Stochastic Analysis of Stock Market Price Models: A Case Study of the Nigerian Stock Exchange (NSE)

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Abstract: In this paper, stochastic analysis of the behaviour of stock prices is considered using a proposed lognormal distribution model. To test this model, stock prices for a period of 19 years were taken from the Nigerian Stock Exchange (NSE) for simulation, and the results reveal that the proposed model is efficient for the prediction of stock prices. Better accuracy of results via this model can be improved upon when the drift and the volatility parameters are structured as stochastic functions of time instead of constants parameters.

Key–Words: Stochastic model, lognormal distribution, random walk, option pricing, stock exchange market

1 Introduction

The stock price is one of the highly volatile variables in a stock exchange market. The unstable property and other considerable factors such as liquidity on stock return [1] call for concern on the part of investors, since the sudden change in share prices occurs randomly and frequently. Researchers are therefore propelled to look into the behavior of the unstable market variable so as to advise investors and owners of cooperation who are looking for convenient ways to raise money by issuing shares of stocks in their cooperation.

The basis of this work lies in the observation of the Scottish botanist Robert Brown in [2]. This is so since the path of the stock price process can be linked to his description of the random collision of some tiny particles with the molecules of the liquid, he introduced what is called Brownian motion. The "Arithmetic" Brownian motion in the Bachelier's model was the first mathematical model of stock [2]. The stock price model proposed by Bachelier assumed that the discount rate is zero while the dynamics of the stock satisfies the following stochastic differential equation (SDE):

$$dS(t) = S_0 \sigma dW(t) \tag{1}$$

where $S(t)$ is the spot price of the underlying assets at time t, $W(t)$ a standard Brownian motion, and σ the volatility of the stock price.

As a result of the shortcoming of Bachelier's model which states that the hypothesis of the absolute Brownian motion in (1) leads to a negative stock price with positive probability, and ignores the discounting which in reality is not visible, this model was refined by Osborne model in [3], who stated that the log return of the stock process should follow a normal distribution with mean zero and variance $\sigma^2\tau$ for any small $\tau > 0$. Shortly, the geometric Brownian Motion was introduced in [4], where the price of the risky stock evolves according to the SDE:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \tag{2}$$

where μ is the expected rate of share price changes in a given trading period, while other parameters are as earlier defined.

In (2), there is an assumption that the option derived from a stock has a constant expected rate of return μ. In [5, 6], Black and Schole contributed to the world of finance via the introduction of Ito calculus to financial mathematics, and also the Black-Scholes formula. Brennan and Schwarz in [7] employed a finite difference method for pricing the American option for the Black-Schole model. This led to the one dimensional parabolic partial differential inequality.

The main shortcoming of the Black-Schole model is its constant volatility assumption. Meanwhile, statistical analysis of the stock market data shows that the volatility of the stock is a time dependent quantity, and also exhibits various random features. Stochastic volatility of models like the Hull-white model in [8] addressed the randomness by assuming that both the stock price and the volatility are stochastic processes affected by the different sources of risk.

The equation in a stochastic model for stock price volatility in financial engineering was combined with the concept of potential energy in econophysics [9]. In the improved equation, random walk term in the stochastic model was multiplied by the absolute value of the difference between the current price and the price in the previous period divided by the square root of a time step raised to the power of a certain value, and then added to a term which is the price difference multiplied by a coefficient.

McNicholas and Rzzo [10] applied the Geometric Brownian Motion (GBM) model to simulate future market prices. Also, the Cox-ingersoll-Ross approach was used by them to derive the integral interest generator and through stochastic simulation, the result was a full array of price outcomes along with their respective probabilities. The model was used to forecast stock prices of the sampled banks in the five weeks in 1999. Since, the stock prices fluctuate as quickly as possible, a determination of an equilibrium price and a dynamic stability is somewhat tasking. Osu [11] determined the equilibrium price of a solution of the stock pricing PDE which is the backward Black-Schole PDE in one variable. This was with regard to different values of the interest rate. For stochastic nature of financial model; Edeki *et al* [12] considered the effect of stochastic capital reserve on actuarial risk analysis. Ugbebor *et al* [13] considered an empirical stochastic model of stock price changes. Fama [14, 15] presented random walks and stock behavior with respect to stock market prices.

In modeling stock prices, Dmouj [16], constructed the GBM and studied the accuracy of the model with detailed analysis of simulated data. Exponential Levy model for application in finance was considered in [17]. On the aspect of parameter estimation and model prediction in stock prices, much work has been done, Sorin [18]. In a similar manner, other aspects of applied and managerial sciences have been attracting the attention of researchers with regard to predictive models [19, 20].

The structure of the remaining part of the paper is as follows: section 2 is on the mathematical formulation of the problem, stock price expected value and model simulation, and parameters estimation - the volatility and the drift; section 3 is on data analysis and application of results; while section 4 deals with discussion of results and concluding remarks.

2 Mathematical Formulation

Let $S(t)$ be the stock price of some assets at a specified time t, and μ, an expected rate of returns on the stock, and dt as the return or relative change in the price during the period of time.

The dynamics of the stock price is:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{3}$$

where $\mu S_t dt$ and $\sigma S_t dW_t$ are the predictable and the unpredictable parts (respectively) of the stock return.

