Wave Equations

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Abstract: A initial-boundary value problem for some nonlinear wave equation with damping and source terms $u_{tt} + Au + u_t + aAu_t = b|u|^{q-1}u$ in a bounded domain is studied, where $A = (-\Delta)^m$, $m \geq 1$ is a nature number, $a \geq 0, b > 0$ and $q > 1$ are real numbers. The existence of global solutions for this problem is proved by constructing the stable sets, and show the exponential decay estimate of global solutions as time goes to infinity by applying the multiplier method. Meanwhile, under the conditions of the nonnegative initial energy and $a = 0$, it is showed that the solution blows up in finite time.

Key–Words: Nonlinear wave equation; Initial boundary value problem; Stable sets; Nonlinear damping and source terms; Exponential decay.

1 Introduction

In this paper we consider the existence and exponential decay estimate of global solutions for the initial-boundary problem of nonlinear wave equation with nonlinear damping and source terms

$$u_{tt} + Au + u_t + aAu_t = b|u|^{q-1}u,\ x \in \Omega,\ t > 0, \quad (1.1)$$

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ x \in \Omega, \quad (1.2)$$

$$D^\alpha u(x,t) = 0,\ |\alpha| \leq m-1,\ x \in \partial\Omega,\ t \geq 0, \quad (1.3)$$

where $A = (-\Delta)^m$, $m \geq 1$ is a nature number, $a \geq 0, b > 0$ and $q > 1$ are real numbers, Ω is a bounded domain of R^N with smooth boundary $\partial\Omega$, Δ is the Laplace operator, and $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_N)$, $|\alpha| = \sum_{i=1}^{N} |\alpha_i|$, $D^\alpha = \prod_{i=1}^{N} \frac{\partial^{\alpha_i}}{\partial x_i^{\alpha_i}}$, $x = (x_1, x_2, \cdots, x_N)$.

For the case $m = 1$, the existence and uniqueness, as well as decay estimates, of global solutions and blow-up of solutions for the initial-boundary value problem or Cauchy problem of the equation (1.1) have been investigated by many people through various approaches and assumptive conditions [1, 4, 5, 7, 8, 15, 17, 22, 23].

M.Nakao [12] has used Galerkin method to present the existence and uniqueness of the bounded solutions, periodic and almost periodic solutions to the problem (1.1)-(1.3) as the dissipative term is a linear function au_t. M.Nakao and H.Kuwahara [13] studied decay estimates of global solutions to the problem (1.1)-(1.3) by using a difference inequality when the dissipative term is a degenerate case $a(x)u_t$. In absence of dissipative term in equation (1.1), P.Brenner and W.Von Wahl [2] proved the existence and uniqueness of classical solutions for the initial-boundary problem of equation (1.1) in Hilbert space. H.Pecher [14] investigated the existence and uniqueness of Cauchy problem for (1.1) by use of the potential well method due to L.Payne and D.H.Sattinger [15] and D.H.Sattinger [16].

When $A = (-\Delta)^m$ is replaced by $P-$Laplace operator $-div(|\nabla u|^m \nabla u)$, S.M.Messaoudi [10] improve the result of reference [19] by giving more precise decay rates. In particular, he shows that for $m = 0$, the decay is exponential. His technique of proof relies on the combination of the perturbed energy and the potential well methods.

When $a = 0$, for the semilinear higher order wave equation (1.1), B.X.Wang [18] show that the scattering operators map a band in H^s into H^s if the nonlinearities have critical or subcritical powers in H^s. C.X.Miao [11] obtain the scattering theory at low energy using time-space estimates and nonlinear estimates. Meanwhile, he also give the global existence and uniqueness of solutions under the condition of low energy.

Recently, Y.J.Ye [20] dealt with the existence and asymptotic behavior of global solutions for (1.1)-(1.3) with nonlinear dissipative term. At the same time, A.B.Aliev and B.H.Lichaei [3] consider the Cauchy problem for equation (1.1), and they found the existence and nonexistence criteria of global solutions using the $L^p - L^q$ estimate for the corresponding linear problem and also established the asymptotic behavior of solutions and their derivatives as $t \to +\infty$.

