nr-Excellent Graphs

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Abstract: Given an k-tuple of vectors, $S = (v_1, v_2, \ldots, v_k)$, the neighbourhood adjacency code of a vertex v with respect to S, denoted by $nc_S(v)$ and defined by $(a_1, a_2, \ldots, a_k)$ where a_i is 1 if v and v_i are adjacent and 0 otherwise. S is called a neighbourhood resolving set or a neighbourhood r-set if $nc_S(u) \neq nc_S(v)$ for any $u, v \in V(G)$. The least(maximum) cardinality of a minimal neighbourhood resloving set of G is called the neighbourhood(upper neighbourhood) resolving number of G and is denoted by $nr(G)$ $(NR(G))$. In this article, we consider the nr-excellent graphs. For any Graph G, G is nr-excellent if every vertex of G is contained in a minimum neighbourhood resolving set of G. We first prove that the union and join of two given nr-excellent graphs is nr-excellent under certain conditions. Also we prove that a non nr-excellent graph G can be embedded in a nr-excellent graph H such that $nr(H)$ = $nr(G)$ + number of nr- bad vertices of G. Some more results are also discussed.

Key–Words: neighbourhood resolving set, neighbourhood resolving number, nr-excellent, nr-bad vertex

1 Introduction

Let $G = (V, E)$ be a simple graph. Given a k-tuple of vertices $(v_1, v_2, \ldots, v_k)$, assign to each vertex $v \in V(G)$, the k-tuple of its distances to these vertices, $f(v) = (d(v, v_1), d(v, v_2), \ldots, d(v, v_k))$. Letting $S = (v_1, v_2, \ldots, v_k)$, the k-tuple $f(v)$ is called the S-location of v. A set S is a locating set for G, if no two vertices have the same S-location and the location number $L(G)$ is the minimum cardinality of a locating set. This concept was introduced by P. J. Slater in **[25] [27] [26]**. It can be easily seen that $L(G) = 1$ if and only if G is a path and any path P_n, $(n \geq 2)$ has two $L(P_n)$-sets, each consisting of an end vertex. Also for any tree T with at least three vertices, a subset S is a locating set if and only if for each vertex u there are vertices in S, contained in at least $deg(u) - 1$ of the $deg(u)$ components of $T - u$.

Slater **[25]** described the usefulness of locating sets when working with U.S. sonar and Coast Guard Loran stations. Independently Harary and Melter **[14]** found these concepts as well but used the term metric dimension, rather than location number, the terminology which has been adopted by subsequent authors. Recently, these concepts were rediscovered by M. A. Johnson **[18] [19]** of the pharmacia company while attempting to develop a capability of large datasets of chemical graphs.

If V is a finite dimensional vector space over a field F and $B = (v_1, v_2, \ldots, v_k)$ is an ordered basis, then every vector v in V can be associated with a unique k-tuple $(\lambda_1, \lambda_2, \ldots, \lambda_k)$ where $v = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_k v_k$. If v and w are distinct vectors, then their associated k-tuples of scalars are distinct. Thus any ordered basis gives rise to a coding for the elements of the vector space. The dimension of the vector space is the order of a basis. Since locating sets in a graph play the same role as bases in a vector space, the locating number is termed as metric dimension. Chartrand et al **[4]** called locating sets as resolving sets and retained the term metric dimension for locating number.

Many papers have appeared in this area since 2000. For example dominating resolving sets, independent resolving sets, acyclic resolving sets, connected resolving sets, resolving partitions and resolving decompositions were studied by many [4], [5], [6],[7], [8], [9], [11], [12], [13], [2], [3], [44].

In all these studies, the graph considered are connected and the codes of the vertices are k-tuples whose entries are positive integers ranging from 0 to the diameter of the graph. An alternate coding of the vertices using 0 and 1 (binary digits) can be thought of, which is possible in disconnected graphs also.

A new type of binary coding for vertices is defined through adjacency. A vertex u in a graph

G with respect to a k-tuple of vertices (say) $S = (v_1, v_2, \ldots, v_k)$ is assigned the code(which is written as $nc_S(u)$) $(a_1, a_2, \ldots, a_k)$ where a_i is 1 if u and v_i are adjacent and 0 otherwise. If $nc_S(u) \neq nc_S(v)$ for any $u \neq v$, $u, v \in V(G)$, then S is called a neighbourhood resolving set of G. If we consider the vector spaces over the field Z_2, then the code of any vector is a binary code. Thus neighbourhood resolving sets correspond to bases in a vector space over Z_2. Obviously neighburhood resolving sets can be defined in disconnected graphs also. But not all graphs can admit neighbourhood resolving sets. For example, a graph G in which $N(u) = N(v)$, for two non-adjacent vertices $u, v \in V(G)$ will not have any neighbourhood resolving set. But these graphs can be embedded in graphs having neighbourhood resolving sets. Obviously a graph with more than one isolated vertex will not have neighbourhood resolving sets. The minimum cardinality of a neighbourhood resolving set in a graph G which admits nieghbourhood resolving sets is called the neighbourhood resolving number of G and is denoted by $nr(G)$. Any nr-set of G with $nr(G) \geq 2$ cannot be independent. Graphs are often used to model different physical networks. nr-sets can be used to detect intruders on models of networks of facilities and it is used to detect the failures on networks of routers or processors.

Suk J. Seo and P. Slater **[34]** defined the same type of problem as an open neighborhood locating dominating set (OLD-set), is a minimum cardinality vertex set S with the property that for each vertex v its open neighborhood $N(v)$ has a unique non-empty intersection with S. But in Neighbourhood resolving sets $N(v)$ may have the empty intersection with S. Clearly every OLD-set of a graph G is a neighbourhood resolving set of G, but the converse need not be true.

M.G. Karpovsky, K. Chakrabarty, L.B. Levitin **[20]** introduced the concept of identifying sets using closed neighbourhoods to resolve vertices of G. This concept was ellaborately studied by A. Lobestein **[21]**.