Theorem 1 *Ito formula Let $(\Omega, \beta, \mu, F(\beta))$ be a filtered probability space, $X = \{X_i, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \mu, F(\beta))$ possessing a quadratic variation $\langle X \rangle$ with an SDE defined as:*

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t), \tag{4}$$

$t \in \mathbb{R}_+$, and for $u = u(t, X(t)) \in C^{1\times 2}(\prod \times \mathbb{R})$. Then

$$\begin{aligned} du(t, X(t)) \quad = \quad & \left[\frac{\partial u}{\partial t} + \frac{g\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right]d\tau \\ & + f\frac{\partial u}{\partial x}dW(t) \end{aligned} \tag{5}$$

Equation (5) whose proof can be found in [6] is a stochastic differential equation (SDE) that can be solved by semi-analytical methods [21, 22] when converted to non SDE of differential type (ODE or PDE). Thus, since the price $S(t)$ of a risky asset such as stock evolves according to the SDE in (4), where $W(t)$ is a standard Brownian motion on the probability space (Ω, β, μ), $\beta = \sigma$ is the algebra generated by $W(t)$, $t \geq 0$, $\mu > 0$ the drift, and σ the volatility of the stock.

Applying Theorem 1 to (3) solves the SDE with the solution as:

$$S(t) = S_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W(t)} \tag{6}$$

According to the properties of standard Brownian motion process for $n \geq 1$ and any sequence of time $0 \leq t_0 \leq t_1 \leq \cdots \leq t_n$, therefore by Euler's method of discretization of the SDE, we have:

$$\ln S_t - \ln S_{t-1} = (\mu - \frac{\sigma^2}{2})\triangle t + \sigma(W_t - W_{t-1}) \tag{7}$$

Remark 2 *The random variables $W_1 - W_{t-1}$ are independent and have the standard normal distribution with mean zero and variance one. Thus, for:*

$$y = \ln S_t - \ln S_{t-1}, \xi = W_t - W_{t-1} \;\; and \;\; \triangle t = 1.$$

Eq. (7) becomes:

$$y_t = \mu - \frac{1}{2}\sigma^2 + \sigma \xi_t \tag{8}$$

In this work, we shall estimate the drift μ, and the volatility σ, in the next section.
Combining (7) and (8) yields:

$$\ln S_t = \ln S_{t-1} + (\mu - \frac{\sigma^2}{2})\triangle t + \sigma\xi(\sqrt{dt}) \quad (9)$$

Eq. (9) has the solution given by:

$$S_t = S_{t-1}e^{(\mu-\frac{\sigma^2}{2})\triangle t+\sigma\xi(\sqrt{dt})} \quad (10)$$

Equation (10) shall be used in this work to develop the simulation of 100 random paths for the stock price for each respective year using the volatility and the drift each stock year.

Eq. (10) is thus referred to as the geometric Brownian motion model (GBMM) of the future stock price S_t from the initial value S_0. Thus, for the time period t, when $dt = t$, (10) will therefore become:

$$S_t = S_0e^{(\mu-\frac{\sigma^2}{2})t+\sigma\xi(\sqrt{t})} \quad (11)$$

2.1 Stock price expected value and simulation model

Let S_t denote the (random) price of the stock at time $t \geq 0$. Then S_t has a normal distribution if $y = \ln S_t$ is normally distributed.

Suppose that $y \sim N(\mu, \sigma^2)$, then the pdf of y is given as,

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-1}{2}\left(\frac{y-\mu}{\sigma}\right)^2},\ y \in (-\infty, \infty), \quad \mu \in (-\infty, \infty),\ \sigma > 0 \quad (12)$$

We obtain the lognormal probability density function (pdf) of the stock price S_t considering the fact that for equal probabilities under the normal and lognormal pdfs, their respective increment areas should be equal.

$$f(y)dy = g(S_t)dS_t \implies g(S_t) = \frac{f(y)dy}{dS_t} \quad (13)$$

Substituting $y = \ln S_t$ & $dy = \dfrac{dS_t}{S_t}$ in (13) defines the pdf of S_t:

$$g(S_t) = \frac{1}{S_t\sigma\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{\ln S_t-\mu}{\sigma}\right)^2} \quad (14)$$

2.2 The Expected value of the stock

After determining the volatility (σ) and the drift (μ) of the stock price of the company, the expected stock price $E(S_t)$ is thus defined and denoted as:

$$\begin{aligned} E(S_t) &= \exp\left(\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma^2 t\right) \\ &= \exp(\ln S_0)\exp\left(\mu + \frac{\sigma^2}{2}\right)t \end{aligned}$$

So

$$E(S_t) = S_0\exp\left(\mu + \frac{\sigma^2}{2}\right)t. \quad (15)$$

Eq (15) holds since,

$$\ln S_t - N(\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)t, \sigma\sqrt{t}).$$

Definition 3 *The Random walk process. For an integer n, $n > 0$ we define the Random Walk process at the time t, $\{W_n(t), t > 0\}$ as follows:*

i. The initial value of the process is:
$W_n(0) = 0$

ii. The layer spacing between two successive jumps is equal to $\frac{1}{n}$;

iii. The "Up" and "down" jumps are equal and of size, $\dfrac{1}{\sqrt{n}}$ with equal probability.

The value of the random walk at the i-th step is defined recursively as follows:

$$W_n\left(\frac{i}{n}\right) = \left(\frac{i-1}{n}\right) + \frac{X_i}{\sqrt{n}},\ \forall\ i \geq 1 \quad (16)$$

For given constants μ and σ the process has the following form:

$$B_t = \mu t + \sigma W_t \quad (17)$$

where t represents time and W_t is a random walk process as described in (17), it can also be expressed as:

$$W_t = \varepsilon\sqrt{t} \quad (18)$$

where ξ is a random number drawn from a standard normal distribution.