The proof of global existence for problem (1.1)-(1.3) is based on the use of the potential well theory [15, 16]. And we study the exponential decay estimate of global solutions by applying the lemma of V.Komornik [6]. Meanwhile, under suitable conditions on the nonnegative initial energy and without strong dissipative term (i.e. $a = 0$), we obtain the blow-up result.

We adopt the usual notation and convention. Let $H^m(\Omega)$ denote the Sobolev space with the usual scalar products and norm. Meanwhile, $H_0^m(\Omega)$ denotes the closure in $H^m(\Omega)$ of $C_0^\infty(\Omega)$. For simplicity of notation, hereafter we denote by $\|\cdot\|_r$ the Lebesgue space $L^r(\Omega)$ norm and $\|\cdot\|$ denotes $L^2(\Omega)$ norm, we write equivalent norm $\|A^{\frac{1}{2}}\cdot\|$ instead of $H_0^m(\Omega)$ norm $\|\cdot\|_{H_0^m(\Omega)}$. Moreover, C_i $(i = 1, 2, 3, \cdots)$ denote various positive constants which depend on the known constants and may be difference at each appearance.

This paper is organized as follows: In the next section, we will study the existence of global solutions of problem (1.1)-(1.3). Then in section 3, we are devoted to the proof of exponential decay estimate. Then in section 4, we are devoted to the proof of global nonexistence of solution for the problem (1.1)-(1.3) without strong dissipative term (i.e. $a = 0$).

We conclude this introduction by stating a local existence result, which can be obtained by a similar way as down in [9, 19].

Theorem 1.1 Suppose that p and q satisfy

$$\begin{aligned} &1 < q < +\infty \; if \; N \leq 2m; \\ &1 < q \leq \frac{N+2m}{N-2m} \; if \; N > 2m, \end{aligned} \tag{1.4}$$

and $(u_0, u_1) \in H_0^m(\Omega) \times L^2(\Omega)$, then there exists $T > 0$ such that the problem (1.1)-(1.3) has a unique local solution $u(t)$ in the class

$$\begin{aligned} &u \in C([0,T); H_0^m(\Omega)), \\ &u_t \in C([0,T); L^2(\Omega)) \cap L^2([0,T); H_0^1(\Omega)). \end{aligned} \tag{1.5}$$

2 The Global Existence

In order to state and prove our main results, we first define the following functionals

$$I(u) = I(u(t)) = \|A^{\frac{1}{2}}u(t)\|^2 - b\|u(t)\|_{q+1}^{q+1},$$

$$J(u) = J(u(t)) = \frac{1}{2}\|A^{\frac{1}{2}}u(t)\|^2 - \frac{b}{q+1}\|u(t)\|_{q+1}^{q+1},$$

for $u \in H_0^m(\Omega)$. Then, for the problem (1.1)-(1.3), we will be able to establish the stability of the set

$$W = \{u \in H_0^m(\Omega),\ I(u) > 0\} \cup \{0\}.$$

We define the total energy related to (1.1) by

$$\begin{aligned} &E(u(t)) = \\ &= \frac{1}{2}\|u_t(t)\|^2 + \frac{1}{2}\|A^{\frac{1}{2}}u(t)\|^2 - \frac{b}{q+1}\|u(t)\|_{q+1}^{q+1} \\ &= \frac{1}{2}\|u_t(t)\|^2 + J(u(t)) \end{aligned}$$

for $u \in H_0^m(\Omega),\ t \geq 0$, and $E(u(0)) = \frac{1}{2}\|u_1\|^2 + J(u_0)$ is the total energy of the initial data.

The following lemmas play an important role to the proof of the global existence of solution for the problem (1.1)-(1.3).

