Research on Supply Chain Coordination for substitutable products in competitive models

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Abstract: This paper studies four substitutable products in a two level supply chain which consists of two suppliers and two manufacturers. We develop two competitive models that decisions are made on the decentralized and centralized decisions respectively. Then we get the optimal pricing for four products in each condition. Through detail numerical analyses, we discuss the price, product demand and the benefit of supply chain under the influences of the price sensitive coefficient, auxiliary material cost and combination of price sensitive coefficient and production cost. Besides that, we also discuss the impact of price adjustment speed parameter on the strategy of pricing for products. We find that the optimal material providing strategies for manufacturers are changing with the value of price sensitive coefficient in the decentralized and centralized decisions. And the competition among substitutable products can slow down the effect of production cost on the benefit of supply chain. We also conclude that the supply chain with coordination mechanism can be more stable under the repeated game between manufacturers.

Key–Words: substitutable product, supply chain coordination, price sensitive coefficient, repeated game, chaos

1 Introduction

Because of the economic globalization, the enterprises are faced with more and more stronger market competitions. They cannot get long-term development only by their own. And with the development of society, the demands for products of customers are gradually diversification. Lots of enterprises learn that they cannot earn more profits only by improving technical level or reducing production cost. They need to develop partnership with other firms which are in the same supply chain so that they can cope with the fierce competitions in the product market effectively. Some enterprises begin to produce substitutable products in order to earn more market share and profit, such as P&G. The products in the P&G occupy more than half of the market share of this field, which strengthens its market status. So a growing number of manufacturers produce substitutable products and establish cooperative partnership with their suppliers and distributors so as to earn more profits.

In economics, two products are substitutable products, in addition to the price, if the increase of one product's price leads to the decrease of another product's demand when all the factors that affect the market demand remain the same, and vice versa. As so far, the researches on the substitutable product have been considered in a variety of papers, which can be broadly classified into two types: one is the firm-driven substitution, the other is the customer-driven substitution[1].

The firm-driven substitution, which is also called one-way substitution, means that enterprises produce products in different grades, the higher grade product can substitute for the lower grade product when the lower grade one is sold out, but it cannot happen in the opposite way. Cai and Chen (2003)[2] focused on the one-way substitution, and analyzed the optimal order quantities of two products when the lower product is sold out and the higher product substitute for it at lower price. Lu, Huang et al.(2011)[3] considered a supply chain with two downward substitutable products. They found that mitigating supply chain disruptions needs not only product substitution, but also multiple sourcing. They also discussed the optimal strategies under the certain and uncertain demand situation respectively.

The customer-driven substitution means that the customer can buy substitutable product according to the quality, price, preferences and so on. And this type can be sorted into two sides. One is the customer switch to another substitution when the product he want to buy is sold out. Su, Xu (2005)[4] analyzed the products' demand transfer between two distributors under the uncertain demand and order quantity, from the view of the distributors' benefits maximization. Huang, Zhou et al. (2011)[1] studied a multi-product competitive newsboy problem with shortage penalty cost and partial product substitution, and got the optimal order quantity with iterative algorithm. This paper also discussed the impacts of product substitution, demand correlation and demand variation on the optimal order quantities and the corresponding expected profits, and contrasted the optimal inventory level in a competitive model with the coordination model's. The other occurs when the customers purchase the ideal one in all the similar products. Hsieh, Wu (2009)[5] developed revenue sharing, return policy and combination of revenue sharing and return policy coordination models in a two level supply chain which consists of two suppliers and one manufacturer. They contrasted them with a basis and uncoordinated model, and they also investigated the effects of retailer's attitude toward risk, product substitutability, and demand and supply uncertainties on supply chain profit. Bish, Suwandechochai (2010)[6] considered that the degree of substitution among the products and operational postponement are two critical factors which affect the multi-product firm's capacity. They studied the impacts of product substitution, price and quantity postponement on the capacity, and analyzed the effect of clearance assumption on the results. Anderson and Bao (2010)[7] studied chain-to-chain competition, and pointed out that each product occupies a certain proportion of the market share and customers can be sorted into switching customers and marginal customers. They concluded that the coefficient of variation of market shares makes decentralized supply chains outperformed integrated supply chains. Xia (2011)[8] considered that buyer's preference for different products is one of the most important factors in substitution competition, and the switching cost happened when buyers switch from one supplier to another is related to the preference stickiness factor and preference location.

Besides that, some scholars extent their studies on the dynamical system, which research on the process of repeated game. Guo and Ma (2013)[9] considered the influences of price adjustment speed parameter on the collecting prices and profits in the decentralized and centralized control respectively. With the nonlinear dynamics theory, Ma and Pu (2013)[10] analyzed the stability of system under the impacts of output and price modification speed. Sun and Ma (2012)[11] took the theory of bifurcations of dynamical system into reality, and analyzed the dynamical behaviors of the market. Ma and Zhang (2012)[12] studied the influences of delayed decision on the dynamical system and provided the effectively control method to control the chaos.

Based on the literatures above, this paper focuses on the second type of customer-driven substitution. We develop the benefit maximization model of the supply chain in the decentralized and centralized decision respectively, and get the optimal sale strategy in each situation. Then, we analyze the effects of price sensitive coefficient,auxiliary material cost and combination of price sensitive coefficient and production cost on price, demand and profits of two supply chains through numerical simulations. At last, we study the process of manufacturer's dynamic price game and the stable range of product market.

The remainder of this paper is organized as follows. In section 2, we illustrate the basic model and assumptions. In section 3, we set up models in decentralized and centralized decisions respectively, and get the optimal pricing for four substitutions in each situation. In section 4, we develop numerical analyses to present the impacts of changing parameters on the performance of supply chain. In section 5, we analyse the influences of price adjustment speed parameter. In section 6, we conclude this paper.

2 Model description

This paper studies a two level supply chain which consists of two suppliers and two manufacturers. The Supplier 1 is the main material supplier, and the Supplier 2 is the auxiliary material supplier. Manufacturer 1 is the leader in the product market. His production capacity is stronger, so he only buys main material from Supplier 1 and chooses to produce auxiliary material by his own. But Manufacturer 2 chooses to purchase both of two materials from outside. The four products which produced by two manufacturers are substitutable products. It occurs price competition in the market and affects each product's demand. So the two manufacturers compete with each other for the customers in the product market.

