Research on the triopoly dynamic game model based on different rationalities and its chaos control

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Abstract: Based on the earlier research results, and combined with the actual competition in Chinese insurance market, the triopoly dynamic game model is built under the hypothesis that the oligarchs make adaptive decision, delayed bounded rational decision and bounded rational decision respectively. The existence of only Nash equilibrium in the system is analyzed theoretically, and its stability and the existence of Hopf bifurcation are also studied. The further numerical simulation result verified the accuracy of theoretical study and displayed more intuitively the dynamic behaviour of the system. From this we can conclude that in the process of considering delayed price game, the insurance company must control the value of delay parameter well and decrease his price adjustment speed properly. Only in this way they can make the system stable to its equilibrium state more rapidly. At last, the variable feedback control method is used to make control to the system, it makes the unstable system to the Nash equilibrium state again.

Key–Words: Insurance market, Oligopoly, Dynamic game, Self-adaptive, Delay, Bounded rationality

1 Introduction

With the rapid development of Chinese insurance market, the number of insurance company is increasing year by year. Although the market concentration has decreased somewhat in recent years, Chinese insurance market is still with the typical oligopoly market structure at present and the price competition is the main competition among oligarchs. The existing literatures focused on the research on static game, the literatures about dynamic game research are relatively less. Han Shufeng and Ding Xiaxing [1] thought that the price competition is the main counter mean in Chinese insurance market, they analyzed the price competition from the point of view of game theory and concluded the insurance company can use the limitation of supply ability of other companies to make pricing strategy of second lowest price to get relatively high profits. Zheng wenzhe and Wu jilin [2] established the price competition model of the oligarches with the difference of service quality, they assumed that the two companies played the static game with incomplete information. T. Dubiel Teleseynski [3] studied the duopoly repeated game model which one made decision with bounded rationality and the other made decision with self-adaption, and explained the diseconomies of scale. Afterward E. Ahmed, H. N. Agiza and S. Z. Hassan [4] expanded Puu's research to the circumstances such as bounded rationality, differentiated products and delayed bounded rationality and so on, they have drawn the conclusion that the delay can increase the stability of the system. Xu Feng [5] takes delayed decisions into a duopoly advertising game model, one player considers delayed decision and the other considers bounded rationality, then the author analyzes the effect of delayed decisions on this system. Ma Junhai [6] supposes all game players consider bounded rationality with delay. In the condition of different parameters, he investigates the stability, Hopf bifurcation and chaos of the system. It concludes that the delayed parameter and other parameter values will change the stability of system. H. N. Agiza[7] analyzed the dynamics of Bowley's model with bounded rationality, studies the existence and stability of the equilibria and the Bowley's model with delayed bounded rationality in monopoly. Gao Qin [8] uses the theory of ecology to analyze the competition and cooperation between two container terminal companies. She improves the Lotka-Volterra model to constitute nonlinear models and investigates the stability and Hopf bifurcation of no-delay and delay models. Woo-Sik Son[9] investigates the dynamical model of a financial system, analyzes the local stability of the system with delays and the effect of delayed feedbacks on the financial model. Su Z. et al.[10] studied the problem of practical asymptotic stability for a class of discrete-time time-delay systems via Razumikhin-type Theorems. Gong Yonghua[11] studied the price competition and differentiation strategy among three oligarchs through building the market competition models with two different game structures. G. H. Wang and J. H. Ma [12] analyzed the output game in a two-level supply chain, and discussed the chaos control of the supply chain system. Ji Weizhuo [13] built the output game model about three oligarchs with different decision rules in power market and used the nonlinear dynamics theory to study the influence which the output adjustment has on the market. In addition, there are many literatures which have done a lot work on this field [14-23].

With the research background of Chinese insurance market, the dynamic price game model with three oligarchs will be built in this paper. We consider that the oligarchs make adaptive decision, delayed bounded rational decision and bounded rational decision respectively, and researches the stability of Nash equilibrium and the existence of Hopf bifurcation. By the numerical simulation analysis, we can find the effect of some parameters on the system. At last, the variable feedback control method is used to control the chaos of the system.

2 Model

With the assumption that an insurance market is a triopoly market, three insurance companies make their own prices for their products. Let $p_i(t), i = 1, 2, 3$ represent the price of i th insurance company, $q_i, i = 1, 2, 3$ represent the quantity of i th insurance company. So the linear inverse demand functions of three insurance companies are as follows:

$$\begin{aligned} q_i &= a - b_i p_i + g_i p_j + h_i p_k \\ &(i, j, k = 1, 2, 3; i \neq j \neq k) \end{aligned} \tag{1}$$

where $a, b_i, g_i, h_i > 0, i = 1, 2, 3$, g_i, h_i represent the product substitutions between the two insurance company. Suppose that insurance companies have no fixed costs, let $c_i > 0, i = 1, 2, 3$ represent the marginal costs of i th insurance company, so the cost function with linear form is $C_i(q_i) = c_i q_i, i = 1, 2, 3$. And accordingly, the profit function of the three insurance companies is given by $\pi_i(p_1, p_2, p_3) = p_i q_i - c_i q_i, i = 1, 2, 3$. We put formula (1) into the profit function and can get:

$$\pi_i(p_i, p_j, p_k) = (p_i - c_i)(a - b_i p_i + g_i p_j + h_i p_k) \tag{2}$$

then the marginal profits of the three insurance companies are:

$$\begin{aligned} &\tfrac{\partial \pi_i(p_i, p_j, p_k)}{\partial p_i} \\ &= a - 2b_i p_i + g_i p_j + h_i p_k + b_i c_i \end{aligned} \tag{3}$$