Lemma 2.1 Let r be a real number with $2 \leq r < +\infty \; if \; N \leq 2m$ and $2 \leq r \leq \frac{2N}{N-2m} \; if \; N > 2m.$ Then there is a constant C depending on Ω and r such that

$$\|u\|_r \leq C\|A^{\frac{1}{2}}u\|,\ \forall u \in H_0^m(\Omega).$$

Lemma 2.2 Let $u(t)$ be a solution of the problem (1.1)-(1.3). Then $E(u(t))$ is a nonincreasing function for $t > 0$ and

$$\frac{d}{dt}E(u(t)) = -\|u_t\|^2 - a\|A^{\frac{1}{2}}u_t(t)\|^2. \tag{2.1}$$

Proof By multiplying equation (1.1) by u_t and integrating over Ω, we get

$$\frac{d}{dt}E(u(t)) = -\|u_t\|^2 - a\|A^{\frac{1}{2}}u_t(t)\|^2 \leq 0.$$

Therefore, $E(u(t))$ is a nonincreasing function of t.

Lemma 2.3 Suppose that (1.4) holds, If $u_0 \in W, u_1 \in L^2(\Omega)$ such that

$$\theta = bC^{q+1}\Big(\frac{2(q+1)}{q-1}E(u_0)\Big)^{\frac{q-1}{2}} < 1, \tag{2.2}$$

then $u(t) \in W$, for each $t \in [0, T)$.

Proof Assume that there exists a number $t^* \in [0, T)$ such that $u(t) \in W$ on $[0, t^*)$ and $u(t^*) \notin W$. Then we have

$$I(u(t^*)) \leq 0,\ u(t^*) \neq 0. \tag{2.3}$$

Since $u(t) \in W$ on $[0, t^*)$, so it holds that

$$\begin{aligned} J(u(t)) &= \frac{1}{2}\|A^{\frac{1}{2}}u(t)\|^2 - \frac{b}{q+1}\|u(t)\|_{q+1}^{q+1} \\ &> \frac{1}{2}\|A^{\frac{1}{2}}u(t)\|^2 - \frac{1}{q+1}\|A^{\frac{1}{2}}u(t)\|^2 \\ &= \frac{q-1}{2(q+1)}\|A^{\frac{1}{2}}u(t)\|^2, \end{aligned} \tag{2.4}$$

it follows from $I(u(t^*)) = 0$ that

$$\begin{aligned} J(u(t^*)) &= \frac{1}{2}\|A^{\frac{1}{2}}u(t^*)\|^2 - \frac{b}{q+1}\|u(t^*)\|_{q+1}^{q+1} \\ &= \frac{q-1}{2(q+1)}\|A^{\frac{1}{2}}u(t^*)\|^2. \end{aligned} \tag{2.5}$$

Therefore, we have from (2.4) and (2.5) that

$$\begin{aligned} &\|A^{\frac{1}{2}}u(t)\|^2 \le \frac{2(q+1)}{q-1}J(u(t)) \\ &\le \frac{2(q+1)}{q-1}E(u(t)) \le \frac{2(q+1)}{q-1}E(u_0), \end{aligned} \tag{2.6}$$

for $\forall t \in [0, t^*]$.

By exploiting Lemma 2.1, (2.2) and (2.6), we easily arrive at

$$\begin{aligned} b\|u\|_{q+1}^{q+1} &\le bC^{q+1}\|A^{\frac{1}{2}}u\|^{q+1} \\ &= bC^{q+1}\|A^{\frac{1}{2}}u\|^{q-1}\|A^{\frac{1}{2}}u\|^2 \\ &\le bC^{q+1}\Big(\frac{2(q+1)}{q-1}E(u(0))\Big)^{\frac{q-1}{2}}\|A^{\frac{1}{2}}u\|^2 \\ &< \|A^{\frac{1}{2}}u\|^2, \end{aligned} \tag{2.7}$$

for all $t \in [0, t^*]$. Therefore, we obtain

$$I(u(t^*)) = \|A^{\frac{1}{2}}u(t^*)\|^2 - b\|u(t^*)\|_{q+1}^{q+1} > 0, \tag{2.8}$$

which contradicts (2.3). Thus, we conclude that $u(t) \in W$ on $[0, T)$.