2.1 Notations

w_i: sales price of Supplier i per unit $(i = 1, 2)$,
Q_i: order quantity of Manufacturer $i(i = 1, 2)$,
c_{s_i}: production cost of Supplier $i(i = 1, 2)$,
p_i: sales price of product i per unit $(i = 1, 2, 3, 4)$,
d_i: market demand for product $i(i = 1, 2, 3, 4)$,
c_i: production cost of product $i(i = 1, 2, 3, 4)$,
Π_{s_i}: expected profit of Supplier $i(i = 1, 2)$,

Π_{m_i}: expected profit of Manufacturer $i(i = 1, 2)$,
Π_i^0: expected profit of the supply chain which consists of Manufacturer $i(i = 1, 2)$.

2.2 Assumptions

Assumption 1. Suppliers can satisfy the demand for materials of two manufacturers. The quantity supplied doesn't beyond the quantity demanded. And the situation of shortage doesn't happen, too.
Assumption 2. We assume that the quantity of products can satisfy the demand for the target market, and it doesn't have shortage and residual. The Supplier 1 and Supplier 2 provide materials for the Manufacturer 2 with the proportion of 1:1, and all the materials are used in production, so $Q_1 = d_1 + d_2$, $Q_2 = d_3 + d_4$.
Assumption 3. We assume that the customer don't have personal preferences on the products, the demand for products are only effected by prices. The market demand for each product is

$$\begin{aligned} d_1 &= \alpha - \beta p_1 + \gamma p_2 + \theta p_3 + \epsilon p_4; \\ d_2 &= \alpha - \beta p_2 + \gamma p_1 + \theta p_4 + \epsilon p_3; \\ d_3 &= \alpha - \beta p_3 + \gamma p_4 + \theta p_1 + \epsilon p_2; \\ d_4 &= \alpha - \beta p_4 + \gamma p_3 + \theta p_2 + \epsilon p_1 \end{aligned}$$

where α is the maximum demand for products, and $\alpha > 0$. β is the own-price sensitive coefficient, and γ, θ and ϵ are the cross-price sensitive coefficients. The own-price sensitive coefficient is larger than the cross-price sensitive coefficient, which means that $\beta > \gamma > 0$, $\beta > \theta > 0$, $\beta > \epsilon > 0$. The differences between β and γ, θ, ϵ present the substitutability in four products. The smaller the difference is, the stronger the substitutability occurs, the more fierce competition will be in the market.

3 Modeling and analysis

3.1 The decentralized models

The suppliers and manufactures make their own decisions independently and take the strategy which can maximize the profits of their owns when they don't coordinate with each other. The Manufacturer 1 is the leader in the product market, he decides the prices of product 1 and product 2 depending on the market demands for them. Then the Manufacturer 2 determines the prices of product 3 and product 4 according to the leader's decision. At last, the two suppliers decide the prices of their own materials respectively.

So, the expected profit functions of two suppliers are as follows

$$\begin{aligned} \Pi_{s_1} &= (w_1 - c_{s_1})(Q_1 + Q_2) \\ \Pi_{s_2} &= (w_2 - c_{s_2})Q_2 \end{aligned}$$

Solving $\frac{\partial \Pi_{s_1}}{\partial w_1} = 0$, we can get

$$w_1 = c_{s_1} + \frac{\alpha}{\beta-\gamma-\theta-\epsilon} - \frac{p_1+p_2+p_3+p_4}{4} \quad (1)$$

and

$$\frac{\partial^2 \Pi_{s_1}}{\partial w_1^2} = 8(\gamma + \theta + \epsilon - \beta).$$

So there exist the optimal w_1^* which can make the expected profit of Supplier 1 maximum when $\gamma + \theta + \epsilon - \beta < 0$.

Solving $\frac{\partial \Pi_{s_2}}{\partial w_2} = 0$, we get

$$\begin{aligned} w_2 &= c_{s_2} + \frac{\alpha}{\beta - \gamma} + \frac{\theta + \epsilon}{2(\beta - \gamma)}(p_1 + p_2) \\ &\quad - \frac{p_3 + p_4}{2}. \end{aligned} \quad (2)$$

Because $\frac{\partial^2 \Pi_{s_2}}{\partial w_2^2} = 4(\gamma - \beta) < 0$, there exist the optimal w_2^* which can make the expected profit of Supplier 2 maximum.

The expected profits of two manufacturers are formulated in (3) and (4)

$$\Pi_{m_1} = (p_1 - c_1 - w_1)d_1 + (p_2 - c_2 - w_1)d_2 \quad (3)$$

$$\begin{aligned} \Pi_{m_2} &= (p_3 - c_3 - w_1 - w_2)d_3 \\ &\quad + (p_4 - c_4 - w_1 - w_2)d_4 \end{aligned} \quad (4)$$

Substituting (1) and (2) into (4), solving $\frac{\partial \Pi_{m_2}}{\partial p_3} = 0$ and $\frac{\partial \Pi_{m_2}}{\partial p_4} = 0$, we can get p_3 and p_4

$$p_3 = A\alpha - Bp_1 - Cp_2 + \frac{7c_3 - 3c_4}{20} + \frac{c_{s_1} + c_{s_2}}{5} \quad (5)$$

$$p_4 = A\alpha - Cp_1 - Bp_2 + \frac{7c_4 - 3c_3}{20} + \frac{c_{s_1} + c_{s_2}}{5} \quad (6)$$

where

$$\begin{aligned} A &= \frac{7}{10(\beta - \gamma)} + \frac{1}{5(\beta - \gamma - \theta - \epsilon)}, \\ B &= \frac{\theta(6\beta + \gamma) + \epsilon(\beta + 6\gamma)}{10(\gamma^2 - \beta^2)} + \frac{1}{20}, \\ C &= \frac{\theta(\beta + 6\gamma) + \epsilon(6\beta + \gamma)}{10(\gamma^2 - \beta^2)} + \frac{1}{20}. \end{aligned}$$

Together with $\frac{\partial^2\Pi_{m_2}}{\partial p_3^2} = \frac{\partial^2\Pi_{m_2}}{\partial p_4^2} = \frac{3\gamma-7\beta}{2} < 0$ and $\frac{\partial^2\Pi_{m_2}}{\partial p_3\partial p_4} = \frac{\partial^2\Pi_{m_2}}{\partial p_4\partial p_3} = \frac{7\gamma-3\beta}{2}$, the determinate of the Hessian

$$\det H = \begin{pmatrix} \frac{\partial^2\Pi_{m_2}}{\partial p_3^2} & \frac{\partial^2\Pi_{m_2}}{\partial p_3\partial p_4} \\ \frac{\partial^2\Pi_{m_2}}{\partial p_4\partial p_3} & \frac{\partial^2\Pi_{m_2}}{\partial p_4^2} \end{pmatrix} = 10(\beta^2-\gamma^2) > 0.$$

We conclude that there exist the optimal p_3^* and p_4^* that can make the expected profit of Manufacturer 2 Π_{m_2} maximum.