Let μ be a parameter of a graph. A vertex $v \in V(G)$ is said to be μ-good if v belongs to a μ-minimum (μ-maximum) set of G according as μ is a super hereditary (hereditary) parameter. v is said to be μ-bad if it is not μ-good. A graph G is said to be μ-excellent if every vertex of G is μ-good. Excellence with respect to domination and total domination were studied in [10], [15], [30],[31], [32], [33] . N. Sridharan and Yamuna [31], [32], [33], have defined various types of excellence. In this paper, definition, examples and properties of nr-excellent graphs is discussed.

2 Neighbourhood Resolving sets in Graphs

Definition 1 *Let G be any graph. Let $S \subset V(G)$. Consider the k-tuple $(u_1, u_2, \ldots, u_k)$ where $S = \{u_1, u_2, \ldots, u_k\}$, $k \geq 1$. Let $v \in V(G)$. Define a binary neighbourhood code of v with respect to the k-tuple $(u_1, u_2, \ldots, u_k)$, denoted by $nc_S(v)$ as a k-tuple $(r_1, r_2, \ldots, r_k)$ where*

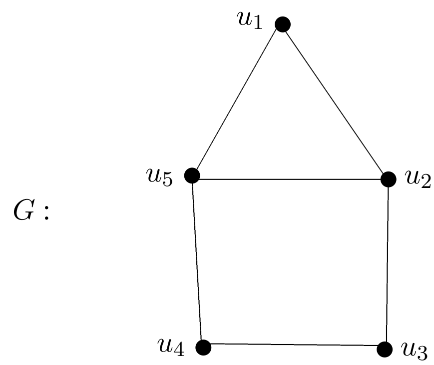
$$r_i = \begin{cases} 1, \ if\ v \in N(u_i), 1 \leq i \leq k \\ 0, \ otherwise \end{cases} .$$

S is called a neighbourhood resolving set or a neighbourhood r-set if $nc_S(u) \neq nc_S(v)$ for any $u, v \in V(G)$.

The least cardinality of a minimal neighbourhood resloving set of G is called the neighbourhood resolving number of G and is denoted by $nr(G)$. The maximum cardinality of a minimal neighbourhood resolving set of G is called the upper neighbourhood resolving number of G and is denoted by $NR(G)$.

Clearly $nr(G) \leq NR(G)$. A neighbourhood resolving set S of G is called a minimum neighbourhood resolving set or nr-set if S is a neighbourhood resolving set with cardinality $nr(G)$.

Example 2



Now $S_1 = \{u_1, u_2, u_5\}$ is a neighbourhood resolving set of G, since $nc_S(u_1) = (0, 1, 1)$; $nc_S(u_2) = (1, 0, 1)$;
$nc_S(u_3) = (0, 1, 0)$; $nc_S(u_4) = (0, 0, 1)$ and
$nc_S(u_5) = (1, 1, 0)$. Also $S_2 = \{u_1, u_3, u_4\}$, $S_3 = \{u_1, u_2, u_4\}$, $S_4 = \{u_1, u_3, u_5\}$ are neighbourhood resolving sets of G. For this graph, $nr(G) = NR(G) = 3$.

Observation 3 *The above definition holds good even if G is disconnected.*

In the following theorem characterisation of connected graphs which admit neighbourhood resolving sets is given.

Theorem 4 **[38]** *Let G be a connected graph of order $n \geq 3$. Then G does not have any neighbourhood resolving set if and only if there exist two non adjacent vertices u and v in $V(G)$ such that $N(u) = N(v)$.*

Definition 5 **[40]** *A subset S of $V(G)$ is called an nr-irredundant set of G if for every $u \in S$, there exist $x, y \in V$ which are privately resolved by u.*

Theorem 6 **[40]** *Every minimal neighbourhood resolving set of G is a maximal neighbourhood resolving irredundant set of G.*

Definition 7 **[40]** *The minimum cardinality of a maximal neighbourhood resolving irredundant set of G is called the neighbourhood resolving irredundance number of G and is denoted by $ir_{nr}(G)$. The maximum cardinality is called the upper neighbourhood resolving irrundance number of G and is denoted by $IR_{nr}(G)$.*

Observation 8 **[40]** *For any graph G,*

$$ir_{nr}(G) \leq nr(G) \leq NR(G) \leq IR_{nr}(G).$$

Theorem 9 **[41]** *For any graph G,*

$$nr(G) \leq n - 1.$$

Theorem 10 **[39]** *Let G be a connected graph of order n such that $nr(G) = k$. Then $log_2 n \leq k$*

Observation 11 **[39]** *There exists a graph G in which $n = 2k$ and there exists a neighbourhood resolving set of cardinality k such that $nr(G) = k$. Hence all the distinct binary k-vectors appear as codes for the n vertices.*

Theorem 12 **[41]** *Let G be a connected graph of order n admitting neighbourhood resolving sets of G and let $nr(G) = k$. Then k = 1 if and only if G is either K_2 or K_1.*

Theorem 13 **[41]** *Let G be a connected graph of order n admitting neighbourhood resolving sets of G. Then $nr(G)$ = 2 if and only if G is either K_3 or K_3 + a pendant edge or $K_3 \cup K_1$ or $K_2 \cup K_1$.*

Result 14 **[40]** *For a complete graph K_n,*

$$nr(K_n) = n - 1, n \geq 2.$$

Result 15 **[40]** *For a path P_n, $n \geq 6$,*

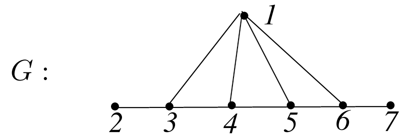
$$nr(P_n) = \lfloor \frac{2n}{3} \rfloor.$$

3 nr-excellent graphs

Definition 16 *Let G = (V, E) be a simple graph. Let $u \in V(G)$. Then u is said to be nr-good if u is contained in a minimum neighbourhood resolving set of G. A vertex u is said to be nr-bad if there exists no minimum neighbourhood resolving set of G containing u.*

Definition 17 *A graph G is said to be nr-excellent if every vertex of G is nr-good.*

Example 18 *Consider the graph G:*



The nr-sets of G containing the vertices $1, 2, 3, 4, 5, 6, 7$ are $\{1, 2, 3, 4\}$, $\{1, 5, 6, 7\}$, $\{1, 3, 4, 6\}$, $\{1, 3, 5, 6\}$, $\{1, 2, 3, 5\}$, $\{1, 4, 6, 7\}$, $\{1, 2, 3, 6\}$ and $\{3, 4, 5, 6\}$.