Theorem 2.1 Assume that (1.5) holds, $u(t)$ is a local solution of problem (1.1)-(1.3) which is obtained in Theorem 1.1. If $u_0 \in W,\ u_1 \in L^2(\Omega)$ satisfy (2.2), then the solution $u(t)$ is a global solution of problem (1.1)-(1.3).

Proof It suffices to show that $\|u_t(t)\|^2 + \|A^{\frac{1}{2}}u(t)\|^2$ is bounded independently of t.

Under the hypotheses in Theorem 2.1, we get from Lemma 2.3 that $u(t) \in W$ on $[0, T)$. So the formula (2.6) in Lemma 2.3 holds on $[0, T)$.

We obtain from (2.6) that

$$\begin{aligned} &E(u_0) \ge E(u(t)) = \frac{1}{2}\|u_t(t)\|^2 + J(u(t)) \\ &\ge \frac{1}{2}\|u_t(t)\|^2 + \frac{q-1}{2(q+1)}\|A^{\frac{1}{2}}u\|^2 \\ &\ge \frac{q-1}{2(q+1)}(\|u_t(t)\|^2 + \|A^{\frac{1}{2}}u\|^2). \end{aligned} \tag{2.9}$$

Therefore

$$\|u_t(t)\|^2 + \|A^{\frac{1}{2}}u\|^2 \le \frac{2(q+1)}{q-1}E(u_0) < +\infty.$$

The above inequality and the continuation principle lead to the global existence of the solution $u(x, t)$ for problem (1.1)-(1.3).

3 Exponential Decay Estimate

The following lemma play an important role in studying the decay estimate of global solutions for the problem (1.1)-(1.3).

Lemma 3.1[6] Let $F : R^+ \to R^+$ be a nonincreasing function and assume that there is a constant $L > 0$ such that

$$\int_S^{+\infty} F(t)dt \le LF(S),\ 0 \le S < +\infty,$$

then $F(t) \le F(0)e^{1-\frac{t}{L}},\ \forall t \ge 0$.

Theorem 3.1 If the hypotheses in Theorem 2.2 are valid, then the global solutions of problem (1.1)-(1.3) have the following exponential decay estimate

$$E(t) \le E(0)e^{1-\frac{t}{M}},$$

where $M > 0$ is a constant.

Proof Let $E(t) = E(u(t))$, if we can prove that the energy of the global solution satisfies the estimate

$$\int_S^T E(t)dt \le ME(S)$$

for all $0 \le S < T < +\infty$, then Theorem 3.1 will be proved by Lemma 3.1.

Multiplying by u on both sides of the equation (1.1) and integrating over $\Omega \times [S, T]$, we obtain that

$$\begin{aligned} 0 &= \int_S^T (\|u_t\|^2 + \|A^{\frac{1}{2}}u\|^2 - \frac{2b}{q+1}\|u\|_{q+1}^{q+1})dt \\ &+ \int_S^T\int_\Omega uu_t dxdt - \int_S^T\int_\Omega [2|u_t|^2 - aA^{\frac{1}{2}}u_tA^{\frac{1}{2}}u]dxdt \\ &+ \Big[\int_\Omega uu_t dx\Big]_S^T + \Big(\frac{2}{q+1} - 1\Big)b\int_S^T \|u\|_{q+1}^{q+1}dt. \end{aligned} \tag{3.1}$$

We get from (2.7) and (2.6) that

$$\begin{aligned}&\left(1-\frac{2}{q+1}\right)b\|u\|_{q+1}^{q+1}\le\frac{(q-1)\theta}{q+1}\|A^{\frac{1}{2}}u\|^2\\&\le\frac{(q-1)\theta}{q+1}\cdot\frac{2(q+1)}{q-1}E(t)=2\theta E(t),\end{aligned}\tag{3.2}$$