Substituting (1), (5) and (6) into (3), solving $\frac{\partial\Pi_{m_1}}{\partial p_1} = 0$ and $\frac{\partial\Pi_{m_1}}{\partial p_2} = 0$, we can get the prices of product 1 and product 2 in the decentralized decision

$$\begin{aligned} p_1 &= \left(\frac{1+A(\theta+\epsilon)}{2D} + \frac{\frac{1}{\beta-\gamma-\theta-\epsilon} - \frac{A}{2}}{3-B-C}\right)\alpha \\ &+ \frac{\beta+\theta B+\epsilon C-E}{(3-B-C)D}c_1 \\ &+ \frac{E-\gamma+\theta C+\epsilon B}{(3-B-C)D}c_2 \\ &+ \frac{\frac{EF}{2} - \frac{D}{20} + M}{(3-B-C)D}c_3 + \frac{N - \frac{EF}{2} - \frac{D}{20}}{(3-B-C)D}c_4 \\ &+ \left(\frac{\theta+\epsilon}{10D} + \frac{9}{10(3-B-C)}\right)c_{s_1} \\ &+ \left(\frac{\theta+\epsilon}{10D} - \frac{1}{10(3-B-C)}\right)c_{s_2} \end{aligned}$$

$$\begin{aligned} p_2 &= \left(\frac{1+A(\theta+\epsilon)}{2D} + \frac{\frac{1}{\beta-\gamma-\theta-\epsilon} - \frac{A}{2}}{3-B-C}\right)\alpha \\ &+ \left[\frac{\beta+\theta B+\epsilon C-E}{(3-B-C)D} - \frac{1}{2}\right]c_1 \\ &+ \left[\frac{E-\gamma+\theta C+\epsilon B}{(3-B-C)D} + \frac{1}{2}\right]c_2 \\ &+ \left[\frac{\frac{EF}{2} - \frac{D}{20} + M}{(3-B-C)D} + \frac{F}{4}\right]c_3 \\ &+ \left[\frac{N - \frac{EF}{2} - \frac{D}{20}}{(3-B-C)D} - \frac{F}{4}\right]c_4 \\ &+ \left(\frac{\theta+\epsilon}{10D} + \frac{9}{10(3-B-C)}\right)c_{s_1} \\ &+ \left(\frac{\theta+\epsilon}{10D} - \frac{1}{10(3-B-C)}\right)c_{s_2} \end{aligned}$$

where

$$\begin{aligned} D &= \beta-\gamma+(\theta+\epsilon)(B+C), \\ E &= \frac{(5-B-C)(\gamma-\theta C-\epsilon B)}{4} \\ &+ \frac{(B+C-1)(\beta+\epsilon C+\theta B)}{4}, \\ F &= \frac{\epsilon-\theta}{\beta+\gamma+(B-C)(\theta-\epsilon)}, \\ M &= \frac{(5-B-C)(7\theta-3\epsilon)}{80} \\ &+ \frac{(1-B-C)(7\epsilon-3\theta)}{80}, \\ N &= \frac{(5-B-C)(7\epsilon-3\theta)}{80} \\ &+ \frac{(1-B-C)(7\theta-3\epsilon)}{80}. \end{aligned}$$

Since

$$\begin{aligned} \frac{\partial^2\Pi_{m_1}}{\partial p_1^2} &= \frac{\partial^2\Pi_{m_1}}{\partial p_2^2} = \frac{B+C-5}{2}(\beta+\theta B+\epsilon C) \\ &+ \frac{1-B-C}{2}(\gamma-\epsilon B-\theta C) < 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2\Pi_{m_1}}{\partial p_1\partial p_2} &= \frac{\partial^2\Pi_{m_1}}{\partial p_2\partial p_1} = \frac{B+C-1}{2}(\beta+\theta B+\epsilon C) \\ &+ \frac{5-B-C}{2}(\gamma-\epsilon B-\theta C) \end{aligned}$$

and the determinate of the Hessian

$$\begin{aligned} \det H &= \left[\frac{29}{5} + \frac{7(\theta+\epsilon)}{5(\beta-\gamma)}\right] \cdot \frac{2(\gamma+\beta)^2 - (\theta-\epsilon)^2}{2(\gamma+\beta)} \cdot \\ &\left[\frac{\theta+\epsilon}{10} + \frac{10(\beta-\gamma)^2 - 7(\theta+\epsilon)^2}{10(\beta-\gamma)}\right] > 0. \end{aligned}$$

So we can find the optimal p_1^* and p_2^* to maximize the Π_{m_1}.

Inserting p_1^* and p_2^* into (5) and (6), we can get the optimal pricing for product 3 and product 4 in the decentralized decision

$$\begin{aligned} &p_3^* = \\ &\left[A-(B+C)\left(\frac{1+A(\theta+\epsilon)}{2D} + \frac{\frac{1}{\beta-\gamma-\theta-\epsilon} - \frac{A}{2}}{3-B-C}\right)\right]\alpha \\ &+ \left[\frac{C}{2} - (B+C)\frac{\beta+\theta B+\epsilon C-E}{(3-B-C)D}\right]c_1 \\ &- \left[\frac{C}{2} + (B+C)\frac{E-\gamma+\theta C+\epsilon B}{(3-B-C)D}\right]c_2 \\ &+ \left[\frac{7}{20} - \frac{CF}{4} - (B+C)\frac{\frac{EF}{2} - \frac{D}{20} + M}{(3-B-C)D}\right]c_3 \\ &+ \left[-\frac{3}{20} + \frac{CF}{4} - (B+C)\frac{N - \frac{EF}{2} - \frac{D}{20}}{(3-B-C)D}\right]c_4 \\ &+ \left[\frac{1}{5} - (B+C)\frac{\theta+\epsilon}{10D} - \frac{9(B+C)}{10(3-B-C)}\right]c_{s_1} \\ &+ \left[\frac{1}{5} - (B+C)\frac{\theta+\epsilon}{10D} + \frac{B+C}{10(3-B-C)}\right]c_{s_2} \end{aligned}$$