Here we assume the first company considers adaptive decision. When $\frac{\partial \pi_1(p_1, p_2, p_3)}{\partial p_1} = 0$, we can compute its price is $\frac{1}{2b_1}(a + g_1 p_2 + h_1 p_3 + b_1 c_1)$. The first companys price game is based on the margin of the above value and $p_1(t - \tau_1)$. If the margin is positive, it will increase its price; otherwise, decrease its price. Let $\alpha_i(p_i), i = 1, 2, 3$ represent the extent of the price variation of ith insurance company according to its marginal profit and they are positive functions. So the dynamic adjustment of the price of the first company is modeled as:

$$\begin{aligned} \dot{p_1}(t) = \alpha_1(p_1)[\tfrac{1}{2b_1}(a + g_1 p_2 + h_1 p_3 \\ + b_1 c_1) - p_1(t - \tau_1)] \end{aligned} \tag{4}$$

We assume the other two companies consider bounded rationality. As we known, the market information of all firms is incomplete. Some firms will take marginal profit into consideration in their price game in order to maximize their profits. If the marginal profit is positive, the firm will increase its price; otherwise, decrease its price. So the dynamic adjustments of the price of the other two companies are modeled as:

$$\dot{p_i}(t) = \alpha_i(p_i) \frac{\partial \pi_i(p_1, p_2, p_3)}{\partial p_i}, i = 2, 3 \tag{5}$$

When the two insurance companies with bounded rationality need to determine their own price, they will consider not only the current marginal profit but also the marginal profit of τ time ago. Here we assume only the second insurance company considers delay decision and the delay parameter is τ_2. So the dynamic model of price game becomes:

$$p_2^d = w p_2(t) + (1 - w) p_2(t - \tau_2) \tag{6}$$

where $0 < w < 1$ represents the weight of the current price, $1 - w$ represents the weight of price of $t - \tau_2$ time. In order to analyze the system easily, here we simplify this model and assume $\tau_1 = \tau_2 = \tau$.

We assume $\alpha_i(p_i)$ has a linear form

$$\alpha_i(p_i) = v_i p_i, \;\; i = 1, 2, 3 \tag{7}$$

where v_i is positive which represents the speed of price adjustment of ith insurance company. Therefore, the final dynamic system of price game is given by

$$\begin{cases} \dot{p}_1(t) = v_1p_1[(a+g_1wp_2(t) + g_1(1-w)p_2(t-\tau) \\ +h_1p_3(t) + b_1c_1)/(2b_1) - p_1(t-\tau)] \\ \dot{p}_2(t) = v_2p_2[a-2b_2wp_2(t) - 2b_2(1-w)p_2(t-\tau) \\ +g_2p_3(t) + h_2p_1(t) + b_2c_2] \\ \dot{p}_3(t) = v_3p_3[a-2b_3p_3(t) + g_3p_1(t) + h_3wp_2(t) \\ +h_3(1-w)p_2(t-\tau) + b_3c_3] \end{cases} \tag{8}$$

3 Equilibrium points and local stability

The equilibrium points of the dynamic system of the triopoly price game must be nonnegative because our model is an economic model. So the eight fixed points of system (8) are

$E_1\,(0,0,0)$, $E_2\left(\frac{a+b_1c_1}{2b_1},0,0\right)$,
$E_3\left(0,\frac{a+b_2c_2}{2b_2},0\right)$, $E_4\left(0,0,\frac{a+b_3c_3}{2b_3}\right)$,
$E_5\left(\frac{2ab_2+2b_1b_2c_1+ag_1+b_2c_2g_1}{4b_1b_2-g_1h_2},\frac{2ab_1+2b_1b_2c_2+ah_2+b_1c_1h_2}{4b_1b_2-g_1h_2},0\right)$,
$E_6\left(\frac{2ab_3+2b_1b_3c_1+ah_1+b_3c_3h_1}{4b_1b_3-g_3h_1},0,\frac{2ab_1+2b_1b_2c_3+ag_3+b_1c_1g_3}{4b_1b_3-g_3h_1}\right)$,
$E_7\left(0,\frac{2ab_3+2b_2b_3c_2+ag_2+b_3c_3g_2}{4b_2b_3-g_2h_3},\frac{2ab_2+2b_2b_3c_3+ah_3+b_2c_2h_3}{4b_2b_3-g_2h_3}\right)$,
$E_8\,(p_1^*,p_2^*,p_3^*)$ where $p_1^* = \frac{e_1+e_2}{e_7-e_8}$, $p_2^* = \frac{e_3+e_4}{e_7-e_8}$, $p_3^* = \frac{e_5+e_6}{e_7-e_8}$ and

$$\begin{aligned} e_1 &= 4ab_2b_3 + 2ab_3g_1 + 2ab_2h_1 + ag_1g_2 - ag_2h_3 \\ &\quad +ah_1h_3 \\ e_2 &= 4b_1b_2b_3c_1 + 2b_2b_3c_2g_1 + 2b_2b_3c_3h_1 \\ &\quad +b_3c_3g_1g_2 - b_1c_1g_2h_3 + b_2c_2h_1h_3 \\ e_3 &= 4ab_1b_3 + 2ab_1g_2 + 2ab_3h_2 + ag_2g_3 - ag_3h_1 \\ &\quad +ah_1h_2 \\ e_4 &= 4b_1b_2b_3c_2 + 2b_1b_3c_3g_2 + 2b_1b_3c_1h_2 \\ &\quad +b_1c_1g_2g_3 - b_2c_2g_3h_1 + b_3c_3h_1h_2 \\ e_5 &= 4ab_1b_2 + 2ab_2g_3 + 2ab_1h_3 + ag_1g_3 - ag_1h_2 \\ &\quad +ah_2h_3 \\ e_6 &= 4b_1b_2b_3c_3 + 2b_1b_2c_1g_3 + 2b_1b_2c_2h_3 \\ &\quad +b_2c_2g_1g_3 - b_3c_3g_1h_2 + b_1c_1h_2h_3 \\ e_7 &= 8b_1b_2b_3 - g_1g_2g_3 - h_1h_2h_3 \\ e_8 &= 2b_1g_2h_3 + 2b_2g_3h_1 + 2b_3g_1h_2 \end{aligned}$$

We know that E_1 means there have no competitors in the insurance market in the end of price game. E_2, E_3, E_4 and E_5, E_6, E_7 mean that there have only one or two insurance companies survived at the insurance market. Obviously, E_1, E_2, E_3, E_4, E_5, E_6, E_7 are boundary equilibria and E_8 is the only Nash equilibrium of system (8).

