Four positive almost periodic solutions to two species parasitical model with impulsive effects and harvesting terms

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Abstract: By applying Mawhin's continuation theorem of coincidence degree theory and some skills of inequalities, sufficient conditions are obtained for the existence of at least four positive almost periodic solutions to two species parasitical system with impulsive effects and harvesting terms.

Key–Words: Almost periodic solutions; Parasitical system; Coincidence degree; Harvesting term; Time delay; Impulsive effects.

1 Introduction

In recent years, the existence of multiple periodic solutions in biological models has been widely studied. For example, systems with harvesting terms are often considered [1-13]. In particular, Zhao and Li [1] considered a non-autonomous two species parasitical model with harvesting terms

$$\begin{cases} \dot{x} = x(t)(a_1(t) - b_1(t)x(t)) - h_1(t), \\ \dot{y} = y(t)(a_2(t) - b_2(t)y(t) + c(t)x(t)) - h_2(t), \end{cases} \tag{1}$$

where, $x(t)$ and $y(t)$ denote the densities of the host and the parasites, respectively; $a_i(t), b_i(t), c(t)$ and $h_i(t)(i = 1, 2)$ are all positive continuous functions and denote the intrinsic growth rate, death rate, obtaining nutriment rate from the host, harvesting rate, respectively. In the system (1), the parasitical influence on its host is negligible.

As we know, in population dynamics, many evolutionary processes experience short-time rapid chance after undergoing relatively long sooth variation. Examples include stocking of species and annual immigration. Incorporating these phenomena gives us impulsive differential equations. In 2013, we studied the existence of multiple positive periodic solutions to two species parasitical model with impulsive effects and harvesting terms [14]:

$$\begin{cases} \dot{x}_1(t) = x_1(t)\big(a_1(t) - b_1(t)x_1(t-\tau_{11}(t))\big) \\ \qquad\qquad -h_1(t), \quad t \neq t_k; \\ \dot{x}_2(t) = x_2(t)\big(a_2(t) - b_2(t)x_2(t-\tau_{22}(t)) \\ \qquad +c(t)x_1(t-\tau_{21}(t))\big) - h_2(t), \ t \neq t_k; \\ x_i(t_k^+) = (1 + g_{ik})x_i(t_k), t = t_k, k \in \mathbb{Z}^+, \end{cases} \tag{2}$$

where $g_{ik} \in (-1, +\infty)(i = 1, 2; k \in \mathbb{N} = \{1, 2\})$. $\{t_k\}_{k\in\mathbb{N}}$ is a strictly increasing sequence with $t_1 > 0$ and $\lim_{t\to\infty} t_k = \infty$. $x_i(t)(i = 1, 2)$ is the ith species population density. $a_i(t) > 0(i = 1, 2)$ denote the intrinsic growth rate, $b_i(t) > 0$ and $h_i(t) > 0(i = 1, 2)$ stand for death rate, obtaining nutriment rate from the host, harvesting rate, respectively. $c(t) > 0$ represents obtaining nutriment rate from the host, $\tau_{21}(t) \geq 0$ stands for the time-lag in the process of transformation from the 1th species to the 2th species. $\tau_{ii}(t) \geq 0(i = 1, 2)$ represents the time-lag in the process of intra-specific competition. For the theory of impulsive differential equations, we refer the reader to [15-30]. In particular, Li and Ye [15] in studied the existence multiple positive almost periodic solutions to an impulsive non-autonomous Lotka-Volterra predator-prey system with harvesting terms and the authors first introduce a new method to discuss the existence multiple positive almost periodic solutions for the system under consideration.

Since biological and environmental parameters are naturally subject to fluctuation in time, the effects of a periodically varying environment are considered as important selective forces on systems in a fluctuating environment. Therefore, on the one hand, models should take into account the seasonality of the periodically changing environment. However, on the other hand, in fact, it is more realistic to consider almost periodic system than periodic system.

By above, this motivates us to investigate the existence of multiple positive almost periodic solutions to system (2). In system (2), non-autonomous parameters $a_i(t), b_i(t), c(t), h_i(t)(i = 1, 2)$ are all positive continuous almost periodic functions, the time delays τ_{11}, τ_{21}, and τ_{22} are all nonnegative continuous almost periodic functions; $g_{ik} > -1, i = 1, 2, k \in \mathbf{Z}^+$ are constants and $0 = t_0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$, are impulse points with $\lim_{k+\infty} t_k = +\infty$.

With system (2) we can take into account the possible exterior effects under which the population densities change very rapidly. For instance, impulsive reduction of the population density of a given species is possible after its partial destruction by catching, a natural constraint in this case is $1 + g_{ik} > 0$ for all $k \in \mathbf{Z}^+$. An impulsive increase of the density is possible by artificial breeding of the species or release some species ($g_{ik} > 0$).

However, to the best of ours knowledge, there are few results on the existence of four positive almost periodic solutions for the delay parasitical with impulsive effects in literatures. The main purpose of this paper is to establish sufficient conditions for the existence of positive almost periodic solutions to system (2) by applying Mawhins continuation theorem of coincidence degree theory [31].

The organization of this paper is as follows. In Section 2, we make some preparations and state some lemmas which are useful in later sections. In Section 3, by applying Mawhins continuation theorem of coincidence degree theory, we establish sufficient conditions for the existence of multiple positive almost periodic solutions to system (2). Conclusion is given in Section 4.