$$
\begin{aligned}
&p_4^* = \\
&[A-(B+C)(\frac{1+A(\theta+\epsilon)}{2D}+\frac{\frac{1}{\beta-\gamma-\theta-\epsilon}-\frac{A}{2}}{3-B-C})]\alpha \\
&+[\frac{B}{2}-(B+C)\frac{\beta+\theta B+\epsilon C-E}{(3-B-C)D}]c_1 \\
&-[\frac{B}{2}+(B+C)\frac{E-\gamma+\theta C+\epsilon B}{(3-B-C)D}]c_2 \\
&+[\frac{BF}{4}-\frac{3}{20}-(B+C)\frac{\frac{EF}{2}-\frac{D}{20}+M}{(3-B-C)D}]c_3 \\
&+[\frac{7}{20}-\frac{BF}{4}-(B+C)\frac{N-\frac{EF}{2}-\frac{D}{20}}{(3-B-C)D}]c_4 \\
&+[\frac{1}{5}-(B+C)\frac{\theta+\epsilon}{10D}-\frac{9(B+C)}{10(3-B-C)}]c_{s_1} \\
&+[\frac{1}{5}-(B+C)\frac{\theta+\epsilon}{10D}+\frac{B+C}{10(3-B-C)}]c_{s_2}
\end{aligned}
$$

3.2 The centralized models

The goals of manufacturers and suppliers are turned into benefits maximization of the supply chain when two manufacturers and their suppliers coordinate with each other and share information at all times. In this condition, two manufacturers deliver the product's demand information to the suppliers directly, and they decide the prices of four products together. We premise that Manufacturer 1 is the leader in the product market, he will coordinate with Supplier 1 and they are the Supply Chain 1. Manufacturer 2 will coordinate with Supplier 1 and Supplier 2, so that they are the Supply Chain 2.

The expected profits of two supply chains in the centralized decision can be expressed as

$$\Pi_1^0 = (p_1 - c_1 - c_{s_1})d_1 + (p_2 - c_2 - c_{s_1})d_2 \quad (7)$$

$$
\begin{aligned}
\Pi_2^0 &= (p_3 - c_3 - c_{s_1} - c_{s_2})d_3 + (p_4 - c_4 \\
&\quad -c_{s_1} - c_{s_2})d_4 \qquad (8)
\end{aligned}
$$

Solving $\frac{\partial \Pi_2^0}{\partial p_3} = 0$ and $\frac{\partial \Pi_2^0}{\partial p_4} = 0$, we can get

$$
\begin{aligned}
p_3 &= \frac{\alpha}{2(\beta-\gamma)} + \frac{\epsilon\gamma+\theta\beta}{2(\beta^2-\gamma^2)}p_1 + \frac{\epsilon\beta+\theta\gamma}{2(\beta^2-\gamma^2)}p_2 \\
&\quad + \frac{c_3}{2} + \frac{c_{s_1}+c_{s_2}}{2} \qquad (9) \\
p_4 &= \frac{\alpha}{2(\beta-\gamma)} + \frac{\epsilon\beta+\theta\gamma}{2(\beta^2-\gamma^2)}p_1 + \frac{\epsilon\gamma+\theta\beta}{2(\beta^2-\gamma^2)}p_2 \\
&\quad + \frac{c_4}{2} + \frac{c_{s_1}+c_{s_2}}{2} \qquad (10)
\end{aligned}
$$

Since the Hessian of Π_2^0 is

$$\det H = \begin{pmatrix} \frac{\partial^2 \Pi_2^0}{\partial p_3^2} & \frac{\partial^2 \Pi_2^2}{\partial p_3 \partial p_4} \\ \frac{\partial^2 \Pi_2^0}{\partial p_4 \partial p_3} & \frac{\partial^2 \Pi_2^2}{\partial p_4^2} \end{pmatrix} = 4(\beta^2-\gamma^2) > 0$$

and $\frac{\partial^2 \Pi_2^0}{\partial p_3^2} = -2\beta < 0$, there exist the optimal p_3^0 and p_4^0 which can make the expected profit of Supply Chain 2 Π_2^0 maximum.

Taking (9) and (10) into (7), solving $\frac{\partial \Pi_1^0}{\partial p_1} = 0$ and $\frac{\partial \Pi_1^0}{\partial p_2} = 0$, we can get prices of product 1 and product 2 in the centralized decision

$$
\begin{aligned}
p_1 &= -[\frac{1}{2(G+H)} + \frac{\theta+\epsilon}{4(\beta-\gamma)(G+H)}]\alpha + \frac{c_1}{2} \\
&\quad + \frac{\epsilon H - \theta G}{4(G^2-H^2)}c_3 - \frac{\epsilon G - \theta H}{4(G^2-H^2)}c_4 \\
&\quad + [\frac{1}{2} - \frac{\theta+\epsilon}{4(G+H)}]c_{s_1} - \frac{\theta+\epsilon}{4(G+H)}c_{s_2}
\end{aligned}
$$

$$
\begin{aligned}
p_2 &= -[\frac{1}{2(G+H)} + \frac{\theta+\epsilon}{4(\beta-\gamma)(G+H)}]\alpha + \frac{c_2}{2} \\
&\quad - \frac{\epsilon G - \theta H}{4(G^2-H^2)}c_3 + \frac{\epsilon H - \theta G}{4(G^2-H^2)}c_4 \\
&\quad + [\frac{1}{2} - \frac{\theta+\epsilon}{4(G+H)}]c_{s_1} - \frac{\theta+\epsilon}{4(G+H)}c_{s_2}
\end{aligned}
$$

where

$$
\begin{aligned}
G &= -\beta + \frac{\theta(\epsilon\gamma+\theta\beta)+\epsilon(\epsilon\beta+\theta\gamma)}{2(\beta^2-\gamma^2)}, \\
H &= \gamma + \frac{\epsilon(\epsilon\gamma+\theta\beta)+\theta(\epsilon\beta+\theta\gamma)}{2(\beta^2-\gamma^2)}.
\end{aligned}
$$

Then,

$$
\begin{aligned}
\frac{\partial^2 \Pi_1^0}{\partial p_1^2} &= \frac{\partial^2 \Pi_1^0}{\partial p_2^2} = \frac{\theta(\epsilon\gamma+\theta\beta)+\epsilon(\epsilon\beta+\theta\gamma)}{\beta^2-\gamma^2} \\
&\quad -2\beta < 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \Pi_1^0}{\partial p_1 \partial p_2} &= \frac{\partial^2 \Pi_1^0}{\partial p_2 \partial p_1} \\
&= 2\gamma + \frac{\epsilon(\epsilon\gamma+\theta\beta)+\theta(\epsilon\beta+\theta\gamma)}{\beta^2-\gamma^2}
\end{aligned}
$$

Because $\gamma + \theta + \epsilon - \beta < 0$, the Hessian of Π_1^0 is

$$
\begin{aligned}
\det H &= \frac{(\epsilon+\theta)^2 - 2(\beta-\gamma)^2}{\beta-\gamma} \cdot \\
&\quad \frac{(\epsilon-\theta)^2 - 2(\beta+\gamma)^2}{\beta+\gamma} > 0.
\end{aligned}
$$

So there exist the optimal p_1^0 and p_2^0 to maximize Π_1^0.

