Long Term Dynamics of Infectious Diseases with Awareness Campaign and Time Delay

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Abstract: An epidemic model with awareness campaign driven by media on the prevalence of infectious diseases is studied considering delay. The aim is to investigate the effect of awareness campaign and delay in disease control. Time delay factor is considered because susceptible individual takes some time to become aware of the disease and take the necessary control methods. The system exhibits two equilibria: the disease-free equilibrium and endemic equilibrium. The disease-free equilibrium is stable if the basic reproduction number is less than unity. The endemic equilibrium exhibits Hopf-bifurcation for a critical value of the delay. Numerical simulations prove the results of analytical analysis and the signif cance of delay in awareness campaigning for preventing and controlling the diseases.

Key–Words: Infectious disease, Mathematical model, Awareness program, Basic reproduction number, Delay Differential Equation, Hopf-bifurcation.

1 Introduction

Pneumonia, Tuberculosis (TB), Diarrheal diseases (Cholera), Malaria, Ebola, HIV/AIDS and more are the major noticeable infectious diseases. More than 11 million people die every year due to infectious disease including premature deaths and deaths of young children. Some of these diseases can be prevented through vaccinations. However this is costly and sometimes the effect is only temporary. On the other hand sometimes disease appropriate awareness in a population can control an infection effectively [1, 2, 3, 4]. Mass media plays an important role in changing behavior related to public health. The government and other health organizations should immediately make people aware about the disease and relevant precautions through awareness campaign [5, 6]. The media not only make the population acquainted with the disease but also suggest the necessary preventive practices such as social distancing, wearing protective masks or vaccination. In general the people who are aware adopt these practices so that their chances of becoming infected are minimized [7, 8].

Many researcher have studies the epidemic outbreak with awareness programs in [12, 13, 27] and the references therein. The impact of information on transmission dynamics of sexually transmitted infections has been studied in [14]. The whole population was assumed as aware of risk but only a certain proportion able to respond by limiting their contact with infected individuals. Epidemic models considering the cumulative density of the awareness program as a separate variable are also studied by some researchers [15, 16, 28]. The model analysis showed that the spread of an infectious disease can be controlled by using awareness programs.

One of the most important problems for epidemiological dynamics is to investigate the effect of time delays on the dynamics of the systems. Time delay is very common phenomena in population dynamics. Misra et al. [17] proposed and analyzed a delay induced mathematical model in the presence of an awareness program. They concluded that the awareness program and time delay plays a crucial role in controlling the spread of disease. Zhao et al. [18] proposed and analyzed an SIRS epidemic model incorporating media coverage with time delay and showed that the time delay in media coverage affects the stability of the endemic equilibrium. Samanta [19] has proposed an epidemic model has been proposed and analyzed the impact of awareness program and reporting delay in the epidemic outbreak.

In this article, a mathematical model has been developed for the prevalence of infectious disease with the inf uence of the awareness program. The main aim of the present work is to study the impact of awareness program and the time delay in the epidemic outbreak.

To attain the goal of the present work, the system of nonlinear delay differential equations has been investigated analytically and numerically.

2 The Mathematical Model

In this section, we provide the formulation of the mathematical model using the following assumptions:

Let $S(t)$ and $I(t)$ be the number of the susceptible and infected population respectively at time t in a particular region and the disease is transmitted from infected to susceptible individuals following a mass action functional form. The number of aware people in that region is $M(t)$. Awareness programs (media campaign) are carried out in the region of the epidemic outbreak to make people aware of the disease.

The total susceptible population is divided into two subclasses: the unaware susceptible population S_u and the aware susceptible population S_a. As the awareness disseminates, people respond to it and eventually will change their behavior to alter their susceptibility. It is also assumed that infected individuals recover through treatment. After recovery, a fraction p of recovered people will join the aware susceptible class, whereas the remaining fraction q will join unaware susceptible class and $p + q = 1$.

Here, it is considered that all the newly recruited individuals are unaware. The rate of being aware is proportional to the number of infected individuals reported by the media and/or health organization, whereas the depletion of the aware state is inversely proportional to the number of cases. Unaware susceptible population becomes aware at the rate α, where α is the rate at which unaware susceptible individual becomes aware susceptible.