Let $u_1(t) = p_1(t) - p_1^*$, $u_2(t) = p_2(t) - p_2^*$, $u_3(t) = p_3(t) - p_3^*$. Then system (8) is written as

$$\begin{cases} \dot{u}_1(t) = v_1(u_1(t) + p_1^*)[-u_1(t-\tau) + \frac{g_1wu_2(t)}{2b_1} \\ +\frac{g_1(1-w)u_2(t-\tau)}{2b_1} + \frac{h_1u_3(t)}{2b_1}] \\ \dot{u}_2(t) = v_2(u_2(t) + p_2^*)[h_2u_1(t) - 2b_2wu_2(t) \\ -2b_2(1-w)u_2(t-\tau) + g_2u_3(t)] \\ \dot{u}_3(t) = v_3(u_3(t) + p_3^*)[g_3u_1(t) + h_3wu_2(t) \\ +h_3(1-w)u_2(t-\tau) - 2b_3u_3(t)] \end{cases} \tag{9}$$

Now we can investigate the stability of point $(0,0)$ instead of the stability of equilibrium point E_8. The linearization of system (9) when $u=0$ is

$$\begin{cases} \dot{u}_1(t) = -v_1p_1^*u_1(t-\tau) + \frac{g_1wv_1p_1^*u_2(t)}{2b_1} \\ +\frac{g_1(1-w)v_1p_1^*u_2(t-\tau)}{2b_1} + \frac{h_1v_1p_1^*u_3(t)}{2b_1} \\ \dot{u}_2(t) = h_2v_2p_2^*u_1(t) - 2b_2wv_2p_2^*u_2(t) \\ -2b_2(1-w)v_2p_2^*u_2(t-\tau) + g_2v_2p_2^*u_3(t) \\ \dot{u}_3(t) = g_3v_3p_3^*u_1(t) + h_3wv_3p_3^*u_2(t) \\ +h_3(1-w)v_3p_3^*u_2(t-\tau) - 2b_3v_3p_3^*u_3(t) \end{cases} \tag{10}$$

Thus, we can obtain the characteristic equation of the system (10)

$$\begin{vmatrix} \lambda + v_1p_1^*e^{-\lambda\tau} & \frac{-g_1wv_1p_1^* - g_1(1-w)v_1p_1^*e^{-\lambda\tau}}{2b_1} & \frac{-h_1v_1p_1^*}{2b_1} \\ -h_2v_2p_2^* & \lambda+2b_2wv_2p_2^*+2b_2(1-w)v_2p_2^*e^{-\lambda\tau} & -g_2v_2p_2^* \\ -g_2v_2p_2^* & -h_3wv_3p_3^* - h_3(1-w)v_3p_3^*e^{-\lambda\tau} & \lambda + 2b_3v_3p_3^* \end{vmatrix} = 0 \tag{11}$$

Compute the determinant (11), we will have

$$\lambda^3 + k_1\lambda^2 + k_2\lambda + k_3 + (k_4\lambda^2 + k_5\lambda + k_6)e^{-\lambda\tau} \\ +(k_7\lambda + k_8)e^{-2\lambda\tau} = 0 \tag{12}$$

where

$$\begin{aligned} k_1 &= 2\,(wb_2v_2p_2^* + b_3v_3p_3^*) \\ k_2 &= wv_2v_3p_2^*p_3^*\,(4b_2b_3 + g_2h_3) \\ &\quad -v_1p_1^*(wg_1h_2v_2p_2^* + g_3h_1v_3p_3^*)/2b_1 \\ k_3 &= -[wv_1v_2v_3p_1^*p_2^*p_3^*(g_1g_2g_3 + h_1h_2h_3 \\ &\quad +2b_2g_3h_1 + 2b_3g_1h_2)]/2b_1 \\ k_4 &= 2(1-w)b_2v_2p_2^* + v_1p_1^* \\ k_5 &= (1-w)v_2v_3p_2^*p_3^*(4b_2b_3 + g_2h_3) \\ &\quad +2v_1p_1^*(wb_2v_2p_2^* + b_3v_3p_3^*) \\ &\quad -(1-w)g_1h_2v_1v_2p_1^*p_2^*/(2b_1) \end{aligned}$$

$$\begin{aligned} k_6 &= wv_1v_2v_3p_1^*p_2^*p_3^*(4b_2b_3+g_2h_3) \\ &\quad -(1-w)v_1v_2v_3p_1^*p_2^*p_3^*(g_1g_2g_3+h_1h_2h_3 \\ &\quad +2b_3g_1h_2+2b_2g_3h_1)/2b_1 \\ k_7 &= 2(1-w)b_2v_1v_2p_1^*p_2^* \\ k_8 &= (1-w)v_1v_2v_3p_1^*p_2^*p_3^*(4b_2b_3+g_2h_3) \end{aligned}$$

When $\tau = 0$, the equation (12) will have the form

$$\lambda^3 + l_1\lambda^2 + l_2\lambda + l_3 = 0 \quad (13)$$

where $l_1 = k_1+k_4$,$l_2 = k_2+k_5+k_7$,$l_3 = k_3+k_6+k_8$. The real parts of all roots are negative if and only if $l_i > 0$ and $l_1l_2 - l_3 > 0$ hold, then the Nash equilibrium of the system is locally asymptotically stable.