It follows from (2.9) that

$$\begin{aligned}&\left|-\left[\int_\Omega uu_tdx\right]_S^T\right|\\&=\left|\int_\Omega u(T)u_t(T)dx-\int_\Omega u(S)u_t(S)dx\right|\\&\le\int_\Omega|u(T)u_t(T)|dx+\int_\Omega|u(S)u_t(S)|dx\\&\le\frac{1}{2}(\|u(T)\|^2+\|u_t(T)\|^2)+\frac{1}{2}(\|u(S)\|^2+\|u_t(S)\|^2)\\&\le\left[\frac{C^2}{2}\|A^{\frac{1}{2}}u(T)\|^2+\frac{1}{2}\|u_t(T)\|^2\right]\\&+\left[\frac{C^2}{2}\|A^{\frac{1}{2}}u(S)\|^2+\frac{1}{2}\|u_t(S)\|^2\right]\\&\le\max\left(\frac{(q+1)C^2}{q-1},1\right)[E(T)+E(S)]\le ME(S).\end{aligned}\tag{3.3}$$

Therefore we conclude from (3.1), (3.2) and (3.3) that

$$\begin{aligned}&2(1-\theta)\int_S^TE(t)dt\\&\le\int_S^T\int_\Omega[2|u_t|^2-aA^{\frac{1}{2}}u_tA^{\frac{1}{2}}u]dxdt\\&+\int_S^T\int_\Omega uu_tdxdt+ME(S).\end{aligned}\tag{3.4}$$

We get from Lemma 2.2 that

$$\begin{aligned}&2\int_S^T\int_\Omega|u_t|^2dxdt=2\int_S^T\|u_t\|^2dt\\&=-2(E(T)-E(S))\le 2E(S).\end{aligned}\tag{3.5}$$

It follows from Young's inequality Lemma 2.1, Lemma 2.2 and (2.9) that

$$\begin{aligned}&\int_S^T\int_\Omega uu_tdxdt\le\int_S^T(\varepsilon_1\|u\|^2+M(\varepsilon_1)\|u_t\|^2)dt\\&\le\int_S^T(\varepsilon_1C^2\|A^{\frac{1}{2}}u(t)\|^2+M(\varepsilon_1)\|u_t(t)\|^2)dt\\&\le\frac{2(q+1)\varepsilon_1}{q-1}\int_S^TE(t)dt+M(\varepsilon_1)(E(S)-E(T))\\&\le\frac{2(q+1)\varepsilon_1}{q-1}\int_S^TE(t)dt+M(\varepsilon_1)E(S).\end{aligned}\tag{3.6}$$

where $M(\varepsilon_1)$ is a positive constant depending on ε_1.

We obtain from Young's inequality, Lemma 2.1, Lemma 2.2 and (2.9) that

$$\begin{aligned}&-a\int_S^T\int_\Omega A^{\frac{1}{2}}uA^{\frac{1}{2}}u_tdxdt\\&\le a\int_S^T(\varepsilon_2\|A^{\frac{1}{2}}u\|^2+M(\varepsilon_2)\|A^{\frac{1}{2}}u_t\|^2)dt\\&\le\frac{2a(q+1)\varepsilon_2}{q-1}\int_S^TE(t)dt+M(\varepsilon_2)(E(S)-E(T))\\&\le\frac{2a(q+1)\varepsilon_2}{q-1}\int_S^TE(t)dt+M(\varepsilon_2)E(S).\end{aligned}\tag{3.7}$$

where $M(\varepsilon_2)$ is a positive constant depending on ε_2.

Choosing ε_1 and ε_2 small enough such that

$$\frac{q+1}{q-1}(\varepsilon_1+a\varepsilon_2)+\theta<1,$$

then, substituting (3.5), (3.6) and (3.7) into (3.4), we get

$$\int_S^TE(t)dt\le ME(S).\tag{3.8}$$

Let $T\to+\infty$, then we have from (3.8) that

$$\int_S^{+\infty}E(t)dt\le ME(S).\tag{3.9}$$

Thus, we obtain from (3.9) and Lemma 3.1 that

$$E(t)\le E(0)e^{1-\frac{t}{M}},\ t\in[0,+\infty).\tag{3.10}$$

4 Blow-up of Solution

In this section, we shall study the blow-up property of solution for the problem (1.1)(1.3) without strong dissipative term (i.e. $a=0$). For this purpose, we need the following Lemma.