2.3 Parameter Estimation (volatility σ and drift μ)

In the course of developing the random walk algorithm, two parameters have to be estimated; the volatility (σ) and the drift (μ) of the stock price for the selected company. Also, for the purpose of the research, the unit of time is chosen to be one day with which both parameters will be calculated. The formulae for the volatility and the drift of the stock price are explained as follows.

2.3.1 The Volatility, σ

Let S_i be the stock price at the end of i-th trading period, $\tau = t, -t_{i-1}, i \geq 1$, the length of time interval between two consecutive trading periods, μ_i the logarithm of the daily return on the stock over the short time interval, τ such that:

$$\mu_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \tag{19}$$

Then the following are defined:

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i \tag{20}$$

$$\nu = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \tag{21}$$

$$\sigma = \frac{\nu}{\sqrt{\tau}} \tag{22}$$

where $\bar{u}$ is the unbiased estimator of the log return, u_i, ν is the standard deviation of the u_i's and σ the volatility of the daily stock return.

2.3.2 The Drift parameter, μ

The drift on the other hand is the expected annual rate of return and is determined from the value of the unbiased estimator as follows.

$$\bar{u} = \left(\mu - \frac{1}{2}\sigma^2\right)\tau \Longrightarrow \mu = \frac{\bar{u}}{\tau} + \frac{1}{2}\sigma^2 \tag{23}$$

3 Data Analysis and Result

The data used for the purpose of this research was from a company listed under the Nigerian Stock Exchange (NSE); available on a web platform [23]. These consist of historical stock data of 4555 closing stock prices from April 12, 1996 to May 16, 2014. The whole data was subdivided into nineteen smaller data samples - each sample containing stock price data for a year each with an average of 240 stock trading days with a minimum and maximum of 96 and 252 days respectively.

It is observed from Table 1 that the trading days for the year 1996 is 183; that of 1997, and 2008 is 253; that of 1998, 1999, 2000, 2002, 2003, 2004, 2005, 2009, 2010, 2011 and 2013 is 252; that of 2001 is 248 and that of 2014 is 96. The trading days can be observed on the first column of Table 1.

For the purpose of this study, the generalized random walk (Brownian motion with drift) model was used in developing the model for the stock price between the years 1997 to the remaining 196 days in 2014 using the volatility and the stock of the previous year from 1996 to 2014 respectively.

Table 1. Stock Price list from April 1996 till May 2014

Date	Stock Price				Volume	Adjusted Close
	Open	High	Low	Close		
12-04-96	25.25	43	24.5	33	409000000	1.38
15-04-96	35.75	36	30	32.25	79219200	1.34
16-04-96	32.25	32.25	28	28.75	48016000	1.2
17-04-96	28.25	28.25	24.75	27	42816000	1.12
18-04-96	30.13	30.13	28	29.25	27268800	1.22
19-04-96	30.13	30.75	28.75	28.87	12913600	1.2
22-04-96	29	29	27.5	28.25	8041600	1.18
23-04-96	28.75	29	28	28	4297600	1.17
24-04-96	28.5	29.12	27.75	29	7795200	1.21
25-04-96	30	32.25	29	31.25	19478400	1.3
26-04-96	32	32.25	31.25	31.75	7561600	1.32
29-04-96	31.5	32	30.5	31	5928000	1.29
30-04-96	31.25	31.5	29.5	29.75	5003200	1.24
01-05-96	30.25	31.75	30	31.63	4881600	1.32
02-05-96	31.5	33.25	31.5	32.87	9731200	1.37
03-05-96	32.25	32.5	31.25	32	6116800	1.33
06-05-96	32.5	32.5	29.38	30.13	8214400	1.26
07-05-96	30	30.75	29.75	30.37	5569600	1.27
08-05-96	30.5	30.75	29.12	30.25	6288000	1.26
09-05-96	30	30.75	29.75	30.75	4032000	1.28
⋮	⋮	⋮	⋮	⋮	⋮	⋮
29-04-14	34.37	35.89	34.12	35.83	28720000	35.83
30-04-14	35.89	36.44	35.25	35.95	23315600	35.95
01-05-14	36.26	36.69	36	36.51	19474700	36.51
02-05-14	36.59	37.12	36.21	36.87	22437600	36.87
05-05-14	36.68	37.05	36.3	36.91	13117400	36.91
06-05-14	36.94	37.17	36.48	36.49	19084100	36.49
07-05-14	35.99	35.99	33.67	34.07	66039200	34.07
08-05-14	33.88	34.57	33.61	33.92	30407700	33.92
09-05-14	34.01	34.1	33.41	33.76	20280700	33.76
12-05-14	33.99	34.6	33.87	34.45	22520600	34.45
13-05-14	34.43	34.69	34.17	34.4	12472200	34.4
14-05-14	34.48	34.65	33.98	34.17	17039000	34.17
15-05-14	34.18	34.19	33.4	33.8	18826600	33.8
16-05-14	33.66	33.66	33.1	33.41	18847100	33.41

Using the values of the volatility and the drift of the stock price of the year 1996; the value of the stock price for the year 1997 will be determined, with that of 1997 used to determine the stock price for the year 1998 etc. Finally, the value of the volatility and the drift for the year 2014 will be used to predict the value of the stock for the remaining days in 2014. All simulations in the work are done using MATLAB.