Therefore every vertex of G belongs to some nr-set of G and hence nr-good. Therefore G is nr-excellent.

Theorem 19 *The following results are obvious from the definition.*

(i) K_n is nr-excellent, for every n.

(ii) P_n is nr-excellent if $n \cong 0, 2, 3, 4 \ (mod\ 6)$ and P_n is non-nr-excellent if $n \cong 1, 5 \ (mod\ 6)$.

(iii) C_n is nr-excellent, for every $n \neq 4$.

Remark 20 *Consider the graph G obtained from two complete graphs K_m and K_n, $m, n \geq 4$ having exactly one vertex in common. Then $nr(G) = m+n-4$. G is not nr-excellent, since there is no nr-set containing the common vertex.*

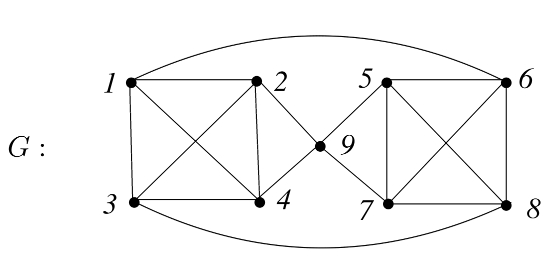
Remark 21 *Suppose G has a unique nr-set. Then G is not nr-excellent.*

Theorem 22 *Any vertex transitive graph is nr-excellent.*

Proof: Let G be a vertex transitive graph. Let S be an nr-set of G. Let $u \in V(G)$. Suppose $u \notin S$. Select any vertex $v \in S$. As G is vertex transitive, there exists an automorphism ϕ of G which maps v to u. Let $S^1 = \{\phi(w)/w \in S\}$. Since S is a nr-set and ϕ is an automorphism, S^1 is also a nr-set. Since $v \in S$, $\phi(v) = u \in S^1$. Hence G is nr-excellent. □

Remark 23 *Any vertex transitive graph is regular.*

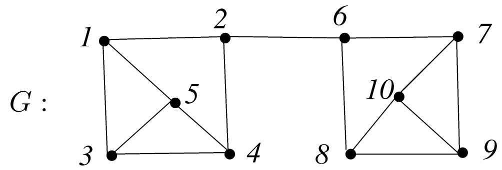
Remark 24 *1. There exists a regular graph which is nr-excellent, but not vertex transitive.*



The nr-sets of G are $\{1, 2, 5, 6, 9\}$ and $3, 4, 7, 8, 9\}$. G is nr-excellent but not vertex transitive.

2. The path P_n where $n \cong 1, 5$ $(mod\ 6)$ are not nr-excellent and not vertex trasitive, but P_n where $n \cong 0, 2, 3, 4$ $(mod\ 6)$ are nr-excellent but not vertex transitive.

3. There are regular graphs which are not vertex transitive without neighbourhood resolving sets. The follwing gaph G is an example.



Since 1 and 4 are non-adjacent and $N(1) = N(4)$, G has no neighbourhood resolving sets.

Theorem 25 *Let G_1 and G_2 be nr-excellent graphs. Then $G_1 \cup G_2$ is nr-excellent if and only if at least one of G_1, G_2 satisfies one of the follwing conditions*

(i) If every nr-set of G_1 (G_2) containing a vertex $x \in V(G_1)(V(G_2))$ admits a 0-code, then there exists an nr-set of $G_2(G_1)$ not allowing 0-code for any vertex of $V(G_2)(V(G_1))$.

(ii) Every nr-set S of G_i, $i = 1, 2$, allows 0-code.

Proof: Let G_1 and G_2 be nr-excellent graphs.

Suppose (i) holds. Then $nr(G_1 \cup G_2) = nr(G_1) + nr(G_2)$. Let $u \in V(G_1)$. Suppose there exists an nr-set S of G_1 containing u and not admitting 0-code. Then for any nr-set T of G_2, $S \cup T$ is an nr-set of $G_1 \cup G_2$ containing u.

Suppose every nr-set of G_1 containing u admits 0-code. Then by condition (i), there exists an nr-set T of G_2 not allowing 0-code. Then $S \cup T$ is an nr-set of $G_1 \cup G_2$ containing u. A similar proof holds for any vertex in $V(G)$. Therefore $G_1 \cup G_2$ is nr-excellent.

Suppose (ii) holds. Then for any nr-set S of G_1 and an nr-set T of G_2, $S \cup T \cup \{x\}$ where $x \in V(G_1 \cup G_2)$ and x is adjacent to at least one of the two vertices which receive 0-code with respect to S or with respect to T, is an nr-set of $G_1 \cup G_2$. Since G_1 and G_2 are nr-excellent, we get that $G_1 \cup G_2$ is nr-excellent.

Conversely, Suppose $G_1 \cup G_2$ is nr-excellent.

Case (i) : $nr(G_1 \cup G_2) = nr(G_1) + nr(G_2)$. Therefore at least one of G_1, G_2 has an nr-set for which there exists a vertex that receives 0-code with respect to S. Without loss of generality, let $S \subseteq V(G_1)$. Suppose there exists a vertex $u \in V(G_1)$ such that every nr-set of G_1 containing u admits 0-code. Since $G_1 \cup G_2$ is nr-excellent, there exists an nr-set of S_1 of $G_1 \cup G_2$ containing u.

Let $S_1^1 = S_1 \cap V(G_1)$ and $S_2^1 = S_1 \cap V(G_2)$. By our assumption, S_1^1 admits 0-code. Since S_1 is an nr-set of $G_1 \cup G_2$, S_2^1 is an nr-set of G_2 which does not allow 0-code in G_2. Hence condition (i) hold.