Substituting p_1^0 and p_2^0 into (9) and (10), we can get the optimal pricing for product 3 and product 4 in the centralized decision

$$p_3^0 =$$

$$[-\frac{\epsilon+\theta}{4(\beta-\gamma)(G+H)}-\frac{(\epsilon+\theta)^2}{8(\beta-\gamma)^2(G+H)}]\alpha$$
$$+\frac{1}{2(\beta-\gamma)}\alpha+\frac{\theta\beta+\epsilon\gamma}{4(\beta^2-\gamma^2)}c_1+\frac{\epsilon\beta+\theta\gamma}{4(\beta^2-\gamma^2)}c_2$$
$$+[\frac{(\epsilon-\theta)(\theta\beta+\epsilon\gamma)}{8(\beta^2-\gamma^2)(G-H)}-\frac{(\epsilon+\theta)(\epsilon G-\theta H)}{8(\beta-\gamma)(G^2-H^2)}]c_3$$
$$-[\frac{(\epsilon+\theta)(\theta G-\epsilon H)}{8(\beta-\gamma)(G^2-H^2)}+\frac{(\epsilon-\theta)(\theta\beta+\epsilon\gamma)}{8(\beta^2-\gamma^2)(G-H)}]c_4$$
$$+\frac{1}{2}c_3+[\frac{1}{2}+\frac{\epsilon+\theta}{4(\beta-\gamma)}-\frac{(\epsilon+\theta)^2}{8(\beta-\gamma)(G+H)}]c_{s_1}$$
$$+[\frac{1}{2}-\frac{(\epsilon+\theta)^2}{8(\beta-\gamma)(G+H)}]c_{s_2}$$

$$p_4^0=$$
$$[-\frac{\epsilon+\theta}{4(\beta-\gamma)(G+H)}-\frac{(\epsilon+\theta)^2}{8(\beta-\gamma)^2(G+H)}]\alpha$$
$$+\frac{1}{2(\beta-\gamma)}\alpha+\frac{\epsilon\beta+\theta\gamma}{4(\beta^2-\gamma^2)}c_1+\frac{\theta\beta+\epsilon\gamma}{4(\beta^2-\gamma^2)}c_2$$
$$+[\frac{(\epsilon-\theta)(\epsilon\beta+\theta\gamma)}{8(\beta^2-\gamma^2)(G-H)}-\frac{(\epsilon+\theta)(\epsilon G-\theta H)}{8(\beta-\gamma)(G^2-H^2)}]c_3$$
$$-[\frac{(\epsilon+\theta)(\theta G-\epsilon H)}{8(\beta-\gamma)(G^2-H^2)}+\frac{(\epsilon-\theta)(\epsilon\beta+\theta\gamma)}{8(\beta^2-\gamma^2)(G-H)}]c_4$$
$$+\frac{1}{2}c_4+[\frac{1}{2}+\frac{\epsilon+\theta}{4(\beta-\gamma)}-\frac{(\epsilon+\theta)^2}{8(\beta-\gamma)(G+H)}]c_{s_1}$$
$$+[\frac{1}{2}-\frac{(\epsilon+\theta)^2}{8(\beta-\gamma)(G+H)}]c_{s_2}$$

4 Numerical analysis

In this section, we use numerical simulation to analyze the effects of parameters changing on the two different models. According to the models which set up in the decentralized and centralized decisions, we analyze the effects of price sensitive coefficient β and γ, auxiliary material cost c_{s_2} and combination of cross-price sensitive coefficient γ and production cost c_2 on the price, demand and profit of supply chain. The parameters need to satisfy $\beta > \gamma+\theta+\epsilon$, so they are setting as follows: $\alpha = 100$, $\beta = 5$, $\gamma = 1.2$, $\theta = 1.8$, $\epsilon = 1.3$, $c_1 = 12$, $c_2 = 9$, $c_3 = 10$, $c_4 = 7$, $c_{s_1} = 5$, $c_{s_2} = 3$.

4.1 The decentralized model

4.1.1 The impact of own-price sensitive coefficient β

When β is varying from 4.4 to 6, the effects of own-price sensitive coefficient β on the price, demand and the expected profits of two supply chains in the decentralized decision are shown from Fig.1 to Fig.4. When β is higher, the product is more sensitive to its own price. From Fig.1, we can find that the prices of four products are declining fast which proves the view. The fall ranges of demands for product 3 and product 4 are larger than product 1 and product 2's. Because Supplier 1 is the main material supplier, the price of main material is also strongly affected by β compared with auxiliary material's price. And the expected profits of two supply chains decrease when own-price sensitive coefficient β is higher. So if customers are more sensitive to product's price, it will decline the profit of the supply chain.

From Fig.4 we find that the expected profit of Supply Chain 1 is larger when the own-price sensitive coefficient β is big enough. So the manufacturer will earn more profit if they produce their own auxiliary material when $4.55 < \beta < 6$. On the other side, it will be profitable to purchase auxiliary material from outside when $4.4 < \beta < 4.55$.

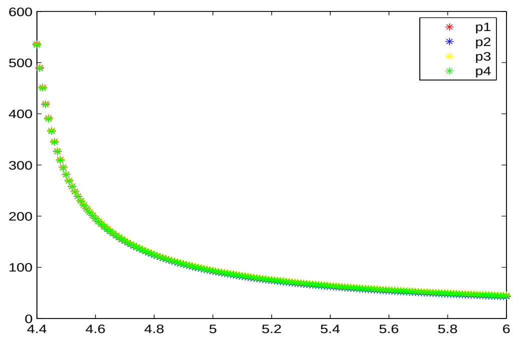

Figure 1: Influence of β on the prices of four products in the decentralized decision