π is the constant recruitment in susceptible population, β is the disease transmission rate, d is the natural mortality rate of the population, e is the disease induced mortality rate of infected population and r is the recovery rate. It is assumed that the disease spreads due to the direct contact between susceptible and infective only.

Awareness programs are implemented proportionally with the change of infected infective individuals at a rate η and cut down at a rate, θ, due to their ineffectiveness. Moreover, people cannot take measures to protect themselves in time after the media campaigns the disease. Generally unaware people cannot become aware instantly whenever awareness campaigns are made. They take some time to become aware. So time delay $\tau > 0$ is introduced that represents the time lag between the awareness campaign and the time of being aware.
The above assumptions lead to the following model:

$$\begin{aligned}
\frac{dS_u}{dt} &= \pi - \beta S_u I - \alpha S_u M(t-\tau) - dS_u + qrI \\
\frac{dS_a}{dt} &= \alpha S_u M(t-\tau) - dS_a + prI \\
\frac{dI}{dt} &= \beta S_u I - (d+e+r)I \\
\frac{dM}{dt} &= \eta I - \theta M. \qquad (1)
\end{aligned}$$

with the initial conditions: $S_u(\lambda) = S_{0u}, S_a(\lambda) = S_{0a}, I(\lambda) = I_0, M(\lambda) = M_0$, where $\lambda \in [-\tau, 0]$. For the analysis of model (2.1), we need the region of attraction, which is given by the set

$$B = \left\{(S_u, S_a, I, M) \in R^4_+ : 0 \le S_u, S_a, I \le \frac{\pi}{d}, M \le \frac{\eta\pi}{\theta d}\right\}$$

Let C denotes the Banach space of continuous functions $\psi : [-\tau, 0] \to \mathbb{R}^4_+$ with norm

$$\|\psi\| = \sup_{-\tau<\lambda<0} \{|\psi_1(\lambda)|, |\psi_2(\lambda)|, |\psi_3(\lambda)|, |\psi_4(\lambda)|\},$$

where $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)$.
The initial conditions of (1) are given as:

$$\begin{aligned}
&S_u(\lambda) = \psi_1(\lambda), S_a(\lambda) = \psi_2(\lambda), I(\lambda) = \psi_3(\lambda), \\
&M(\lambda) = \psi_4(\lambda), \lambda \in [-\tau, 0]. \qquad (2)
\end{aligned}$$

where the initial function $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)$ belongs to the Banach space $C = C([-\tau, 0], \mathbb{R}^4_+)$ of continuous functions mapping the initial $[-\tau, 0]$ into $\mathbb{R}^4$. For biological reasons, the initial functions are assumed as:

$$\psi_i(\lambda) \ge 0, \lambda \in [-\tau, 0], \psi_i(0) > 0, i = 1, 2, 3, 4.$$

The system (1) can be written as follows:

$$\frac{dX}{dt} = f(X), \qquad (3)$$

where, $X(\lambda) = col(S_u(\lambda), S_a(\lambda), I(\lambda), M(\lambda)) \in C$ and $\psi_i(0) > 0, i = 1, 2, 3, 4$, $f = (f_1, f_2, f_3, f_4)^T$, are the right sides of system (1). By the fundamental theory of functional differential equations [20], we know that there is a unique solution $(S_u(t), S_a(t), I(t), M(t))$ to system (1) with initial conditions given in (2).

2.1 Positive invariance

Biologically, positivity implies the survival of the populations. To proof this, the methods of Bodnar [21] and Yang et al. [22] have been followed, and provide the following theorem:

Theorem 1. *All the solution of (1) with initial conditions (2) are positive.*

Proof. Using the lemma in [22, 21], we show that the solution of the system of equations (1) exists in the region $\mathbb{R}^4_+$ and all solutions remain non-negative for all $t > 0$.

It is easy to check in system (2.1) that whenever choosing $X(\lambda) \in R_+$ such that $S_u = 0, S_a = 0, I = 0, M = 0$, then

$$f_i(X)|_{x_i=0, X\in R^4_+} \geq 0,$$

with $x_1(t) = S_u(t), x_2(t) = S_a(t), x_3(t) = I(t), x_4(t) = M(t)$.