Case (ii) : $nr(G_1 \cup G_2) = nr(G_1) + nr(G_2) + 1$. Therefore every nr-set of G_1 as well as that of G_2, allows 0-code. Hence (ii) holds. □

Definition 26 *A graph G is said to be of type-I if there exists an nr-set S in G such that no vertex in $V - S$ is adjacent to every vertex of S. (That is every vertex in $V - S$ is not adjacent to at least one vertex of S.)*

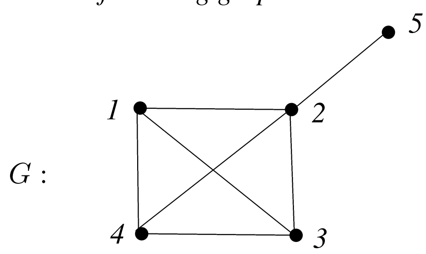
Definition 27 *A graph G is said to be of type-II if there exists an nr-set S in G such that there exists a vertex in $V - S$ which is adjacent to every vertex of S.*

Definition 28 *(i) A graph G is of type-I nr-full if for every nr-set S in G, every vertex outside S is not adjacent to at least one vertex in S.*

(ii) A graph G is type-II nr-full if for every nr-set S in G, there exists a vertex outside S adjacent to every vertex of S.

Example 29 *(i) K_n, $n \geq 3$ is of type-II nr-full, but type-I non-nr-full.*

(ii) Consider the following graph G.

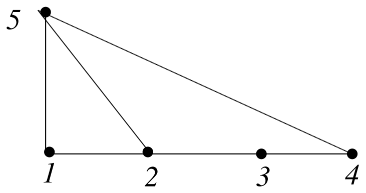


The nr-sets of G are $\{1, 3, 4\}$, $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{2, 3, 4\}$, $1, 4, 5\}$, $\{3, 4, 5\}$ and $\{1, 3, 5\}$. For an nr-set $S = \{1, 3, 4\}$ there exists a vertex 2 outside S such that 2 is adjacent to 1,3,4. Similarly the property holds for other nr-sets also.

Therefore, G is type-II nr-full, but not type-I nr-full.

(iii)

G :



The nr-sets of G are $\{1, 2, 4\}$, $\{1, 3, 4\}$, $\{1, 3, 5\}$, $\{1, 2, 5\}$.

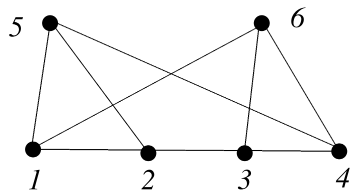
The type-I nr-sets are $\{1, 3, 4\}$, $\{1, 3, 5\}$ and $\{1, 2, 5\}$, since every vertex outside the nr-set is not adjacent to at least one vertex of the set.

The only type-II nr-set is $\{1, 2, 4\}$, since there exists a vertex 5 outside the set $\{1, 2, 4\}$ such that 5 is adjacent to 1,2 and 4.

Therefore G is type-II non-nr-full and type-I non-nr-full.

(iv)

G :



The type-II nr-sets of G are $\{1, 2, 4\}$, $\{1, 3, 4\}$, $\{1, 5, 3\}$, $\{2, 5, 6\}$, $\{3, 5, 6\}$ and $\{2, 4, 6\}$.

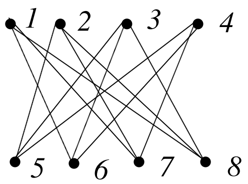
The type-I nr-sets are $\{3, 4, 6\}$ and $\{1, 2, 5\}$.

Therefore G is type-II non-nr-full and type-I non-nr-full.

(v) P_n and C_n are type-I nr-full, but type-II non-nr-full.

(vi) Consdier the graph G.

G :



The nr-sets of G are $S_1 \cup S_2$ where S_1 and S_2 are 3-element subsets of $\{1, 2, 3, 4\}$ and $\{5, 6, 7, 8\}$ respectively.

G is type-I nr-full and type-II non-nr-full.

Remark 30 *If G is type-I nr-full then it is type-II non-nr-full and vice versa.*

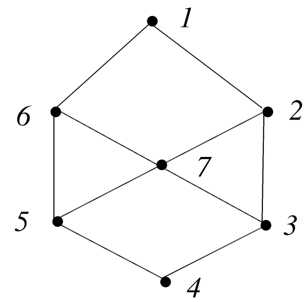
Definition 31 *Let G be a graph. A vertex $u \in V(G)$ is said to be type-I (type-II) nr-good if u belongs to some type-I(type-II) nr-set. Otherwise u is type-I(type-II) nr-bad.*

A graph G is type-I (type-II) nr-excellent if every vertex of G is type-I (type-II) nr-good.

Example 32 *(i) K_n is type-II nr-excellent, but not type-I nr-excellent.*

(ii) Consider the following graph G.

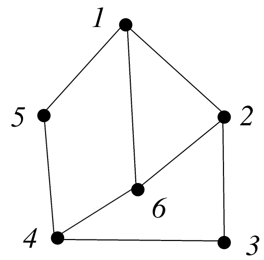
G :



The type-I ne-sets of G are $\{1, 2, 4, 5\}$, $\{1, 3, 4, 6\}$, $\{1, 2, 3, 4\}$, $\{1, 2, 5, 7\}$, $\{1, 4, 5, 6\}$, $\{1, 2, 3, 6\}$, $\{1, 2, 5, 6\}$, $\{1, 2, 3, 7\}$, $\{1, 3, 6, 7\}$, $\{1, 5, 6, 7\}$, $\{2, 4, 5, 7\}$, $\{3, 4, 6, 7\}$, $\{2, 3, 4, 5\}$, $\{3, 4, 5, 6\}$, $\{2, 3, 4, 7\}$, $\{4, 5, 6, 7\}$, $\{2, 3, 5, 7\}$, $\{3, 5, 6, 7\}$, $\{2, 3, 6, 7\}$ and $\{2, 5, 6, 7\}$. The only type-II nr-set is $\{2, 3, 5, 6\}$. Note that G is both type-II and type-I non-nr-full. Therefore G is type-I nr-excellent but not type-II nr-excellent.

(iii) Consider the following graph G:

G :



The only type-I nr-set of G is $\{1, 2, 6\}$ and the only type-II nr-set of G is $\{1, 2, 4\}$. Therefore G is neither type-I nr-excellent nor type-II nr-excellent. Note that G is not nr-full.