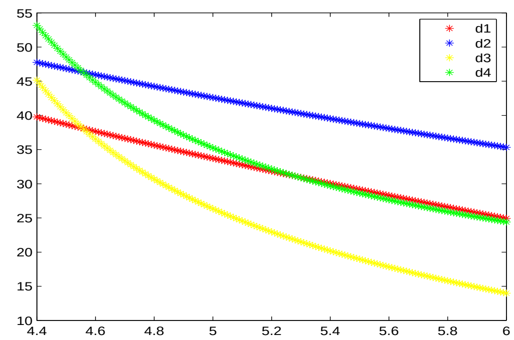

Figure 2: Influence of β on the demand in the decentralized decision

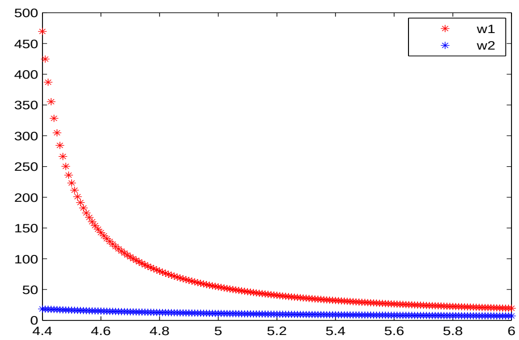

Figure 3: Influence of β on the prices of two materials in the decentralized decision

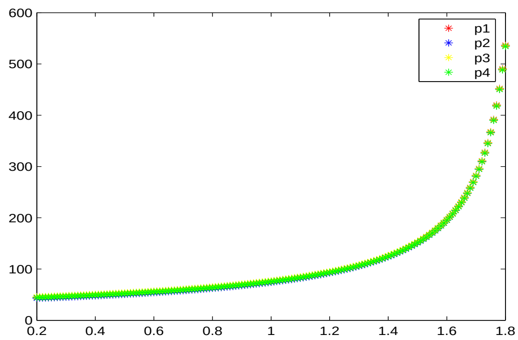

Figure 5: Influence of γ on the prices of four products in the decentralized decision

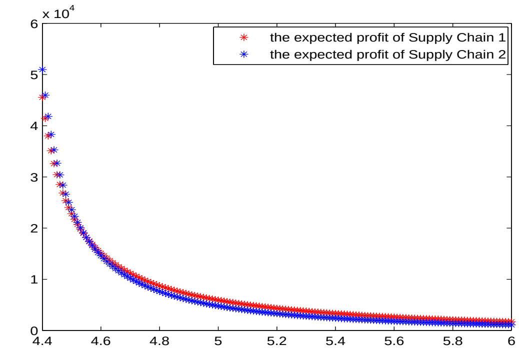

Figure 4: Influence of β on the expected profits of two supply chains in the decentralized decision

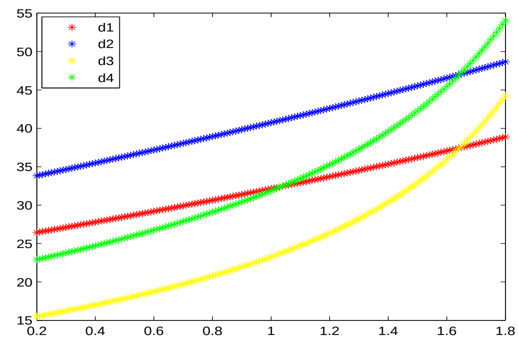

Figure 6: Influence of γ on the demand in the decentralized decision

4.1.2 The impact of cross-price sensitive coefficient γ

In this part, we change γ from 0.2 to 1.8, when other parameters remain the same. We can observe the influence of γ on the decentralized model from Fig.5 to Fig.8. When the cross-price sensitive coefficient γ is higher, the competitions among substitutable products are serious, and the product is less sensitive to its own price. Through the figures, we can find that the prices of four products and two materials, market demands and the profits of two supply chains are increasing when γ is increasing. So higher competition among products can increase the profits of members in the supply chain. The situation is similar when cross-price sensitive coefficient θ or ϵ is changing.

Fig.8 shows that when the cross-price sensitive coefficient is higher, the expected profit of Supply Chain 2 is more than the Supply Chain 1's. So when the cross-price sensitive coefficient $0.2 < \gamma < 1.65$, it's wiser to produce their own auxiliary material for manufacturers. And when $1.65 < \gamma < 1.8$, purchasing auxiliary material from outside is more profitable.

4.1.3 The impact of auxiliary material cost c_{s_2}

The parameters in this part are set as before, instead of changing c_{s_2} from 3 to 9. The effects of auxiliary material cost c_{s_2} on the prices, demands and profits of two supply chains are illustrated in Fig.9 to Fig.12. When auxiliary material cost c_{s_2} is increasing, Fig.11 shows that the rise ranges of prices of product 3 and product 4 are bigger than product 1 and product 2's. The demands for product 3 and product 4 are decreasing while product 1 and product 2's demands are increasing. As expected, the price of the auxiliary material is higher, and the expected profit of Supply Chain 2 is decreasing. And the price of main material declines a little. Because Manufacturer 1 have the ability to produce the auxiliary material, he has initiative in the product market. So he increases

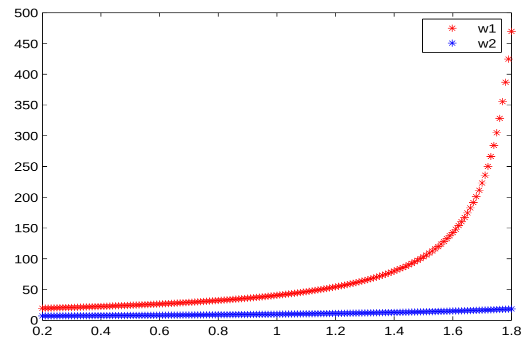

Figure 7: Influence of γ on the prices of two materials in the decentralized decision

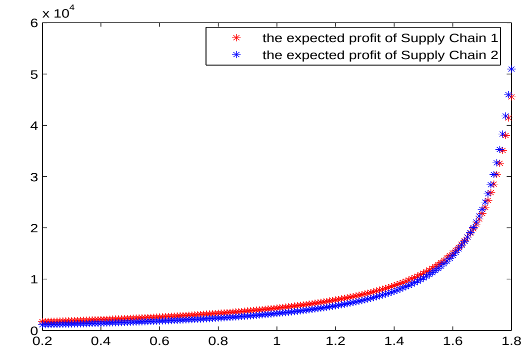

Figure 8: Influence of γ on the expected profits of two supply chains in the decentralized decision

the prices of product 1 and product 2 and earns more profits.