2 Preliminaries

Now we first introduce some basic notations. Let $AP(\mathbb{R}) = \{p(t) : p(t)$ is a real valued almost periodic function on $\mathbb{R}\}$. Suppose that $f(t, \phi)$ is almost periodic in t, uniformly with respect to $\phi \in C([-\sigma, 0], R)$. $T(f, \epsilon, S)$ will denote the set of ϵ-almost periods with respect to $S \subset C([-\sigma, 0], R), l(\epsilon, S)$ the inclusion interval, $\Lambda(f)$ the set of Fourier exponents, mod(f) the module of f, and $m(f)$ the mean value.

Lemma 1 (14). *If $f(t) \in AP(\mathbb{R})$, then there exists $t_0 \in R$ such that $f(t_0) = m(f)$.*

Lemma 2 (14). *Assume that $p(t) \in AP(\mathbb{R})$, then $p(t)$ is bounded on $\mathbb{R}$.*

Lemma 3 (14). *Assume that $x(t) \in AP(\mathbb{R}) \cap C^1(\mathbb{R}, \mathbb{R})$, then there exist two points sequences $\{\xi_k\}_{k=1}^{\infty}, \{\eta_k\}_{k=1}^{\infty}$ such that $N'(\xi_k) = N'(\eta_k) = 0, \lim_{k\to\infty} \xi_k = \infty$ and $\lim_{k\to\infty} \eta_k = -\infty$.*

Lemma 4 (14). *Assume that $N(t) \in AP(\mathbb{R}) \cap C^1(\mathbb{R}, \mathbb{R})$, then $N(t)$ falls into one of the following four case:*

(i) There are $\xi, \eta \in R$ such that $N(\xi) = \sup_{t\in\mathbb{R}} N(t)$ and $N(\eta) = \inf_{t\in\mathbb{R}} N(t)$. In this case, $N'(\xi) = N'(\eta) = 0$

(ii) There are no $\xi, \eta \in \mathbb{R}$ such that $N(\xi) = \sup_{t\in\mathbb{R}} N(t)$ and $N(\eta) = \inf_{t\in\mathbb{R}} N(t)$. In this case, for any $\epsilon > 0$, there are exist two points $\xi, \eta \in \mathbb{R}$ such that $N'(\xi) = N'(\eta) = 0, N(\xi) > \sup_{t\in\mathbb{R}} N(t) - \epsilon$ and $N(\eta) < \inf_{t\in\mathbb{R}} N(t) + \epsilon$

(iii) There is a $\xi \in \mathbb{R}$ such that $N(\xi) = \sup_{t\in\mathbb{R}} N(t)$ and There is no $\eta \in \mathbb{R}$ such that $N(\eta) = \inf_{t\in\mathbb{R}} N(t)$. In this case, $N'(\xi) = 0$ and for any $\epsilon > 0$, there exists an ξ such that $N'(\xi) = N'(\eta) = 0$ and $N(\eta) < \inf_{t\in\mathbb{R}} N(t) + \epsilon$.

(iv) There is a $\eta \in \mathbb{R}$ such that $N(\eta) = \inf_{t\in\mathbb{R}} N(t)$ and There is no $\xi \in \mathbb{R}$ such that $N(\xi) = \sup_{t\in\mathbb{R}} N(t)$. In this case, $N'(\eta) = 0$ and for any $\epsilon > 0$, there exists an ξ such that $N'(\xi) = N'(\eta) = 0$ and $N(\xi) > \sup_{t\in\mathbb{R}} N(t) - \epsilon$.

Let $PC(\mathbb{R}, \mathbb{R}^2) = \{\varphi : \mathbb{R} \to \mathbb{R}, \varphi$ is a piecewise continuous function with points of discontinuity of the first kind at $t_k, k = 1, 2, \ldots$, at which $\varphi(t_k^-)$ and $\varphi(t_k^-) = \varphi(t_k)\}$.

Since the solutions of (2) belong to the space $PC(\mathbb{R}, \mathbb{R}^2)$, we adopt the following definitions for almost periodicity.

Definition 5 (27). *The family of sequences $\{t_k^j = t_{k+j} - t_k, k, j \in \mathbb{Z}\}$ is said to be equipoltentially almost periodic if for arbitrary $\epsilon > 0$, there exists a relatively dense set of ϵ-almost periods, that are common for any sequences.*

Definition 6 (27). *The function $\varphi \in PC(\mathbb{R}, \mathbb{R})$ is said to be almost periodic, if the following conditions hold:*

(1) the set of sequences $\{t_k^j = t_{k+j} - t_k, k, j \in \mathbb{Z}\}$ is equipotentially almost periodic;

(2) for any $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that if the points t_1 and t_2 belong to the same interval of continuity of $\varphi(t)$ and $|t_1 - t_2| < \delta$, then $|\varphi(t_1) - \varphi(t_2)| < \epsilon$;

(3) for any $\epsilon > 0$ there exists a relatively dense set T of ω-almost periodic such that if $\tau \in T$, then $|\varphi(t + \tau) - \varphi(t)| < \epsilon$ for all $t \in \mathbb{R}$ which satisfy the condition $|t - t_k| > \epsilon, k \in \mathbb{Z}$.

Consider the following system

$$\begin{cases} \dot{N}_1(t) = N_1(t)\big(a_1(t) - \bar{b}_1(t)N_1(t - \tau_{11}(t))\big) \\ \qquad\quad -\bar{h}_1(t), \\ \dot{N}_2(t) = N_2(t)\big(a_2(t) - \bar{b}_2(t)N_2(t - \tau_{22}(t)) \\ \qquad\quad +\bar{c}(t)N_1(t - \tau_{21}(t))\big) - \bar{h}_2(t), \end{cases} \tag{3}$$

where

$$\bar{b}_i(t) = b_i(t) \prod_{0<t_k<t} (1+g_{ik});$$

$$\bar{c}(t) = c(t) \prod_{0<t_k<t} (1+g_{2k});$$

$$\bar{h}_i(t) = h_i(t) \prod_{0<t_k<t} \frac{1}{1+g_{ik}}, i = 1, 2.$$

Lemma 7. *For systems (2) and system (3), the following results hold:*

(1)if $(N_1(t), N_2(t))^T$ is a solution of (3), then

$$\begin{aligned}&(x_1(t), x_2(t))^T\\ =& \Big(N_1(t) \prod_{0<t_k<t} (1+g_{1k}),\\ &N_2(t) \prod_{0<t_k<t} (1+g_{2k})\Big)^T\end{aligned}$$

is a solution of (2).