3.1 Analysis and Result of Parameters

The volatility and drift were calculated for each year using the stock returns over a period of one year each (see Table 2). Although, the unit period time has an average of 240 days per stock year-each respective year was allocated its own unit period given the number of days stated for the stock year; for example a stock year of 252 days will have a period, $\tau = \frac{1}{252}$ years. Hence, the values of the volatility and drift of each respective year was determined.

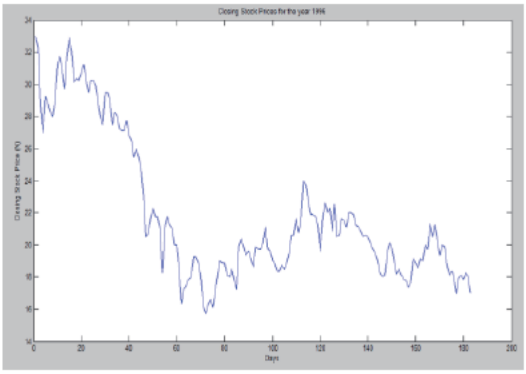

Figure 1: Stock price distributions for the year 1996

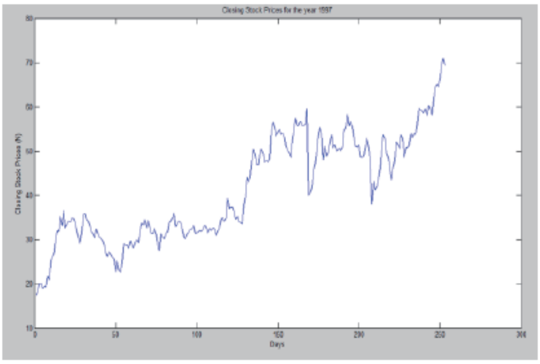

Figure 2: Stock price distributions for the year 1997

3.1.1 Results of volatility, σ

According to Table 2; the distribution of the daily volatility of each stock is stated in the 7^{th} column for the years 1996 till 2014. The highest volatility was observed for the year 1998 with a value of 66% while other years range between 0.2% for 2013, 2% for the years 2005, 2007, 2010, 2011 and 2014, 3% for the years 2003, 2006 and 2009, 4% for the year 1996; 5% for the years 2002 and 2008; 6% for the years 1997 and 2001 and 7% for the years 1998 and 1999 (see Fig 7 for the plot of the volatility).

3.1.2 Results of the drift, μ

According to the data shown in Table 2, the distribution of the daily drift for the stock price stated for the period of 1996 till 2014 is stated in 3^{rd} column. The drift which is defined as the daily expected rate of return of the stock for each year shows a varying distribution of the return rate. The highest drift was observed for the years 2013, 1998, 1999 and 2003 with a value of 70%, 184%, 123% and 102% respectively (see Fig 6).

The lowest drifts were also recorded for the year 2000 with a value of -212% followed by the years

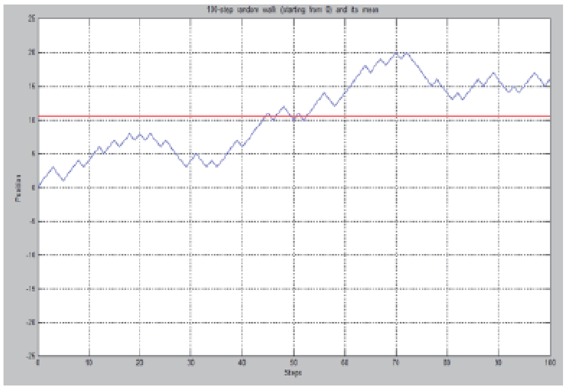

Figure 3: 100-step random walk and its mean (starting from zero)

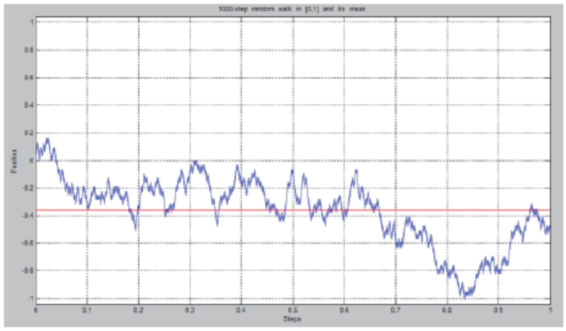

Figure 4: 1000-step random walk in [0, 1] and its mean

1996, 2006, 2008 and 2014 with a value of -49%, -39%, -38% and -14.6% respectively. The remaining years have drifts falling within the range of 2% and 4%.

The distribution of the remaining parameters which were also used in determining the drift and the volatility of the stock is shown in Fig 7 for the estimator of standard deviation/volatility, v and Fig 8 for the unbiased estimator, $\bar{\mu}$.

3.1.3 Results and discussion of the simulation of the stock price for the period

For the simulation of the stock price of the year 1997; the drift and volatility of the year 1996 (see Fig 1 for the stock price of 1996) which had a value of -0.4908 and 0.593452 respectively were used in developing the simulation of the 100 random walks for the year 1996 (see Fig 2). Majority of the simulated plots can be observed to produce lower stock prices compared to the actual stock price which shows an increased s-tock distribution towards the end of the year.