Theorem 33 *Let G and H be nr-excellent graphs. Then $G + H$ is nr-excellent if and only if*

(i) G and H are type-I nr-full. (or)

(ii) G and H are type-II nr-full. (or)

(iii) G is type-I nr-full and H is not nr-full. (or)

(iv) H is type-I nr-full and G is not nr-full. (or)

(v) G is type-II nr-full, H is not nr-full and for every vertex $u \in V(H)$, there exists a type-I nr-set in H containing u. (or)

(vi) G is not nr-full, H is type-II nr-full and for every vertex u in $V(G)$, there exists a type-I nr-set in G containing u. (or)

(vii) G and H are not nr-full and for every vertex u in $V(G)$, there exists a type-I nr-set in G containing u and for every vertex $v \in V(H)$, there exists a type-I nr-set in H containing v.

Proof: Suppose G and H are nr-full.

Case (i) : G and H are type-I nr-full.

Then for any nr-set S_1 of G and any nr-set S_2 of H, $S_1 \cup S_2$ is an nr-set of $G + H$. Therefore $G + H$ is nr-excellent.

Case (ii) : G and H are type-II nr-full.

Then for any nr-set S_1 of G and any nr-set of S_2 of H, $S_1 \cup \{u\} \cup S_2$ and $S_1 \cup S_2 \cup \{v\}$ is an nr-set of $G + H$, where $u \in V(G) - S_1$ is adjacent with every vertex of S_1 and $v \in V(H) - S_2$ is adjacent with every vertex of S_2.

Therefore $nr(G + H) = nr(G) + nr(H) + 1$. Clearly $G + H$ is nr-excellent.

Case (iii) : G is not nr-full and H is nr-full.

Therefore there exist nr-sets S_1 and S_2 in G such that there exists a vertex $u \in V(G) - S_1$ which is adjacent to every vertex of S_1 and for any vertex $v \in V(G) - S_2$, v is non-adjacent with some vertex of S_2. Let S_3 be any nr-set of H.

Subcase (i) : H is type-I nr-full.

Then for every vertex $w \in V(H) - S_3$, w is not adjacent to some vertex of S_3. In this case for any nr-set S of G and any nr-set S of H, $S \cup S^!$ is an nr-set of $G + H$. Therefore $G + H$ is nr- excellent.

Subcase (ii) : H is type-II nr-full.

Let S^1 be any nr-set of H. Then $S_2 \cup S^1$ is an nr-set of $G+H$ and $S_1 \cup S^1$ is not an nr-set of $G+H$.

If for every vertex u in G, there exists a type- I nr-set T_u in G containing u, then $G + H$ is nr-excellent.

If there exists a vertex u in G such that if it is contained only in type-II nr-sets, then $G + H$ is not nr-excellent.

Case (iv) : G is nr-full and H is not nr-full.

If G is type-I then arguing as in Subcase (i) of Case (iii), we get that $G + H$ is nr-excellent. If G is type-II, then arguing as in Subcase (ii) of Case (iii), if for every vertex u in H, there exists a type-I nr-set T_u in H containing u, then $G + H$ is nr-excellent.

Case (v) : G and H are not nr-full.

Then the union of any two type-II nr-sets in G and H will not be an nr-set in $G+H$. If there exists a vertex $u \in V(G)$ such that it is contained only in type-II nr-sets of G (or) there exists a vertex $v \in V(H)$ such that it is contained only in type-II nr-sets of H, then $G + H$ is not nr-excellent. Otherwise $G + H$ is nr-excellent. Converse is obviously true. □

The above theorem can be restated as follows:

Theorem 34 *Let G and H be nr-excellent graphs. Then $G + H$ is nr-excellent if and only if*

(i) G and H are type-II nr-full. (or)

(ii) G or H is type-I nr-full. (or)

(iii) G is type-II nr-full and H is type-I nr-excellent or vice versa. (or)

(iv) G and H are non-nr-full and both are type-I nr-excellent.

Remark 35 *Let G be a non-nr-excellent graph. Then the number of nr-bad vertices of G $\leq n - nr(G)$.*

When $nr(G) = 1$, $G = K_1$ or K_2 and hence G is nr-excellent. Therefore $nr(G) \geq 2$ and number of nr-bad vertices of G $\leq n - 2$.

When $nr(G) = 2$, G is K_3 or K_3 + a pendant edge or $K_3 \cup K_1$ or $K_2 \cup K_1$.

When $G = K_3$ + pendant edge or $K_3 \cup K_1$ or $K_2 \cup K_1$, then G is not nr-excellent.

The number of nr-bad vertices of G is 2 or 1.

Therefore the number of nr-bad vertices when G is K_3 + pendant edge = 2 = $n - 2$, since $n = 4$.

Theorem 36 *Let G be a non nr-excellent graph. Then G can be embedded in a nr-excellent graph (say) H such that $nr(H) = nr(G)$ + number of nr-bad vertices of G.*

Proof: Let G be a non-nr-excellent graph. Let $B = \{u_1, u_2, \ldots, u_k\}$ be the set of all nr-bad vertices of G. Add a vertex v_1 to $V(G)$ and join v_1 with every vertex of $N(u_1)$.

Let the resulting graph be G_1. Let S be an nr-set of G. Then $N_S(u_1) = N_S(v_1)$. Therefore S is not a neighbourhood resolving set of G.

Let $S_{11} = S \cup \{u_1\}$ and $S_{12} = S \cup \{v_1\}$. Let $x, y \in V(G)$. Then $nc_{S_{11}}(x) = (a_1, a_2, \ldots, a_r, l_1)$; $nc_{S_{11}}(y) = (b_1, b_2, \ldots, b_r, l_2)$; $nc_{S_{11}}(u_1) = (c_1, c_2, \ldots, c_r, 0)$; $nc_{S_{11}}(v_1) = (c_1, c_2, \ldots, c_r, 1)$.

Since S is an nr-set of G, there exist i, j, k such that $1 \leq i \leq r$ and $a_i \neq b_i$, $1 \leq j \leq r$ and $a_j \neq c_j$ and $1 \leq k \leq r$ and $b_k \neq c_k$. Therefore S_{11} is a neighbourhood resolving set of G_1. Similarly S_{12} is a neighbourhood resolving set of G_1. Therefore $nr(G_1) \leq nr(G) + 1$.

Let D be an nr-set of G_1. If $D \subseteq V(G)$, then $nc_D(u_1) = nr_D(v_1)$, a contradiction, since D is an nr-set of G. Therefore D must contain either u_1 or v_1.