(2) if $(x_1(t), x_2(t))^T$ is a solution of (2), then

$$\begin{aligned}&(N_1(t), N_2(t))^T\\ =& \Big(x_1(t) \prod_{0<t_k<t} (1+g_{1k})^{-1},\\ &x_2(t) \prod_{0<t_k<t} (1+g_{2k})^{-1}\Big)^T\end{aligned}$$

is a solution of (3).

Proof. Suppose that $(N_1(t), N_2(t))^T$ is a solution of (3). Let

$$x_i(t) = \prod_{0<t_k<t} (1+g_{ik}) N_i(t), i = 1, 2,$$

then for any $t \neq t_k, k \in \mathbb{Z}$, by substituting

$$N_i(t) = \prod_{0<t_k<t} (1+g_{ik})^{-1} x_i(t), i = 1, 2$$

into system (3), we can easily verify that the first and the second equations of system (2) holds.

For $t = t_k, k \in \mathbb{Z}^+, i = 1, 2$, we obtain

$$\begin{aligned} x_i(t_k^+) &= \lim_{t\to t_k^+} \prod_{0<t_k<t} (1+g_{ik}) N_i(t)\\ &= \prod_{0<t_s<t_k} (1+g_{is}) N_i(t_k)\\ &= (1+g_{ik}) \prod_{0<t_s<t_k} (1+g_{is} N_i(t_k))\\ &= (1+g_{ik}) N_i(t_k).\end{aligned}$$

Hence, the second equation of system (2) also holds. Thus $(x_1(t), x_2(t))^T$ is a solution of system (2).

(2) We first show that $N_i(t), i = 1, 2$ are continuous. Since $N_i(t), i = 1, 2$ are continuous on each interval $(t_k, t_{k+1}]$, it is sufficient to check the continuity of $N_i(t)$ at the impulse points $t_k, k \in \mathbb{Z}^+$. Since $N_i(t) = \prod_{0<t_k<t}(1+g_{ik})^{-1} x_i(t), i = 1, 2$ we have

$$\begin{aligned} N_i(t_k^+) &= \prod_{0<t_k<t} (1+g_{is})^{-1} x_i(t_k^+)\\ &= \prod_{0<t_s<t_k} (1+g_{is})^{-1} x_i(t_k) = N_i(t_k),\\ N_i(t_k^-) &= \prod_{0<t_k<t} (1+g_{is})^{-1} x_i(t_k^-)\\ &= \prod_{0<t_s<t_k} (1+g_{is})^{-1} x_i(t_k) = N_i(t_k).\end{aligned}$$

Thus $N_i(t), i = 1, 2$ is continuous on $[0, \infty)$. It is easy to check that $(N_1(t), N_2(t))^T$ satisfies system (3). Therefore, it is a solution of system (3). This completes the proof of Lemma 7. □

Lemma 8. *[10] Assume that $x(t) \in AP(\mathbb{R})$, then $x(t)$ is bounded on $\mathbb{R}$.*

For the sake of convenience, we introduce notations as follows:

$$f^l = \inf_{t\in\mathbb{R}} f(t), \quad f^M = \sup_{t\in\mathbb{R}} f(t),$$

where $f(t)$ is a positive continuous almost periodic function. For simplicity, we need to introduce some notations as follows.

$$l_1^{\pm} = \frac{a_1^M \pm \sqrt{(a_1^M)^2 - 4\bar{b}_1^l \bar{h}_1^l}}{2\bar{b}_1^l},$$

$$l_2^{\pm} = \frac{a_2^l \pm \sqrt{(a_2^l)^2 - 4\bar{b}_2^M \bar{h}_2^M}}{2\bar{b}_2^M},$$

where $i = 1, 2$.

Throughout this paper, we need the following assumptions.

(C_1): $a_1^l > 2\sqrt{\bar{b}_1^M \bar{h}_1^M}$;

(C_2): $a_2^l > 2\sqrt{\bar{b}_2^M \bar{h}_2^M}$;

(C_3): $\bar{c}^M l_1^+ > \sqrt{(a_2^l)^2 - 4\bar{b}_2^M \bar{h}_2^M}$.

Lemma 9 (4)**.** *Let $x > 0, y > 0, z > 0$ and $x > 2\sqrt{yz}$, for the functions*

$$f(x, y, z) = \frac{x + \sqrt{x^2 - 4yz}}{2z}$$

and

$$g(x,y,z) = \frac{x - \sqrt{x^2 - 4yz}}{2z},$$

the following assertions hold.

(1) $f(x,y,z)$ *and* $g(x,y,z)$ *are monotonically increasing and monotonically decreasing on the variable* $x \in (0,\infty)$, *respectively.*

(2) $f(x,y,z)$ *and* $g(x,y,z)$ *are monotonically decreasing and monotonically increasing on the variable* $y \in (0,\infty)$, *respectively.*

(3) $f(x,y,z)$ *and* $g(x,y,z)$ *are monotonically decreasing and monotonically increasing on the variable* $z \in (0,\infty)$, *respectively.*

Lemma 10. *Assume that* (C_1),(C_2) *and* (C_3) *hold, then we have the following inequalities:*

(1) $\ln l_1^+ > \ln A^+, \ \ln A^- > \ln l_1^-$;

(2) $\ln l_2^+ < \ln H_1, \ \ln l_2^- > \ln H_2$.