For the simulation of the stock price of the year 1998; the drift and volatility of the year 1997 which

Table 2: Values of the drift, volatility, standard deviation and unbiased estimator for each stock year

Year	Trading days	Stock drift, μ	Estimator for Std.deviation,υ	Time interval τ(years)	Volatility Std deviation,σ	Unbiased estimation,$\bar{u}$
1996	183	-0.49084601	0.04386924	0.005464481	0.593452079	-0.00364447
1997	253	1.7862	0.05698	0.003952569	0.906322383	0.00543687
1998	252	1.84288	0.66871	0.003968254	10.61544216	0.005077177
1999	252	1.232176388	0.073224139	0.003968	1.162434369	0.002208702
2000	252	-2.121176075	0.07228273	0.003968	1.147489486	-0.010996492
2001	248	0.111562421	0.061873163	0.003968	0.982237445	-0.001867521
2002	252	0.148347748	0.047091024	0.003968	0.747570754	-0.000520101
2003	252	1.028327383	0.025996734	0.003968	0.412698565	0.003742749
2004	252	0.120409721	0.049403982	0.003968	0.784288999	-0.00074256
2005	252	0.0683571	0.018344044	0.003984064	0.290624313	0.000103006
2006	251	-0.39056469	0.025631235	0.003984064	0.406075131	-0.001884515
2007	251	-0.028299874	0.023334203	0.003984064	0.3696833	-0.000184991
2008	253	-0.380322227	0.047648152	0.003952569	0.757890254	-0.002638423
2009	252	0.356700294	0.026546348	0.003968254	0.42141021	0.001063123
2010	252	0.015570308	0.018591583	0.003968	0.295141675	-0.000111037
2011	252	0.040694719	0.024970197	0.003968	0.396402274	-0.000150268
2012	251	0.22112019	0.012695823	0.003984064	0.201139664	0.000803889
2013	252	0.703864574	0.002789224	0.003968254	0.044277558	0.002789224
2014	96	-0.14594705	0.023333683	0.010416667	0.228622469	-0.00182496

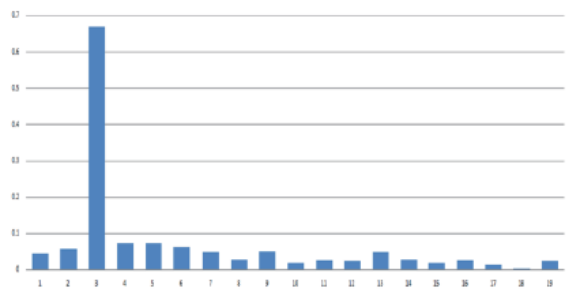

Figure 5: Estimator of the standard deviation, ν of the stock price for the year 1996 to 2014

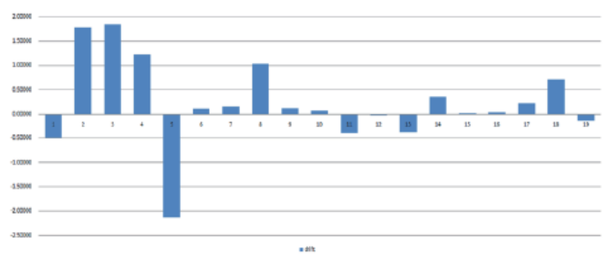

Figure 6: Drift, μ of the stock price for the year 1996 to 2014

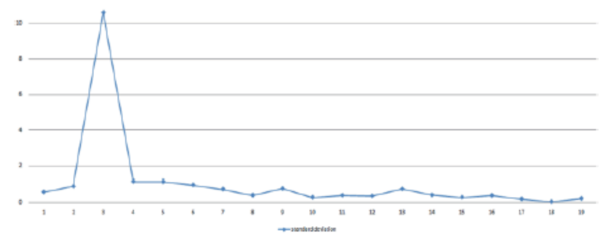

Figure 7: Volatility/standard deviation, σ of the stock price for the year 1996 to 2014

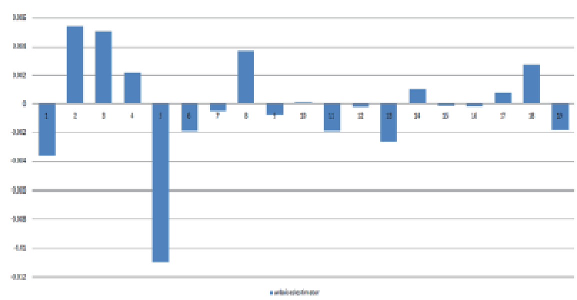

Figure 8: Unbiased estimator, $\bar{u}$ of the stock price for the year 1996 to 2014

has a value of 1.7862 and 0.906322 respectively were used in developing the simulation of the 100 random walks for the year 1998 (see Fig 9) using the initial stock price of 66.25. This can be observed to be as a result of the high volatility of 1998 with a value of 10.615442 the year with the highest volatility compared to a volatility of 0.906322 for the year 1997.

For the simulation of the stock price of the year 1999; the drift and volatility of the year 1998 which has a value of 1.8429 and 10.615442 respectively were used in developing the simulation of the 100 random walks for the year 1999 (see Fig 10) using the initial stock price of 24.80. The simulation of the stock price for the year were out of place due to the difference in the volatility of the stock year with a value of 10.615442 for the previous year (1998) while that of 1999 is 1.162434. The simulation of the stock price for the year 1999 was not a good interpretation of the actual stock price for the year.