Using the lemma in [22], theorem in [21], we can conclude that any solution of (2.1) with $X(\lambda) \in C$, say $X(t) = X(t, X(\lambda))$, is such that $X(\lambda) \in R^4_+$ for all $t \geq 0$. Hence the solution of the system of system (2.1) exist in the region R^4_+ and all solutions remain non-negative for all $t > 0$. Therefore, the positive octant $\mathbb{R}^4_+$ is an invariant region. □

3 The system without delays

In this section, we analyse the equilibria and stability of the non-delayed system.

3.1 Equilibria

The system, without delay, has two equilibria viz.

i. the disease-free equilibrium, E^0, which always exists and is in the form:

$$E^0(S^0_u, S^0_a, I^0, M^0) = (\tfrac{\pi}{d}, 0, 0, 0, 0).$$

ii. the endemic equilibrium, E^*, is of the form: $E^*(S^*_u, S^*_a, I^*, M^*)$, in which

$S^*_u = \frac{d+e+r}{\beta}$,
$S^*_a = \frac{M^*[\beta pr\theta + \eta\alpha(d+e+r)]}{\beta\eta}$,
$I^* = \frac{\theta M^*}{\eta}$, and
$M^* = \frac{\beta\pi - d(d+e+r)}{(d+e+r)\eta\alpha + \beta(d+e)}$.

The endemic equilibria E^* exists if

$$\beta\pi - d(d+e+r) > 0. \tag{4}$$

The Jacobian matrix at any equilibrium point is given by:

$$J = \begin{pmatrix} J_1 & 0 & J_3 & J_4 \\ J_5 & J_6 & J_7 & J_8 \\ J_9 & 0 & J_{10} & 0 \\ 0 & 0 & J_{11} & J_{12} \end{pmatrix},$$

where,
$J_1 = -\beta I - \alpha M - d$, $J_3 = qr - \beta S_u$, $J_4 = -J_8 = -\alpha S_u$, $J_5 = \alpha M$, $J_6 = -d$, $J_7 = pr$, $J_9 = \beta I$, $J_{10} = \beta S_u - (d+e+r)$, $J_{11} = \eta$, $J_{12} = -\theta$.

3.2 Stability

At the disease free equilibria, E^0, the characteristic equation of the Jacobian matrix has only negative eigenvalues if $\beta\pi - d(d+e+r) < 0$. Thus we have the following theorem:

Theorem 2. *Define $R_0 = \frac{\beta\pi}{d(d+e+r)}$ as the basic reproduction number for model system. Then the disease free equilibrium E^0 is stable if $R_0 < 1$ and unstable for $R_0 > 1$. For $R_0 = 0$, a transcritical bifurcation occurs. Moreover, the endemic equilibrium E^* exists for $R_0 > 1$.*

At E^* the characteristic equation is given by:

$$\rho^4 + \sigma_1\rho^3 + \sigma_2\rho^2 + \sigma_3\rho + \sigma_4 = 0, \tag{5}$$

where,

$$\begin{aligned}
\sigma_1 &= -[J^*_1 + J^*_6 + J^*_{10} + J^*_{12}], \\
\sigma_2 &= J^*_1J^*_6 - J^*_3J^*_9 + J^*_1J^*_{10} + J^*_6J^*_{10} + J^*_1J^*_{12} \\
&\quad + J^*_6J^*_{12} + J^*_{10}J^*_{12}, \\
\sigma_3 &= +J^*_3J^*_6J^*_9 - J^*_1J^*_6J^*_{10} - J^*_4J^*_9J^*_{11} \\
&\quad - J^*_1J^*_6J^*_{12} + J^*_3J^*_9J^*_{12} - J^*_1J^*_{10}J^*_{12} - J^*_6J^*_{10}J^*_{12}, \\
\sigma_4 &= J^*_4J^*_6J^*_9J^*_{11} - J^*_3J^*_6J^*_9J^*_{12} + J^*_1J^*_6J^*_{10}J^*_{12}.
\end{aligned} \tag{6}$$