Proof. Since

$$\begin{aligned} l_2^+ &= \frac{a_2^l + \sqrt{(a_2^l)^2 - 4\bar{b}_2^M \bar{h}_2^M}}{2\bar{b}_2^M} \\ &< \frac{a_2^l}{\bar{b}_2^M} < \frac{a_2^M}{\bar{b}_2^l} \\ &< \frac{a_2^M + \bar{c}^M l_1^+}{\bar{b}_2^l} = H_1, \end{aligned}$$

$$\begin{aligned} l_2^- &= \frac{a_2^l - \sqrt{(a_2^l)^2 - 4\bar{b}_2^M \bar{h}_2^M}}{2\bar{b}_2^M} \\ &= \frac{2\bar{h}_2^M}{a_2^l + \sqrt{(a_2^l)^2 - 4\bar{b}_2^M \bar{h}_2^M}} \\ &> \frac{\bar{h}_2^M}{a_2^l} > \frac{\bar{h}_2^l}{a_2^M + \bar{c}^m l_1^+} = H_2. \end{aligned}$$

Applying Lemma 9, we have

$$\begin{aligned} l_1^+ &= \frac{a_1^M + \sqrt{(a_1^M)^2 - 4\bar{b}_1^l \bar{h}_1^l}}{2\bar{b}_1^l} \\ &= f(a_1^M, \bar{b}_1^l, \bar{h}_1^l) \\ &> f(a_1^l, \bar{b}_1^M, \bar{h}_1^M) \\ &= \frac{a_1^l + \sqrt{(a_1^l)^2 - 4\bar{b}_1^M \bar{h}_1^M}}{2\bar{b}_1^M} = A^+, \end{aligned}$$

$$\begin{aligned} l_1^- &= \frac{a_1^M - \sqrt{(a_1^M)^2 - 4\bar{b}_1^l \bar{h}_1^l}}{2\bar{b}_1^l} \\ &= g(a_1^M, \bar{b}_1^l, \bar{h}_1^l) \\ &< g(a_1^l, \bar{b}_1^M, \bar{h}_1^M) \\ &= \frac{a_1^l - \sqrt{(a_1^l)^2 - 4\bar{b}_1^M \bar{h}_1^M}}{2\bar{b}_1^M} = A^-. \end{aligned}$$

Thus, we have $\ln l_1^+ > \ln A^+$, $\ln A^- > \ln l_1^-$ and $\ln l_2^+ < \ln H_1$, $\ln l_2^- > \ln H_2$ hold. The proof of Lemma 10 is complete. □

3 Existence of at least four positive almost periodic solutions

We first summarize serval concepts from the book by Gaines and Mawhin [31].

Let X and Z be real normed vector spaces. Let $L : \mathrm{Dom}\, L \subset X \to Z$ be a linear mapping and $N : X \times [0,1] \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if dim Ker L = codim Im $L < \infty$ and ImL is closed in Z. If L is a Fredholm mapping of index zero, then there exists continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\mathrm{Im}\, P = \mathrm{Ker}\, L$ and $\mathrm{Ker}\, Q = \mathrm{Im}\ L = \mathrm{Im}\,(I-Q)$, and $X = \mathrm{Ker}\, L \bigoplus \mathrm{Ker}\ P, Z = \mathrm{Im}\ L \bigoplus \mathrm{Im}\, Q$. It follows that $L|_{\mathrm{Dom}\ L \cap \mathrm{Ker}\ P} : (I-P)X \to \mathrm{Im}\ L$ is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\bar{\Omega} \times [0,1]$, if $QN(\bar{\Omega} \times [0,1])$ is bounded and $K_P(I-Q)N : \bar{\Omega} \times [0,1] \to X$ is compact. Because Im Q is isomorphic to Ker L, there exists an isomorphism $J : \mathrm{Im}\, Q \to \mathrm{Ker}\ L$.

Lemma 11. *[31] Let L be a Fredholm mapping of index zero and let N be L-compact on* $\bar{\Omega} \times [0,1]$. *Assume*

(a) *for each* $\lambda \in (0,1)$, *every solution* x *of* $Lx = \lambda N(x,\lambda)$ *is such that* $x \notin \partial\Omega \cap \mathrm{Dom}\ L$*;*

(b) $QN(x,0)x \neq 0$ *for each* $x \in \partial\Omega \cap \mathrm{Ker}\ L$*;*

(c) $\deg(JQN(x,0), \Omega \cap \mathrm{Ker}\, L, 0) \neq 0$.

Then $Lx = N(x,1)$ *has at least one solution in* $\overline{\Omega} \cap \mathrm{Dom}\, L$.

Let T be a given positive constant and a finite number of points of the sequence $\{\tau_k\}$ lies in the interval $[0,T]$. Let $PC([0,T],\mathbb{R}^n)$ be the set of functions $x : [0,T] \to \mathbb{R}^n$ which are piecewise continuous in $[0,T]$ and have points of discontinuous

$\tau_k \in [0,T]$, where they are continuous from the left. In the set $PC([0,T],\mathbb{R}^n)$ introduce the norm $\|x\| = \sup|x(t)| : t \in [0,T]$ with which $PC([0,T],\mathbb{R}^n)$ becomes a Banach space with the uniform convergence topology.