When $\tau > 0$, multiply $e^{\lambda\tau}$ on both sides of equation (12), we will have

$$\begin{aligned} &(\lambda^3 + k_1\lambda^2 + k_2\lambda + k_3)e^{\lambda\tau} + (k_4\lambda^2 + k_5\lambda + k_6) \\ &+(k_7\lambda + k_8)e^{-\lambda\tau} = 0 \end{aligned} \quad (14)$$

If $\lambda = i\omega(\omega > 0)$ is a root of equation (14), separating the real and imaginary parts, we can obtain

$$\begin{cases} (k_3+k_8-k_1\omega^2)\cos(\omega\tau) \\ -\omega(k_2-k_7-\omega^2)\sin(\omega\tau) = k_4\omega^2 - k_6 \\ (k_3-k_8-k_1\omega^2)\sin(\omega\tau) \\ +\omega(k_2+k_7-\omega^2)\cos(\omega\tau) = -k_5\omega. \end{cases} \quad (15)$$

According to the equation (15), we can compute

$$\begin{cases} \sin(\omega\tau) = \frac{m_1\omega^5+m_2\omega^3+m_3\omega}{\omega^6+m_7\omega^4+m_8\omega^2+m_9} \\ \cos(\omega\tau) = \frac{m_4\omega^4+m_5\omega^2+m_6}{\omega^6+m_7\omega^4+m_8\omega^2+m_9} \end{cases} \quad (16)$$

where $m_1 = k_4$, $m_2 = k_1k_5 - k_2k_4 - k_4k_7 - k_6$, $m_3 = k_2k_6 + k_6k_7 - k_3k_5 - k_5k_8$, $m_4 = k_5 - k_1k_4$, $m_5 = k_1k_6 + k_3k_4 + k_5k_7 - k_2k_5 - k_4k_8$, $m_6 = k_6k_8 - k_3k_6$, $m_7 = k_1^2 - 2k_2$, $m_8 = k_2^2 - k_7^2 - 2k_1k_3$, $m_9 = k_3^2 - k_8^2$.

Since $\sin^2(\omega\tau) + \cos^2(\omega\tau) = 1$, combining with equation (16), we have

$$\omega^{12} + n_1\omega^{10} + n_2\omega^8 + n_3\omega^6 + n_4\omega^4 + n_5\omega^2 + n_6 = 0 \quad (17)$$

where $n_1 = 2m_7 - m_1^2$, $n_2 = m_7^2 - m_4^2 + 2m_8 - 2m_1m_2$, $n_3 = 2m_7m_8 + 2m_9 - m_2^2 - 2m_1m_3 - 2m_4m_5$, $n_4 = m_8^2 - m_5^2 + 2m_7m_9 - 2m_2m_3 - 2m_4m_6$, $n_5 = 2m_8m_9 - 2m_5m_6 - m_3^2$, $n_6 = m_9^2 - m_6^2$.

Denote $s = \omega^2$, the equation (17) becomes

$$s^6 + n_1s^5 + n_2s^4 + n_3s^3 + n_4s^2 + n_5s + n_6 = 0 \quad (18)$$

Assume the equation (18) has six real positive roots $s_k(k = 1, 2, \cdots, 6)$, then the equation (17) will has six positive roots $\omega_k = \sqrt{s_k}$, $k = 1, 2, \cdots, 6$, substituting them into equation (16), we have

$$\tau_k^j = \frac{1}{\omega_k}\arccos\left[\frac{m_4\omega_k^4 + m_5\omega_k^2 + m_6}{\omega_k^6 + m_7\omega_k^4 + m_8\omega_k^2 + m_9}\right] + \frac{2j\pi}{\omega_k} \quad (19)$$

where $k = 1, 2, \cdots, 6, j = 0, 1, 2, \cdots$

We can denote $\tau_0 = \min(\tau_k^j)$. Taking the derivative with respect to τ in equation (14), we have

$$\left[\frac{d\lambda(\tau)}{d\tau}\right]^{-1} = \frac{(3\lambda^2+2k_1\lambda+k_2)e^{\lambda\tau}+2k_4\lambda+k_7e^{-\lambda\tau}+k_5}{k_4\lambda^3+k_5\lambda^2+k_6\lambda+2\lambda(k_7\lambda+k_8)e^{-\lambda\tau}} - \frac{\tau}{\lambda} \quad (20)$$

Together with (20), it leads to

$$\left[\frac{d\lambda(\tau)}{d\tau}\right]^{-1}\Big|_{\tau=\tau_0} = \frac{r_1r_3 - r_2r_4}{r_3^2 + r_4^2}$$

where

$$\begin{aligned} r_1 &= (-3\omega^2 + k_2 + k_7)\cos(\omega\tau) - 2\omega k_1\sin(\omega\tau) + k_5, \\ r_2 &= (-3\omega^2 + k_2 - k_7)\sin(\omega\tau) + 2\omega k_1\cos(\omega\tau) + 2k_4\omega, \\ r_3 &= -k_5\omega^2 + 2\omega(k_7\omega + k_8)\sin(\omega\tau), \\ r_4 &= -k_4\omega^3 + k_6\omega + 2\omega k_8\cos(\omega\tau). \end{aligned}$$

Thus, we have

$$\begin{aligned} &sign\left[\mathrm{Re}\left[\frac{d\lambda(\tau)}{d\tau}\right]\Big|_{\tau=\tau_0}\right] \\ &= sign\left[\mathrm{Re}\left[\frac{d\lambda(\tau)}{d\tau}\right]^{-1}\Big|_{\tau=\tau_0}\right] \neq 0 \end{aligned}$$

Therefore, we will have the following results based on the above research. When $\tau \in [0, \tau_0)$, the equilibrium of system (8) is asymptotically stable, and when $\tau = \tau_0$ the system (8) will have a Hopf bifurcation at Nash equilibrium.