where, $J^*_1 = -\beta I^* - \alpha M^* - d$, $J^*_3 = qr - \beta S^*_u$, $J^*_4 = -J^*_8 = -\alpha S^*_u$, $J^*_5 = \alpha M^*$, $J^*_6 = -d$, $J^*_7 = pr$, $J^*_9 = \beta I^*$, $J^*_{10} = \beta S^*_u - (d+e+r)$, $J^*_{11} = \eta$, $J^*_{12} = -\theta$
According to Routh-Hurwitz criterion, all the eigenvalues of the Jacobian matrix at E^* are negative or have negative real part if:

$$\begin{aligned}
\sigma_1 > 0, \quad \sigma_4 > 0, \\
\sigma_1\sigma_2 - \sigma_3 > 0, \\
(\sigma_1\sigma_2 - \sigma_3)\sigma_3 - \sigma_1^2\sigma_4 > 0.
\end{aligned} \tag{7}$$

Proposition 3. *The endemic equilibrium point E^* is stable if the conditions given in (7) are satisfied.*

Table 1: List of parameters used for numerical simulations [17, 24, 25] .

Parameter	Definition	Value (day^{-1})
π	Constant recruitment rate	5
β	Disease transmission rate	0.0005
α	Contact rate between unaware with media	0.02
d	Natural death rate	0.005
e	Additional death rate	0.007
η	Rate of implementation of awareness programs	0.0025
θ	Fading away rate of interest	0.015

4 The system with delay

In this section, the local stability of the delayed system (2.1) is studied around the coexisting equilibrium point only. Without loss of generality it is assumed that E^* be the interior equilibrium point of the system (2.1). The expressions of S_u^*, S_a^*, I^*, M^* are already obtained in the previous section. We are now interested the local asymptomatic stability of the infected steady state E^* for the delayed system. Linearizing the system (1) about E^*, we get

$$\frac{dX}{dt} = MX(t) + GX(t-\tau), \tag{8}$$

where, M, G are 4×4 matrices, given as below:

$$M = [M_{ij}] = \begin{pmatrix} -\beta - \alpha M^* - d & 0 & -\beta S_u^* + qr & 0 \\ \alpha M^* & -d & pr & 0 \\ \beta I^* & 0 & \beta S_u^* - (d+e+r) & 0 \\ 0 & 0 & \eta & -\theta \end{pmatrix},$$

$$G = [G_{ij}] = \begin{pmatrix} 0 & 0 & 0 & -\alpha S_u^* \\ 0 & 0 & 0 & \alpha S_u^* \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

The characteristic equation of system (1) is given by,

$$\triangle(\xi) = \mid \xi I - M - e^{-\xi\tau} G \mid = 0.$$

This equation can be written as,

$$\psi(\xi, \tau_1, \tau_2) = \xi^4 + a_1\xi^3 + a_2\xi^2 + a_3\xi + a_4 + e^{-\xi\tau}[b_1\xi + b_2] = 0. \tag{9}$$

where

$$\begin{aligned} a_1 &= -[M_{11} + M_{22} + M_{33} + M_{44}], \\ a_2 &= M_{11}M_{22} - M_{13}M_{31} + M_{11}M_{33} \\ &\quad + M_{22}M_{33} + M_{11}M_{44}, \\ &\quad + M_{22}M_{44} + M_{33}M_{44} \\ a_3 &= M_{13}M_{22}M_{31} - M_{11}M_{22}M_{33} \\ &\quad - M_{11}M_{22}M_{44} + M_{13}M_{31}M_{44} \\ &\quad - M_{11}M_{33}M_{44} - M_{22}M_{33}M_{44} \\ a_4 &= -M_{13}M_{22}M_{31}M_{44} \\ &\quad + M_{11}M_{22}M_{33}M_{44} \\ b_1 &= -G_{14}M_{31}M_{43}, \\ b_2 &= G_{14}M_{22}M_{31}H_{43}. \end{aligned} \tag{10}$$

The coexisting equilibrium point E^* will be locally asymptotically stable if all the roots of the corresponding characteristic equation (9) are negative or having negative real parts. The classical Routh-Hurwitz criterion cannot be used to investigate the stability of the system as the equation (9) is a transcendental equation in ξ. It is known that E^* is locally asymptotically stable if all the roots of the corresponding characteristic equation have negative real parts and unstable if purely imaginary roots exist. The following cases may arise.