For the simulation of the stock price of the year 2000; the drift and volatility of the year 1999 which has a value of 1.2322 and 1.162434 respectively were used in developing the simulation of the 100 random walks for the year 2000 (see Fig 11) using the initial stock price of 475. The volatility of the stock shows almost equal values with 1.162434 for 1999 and 1.147489 for 2000.

For the simulation of the stock price of the year 2001; the drift and volatility of the year 2000 which has a value of -2.1212 and 1.147489 respectively were used in developing the simulation of the 100 random walks for the year 2001 (see Fig 12) using the initial stock price of 28.19. Although the volatility for the years 2000 and 2001 are almost equal their drift shows a significant difference - the stock return for the year 2001 is much larger than that of 2000 with a value of 0.1112 a positive value unlike the negative value observed for the year 2000 used in performing the 100 random paths.

4 Discussion of Result and Concluding Remark

This paper stochastically analyzes stock market prices via a proposed lognormal model. To test this, stock prices for a period of 19 years (from the nigerian stock Exchange) were simulated. As indicated in Figs 13 to 24, the simulation of the annual stock price of the years: 2002 to 2013 respectively with respect to their initial prices is showed; for the simulation purpose, the drift and volatility parameters of the previous years: 2001 to 2012 respectively were used. Similarly, Fig 25 shows the simulation of the stock price

Table 3: Values of the initial stock price, drift, volatility and trading days used in simulating the 100 random walks for each year

Year	(S_0)	Drift (μ)	Volatility σ	Trading days, n
1997	17.50	-0.4908	0.593452	253
1998	66.25	1.7862	0.906322	252
1999	24.80	1.8429	10.615442	252
2000	475.00	1.2322	1.162434	252
2001	28.19	-2.1212	1.147489	252
2002	18.63	0.1112	0.982237	252
2003	17.60	0.1483	0.747571	252
2004	45.40	1.0283	0.412699	252
2005	36.18	0.1204	0.784289	252
2006	40.91	0.0684	0.290624	251
2007	25.61	-0.3906	0.406075	251
2008	23.72	-0.0283	0.369683	253
2009	12.85	-0.3803	0.75789	252
2010	17.10	0.3567	0.42141	252
2011	16.75	0.0156	0.295142	252
2012	16.29	0.0407	0.396402	251
2013	20.08	0.2211	0.20114	252
2014	39.59	0.7039	0.44278	96
2014 $(remain)$	33.41	-0.1459	0.228622	156

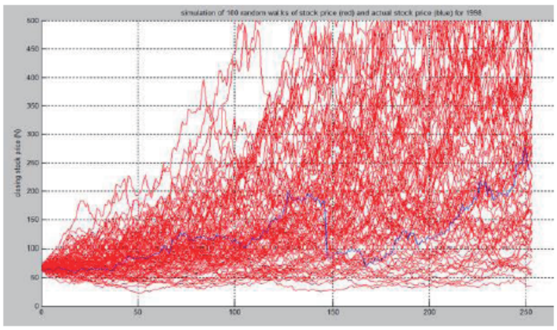

Figure 9: Simulation of 100 random walks for 1998 and actual stock price (blue)

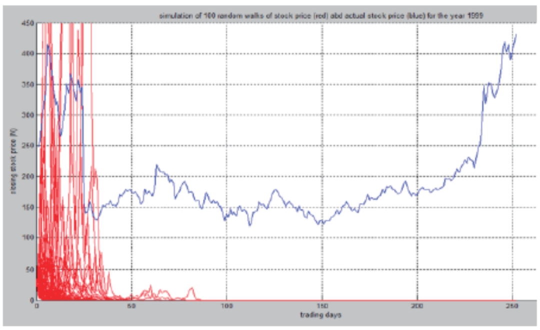

Figure 10: Simulation of 100 random walks for 1999 and actual stock price (blue)

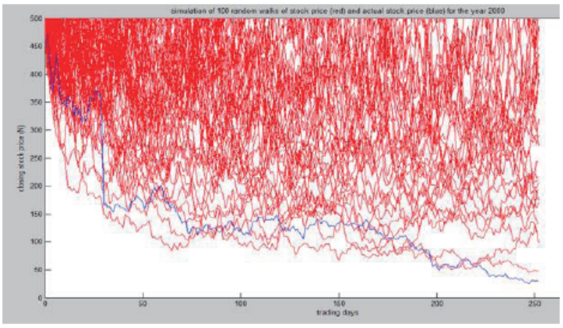

Figure 11: Simulation of 100 random walks for 2000

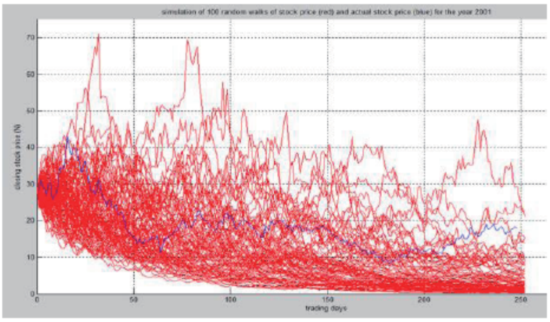

Figure 12: Simulation of 100 random walks for 2001

for the first 96 days of 2014 using the drift and volatility of the year 2013 but the initial price of the stock year of 2014 as at the beginning of the year. From the diagram, it can be observed that the stock price for the year 2014 - the actual plot lies within the range of 35 to 40 but most of the simulations of the 100 random paths lie more in the range of 20 to 140. For the first 20 days of simulation, it can be observed that the random paths cluster around the actual stock but as the simulation progresses further most of the random paths tend to fall out of place with just a little percentage lying within the area of the actual stock price for the year 2014.