4 Numerical simulations

Numerical simulations are carried to show the stability of system (8). In order to investigate the local stability properties of the Nash equilibrium conveniently, here we use certain value data to simulate the dynamics of the system. The parameters are taken to be

$$\begin{aligned} &a = 5, && b_1 = 3.8, && b_2 = 3.5, && b_3 = 3.1, \\ &g_1 = 0.4, && g_2 = 0.5, && g_3 = 0.6, && h_1 = 0.45, \\ &h_2 = 0.55, && h_3 = 0.65, \end{aligned}$$

the marginal costs of three insurance companies are $c_1 = 0.0016$, $c_2 = 0.0014$, $c_3 = 0.0012$, the initial prices of their products are $p_1(0) = 0.5$, $p_2(0) = 0.4$, $p_3(0) = 0.3$, the speeds of price adjustment are $v_1 = v_2 = v_3 = 0.5$, the weight of price is $w =$

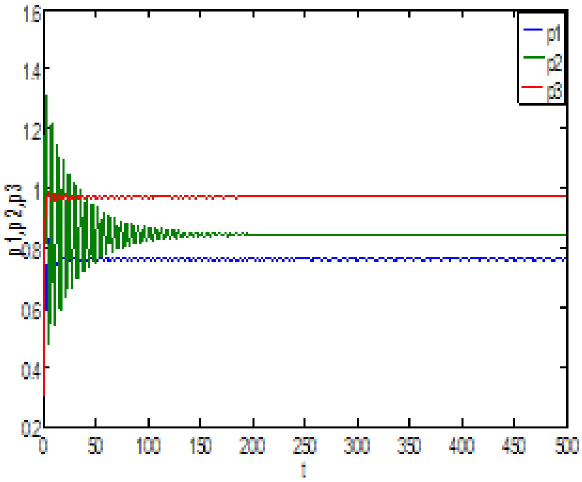

Figure 1: The orbits of p_1 and p_2 ($\tau = 2.1$)

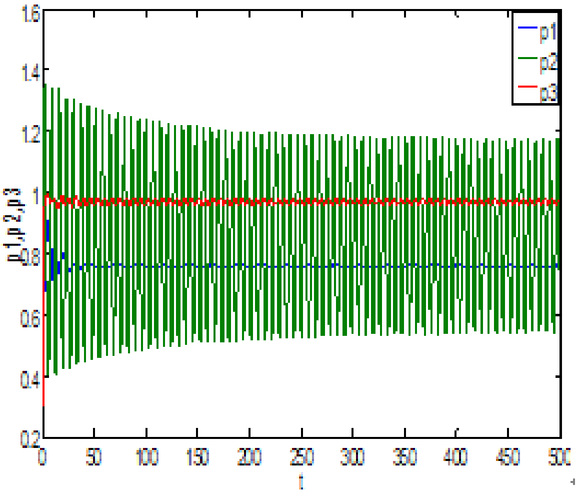

Figure 2: The orbits of p_1 and p_2 ($\tau = 2.8$)

0.45. According to (19), we will obtain $\tau_0 = 2.6863$. Figs.1, and 2 show the dynamics of system (8) for $\tau = 2.1, 2.8$, respectively.

From Fig.1 and Fig. 2, we see that if $\tau < \tau_0$, the Nash equilibrium point of system (8) is stable. As τ increases, it will take longer time to be stable. If $\tau > \tau_0$, Hopf bifurcation and period orbits are appeared and the Nash equilibrium becomes unstable. If the system is stable, we can compute the value of the Nash equilibrium is $p_1^* = 0.7605$, $p_2^* = 0.8440$, $p_3^* = 0.9691$. According to (2), we can obtain the three players profits $\pi_1 = 2.1885$, $\pi_2 = 2.4847$, $\pi_3 = 2.9043$.

If we change the 2th company's speed of price adjustment, Fig.3 and Fig. 4 are for $\upsilon_2 = 0.3$. From the above assumption, the second company has considered delay decision. Its price will influenced by delay parameter and other parameters of the system. Compare the two figures with Fig.1 and Fig. 2, when decrease the second company's speed of price adjustment, the system will need less time to be stable, or unstable system may become stable.

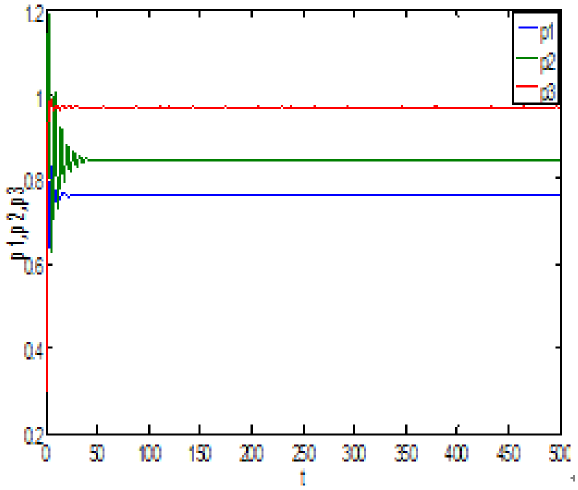

Figure 3: The orbits of p_1 and p_2 ($\upsilon_2 = 0.3, \tau = 2.1$)