Case I: When $\tau = 0$

In absence of both the delays the characteristics equation (9) becomes,

$$\xi^4 + a_1\xi^3 + a_2\xi^2 + (a_3 + b_1)\xi + (a_4 + b_2) = 0. \tag{11}$$

Employing Routh-Hurwitz criteria for sign of roots, we can get the same results as in non delayed system analysis.

Case II : $\tau > 0$

A necessary condition for a stability changes of E^* is that the characteristic equation (9) should have purely imaginary solutions. Let $i\theta$ be a root of equation (9) and from which we get,

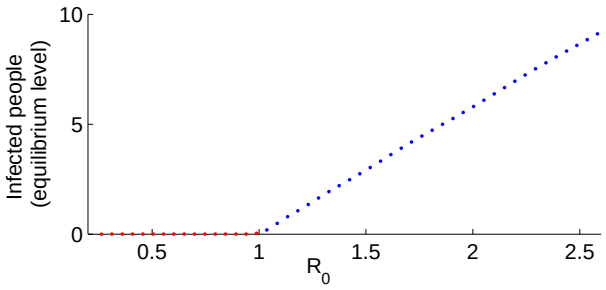

Figure 1: Transcritical bifurcation: system is disease free for $R_0 < 1$ and endemic for $R_0 > 1$.

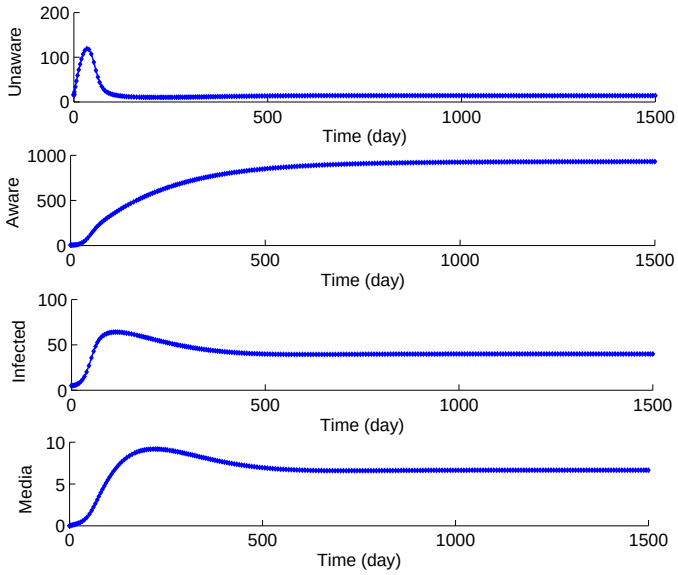

Figure 2: Time series solution of the system with no delay is plotted. The system is stable.

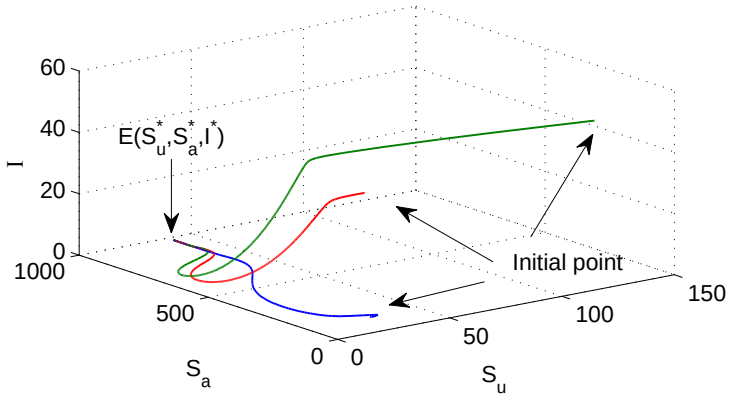

Figure 3: Nonlinear stability of the non delayed system.