In our case, we shall consider $\mathbb{X} = \mathbb{Z} = V_1 \bigoplus V_2$, where

$$V_1 = \left\{ \begin{array}{l} z(t) = (z_1(t), z_2(t))^T \mid z_i(t) \in AP(\mathbb{R}) : \\ \text{mod}(z_i(t)) \subset \text{mod}(F_i), \forall \mu \in \Lambda(z_i(t)) \\ \text{ satisfies } |\mu| > \alpha, i = 1,2 \end{array} \right\}$$

satisfies that $V_1 \cup \{a_i(t), b_i(t), c(t), h_i(t), \bar{a}_i(t), \bar{b}_i(t), \bar{h}_i(t), i = 1,2\}$ is equi-almost-periodic,

$$V_2 = \{z(t) \equiv (c_1, c_2) \in \mathbb{R}^2\},$$

where

$$\begin{aligned} F_1 &= F(t, \phi_1, \phi_2) \\ &= a_1(t) - \bar{b}_1(t)e^{\phi_1(0)} - \bar{h}_1(t)e^{-\phi_1(0)}, \\ F_2 &= F(t, \phi_1, \phi_2) \\ &= a_2(t) - \bar{b}_2(t)e^{\phi_2(0)} + \bar{c}(t)e^{\phi_1(0)} \\ &\quad -\bar{h}_2(t)e^{-\phi_2(0)}, \end{aligned}$$

in which $\phi_i \in C([-\sigma,0],\mathbb{R}), i = 1,2, \sigma = \max\{\tau\}$ and α is a given positive constant. Define the norm

$$\|z\| = \sum_{i=1}^{2} \sup_{t\in\mathbb{R}} |z_i(t)| \text{ for all } z \in \mathbb{X} = \mathbb{Z}).$$

By making the substitution

$$N_i(t) = e^{u_i(t)},\ i = 1,2,$$

system (2) can be reformulated as

$$\left\{ \begin{array}{l} \dot{u}_1(t) = a_1(t) - \bar{b}_1(t)e^{u_1(t-\tau_{11}(t))} \\ \qquad\qquad -\bar{h}_1(t)e^{-u_1(t)}, \\ \dot{u}_2(t) = a_2(t) - \bar{b}_2(t)e^{u_2(t-\tau_{22}(t))} \\ \qquad +\bar{c}(t)e^{u_1(t-\tau_{21}(t))} - \bar{h}_2(t)e^{-u_2(t)}. \end{array} \right. \tag{4}$$

Similar to the proofs of Lemma 2 and Lemma 7 in [6], one can easily prove the following three Lemmas, respectively.

Lemma 12. *$\mathbb{X}$ and $\mathbb{Z}$ are Banach spaces equipped with the norm $\|\cdot\|$.*

Lemma 13. *Let $L : \mathbb{X} \to \mathbb{Z}, Lu = u' = (u_1', u_2')^T$. Then L is a Fredholm mapping of index zero.*

Lemma 14. *Let $N : \mathbb{X} \times [0,1] \to \mathbb{Z}$,,*

$$N(u(t), \lambda) = (N_1(u(t), \lambda), N_2(u(t), \lambda))^T,$$

where

$$\begin{aligned} N_1(u(t), \lambda) &= a_1(t) - \bar{b}_1(t)e^{u_1(t-\tau_{11}(t))} \\ &\quad -\bar{h}_1(t)e^{-u_1(t)}, \\ N_2(u(t), \lambda) &= a_2(t) - \bar{b}_2(t)e^{u_2(t-\tau_{22}(t))} \\ &\quad +\lambda\bar{c}(t)e^{u_1(t-\tau_{21}(t))} - \bar{h}_2(t)e^{-u_2(t)} \end{aligned}$$

and $P : \mathbb{X} \to \mathbb{X}, Px = m(x); Q : Z \to \mathbb{Z}, Qu = m(u)$. Then N is L-compact on $\bar{\Omega}$ (Ω is a open bounded subset of $\mathbb{X}$).

Theorem 15. *Assume that (C_1),(C_2) and (C_3) hold. Then system (2) has at least four positive almost periodic solutions.*

Proof. In order to use Lemma 11, we have to find at least four appropriate open bounded subsets in $\mathbb{X}$. Corresponding to the operator $Lu = \lambda N(u,\lambda), \lambda \in (0,1)$, we have

$$\left\{ \begin{array}{l} \dot{u}_1(t) = \lambda(a_1(t) - \bar{b}_1(t)e^{u_1(t-\tau_{11}(t))} \\ \qquad -\bar{h}_1(t)e^{-u_1(t)}), \\ \dot{u}_2(t) = \lambda(a_2(t) - \bar{b}_2(t)e^{u_2(t-\tau_{22}(t))} \\ \qquad +\lambda\bar{c}(t)e^{u_1(t-\tau_{21}(t))} - \bar{h}_2(t)e^{-u_2(t)}). \end{array} \right. \tag{5}$$

Assume that $u \in X$ is an almost periodic solution of system (5) for some $\lambda \in (0,1)$. By Lemma 7, for any $\epsilon > 0$, there exist $\xi_i, \eta_i \in \mathbb{R}$ such that $u_i(\xi_i) > u_i^M - \epsilon, u_i(\eta) < u_i^l + \epsilon$ and $\dot{u}_i(\xi_i) = 0, \dot{u}_i(\eta_i) = 0, i = 1,2$. From this and system (5), we obtain

$$\left\{ \begin{array}{l} 0 = (1-\lambda)a_1(\xi_1) - \bar{b}_1(\xi_1)e^{u_1(\xi_1-\tau_{11}(\xi_1))} \\ \quad -\bar{h}_1(\xi_1)e^{-u_1(\xi_1)}, \\ 0 = a_2(\xi_2) - \bar{b}_2(\xi_2)e^{u_2(\xi_2-\tau_{22}(\xi_2))} \\ \quad +\lambda\bar{c}(\xi_2)e^{u_1(\xi_2-\tau_{21}(\xi_2))} - \bar{h}_2(\xi_2)e^{-u_2(\xi_2)}, \end{array} \right. \tag{6}$$