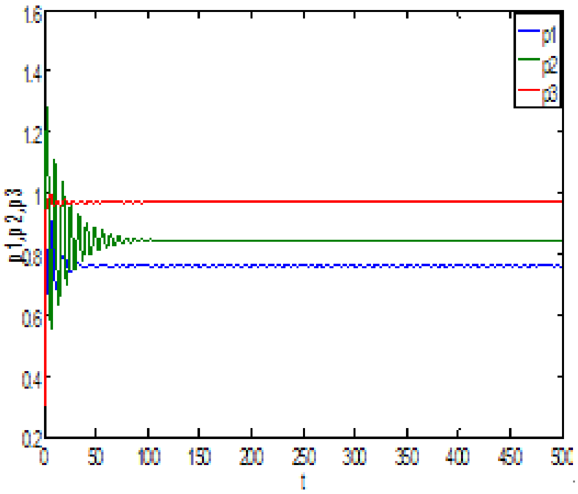

Figure 4: The orbits of p_1 and p_2 ($\upsilon_2 = 0.3, \tau = 2.8$)

In order to avoid the confusion of the insurance market, the second company need to reduce the value of delay parameter and appropriately decrease the speed of price adjustment, then all companies will obtain profit easily. The company who considers delay decision need to get the latest market information of price, so we can minimize the impact of the lag on the stability of the system. At the same time, we need to reduce the price sensitivity of the profit margin, eventually the whole insurance market will be stable.

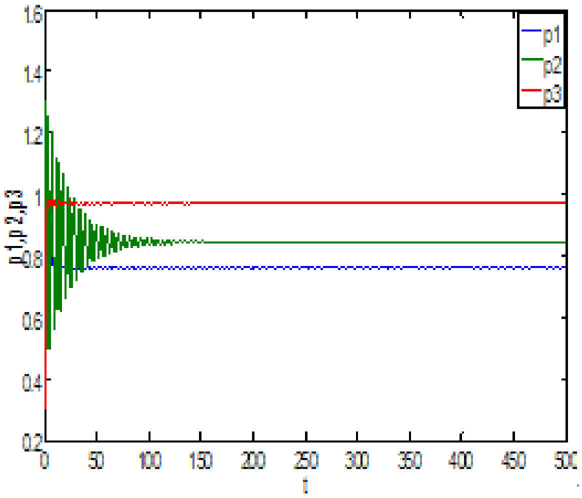

Figure 5: The orbits of p_1 and p_2 ($\tau_1 = 2.1, \tau_2 = 2$)

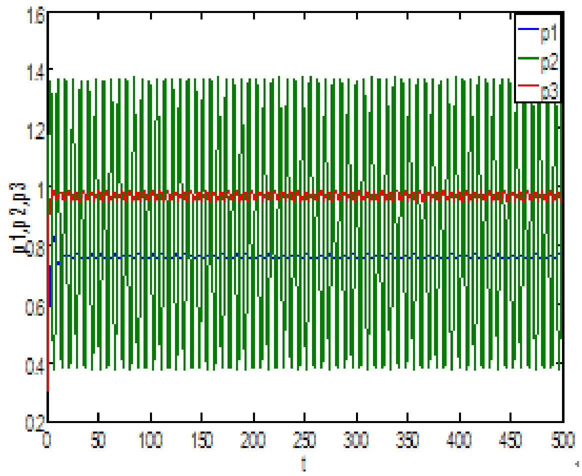

Figure 6: The orbits of p_1 and p_2 ($\tau_1 = 2.1, \tau_2 = 3$)

Fix the value of other parameters and change the values of the delay parameter of the first and second company, numerical simulations are as follows: Fig.5 – Fig.10.

From Fig.5–Fig.7, when the value of the delay parameter of the first company is fixed, if we increase the value of the delay parameter of the second company, the stable system will become unstable and the periods of solutions of the system will also increase. A t this time, delay parameter has more obvious impact on the second company than the other two companies. From Fig.8 – Fig.10, if we fix the value of the delay parameter of the first company and increase the value of the delay parameter of the first company, the system will become unstable and period orbits are appeared. A t this time, delay parameter will significantly affect the price of the second company and also has impact

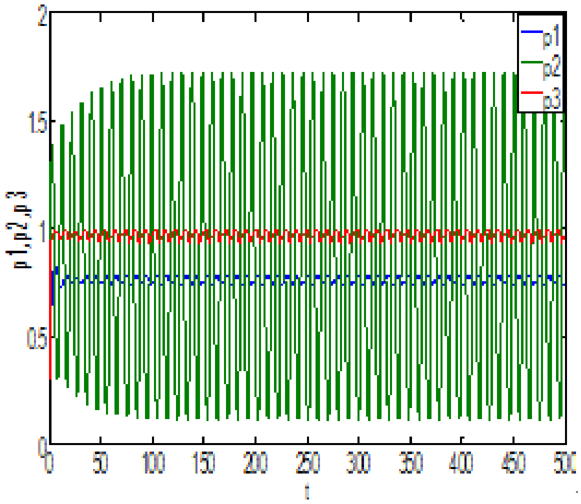

Figure 7: The orbits of p_1 and p_2 ($\tau_1 = 2.1, \tau_2 = 4$)

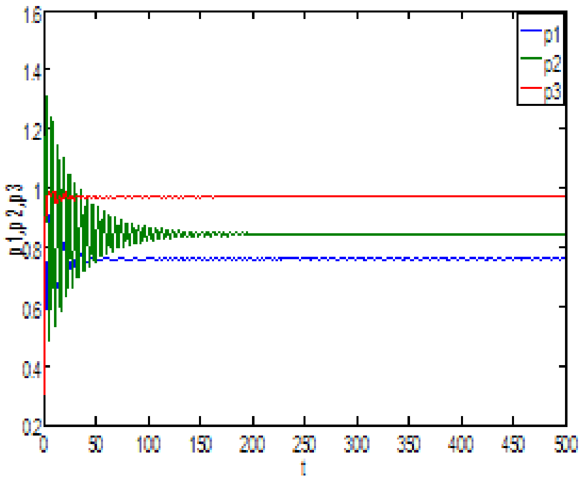

Figure 8: The orbits of p_1 and p_2 ($\tau_1 = 3, \tau_2 = 2.1$)