$$b_1 \sin\theta\tau + b_2 \cos\theta\tau = -\theta^4 + a_2\theta^2 - a_4 \quad (12)$$
$$b_1 \cos\theta\tau - b_2 \cos\theta\tau = a_1\theta^3 \quad (13)$$

Squaring and adding above two equations,

$$\theta^8 + \alpha_1\theta^6 + \alpha_2\theta^4 + \alpha_3\theta^2 + \alpha_4 = 0. \quad (14)$$

Simplifying and substituting $\theta^2 = l$ in equation (14) we get the following equation

$$l^4 + \alpha_1 l^3 + \alpha_2 l^2 + \alpha_3 l + \alpha_4 = 0. \quad (15)$$

Where

$$\begin{aligned} \alpha_1 &= {a_1}^2 - 2a_2, \\ \alpha_2 &= a_2^2 + 2a_4, \\ \alpha_3 &= -2a_2a_4, \\ \alpha_4 &= a_4^2 - (b_1^2 + b_2^2) \end{aligned}$$

It may be noted that equation (15) will have negative real part if and only if Routh-Hurwitz criterion is satisf ed and hence equation (14) will have no purely imaginary root. Thus, we have the following proposition.

Proposition 4. *In case of delay model system (2.1), the infected steady state E^* will be locally asymptotically stable for all $\tau > 0$, if the following conditions are satisfied:*

$\alpha_1 > 0, \alpha_4 > 0, \alpha_1\alpha_2 - \alpha_3 > 0$ *and* $(\alpha_1\alpha_2 - \alpha_3)\alpha_3 - \alpha_1^2\alpha_4 > 0$.

If $\alpha_4 < 0$ holds then equation (15) will admit at least one positive root. If θ_0 be a positive root of (15), then equation (14) will have a purely imaginary root $\pm i\theta_0$ corresponding to the delay τ. By Butler's lemma, [23] the endemic equilibrium E^* remains stable for $\tau < \tau^*$. Now we evaluate the critical value of τ for which the delayed system (2.1) remain stable.
From equation (12),

$$\tau^* = \frac{1}{\theta_0} cos^{-1}\left[\frac{b_2(-\theta_0^4 + a_2\theta_0^2 - a_4) + b_1 a_1 \theta_0^3}{b_1^2 + b_2^2}\right] + \frac{2\pi n}{\theta_0}, n = 0, 1, 2, 3, ... \quad (16)$$

From the above analysis we have the following theorem.

Theorem 5. *If $\alpha_4 < 0$ is satisfied then the steady state E^* is locally asymptotically stable for $\tau < \tau^*$ and becomes unstable for $\tau > \tau^*$. Furthermore, the system will undergo a Hopf-bifurcation at E^* when $\tau = \tau^*$ provided $4\theta_0^6 + A_1\theta_0^4 + A_2\theta_0^2 + A_3 > 0$, were,*

$$\begin{aligned} A_1 &= 3a_1 - 6a_2, \\ A_2 &= 2a_2 + 4a_4 - 4a_1a_3, \\ A_3 &= a_3^2 - 2a_2a_4 - b_1^2. \end{aligned}$$

Proof. Differentiating (9) with respect to τ we get:

$$\frac{d\tau}{d\xi} = \frac{4\xi^3 + 3a_1\xi^2 + 2a_2\xi + a_3}{b_1\xi^2 + b_2} e^{\xi\tau} + \frac{b_1}{b_1\xi^2 + b_2\xi} - \frac{\tau}{\xi}.$$

Now, using the relation (12) one can obtain:

$$\begin{aligned} Sgn[\frac{d(Re\xi}{d\tau}]_{\tau=\tau^*} &= Sgn[Re(\frac{d\xi}{d\tau})^{-1}]_{\xi=i\theta_0}, \\ &= Sgn\left[\frac{4\theta_0^6 + A_1\theta_0^4 + A_2\theta_0^2 + A_3}{b_1\theta_0^2 + b_2^2}\right]. \end{aligned}$$

Now $Sgn[\frac{d(Re\xi}{d\tau}]_{\tau=\tau^*} > 0$ if $4\theta_0^6 + A_1\theta_0^4 + A_2\theta_0^2 + A_3 > 0$ i.e. the transversality condition holds and the system undergoes Hopf bifurcation at $\tau = \tau^*$.