For the simulation of the remaining 196 days of the stock price for the year 2014; the drift and volatility of the year 2014 for the 96 days for which information on the stock price is made available is determined and used with the value of the stock price at the 96^{th} trading day as the initial stock price for the remaining 196 trading days of the year 2014.

100 random paths were simulated for the period and the simulations show a close resemblance to the observed actual stock price for the first 96 trading days. The trend shows that most of the 100 random paths simulation of the expected stock price for the remaining part of the year falls within the range of a minimum drop to 15 and a maximum step to 45. This

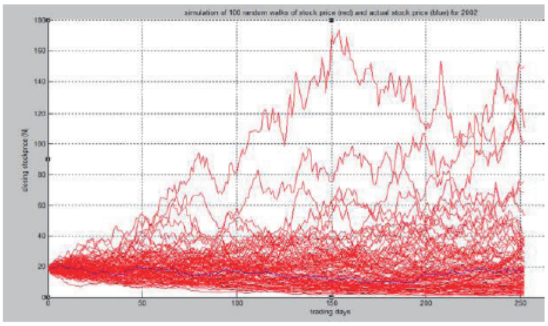

Figure 13: Simulation of 100 random walks for 2002 and actual stock price (blue)

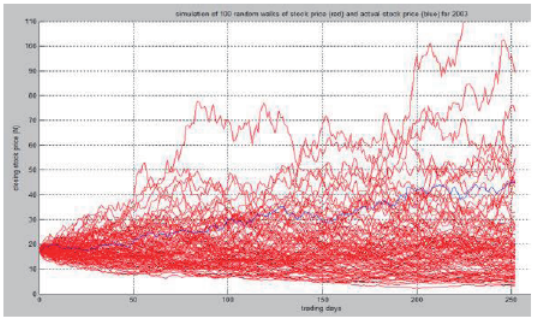

Figure 14: Simulation of 100 random walks for 2003 and actual stock price (blue)

behavior is almost acceptable since most of the stock price for the early part of the year lie within the range of a drop to 20 and step to 40.

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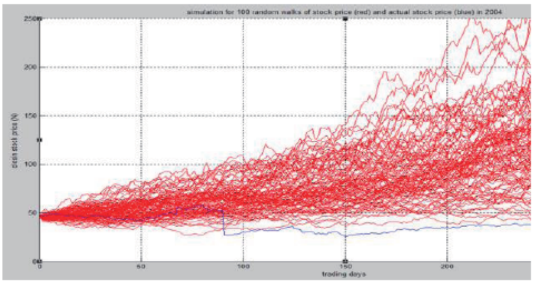

Figure 15: Simulation of 100 random walks for 2004 and actual stock price (blue)

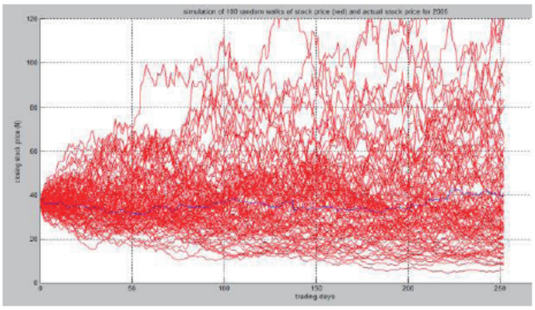

Figure 16: Simulation of 100 random walks for 2005 and actual stock price (blue)

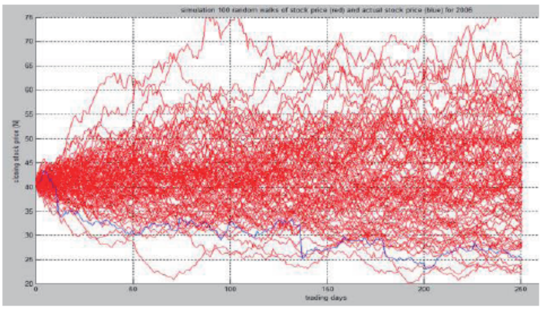

Figure 17: Simulation of 100 random walks for 2006 and actual stock price (blue)

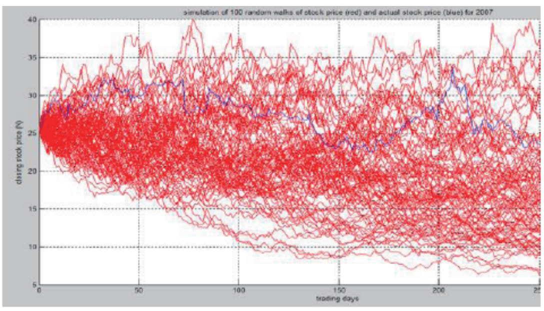

Figure 18: Simulation of 100 random walks for 2007 and actual stock price (blue)