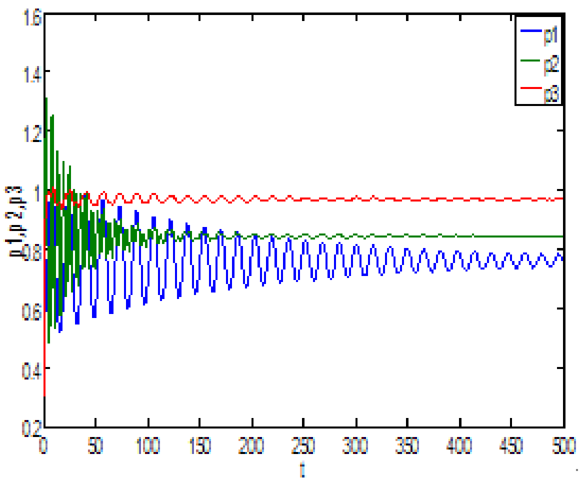

Figure 9: The orbits of p_1 and p_2 ($\tau_1 = 4, \tau_2 = 2.1$)

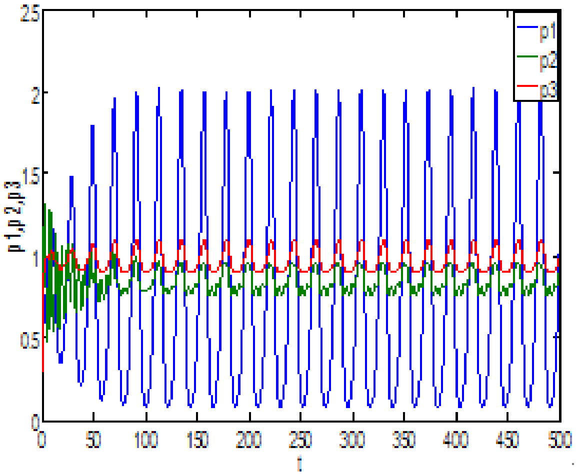

Figure 10: The orbits of p_1 and p_2 ($\tau_1 = 5, \tau_2 = 2.1$)

on other companies. Therefore, the delay parameter of the adaptive player has more significant effect on the system than the player with delay decision. In order to make the price of the entire insurance market to be able to stabilize, all the companies with different decisions need to reduce the values of delay parameter properly, then the system will take less time to be stable and the price will be rational.

5 Chaos control

According to the preceding numerical simulation and analysis, we know that the value of delay parameter and the control of price adjustment speed have a direct effect to the price setting of each oligarch and the changing of price movements. For example, if oligarch 2 increases his delay parameter value or improves his price adjustment speed, the instability of system (8) will be increased, that is, the competitive chaos of insurance market is aggravated. However this is not the desired situation in reality, because the instability means unordered competition in the market, and this kind of competition is harmful whether to the oligarch himself or to the whole insurance market, so the related theoretical research and application on chaos control arisen. In all the research methods on chaos control, the variable feedback control method is more widely used in practice and has get an excellent effect. In this paper, this method will be used to control the chaos of system (8). First we have a basic introduction to this method.

Considering a nonlinear dynamic system of n dimension,

$$\dot{x} = f(x, t) \tag{21}$$

If its desired target is $\bar{x} = (\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n)$, the feedback control is as follows:

$$\dot{x} = f(x, t) - K(x - \bar{x}) \tag{22}$$

where $K = [K_{i,j}]$ is the feedback gain matrix.

Let

$$G = Df - K \tag{23}$$

where $Df_{i,j} = \left.\frac{\partial f_i}{\partial x_j}\right|_{x=x}$. If all the eigenvalues of G are less than zero, it is in a partial stable state.

In the n^2 adjustable parameters of $K_{i,j}$, the control to system can be achieved only by adjusting n parameters which are $\lambda_1, \lambda_2, \cdots, \lambda_n$. That is,

$$\frac{dx_i}{dt} = f_i - \lambda_i(x_i - \bar{x}_i)\lambda_i \geq 0, \tag{24}$$

for $i = 1, 2, \cdots n$.

In formula (24), a big-enough λ_i can make the target state of nonlinear dynamic system be stable.

Based on the previous analysis, we first set system (8) as the following form:

$$\begin{cases} \dot{p}_1(t) = g_1(p_1(t), p_1(t-\tau), p_2(t), p_2(t-\tau), p_3(t)) \\ \dot{p}_2(t) = g_2(p_1(t), p_1(t-\tau), p_2(t), p_2(t-\tau), p_3(t)) \\ \dot{p}_3(t) = g_3(p_1(t), p_1(t-\tau), p_2(t), p_2(t-\tau), p_3(t)) \end{cases} \tag{25}$$

Then the variable feedback control method is used to control the price of oligarch 2,

$$\begin{cases} \dot{p}_1(t) = g_1(p_1(t), p_1(t-\tau), p_2(t), p_2(t-\tau), p_3(t)) \\ \dot{p}_2(t) = g_2(p_1(t), p_1(t-\tau), p_2(t), p_2(t-\tau), p_3(t)) \\ -Kp_2(t) \\ \dot{p}_3(t) = g_3(p_1(t), p_1(t-\tau), p_2(t), p_2(t-\tau), p_3(t)) \end{cases} \tag{26}$$

The specific form of system (26) is,

$$\begin{cases} \dot{p}_1(t) = v_1 p_1[(a + g_1 w p_2(t) + g_1(1-w)p_2(t-\tau) \\ \quad +h_1 p_3(t) + b_1 c_1)/2b_1 - p_1(t-\tau)] \\ \dot{p}_2(t) = v_2 p_2[a - 2b_2 w p_2(t) - 2b_2(1-w)p_2(t-\tau) \\ \quad +g_2 p_3(t) + h_2 p_1(t) + b_2 c_2] - Kp_2(t) \\ \dot{p}_3(t) = v_3 p_3[a - 2b_3 p_3(t) + g_3 p_1(t) + h_3 w p_2(t) \\ \quad +h_3(1-w)p_2(t-\tau) + b_3 c_3\,] \end{cases} \tag{27}$$

By setting proper value for feedback parameter K in system (27), it can have obvious control on the chaos and instability, and shorten the time from the price instability to equilibrium state.