□

5 Numerical results and Discussion

In this section, numerical simulations are performed to investigate the dynamics of the system. We visualize the effect of increasing time delay, τ, on the dynamics of the system through numerical simulations. The numerical experiments are performed on the systems (with delay and without delay) to conf rm our theoretical f ndings. Parameters are taken from Table 1 and $S_a(0) = 15, S_u(0) = 5, I(0) = 5, M(0) = 0$.

The steady-state values of infected population, of the system (1) without delay, is plotted versus R_0 in Figure 1. Stable steady states y^* are indicated by blue dotted line, unstable steady states by red doted line. For $R_0 > 1$, the uninfected steady state with $I = 0$ is stable. At $R_0 = 1$, this disease free steady state loses its stability, and the system becomes endemically infected and the endemically infected state can be either stable, or unstable and surrounded by a stable limit cycle.

For the system without delay, the system is a asymptotically stable (Figure 2) for the values of the

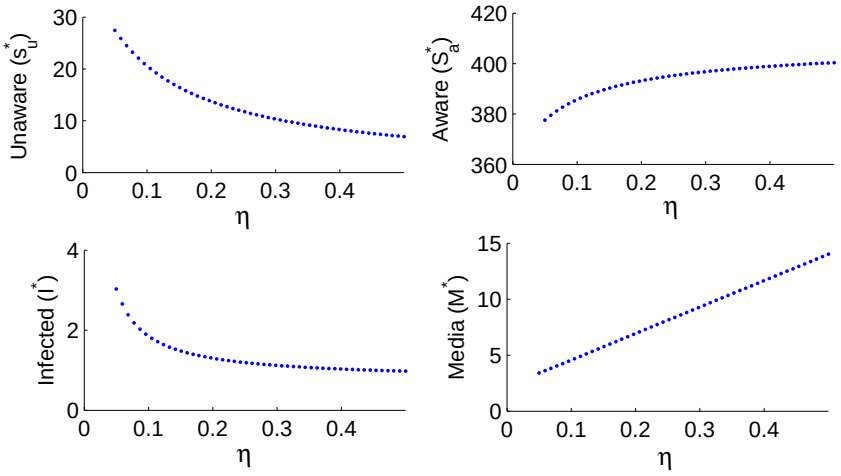

Figure 4: Time series solution of the system with no delay is plotted. The system is stable.

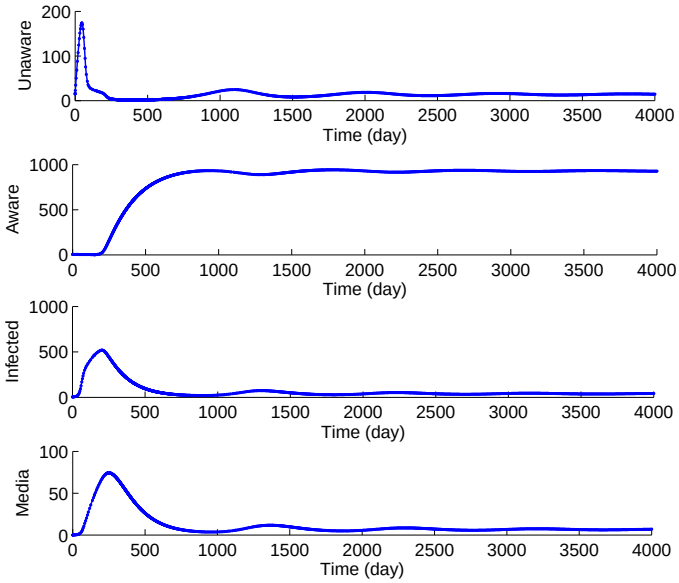

Figure 5: Time series solution of the system for $\tau = 150$, is plotted. Here, system is stable.