Case (i) : $u_1 \in D$ and $v_1 \notin D$. Then $D \subseteq V(G)$. Clearly D is neighbourhood resolving set of G.

Since u_1 is a nr-bad vertex of G, D is not an nr-set of G. Therefore $nr(G) < |D| = nr(G_1)$ and hence $nr(G) + 1 \leq nr(G_1)$. Therefore $nr(G_1) = nr(G) + 1$.

Case (ii) : $v_1 \in D$ and $u_1 \notin D$.

Let $x, y \in V(G)$. If $x, y \in N[u_1]$, $x, y \neq u_1$ or $x, y \notin N[u_1]$, then x and y have the same code with respect to v_1 and hence v_1 does not resolve x and y. Let $x = u_1$ and $y \notin N[u_1]$. Then v_1 resolves u_1 and y. Suppose v_1 privately resolves u_1 and y where $y \notin N[u_1]$. Then $nc_{D-\{v_1\}}(u_1) = nc_{D-\{v_1\}}(y) =$

$nc_{D-\{v_1\}}(v_1)$. Since u_1 is not adjacent to y, v_1 is not adjacent to y. Therefore $nc_D(y) = nc_D(v_1)$, a contradiction, since D is an nr-set of G_1. Therefore v_1 resolves only u_1 and v_1 in G_1. Thus $D - \{v_1\}$ is a neighbourhood resolving set of G and $nr(G) \leq |D| - 1 = nr(G_1) - 1$ which implies that $nr(G_1) \geq nr(G) + 1$. Therefore $nr(G_1) = nr(G) + 1$.

Case (iii) : $u_1, v_1 \in D$.

As in Case(ii), v_1 resolves only u_1 and v_1 in G. Since $u_1 \in D$, u_1 resolves u_1 and v_1 in G_1. Therefore $D - \{u_1\}$ is a neighbourhood resolving set of G_1, a contradiction. Thus $S_{11} = S \cup \{u_1\}$ and $S_{12} = S \cup \{v_1\}$ are nr-sets of G_1, which means u_1 and v_1 are nr-good in G_1.

Since any nr-good vertices of G belongs to an nr-set of G, these vertices are also nr-good in G_1. More over G is an induced subgraph of G_1 and $nr(G_1) = nr(G) + 1$. Let B_1 be the set of all nr-bad vertices of G_1. Then $B_1 = \{u_2, u_3, \ldots, u_k\}$. Proceeding as before, construct a graph G_2 in which there is a new vertex $v_2 \notin V(G_1)$ such that $N_{G_2}[v_2] = N_{G_1}[u_2] = N_G[u_2]$ and $S_{21}^{(1)} = S_{11} \cup \{u_2\}$; $S_{22}^{(1)} = S_{12} \cup \{v_2\}$ $S_{21}^{(2)} = S_{11} \cup \{v_2\}$ $S_{22}^{(2)} = S_{12} \cup \{u_2\}$ are nr-sets of G_2. Therefore u_2 and v_2 are nr-good in G_2 and all nr-good vertices in G_1 are also nr-good in G_2 and G_1 and hence G is an induced subgraph of G_2. Also $nr(G_2) = nr(G) + 2$.

Proceeding in this way, the k^{th} stage yields a graph G_k such that G_k is nr-excellent, G is an induced subgraph of G_k and $nr(G_k) = nr(G) + k$. □

Corollary 37 *Let G be a non nr-excellent graph and let H be a nr-excellent graph containing G as an induced subgraph. Then*

$$nr(G) < nr(H) \leq nr(G) + n - 2.$$

Theorem 38 *Let G be a connected non-nr-excellent graph. Let $\{u_1, u_2, \ldots, u_k\}$ be the set of all nr-bad vertices of G. Add vertices v_1, v_2, v_3, v_4 with $V(G)$. Join v_i with v_j, $1 \leq i, j \leq 4$, $i \neq j$. Join u_i with v_1, $1 \leq i \leq k$. Let H be the resulting graph. Suppose there exists no nr-set T of H such that v_1 privately resolves nr-good vertices and nr-bad vertices of G. Then H is nr-excellent, G is an induced subgraph of H and $nr(H) = nr(G) + 3$.*

Proof: Let G be a connected non-nr-excellent graph. Let $\{u_1, u_2, \ldots, u_k\}$ be the set of all nr-bad vertices of G. Add vertices v_1, v_2, v_3, v_4 with $V(G)$. Join v_i with v_j, $1 \leq i, j \leq 4$, $i \neq j$. Join u_i with v_1, $1 \leq i \leq k$. Let H be the resulting graph.

Suppose there exists no nr-set of T of H such that v_1 privately resolves nr-good vertices and nr-bad vertices of G. Let S be an nr-set of G. Let $A = \{v_1, v_2, v_3, v_4\}$. Let $S_1 = S \cup T$ where T is a three element subset of A containing v_1. Let $S_2 = S \cup \{u_i, v_{i_1}, v_{i_2}\}$ where $v_{i_1}, v_{i_2} \in \{v_2, v_3, v_4\}$, $1 \leq i \leq k$ and $S_3 = S \cup \{v_2, v_3, v_4\}$. Let $x, y \in G$ where $x, y \neq u_i$, $1 \leq i \leq k$.

Now $nc_{S_1}(x) = (a_1, a_2, \ldots, a_r, 0, 0, 0)$; $nc_{S_1}(y) = (b_1, b_2, \ldots, b_r, 0, 0, 0)$. $nc_{S_1}(u_i) = (c_1, c_2, \ldots, c_r, 1, 0, 0)$; $nc_{S_1}(u_j) = (d_1, d_2, \ldots, d_r, 1, 0, 0)$. $nc_{S_1}(v_1) = (0, 0, \ldots, 0, 0, 1, 1)$; $nc_{S_1}(v_2) = (0, 0, \ldots, 0, 1, l_1, l_2)$. $nc_{S_1}(v_3) = (0, 0, \ldots, 0, 0, l_1^1, l_2^1)$; $nc_{S_1}(v_4) = (0, 0, \ldots, 0, 1, l_1^{11}, l_1^{11})$. l_1, l_2 are not both 0. similarly l_1^1, l_2^1 and l_1^{11}, l_2^{11} are not both 0. Therefore S_1 is a neighbourhood resolving set of H.