Compare Figure 11 to Figure 2 and Figure 12 to Figure 6 respectively, it can be seen that after the feedback parameter K is added to system (8), the original system with periodical change tends to be the equilibrium state, the stability increases. Compare Figure 13

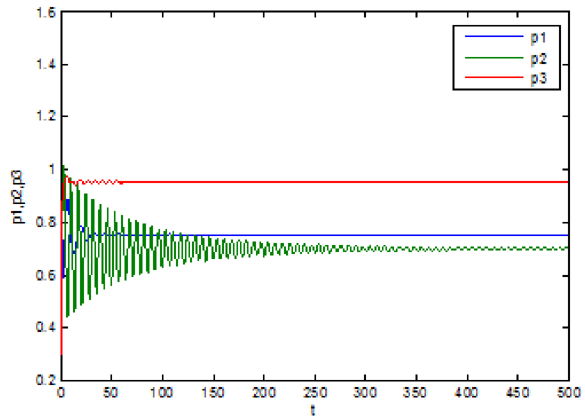

Figure 11: The changes of price when $K = 0.5, \tau = 2.8$

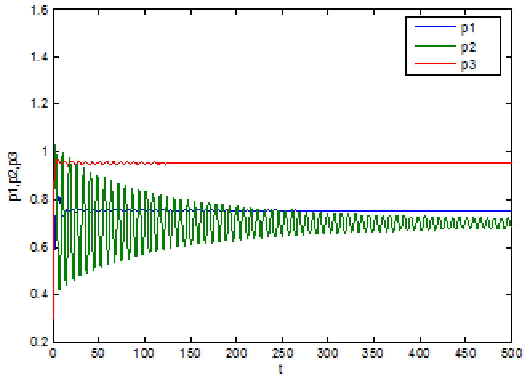

Figure 12: The changes of price when $K = 0.5, \tau_1 = 2.1, \tau_2 = 3$

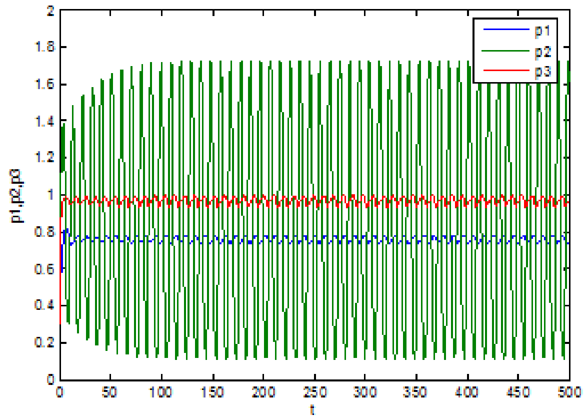

Figure 13: The changes of price when $K = 0.5, \tau_1 = 2.1, \tau_2 = 4$

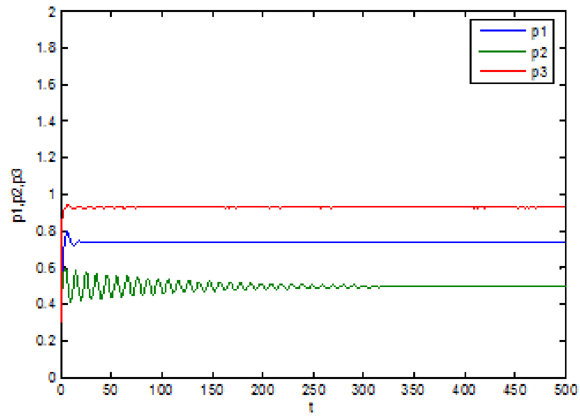

Figure 14: The changes of price when $K = 1.2, \tau_1 = 2.1, \tau_2 = 4$

to Figure 12 and Figure 15, Figure 14 to Figure 7 respectively, it can be seen that with the increasing of K, the time from the instability to the equilibrium state of each oligarch is greatly reduced and the adjusting intensity from the instability to stability is gradually increased. The numerical simulation results demonstrate that it can change the speed which the system reaches to stable period or equilibrium state by adjust the value of K, even if K is very small, it can control the chaos and have a better control effect. The using of the variable feedback control method will provide theoretical direction for the policy adjustment when the insurance market is in unordered competition state, so it is of great economic significance.

6 Conclusions

In this paper, we investigated the dynamic model of a triopoly price game in insurance market. In this model, we have considered adaptive, bounded rationality with delay and bounded rationality. According to theoretical analysis and numerical simulation of the differential equation with delay, the players must properly reduce the value of delay parameter to make the system stable as soon as quickly. The insurance market cannot accelerate the speed of price adjustment and ignore its negative impact, because it may lead to the disordering of the insurance market. In addition, the variation of the different companies' values of delay parameter will have different effects on the entire insurance market. Therefore, insurance companies in competition must reasonably control the values of all parameters, the entire insurance market will have a healthy and stable development. At the same time, when the market is in unstable state, it can also take the measure with variable feedback control to change the speed with which the system enters into stable period by adjusting the value of feedback intensity. Even if the feedback parameter is very small, the chaos control can be achieved.

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