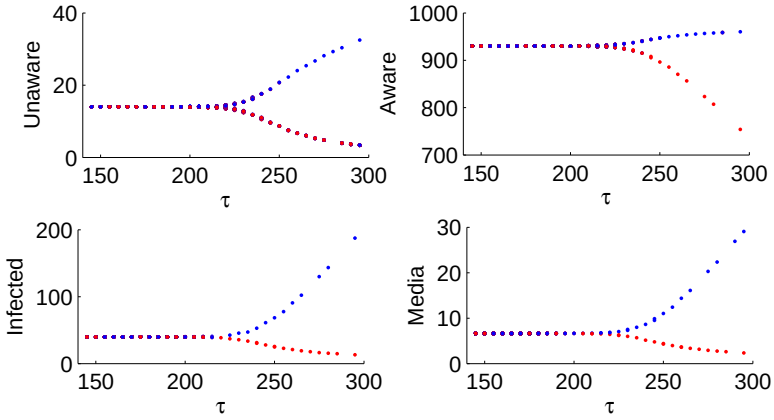

Figure 6: System populations are plotted as function of τ, the aware population growth rate.

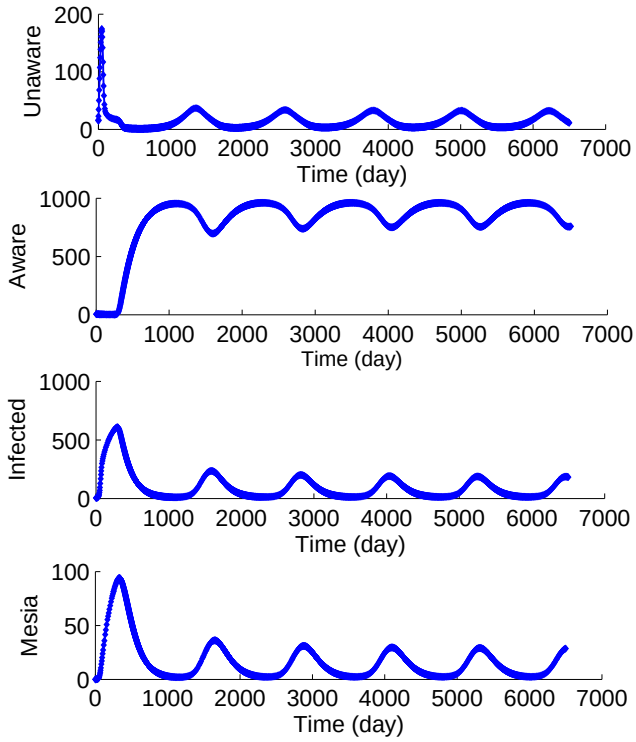

Figure 7: Time series solution of the system for $\tau = 250 > \tau^*$ is plotted and periodic solution is observed.

parameters as given in Table 1. Number of unaware people is decreased and aware people increased. This is due to the fact of awareness campaign. Figure 3 is the solution trajectory of the system (without delay) in (S_u, S_a, I) space. Trajectories from different initial conditions converge to the same steady state. Thus the system without delay is nonlinearly stable.

In Figure 4, is observed that if the rate of awareness (i.e. η) increases, the equilibrium number of infected individuals reduces and aware individuals increases. Considering delay ($\tau = 150$), the system initially oscillates but ultimately becomes stable (Figure 5) at E^*.

Figure 6 and 7 show that as value of delay τ exceeds its critical value τ^*, all variables bifurcate into periodic solution at the endemic equilibria E^*. This indicates that sometimes the number of infective will be high and sometimes low and it may be diff cult to make the prediction regarding the size of epidemic. The system takes more time to become stable. These f gures indicate that equilibrium E^* of model system (1) is stable for $\tau < \tau^*$ and unstable if the inequality is reversed.

6 Conclusion

In this paper, the effect of awareness programs and time delay on infectious disease dynamics have been studied. Time delay is considered due to time taken by unaware people to become aware. For the model system, suff cient conditions for the stability of the disease free and endemic equilibrium and the existence of Hopf-bifurcations are obtained for different sets of values of the delay parameters. However, sustained oscillation may arise if the time lag increases over a threshold value which could possibly pose a challenge in controlling the epidemic.

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