Lemma 4.1 [21] Suppose that $\Psi(t)$ is a twice continuously differential satisfying

$$\Psi''(t)+\Psi'(t)\ge C_0\Psi^{1+\alpha},\ \Psi(0)>0,\ \Psi''(0)\ge 0,\tag{4.1}$$

for $t>0$, where $C_0>0$ and $\alpha>0$ are constants. Then $\Psi(t)$ blows up in finite time.

The result of this section reads as follows:

Theorem 4.1 Let the assumptions of Theorem 1.1 hold. Then the local solution for problem (1.1) -(1.3) with initial conditions satisfying

$$E(0)\le 0,\ \int_\Omega u_0u_1dx\ge 0\tag{4.2}$$

blows up in finite time. In other words, there exists T^* such that $\lim\limits_{t\to T^*}\|u\|^2=+\infty$.

Proof We define the following function

$$\Psi(t) = \frac{1}{2}\int_\Omega |u(t)|^2 dx. \tag{4.3}$$

Then we have

$$\begin{aligned}\Psi'(t) &= \int_\Omega u(t)u_t(t)dx,\\ \Psi''(t) &= \|u_t(t)\|^2 + \int_\Omega uu_{tt}dx.\end{aligned} \tag{4.4}$$

We obtain from (1.1) and (4.4) that

$$\begin{aligned}\Psi''(t) &= \|u_t(t)\|^2 - \|A^{\frac{1}{2}}u(t)\|^2\\ &- \textstyle\int_\Omega uu_t dx + b\|u(t)\|_{q+1}^{q+1}.\end{aligned} \tag{4.5}$$

We conclude from (4.4) and (4.5) that

$$\Psi''(t)+\Psi'(t) = \|u_t(t)\|^2-\frac{1}{2}\|A^{\frac{1}{2}}u(t)\|^2+b\|u(t)\|_{q+1}^{q+1}. \tag{4.6}$$

It follows from the definition of $E(t)$, Lemma 2.1, (4.2) and (4.6) that

$$\begin{aligned}&\Psi''(t) + \Psi'(t)\\ &= -E(t) + \frac{3}{2}\|u_t(t)\|^2 + \frac{qb}{q+1}\|u(t)\|_{q+1}^{q+1}\\ &\geq -E(0) + \frac{qb}{q+1}\|u(t)\|_{q+1}^{q+1} \geq \frac{qb}{q+1}\|u(t)\|_{q+1}^{q+1}.\end{aligned} \tag{4.7}$$

By Hölder inequality, we have

$$\int_\Omega |u|^2 dx \leq |\Omega|^{\frac{q-1}{q+1}}\|u\|_{q+1}^2,$$

where $|\Omega|$ is the measures of the bounded domain. Therefore,

$$\|u\|_{q+1}^{q+1} \geq 2^{\frac{q+1}{2}}|\Omega|^{\frac{1-q}{q+1}}\Psi^{\frac{q+1}{2}}(t). \tag{4.8}$$

We obtained from (4.7) and (4.8) that

$$\Psi''(t) + \Psi'(t) \geq C_0\Psi^{1+\alpha}(t), \tag{4.9}$$

where $C_0 = 2^{\frac{q+1}{2}}\frac{qb}{q+1}|\Omega|^{\frac{1-q}{q+1}} > 0,\ \ \alpha = \frac{q-1}{2} > 0$. Thus, we prove that the solution blows up in the sense of L^2 norm.

5 Conclusion

In this paper, the initial boundary value problem for a class of nonlinearly damped Petrovsky equation in a bounded domain is considered. At first, the existence of global solutions which is not related to the parameters p and r is proved by constructing a stable set in $H_0^2(\Omega)$. Moreover, we have the other global existence result which is related to parameters p and r, i.e. $p \leq r$. Secondly, we obtain the energy decay estimate through the use of an important lemma of V.Komornik. At last, under the conditions of the positive initial energy, it is proved that the solution blows up in the finite time and the lifespan estimates of solutions are also given.

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