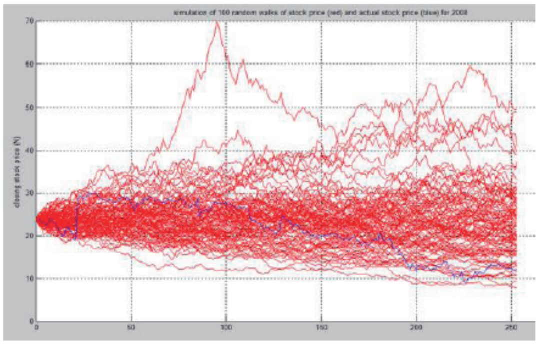

Figure 19: Simulation of 100 random walks for 2008 and actual stock price (blue)

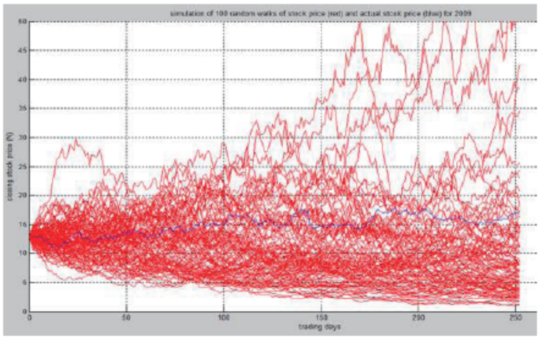

Figure 20: Simulation of 100 random walks for 2009 and actual stock price (blue)

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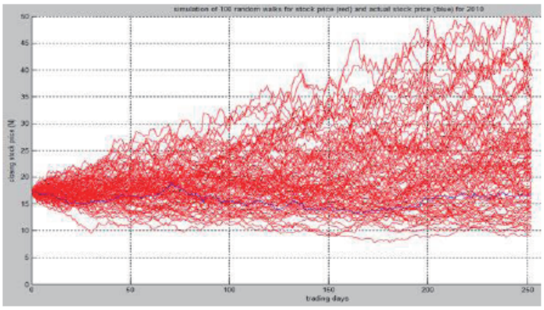

Figure 21: Simulation of 100 random walks for 2010 and actual stock price (blue)

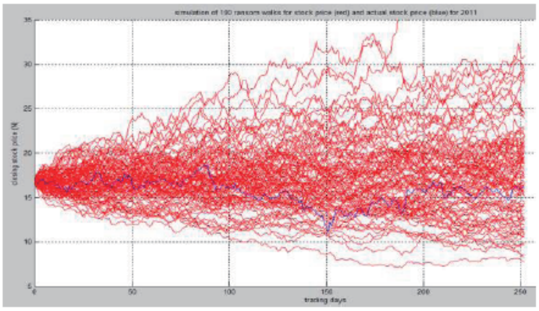

Figure 22: Simulation of 100 random walks for 2011 and actual stock price (blue)

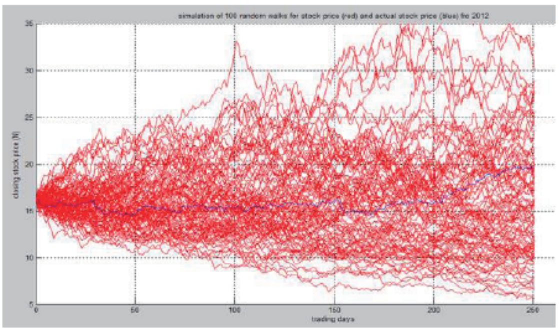

Figure 23: Simulation of 100 random walks for 2012 and actual stock price (blue)

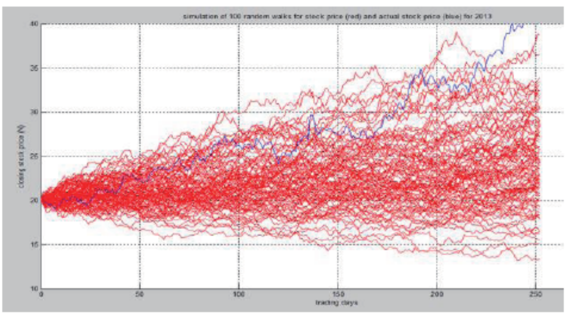

Figure 24: Simulation of 100 random walks for 2013 and actual stock price (blue)

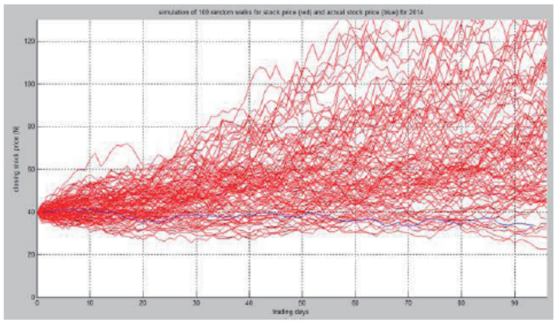

Figure 25: Simulation of 100 random walks for 2014 and actual stock price (blue)

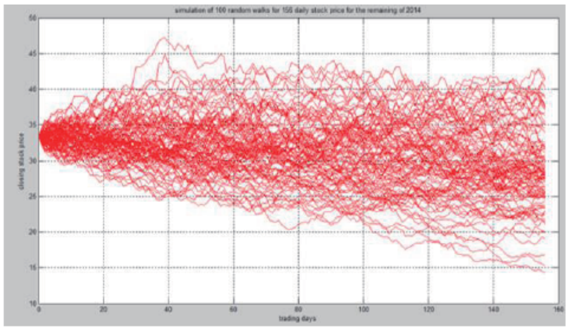

Figure 26: Simulation of the expected stock price for the remaining 196 trading days in 2014

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