Suppose S_2 contains v_2 and v_3. Then $nc_{S_2}(x) = (a_1, a_2, \ldots, a_r, a_{r+1}, 0, 0)$; $nc_{S_1}(y) = (b_1, b_2, \ldots, b_r, b_{r+1}, 0, 0)$. $nc_{S_1}(u_i) = (c_1, c_2, \ldots, c_r, 0, 0, 0)$; $nc_{S_1}(u_j) = (d_1, d_2, \ldots, d_r, d_{r+1}, 0, 0)$. $nc_{S_1}(v_1) = (0, 0, \ldots, 0, 1, 1, 1)$; $nc_{S_1}(v_2) = (0, 0, \ldots, 0, 0, 0, 1)$. $nc_{S_1}(v_3) = (0, 0, \ldots, 0, 0, 1, 0)$; $nc_{S_1}(v_4) = (0, 0, \ldots, 0, 0, 1, 1)$. Therefore S_2 is a neighbourhood resolving set of H.

Now $nc_{S_2}(x) = (a_1, a_2, \ldots, a_r, 0, 0, 0)$; $nc_{S_1}(y) = (b_1, b_2, \ldots, b_r, 0, 0, 0)$. $nc_{S_1}(u_i) = (c_1, c_2, \ldots, c_r, 0, 0, 0)$; $nc_{S_1}(u_j) = (d_1, d_2, \ldots, d_r, 0, 0, 0)$. $nc_{S_1}(v_1) = (0, 0, \ldots, 0, 1, 1, 1)$; $nc_{S_1}(v_2) = (0, 0, \ldots, 0, 0, 1, 1)$. $nc_{S_1}(v_3) = (0, 0, \ldots, 0, 1, 0, 1)$; $nc_{S_1}(v_4) = (0, 0, \ldots, 0, 1, 1, 0)$. Therefore S_3 is a neighbourhood resolving set of H. Therefore $nr(H) \leq nr(G) + 1$. Let D be an nr-set of H.

Case (A) : D contains at most two elements from $\{u_1, u_2, \ldots, u_k, v_1, v_2, v_3, v_4\}$.

Subcase (i) : D contains u_i, u_j, $1 \leq i, j \leq k$, $i \neq j$. Then $nc_D(v_2) = nc_D(v_3) = nc_D(v_4) = (0, 0, 0)$, a contradiction.

Subcase (ii) : D contains u_i, v_j, $1 \leq i \leq k$, $2 \leq j \leq 4$. Then $nc_D(v_r) = nc_D(v_s) = (0, 0, \ldots, 0, 1)$ where $r, s \neq j$, $r \neq s$, $2 \leq r, s \leq 4$, a contradiction.

Subcase (iii) : D contains v_i, v_j, $1 \leq i, j \leq 4$, $i \neq j$. Then $nc_D(v_r) = nc_D(v_s) = (0, 0, \ldots, 1, 1)$ where $r, s \neq i, j$, $r \neq s$, $1 \leq r, s \leq 4$, a contradiction.

Subcase (iv) : D contains u_i, v_1, $1 \leq i \leq k$. Then $nc_D(v_2) = nc_D(v_3) = (0, 0, \ldots, 0, 1)$, a contradiction.

Therefore D contains at least three vertices from $\{u_1, u_2, \ldots, u_k, v_1, v_2, v_3, v_4\}$.

Case (B) : D contains more than three elements from $\{u_1, u_2, \ldots, u_k, v_1, v_2, v_3, v_4\}$.

Subcase (i) : $D \subset \{u_1, u_2, \ldots, u_k\}$. Then v_2, v_3 and v_4 have 0-code with respect to D, a contradiction.

Subcase (ii) : D contains v_2, v_3, v_4 and the remaining vertices from $\{u_1, u_2, \ldots, u_k\}$. Then $D -$

$\{v_2, v_3, v_4\}$ is a neighbourhood resolving set of G. Since D contains nr-bad vertices of G, $nr(G) < |D| - 3 = nr(H) - 3$. That is $nr(H) \geq nr(G) + 4$, a contradiction.

Subcase (iii) : D contains two of the vertices from $\{v_1, v_2, v_3, v_4\}$ and at least two vertices from $\{u_1, u_2, \ldots, u_k\}$.

Subcase (iiia) : D contains v_2, v_3 and at least two vertices from $\{u_1, u_2, \ldots, u_k\}$. Then $D^1 = D - \{v_2, v_3\}$ is a neighbourhood resolving set of G. Since D^1 contains nr-bad vertices of G, $nr(G) + 1 \leq |D^1| = nr(H) - 2$ and $nr(H) \geq nr(G) + 3$. Therefore $nr(H) = nr(G) + 3$. If D contains v_3, v_4; v_2, v_4, similar proof as in Subsubcase(i) can be given.

Subcase (iiib) : D contains v_1, v_2 and at least two vertices from $\{u_1, u_2, \ldots, u_k\}$. Then $nc_D(v_3) = nc_D(v_4) = (0, 0, \ldots, 0, 1, 1)$, a contradiction.

Similarly if D contains v_1, v_3; v_1, v_4, then it leads to a contradiction.

Subcase (iiic) : D contains v_1, v_2, v_3 and at least one vertex from $\{u_1, u_2, \ldots, u_k\}$. Then $D^1 = D - \{v_2, v_3\}$ is a neighbourhood resolving set of G.

As in Subcase (iiia), $nr(H) = nr(G) + 3$. If D contains v_1, v_3, v_4; v_1, v_2, v_4, similar result is arrived at.

Case (C) : D contains exactly three vertices from $\{u_1, u_2, \ldots, u_k, v_1, v_2, v_3, v_4\}$.

Subcase (i) : D contains $u_{i_1}, u_{i_2}, u_{i_3}$, $1 \leq i_1, i_2, i_3 \leq k$. Then $nc_D(v_2) = nc_D(v_3) = nc_D(v_4) = (0, 0, \ldots, 0)$, a contradiction.