4.1.4 The combined effect of cross-price sensitive coefficient γ and production cost c_2

Setting γ in 0.5 and 1.2 respectively and changing c_2 from 6 to 11 when other parameters remain the same. We can observe from Fig.13 and Fig.14 that the combined influences of cross-price sensitive coefficient γ and production cost c_2 on the profits of two supply chains. The expected profit of Supply Chain 1 is decreasing when the production cost of product 2 is increasing. Manufacturer 1 is the leader in the product market, so the expected profit of Supply Chain 1 is more in the decentralized decision. And the fall range of Supply Chain 1's profit is smaller when the cross-price sensitive coefficient γ is higher. So the higher substitutability among products can slow down the influence of production cost on the profit of supply chain.

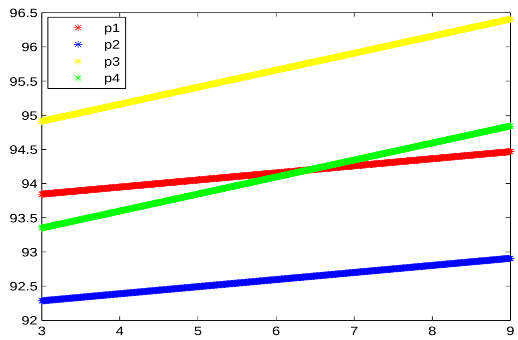

Figure 9: Influence of c_{s_2} on the prices of four products in the decentralized decision

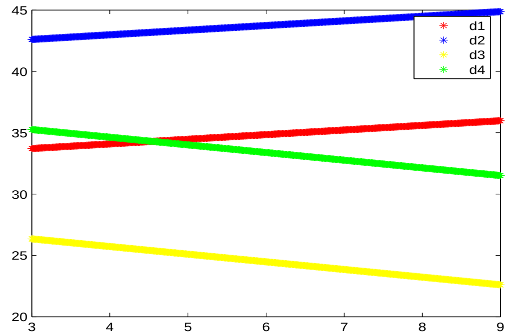

Figure 10: Influence of c_{s_2} on the demand in the decentralized decision

4.2 The centralized model

4.2.1 The impact of own-price sensitive coefficient β

The parameters are set just as the first part of the decentralized model. Fig.15 to Fig.17 show that the effects of own-price sensitive coefficient β on the centralized model when β is varying from 4.4 to 6. The prices and demands of four products and the expected profits of two supply chains are decreasing in the centralized decision, just like the situation in the decentralized one. But the fall range of four products' prices and demands are smaller than before. If $4.4 < \beta < 5.66$, the expected profit of Supply Chain 2 is higher than Supply Chain 1 when Manufacturer 2 coordinates with two suppliers. So it's wiser to purchase auxiliary material from outside. On the other side, if $5.66 < \beta < 6$, the Supplier Chain 1's profit is more higher. So it's a good choice to produce it by own.

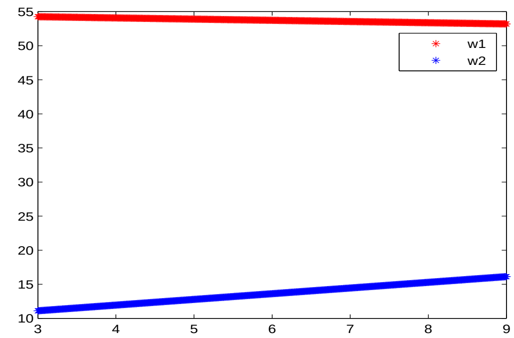

Figure 11: Influence of c_{s_2} on the price of two materials in the decentralized decision

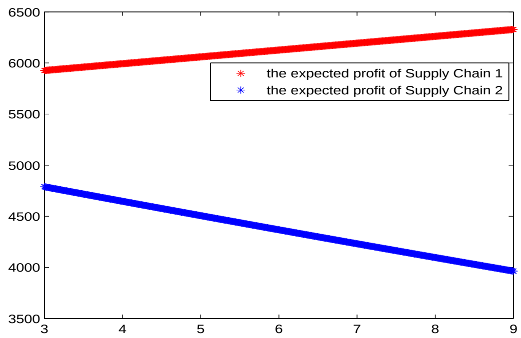

Figure 12: Influence of c_{s_2} on the expected profits of two supply chains in the decentralized decision

4.2.2 The impact of cross-price sensitive coefficient γ

From Fig.18 to Fig.20, we can see that the effects of cross-price sensitive coefficient γ on the centralized model are similar with the situation in the decentralized model when γ is varying from 0.2 to 1.8. Contrast with the decentralized decision, we can find that the prices of four products are lower, the demands are higher, and the increasing of two supply chains' profits are smaller in the centralized decision. Because manufacturers share profits with suppliers in the centralized decision, the enthusiasm of them decreases, so the profits of two supply chains in the centralized decision are less than the decentralized decision's. So when the manufacturers can forecast the demand for products exactly, the manufacturers prefer not to cooperate with the suppliers.

When Manufacturer 2 coordinates with Supplier 1 and Supplier 2, the prices and demands of two products are more competitive than Manufacture 1's.

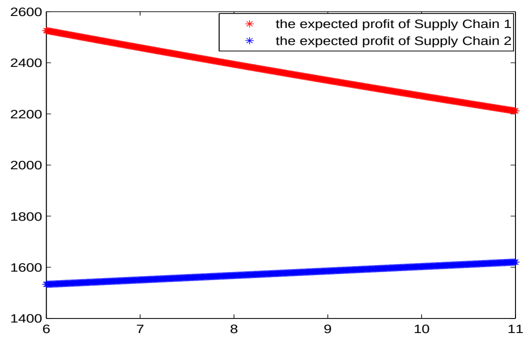

Figure 13: Combined influence on the expected profits of two supply chains in the decentralized decision when $\gamma = 0.5$

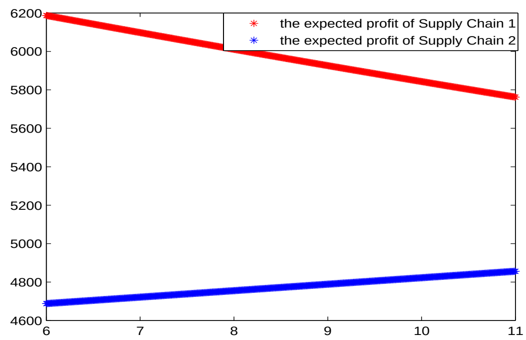

Figure 14: Combined influence on the expected profits of two supply chains in the decentralized decision when $\gamma = 1.2$

It illustrates that purchasing auxiliary material from outside is a wiser choice in the centralized decision when cross-price sensitive coefficient $0.54 < \gamma < 1.8$. On the contrary, if $0.2 < \gamma < 0.54$, it's a good choose to produce auxiliary material by their own.