and

$$\left\{ \begin{array}{l} 0 = (1-\lambda)a_1(\eta_1) - \bar{b}_1(\eta_1)e^{u_1(\eta_1-\tau_{11}(\eta_1))} \\ \quad -\bar{h}_1(\eta_1)e^{-u_1(\eta_1)}, \\ 0 = a_2(\eta_2) - \bar{b}_2(\eta_2)e^{u_2(\eta_2-\tau_{22}(\eta_2))} \\ \quad +\lambda\bar{c}(\eta_2)e^{u_1(\eta_2-\tau_{21}(\eta_2))} - \bar{h}_2(\eta_2)e^{-u_2(\eta_2)}. \end{array} \right. \tag{7}$$

On the one hand, according to the first equation of (6) and (7),

$$\begin{aligned} a_1^M &> (1-\lambda)a_1(\xi_1) \\ &= \bar{b}_1(\xi_1)e^{\xi_1-\tau_{11}(\xi_1)} + \bar{h}_1(\xi_1)e^{-u_1(\xi_1)} \\ &\geq \bar{b}_1^l e^{u_1(\xi_1)} + \bar{h}_1^l e^{-u_1(\xi_1)} > 0, \end{aligned}$$

namely,

$$\bar{b}_1^l e^{2u_1(\xi_1)} - a_1^M e^{u_1(\xi_1)} + \bar{h}_1^l < 0,$$

which imply that

$$\ln l_1^- < u_1(\xi_1) < \ln l_1^+. \qquad (8)$$

Similarly, by the first equation of (7), we obtain

$$\ln l_1^- < u_1(\eta_1) < \ln l_1^+. \qquad (9)$$

The second equation of (6) gives

$$\begin{aligned} & \bar{b}_2^M e^{2u_2(\xi_2)} + \bar{h}_2^M \\ \geq\ & \bar{b}_2(\xi_2) e^{2u_1(\xi_2)} + \bar{h}_2(\xi_2) \\ =\ & \left[a_2(\xi_2) + \lambda \bar{c}(\xi_2) e^{u_1(\xi_2)}\right] e^{u_2(\xi_2)} \\ >\ & a_2(\xi_2) e^{u_2(\xi_2)} > a_2^l e^{u_2(\xi_2)}. \end{aligned}$$

That is

$$\bar{b}_2^M e^{2u_2(\xi_2)} - a_2^l e^{u_2(\xi_2)} + \bar{h}_2^M > 0,$$

which imply

$$u_2(\xi_2) > \ln l_2^+ \ \text{or} \ u_2(\xi_2) < \ln l_2^-. \qquad (10)$$

Similarly, by the second equation of (7), we get

$$u_2(\eta_2) > \ln l_2^+ \ \text{or} \ u_2(\eta_2) < \ln l_2^-. \qquad (11)$$

Moreover, from the second equation of (6), we have

$$\begin{aligned} & \bar{b}_2^l e^{u_2(\xi_2)} \\ \leq\ & \bar{b}_2 e^{u_2(\xi_2)} < \bar{b}_2 e^{u_2(\xi_2)} + \bar{h}_2(\xi_2) e^{-u_2(\xi_2)} \\ =\ & a_2(\xi_2) + \lambda \bar{c}(\xi_2) e^{u_1(\xi_2)} \\ <\ & a_2(\xi_2) + \bar{c}(\xi_2) e^{u_1(\xi_1)} \leq a_2^M + \bar{c}^M e^{u_1(\xi_1)} \\ <\ & a_2^M + \bar{c}^M l_1^+, \end{aligned}$$

which imply that

$$u_2(\xi_2) < \ln \frac{a_2^M + c^M l_1^+}{b_2^l} := H_1. \qquad (12)$$

Similarly, from the second equation of (7), we obtain

$$\begin{aligned} & \bar{h}_2^l e^{-u_2(\eta_2)} \\ \leq\ & \bar{h}_2 e^{-u_2(\eta_2)} < \bar{b}_2(\eta_2) e^{u_2(\eta_2)} + \bar{h}_2(\eta_2) e^{-u_2(\eta_2)} \\ =\ & a_2(\eta_2) + \lambda \bar{c}(\eta_2) e^{u_1(\eta_2)} \\ <\ & a_2(\eta_2) + \bar{c}(\eta_2) e^{u_1(\eta_1)} \leq a_2^M + \bar{c}^M e^{u_1(\eta_1)} \\ <\ & a_2^M + \bar{c}^M l_1^+, \end{aligned}$$

which imply that

$$u_2(\eta_2) > \ln \frac{\bar{h}_2^l}{a_2^M + c^M l_1^+} := H_2. \qquad (13)$$

According to the first equation of (6), we have

$$\begin{aligned} a_1^l &\leq (1-\lambda) a_1(\xi_1) \\ &= \bar{b}_1(\xi_1) e^{\xi_1} + \bar{h}_1(\xi_1) e^{-u_1(\xi_1)} \\ &< \bar{b}_1^M e^{u_1(\xi_1)} + \bar{h}_1^M e^{-u_1(\xi_1)}, \end{aligned}$$

namely,

$$\bar{b}_1^M e^{2u_1(\xi_1)} - a_1^l e^{u_1(\xi_1)} + \bar{h}_1^M > 0,$$

which imply that

$$u_1(\xi_1) > \ln A^+, \ \text{or} \ u_1(\xi_1) < \ln A^-. \qquad (14)$$

Similarly, by the first equation of (7), we obtain

$$u_1(\eta_1) > \ln A^+, \ \text{or} \ u_1(\eta_1) < \ln A^-. \qquad (15)$$

From (8), (9), (14) and (15), we obtain

$$\begin{aligned} & \ln l_1^- < u_1(\eta_1) < u_1(\xi_1) < \ln A^- \ \text{or} \\ & \ln A^+ < u_1(\eta_1) < u_1(\xi_1) < \ln l_1^+. \end{aligned} \qquad (16)$$