Subcase (ii) : D contains u_1, u_2, v_1. Then $nc_D(v_2) = nc_D(v_3) = nc_D(v_4) = (0, 0, \ldots, 0, 1)$, a contradiction. Similarly if D contains $\{u_1, u_2, v_2\}$ or $\{u_1, u_2, v_3\}$ or $\{u_1, u_2, v_4\}$, it gives rise to a contradiction.

Subcase (iii) : D contains u_1, v_1, v_2. Then $nc_D(v_3) = nc_D(v_4) = (0, 0, \ldots, 0, 1, 1)$, a contradiction. If D contains $\{u_1, v_1, v_3\}$ or $\{u_1, v_1, v_4\}$, then these cases also lead to a contradiction.

Subcase (iv) : D contains u_1, v_2, v_3. Then $D^1 = D - \{v_2, v_3\}$ is a neighbourhood resolving set of G. Since D^1 contains nr-bad vertex u_1 of G, then $nr(G) + 1 \leq |D^1| = nr(H) - 2$. Hence $nr(H) \geq nr(G) + 3$. Therefore $nr(H) = nr(G) + 3$. The same result is true if D contains $\{u_1, v_3, v_4\}$ or $\{u_1, v_2, v_4\}$.

Subcase (v) : D contains v_2, v_3, v_4. Then $D^1 = D - \{v_2, v_3, v_4\}$ is a neighbourhood resolving set of G. Therefore $nr(G) \leq |D^1| = |D| - 3 = nr(H) - 3$. That is $nr(H) \geq nr(G) + 3$. Therefore $nr(H) = nr(G) + 3$.

Subcase (vi) : D contains v_1, v_2, v_3. Since D is an nr-set of H, by hyphothesis v_1 does not resolve privately any u_i, x_i where $x_i \neq u_i$ and $x_i \in V(G)$, $1 \leq i \leq k$. Therefore $D - \{v_1, v_2, v_3\}$ is a neighbourhood resolving set of G. Therefore $nr(G) \leq |D^1| = |D| - 3 = nr(H) - 3$. That is $nr(H) \geq nr(G) + 3$. Therefore $nr(H) = nr(G) + 3$. Hence in all cases, $nr(H) = nr(G) + 3$.

Since $|S_1| = |S_2| = |S_3| = nr(G) + 3$, S_1, S_2 and S_3 are nr-sets of H containing u_i, $1 \leq i \leq k$, v_1, v_2, v_3, v_4. Therefore u_i, $1 \leq i \leq k$, v_1, v_2, v_3, v_4 are nr-good in H, also all nr-good vertices in G are also nr-good in H and G is an induced subgraph of H. Therefore H is nr-excellent graph containing G as an induced subgraph and also $nr(H) = nr(G) + 3$. □

Theorem 39 *Let G be a graph with $|G| = 2^{nr(G)}$. Then $\Delta(G) \geq 2^{nr(G)-1}$.*

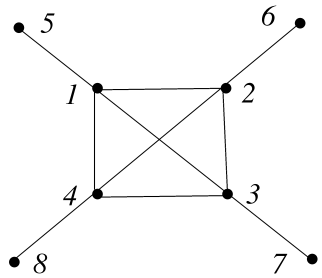
Proof: Let S be an nr-set of G. Let u_1 be the first vertex in the ordered set S. Since $2^{nr(G)}$ distinct codes are associated with the vertices of G, any vertex of G with the first element of its code with respect to S is 1, is adjacent to u_1. Since there are $2^{nr(G)-1}$ vertices with first element of its code 1, u has degree exactly $2^{nr(G)-1}$. Hence $\Delta(G) \geq 2^{nr(G)-1}$. □

Remark 40 *Every vertex of S has degree $2^{nr(G)-1}$.*

Remark 41 *There exists a graph G with $|G| = 2^{nr(G)}$ and $\Delta(G) = 2^{nr(G)-1}$.*

Consider the graph G :

G :



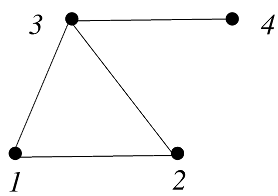
$S = \{1, 2, 3\}$ is an nr-set of G. $nc_S(1) = (0, 1, 1)$; $nc_S(2) = (1, 0, 1)$; $nc_S(3) = (1, 1, 0)$; $nc_S(4) = (1, 1, 1)$; $nc_S(5) = (1, 0, 0)$; $nc_S(6) = (0, 1, 0)$; $nc_S(7) = (0, 0, 1)$; $nc_S(8) = (0, 0, 0)$.

Therefore G is a graph with $|G| = 2^{nr(G)} = 2^3 = 8$ and $\Delta(G) = 4 = 2^{nr(G)-1} = 2^2$.

Remark 42 *There exists a graph G with $|G| = 2^{nr(G)}$ and $\Delta(G) > 2^{nr(G)-1}$.*

Consider the graph G:

$G =$



$S = \{1, 2\}$ *is an* nr*-set of* G. $nc_S(1) = (0, 1)$ *;* $nc_S(2) = (1, 0)$ *;* $nc_S(3) = (1, 1)$*;* $nc_S(4) = (0, 0)$ *. Therefore* G *is a graph with* $|G| = 2^{nr(G)} = 2^2 = 4$ *and* $\Delta(G) = 3 > 2^{nr(G)-1} = 2$*.*

Remark 43 *Let* G *be a* nr*-excellent graph with* $|G| = 2^{nr(G)}$*. Then* G *is regular with degree of regularity* $2^{nr(G)-1}$*.*

Proof: Suppose G is a nr-excellent graph with $|G| = 2^{nr(G)}$. Then every vertex belongs to an nr-set of G. Therefore degree of every vertex is $2^{nr(G)-1}$. □

4 Conclusion

Using the concept of excellence in graphs and neighbourhood resolving set of a graph $nr(G)$, we have defined nr-excellent graphs. we have characterized graphs G and H for which $G \cup H$ and $G + H$ are nr-excellent, when G and H are nr-excellent. We have also proved that a non nr-excellent graph G can be embedded in a nr-excellent graph H such that $nr(H) = nr(G)+$ number of nr-bad vertices of G. Also a new graph H can be constructed from a connected non nr-excellent graph G such that H is nr-excellent, G is an induced subgraph of H and $nr(H) = nr(G) + 3$. Some more results are also discussed.

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