Similarly, from (10), (11), (12) and (13), we obtain

$$\begin{aligned} & \ln l_2^+ < u_2(\eta_2) < u_2(\xi_2) < \ln H_1 \quad \text{or} \\ & \ln H_2 < u_2(\eta_2) < u_2(\xi_2) < \ln l_2^-. \end{aligned} \qquad (17)$$

By (16) and (17), we have for all $t \in \mathbb{R}$

$$\begin{aligned} & \ln l_1^- < u_1(t) < \ln A^- \ \text{or} \\ & \ln A^+ < u_1(t) < \ln l_1^+, \end{aligned} \qquad (18)$$

and

$$\begin{aligned} & \ln l_2^+ < u_2(t) < \ln H_1 \ \text{or} \\ & \ln H_2 < u_2(t) < \ln l_2^-. \end{aligned} \qquad (19)$$

Clearly, $\ln l_1^\pm, \ln l_2^\pm, \ln A^\pm, \ln H_1$ and $\ln H_2$ are independent of λ. Now let

$$\Omega_1 = \left\{ u = (u_1, u_2)^T \in X \,\middle|\, \begin{array}{l} \ln l_1^- < u_1(t) < \ln A^-, \\ \ln l_2^+ < u_2(t) < \ln H_1 \end{array} \right\}$$

$$\Omega_2 = \left\{ u = (u_1, u_2)^T \in X \,\middle|\, \begin{array}{l} \ln l_1^- < u_1(t) < \ln A^-, \\ \ln H_2 < u_2(t) < \ln l_2^- \end{array} \right\}$$

$$\Omega_3 = \left\{ u = (u_1, u_2)^T \in X \,\middle|\, \begin{array}{l} \ln A^- < u_1(t) < \ln l_1^+, \\ \ln l_2^+ < u_2(t) < \ln H_1 \end{array} \right\}$$

and

$$\Omega_4 = \left\{ u = (u_1, u_2)^T \in X \,\middle|\, \begin{array}{l} \ln A^- < u_1(t) < \ln l_1^+, \\ \ln H_2 < u_2(t) < \ln l_2^- \end{array} \right\}$$

Then $\Omega_i(i = 1, 2, 3, 4)$ are bounded open subsets of $X, \Omega_i \cap \Omega_j = \phi$. Thus $\Omega_i(i = 1, 2, 3, 4)$ satisfies the requirement (a) in Lemma 11.

Now we show that (b) of Lemma 11 holds, i.e., we prove when $u \in \partial\Omega_i \cap \ker L = \partial\Omega_i \cap R^2, QN(u, 0) \neq (0, 0)^T, i = 1, 2, 3, 4$. If it is not true, then when $u \in \partial\Omega_i \cap \ker L = \partial\Omega_i \cap R^2, i = 1, 2, 3, 4$, constant vector $u = (u_1, u_2)^T$ with $u \in \partial\Omega_i, i = 1, 2, 3, 4$ satisfies

$$m(a_1(t) - \bar{b}_1(t)e^{u_1} - \bar{h}_1 e^{-u_1}) = 0,$$

and

$$m(a_2(t) - \bar{b}_2(t)e^{u_2} - \bar{h}_2 e^{-u_2}) = 0.$$

In view of the mean value theorem, there exist two points $\zeta_i(i = 1, 2)$ such that

$$a_1(\zeta_1) - \bar{b}_1(\zeta_1)e^{u_1} - \bar{h}_1(\zeta_1)e^{-u_1} = 0, \tag{20}$$

and

$$a_2(\zeta_2) - \bar{b}_2(\zeta_2)e^{u_2} - \bar{h}_2(\zeta_2)e^{-u_2} = 0. \tag{21}$$

By (20) and (21), we have

$$\nu_i^{\pm} = \frac{a_i(\zeta_i) \pm \sqrt{(a_i(\zeta_i))^2 - 4\bar{b}_i(\zeta_i)\bar{h}_i(\zeta_i)}}{2\bar{b}_i(\zeta)}$$

with $i = 1, 2$.

According to Lemma 10,we obtain

$$\ln l_1^- < \ln \nu_1^- < \ln A^- < \ln A^+ < \ln \nu_1^+ < \ln l_1^+,$$

$$\ln H_2 < \ln \nu_2^- < \ln l_2^- < \ln l_2^+ < \ln \nu_2^+ < \ln H_1.$$

Then $u \in \Omega_1 \cap R^2$ or $u \in \Omega_2 \cap R^2$ or $u \in \Omega_3 \cap R^2$ or $u \in \Omega_4 \cap R^2$. This contradicts the fact that $u \in \partial\Omega_i \cap R^2, i = 1, 2, 3, 4$. This proves (b) in Lemma 11 holds. Finally, we show that (c) in Lemma 11 holds. Note that the system of algebraic equations:

$$\begin{cases} a_1(\zeta_1) - \bar{b}_1(\zeta_1)e^{u_1} - \bar{h}_1(\zeta_1)e^{-u_1} = 0, \\ a_2(\zeta_2) - \bar{b}_2(\zeta_2)e^{u_2} - \bar{h}_2(\zeta_2)e^{-u_2} = 0 \end{cases}$$

has four distinct solutions since $(C_1), (C_2)$ and (C_3) hold,

$$(u_1^*, u_2^*) = (\ln \hat{u}_1, \ln \hat{u}_2),$$

where $\hat{u}_i = u_i^-$ or $\hat{u}_i = u_i^+$, and

$$u_i^{\pm} = \frac{a_i(\zeta_i) \pm \sqrt{(a_i(\zeta_i))^2 - 4\bar{b}_i(\zeta_i)\bar{h}_i(\zeta_i)}}{2\bar{b}_i(\zeta)}$$

$(i = 1, 2)$. By Lemma 9, it is easy to verify that

$$\ln l_1^- < \ln \nu_1^- < \ln A^- < \ln A^+ < \ln \nu_1^+ < \ln l_1^+,$$

and

$$\ln H_2 < \ln \nu_2^- < \ln l_2^- < \ln l_2^+ < \ln \nu_2^+ < \ln H_1.$$

Therefore, (u_1^*, u_2^*) uniquely belongs to the corresponding Ω_i. Since $\mathrm{Ker}L = \mathrm{Im}Q$, we can take $J = I$. A direct computation gives, for $i = 1, 2, 3, 4$,

$$\begin{aligned} & \deg\Big\{JQN(z, 0), \Omega_i \cap \mathrm{Ker}L, (0, 0)^T\Big\} \\ = \; & \mathrm{sign}\begin{vmatrix} -\bar{b}_1(\zeta_1)u_1^* + \frac{\bar{h}_1(\zeta_1)}{u_1^*} & 0 \\ 0 & -\bar{b}_2(\zeta_2)u_2^* + \frac{\bar{h}_2(\zeta_2)}{u_2^*} \end{vmatrix} \\ = \; & \mathrm{sign}\Big[\big(-\bar{b}_1(\zeta_1)u_1^* + \frac{\bar{h}_1(\zeta_1)}{u_1^*}\big)\big(-\bar{b}_2(\zeta_2)u_2^* \\ & + \frac{\bar{h}_2(\zeta_2)}{u_2^*}\big)\Big]. \end{aligned}$$

Since

$$a_1(\zeta_1) - \bar{b}_1(\zeta_1)u_1^* - \frac{\bar{h}_1(\zeta_1)}{u_1^*} = 0,$$
$$a_2(\zeta_2) - \bar{b}_2(\zeta_2)u_2^* - \frac{\bar{h}_2(\zeta_2)}{u_2^*} = 0,$$

then

$$\begin{aligned} & \deg\Big\{JQN(u, 0), \Omega_i \cap \ker L, (0, 0)^T\Big\} \\ = \; & \mathrm{sign}\Big[\big(a_1(\zeta_1) - 2\bar{b}_1(\zeta_1)u_1^*\big)\big(a_2(\zeta_2) \\ & -2\bar{b}_2(\zeta_2)u_2^*\big)\Big] \\ = \; & \pm 1. \end{aligned}$$

So far, we have prove that $\Omega_i(i = 1, 2, 3, 4)$ satisfies all the assumptions in Lemma 11. Hence, system (4) has at least four different almost periodic solutions. If $u^*(t) = (u_1^*, u_2^*)^T$ is an almost periodic solution of system (3), by applying Lemma 7, we known that

$$(x_1(t), x_2(t))^T = \Big(e^{u_1^*(t)} \prod_{0<t_k<t}(1 + g_{1k}), \; e^{u_2^*(t)} \prod_{0<t_k<t}(1 + g_{2k})\Big)^T$$

is almost periodic solution of system (2). Since conditions (C_1), (C_2) and (C_3) hold, similar to the proofs of Lemma 31 and Theorem 79 in Ref [27], we can prove that $\bar{x}_i(t) = \prod_{0<t_k<t}(1 + g_{ik})e^{\bar{z}_i(t)}$ is almost

periodic in the sense of Definition 2. Therefore, system (2) has at least four different positive almost periodic solutions. This completes the proof of Theorem 15. □

Consider the following non-autonomous two species parasitical model with harvesting terms

$$\begin{cases} \dot{x}_1(t) = x_1(t)\big(a_1(t) - b_1(t)x_1(t-\tau_{11}(t))\big) \\ \qquad\quad -h_1(t), \\ \dot{x}_2(t) = x_2(t)\big(a_2(t) - b_2(t)x_2(t-\tau_{22}(t)) \\ \qquad\quad +c(t)x_1(t-\tau_{21}(t))\big) - h_2(t), \end{cases} \tag{22}$$

where $a_i(t), b_i(t), c(t), h_i(t)(i = 1,2)$,$\tau_{11}, \tau_{21}$, and τ_{22} are all nonnegative continuous almost periodic functions.

Similar to the proof of Theorem 15, one can easily obtain, here we omit it.

Corollary 16. *Assume that the following condition holds*

$$(H_1') \qquad a_i^l > 2\sqrt{b_i^M h_1^M};$$
$$c^M l_1^+ > \sqrt{(a_2^l)^2 - 4b_2^M h_2^M}.$$

Then system (22) has at least four different positive almost periodic solutions.

4 Conclusion

By applying Mawhins continuation theorem of coincidence degree theory, we study an impulsive non-autonomous two species parasitical model with harvesting terms and obtain some sufficient conditions for the existence of four positive almost periodic solutions for the system (2).

Remark 17. *From the proof of Theorem 15, we can see that if the harvesting terms $h_1(t) = h_2(t) = 0$, system (2) has at least one positive almost periodic solution, but we could not conclude that system (2) has at least four almost positive periodic solutions because we could not construct $\Omega_i, i = 1,2,3,4$, satisfying $\Omega_i \cap \Omega_j = \phi$.*

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