4.2.3 The impact of auxiliary material cost c_{s_2}

The impacts of auxiliary material cost c_{s_2} on the centralized model are shown from Fig.21 to Fig.23. The prices of product 3 and product 4 are increasing and the demands of them are decreasing when the auxiliary material cost c_{s_2} is increasing from 3 to 9, just like the situation in the decentralized model. But the prices of product 3 and product 4 doesn't beyond the price of product 1, and the demands of them are not the lowest. So when Manufacturer 2 cooperates with Supplier 1 and Supplier 2, the influences of auxiliary material cost c_{s_2} on the Supply Chain 2 are

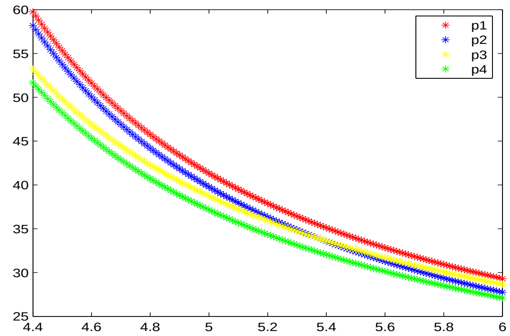

Figure 15: Influence of β on the prices of four products in the centralized decision

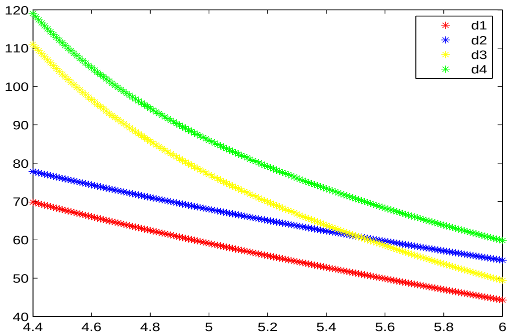

Figure 16: Influence of β on the demand in the centralized decision

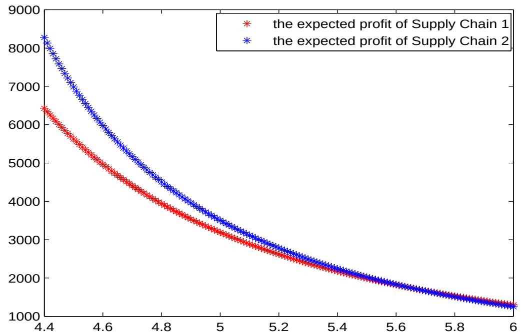

Figure 17: Influence of β on the expected profits of two supply chains in the centralized decision

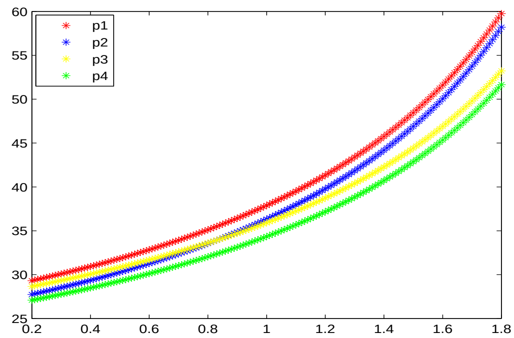

Figure 18: Influence of γ on the pricing in the centralized decision

smaller than before. So the coordination mechanism can weaken the negative effect of cost on the supply chain.

4.2.4 The combined effect of cross-price sensitive coefficient γ and production cost c_2

The parameters are set as the third part of the decentralized decision. The combined effects of cross-price sensitive coefficient γ and product cost c_2 on the centralized model are shown in Fig.24 and Fig.25. The expected profit of Supply Chain 1 is not far beyond the Supply Chain 2's in the centralized decision. And the profit of Supply Chain 2 is higher than Supply Chain 1's when $\gamma = 1.2$. So when Manufacturer 2 coordinates with two suppliers, the effect of production cost on the supply chain is smaller. And under the same cross-price sensitive coefficient γ, the decreasing of supply chains' expected profits are lower in the centralized decision in contrast with the decentralized decision's. So the competitive model in the centralized decision is more stable.

5 Pricing game analysis

In the reality, the game between enterprises is dynamic and repeated. The strategies of pricing for products which the firms make are not just depends on the current market environment. They also consider the pricing strategy of previous time period. If the profit is positive at this time period, the firm will continue to use this pricing strategy at the next time period. However, if the profit is negative, the firm will change the strategy of pricing for products.

5.1 The decentralized decision

In this part, we assume that the wholesale prices of products are known. The Manufacturer 1 decides the prices of product 1 and product 2. Then, according to Manufacturer 1's strategy, Manufacturer 2 makes decisions on the retail prices of product 3 and product

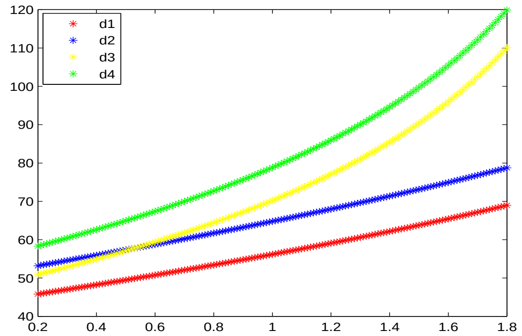

Figure 19: Influence of γ on the demands in the centralized decision

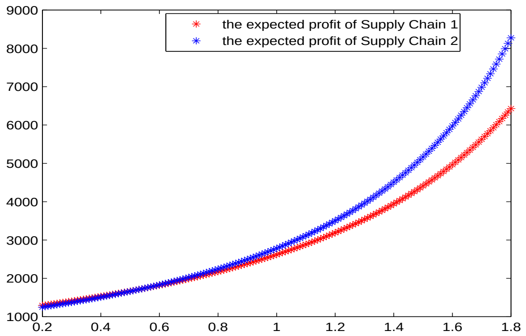

Figure 20: Influence of γ on the expected profits of two supply chains in the centralized decision

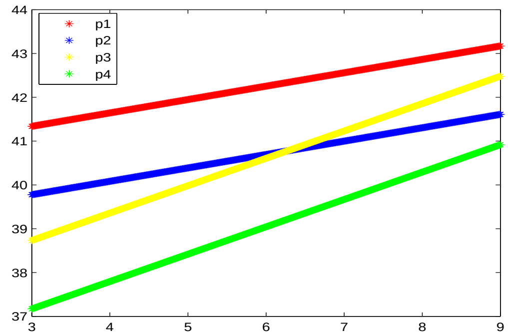

Figure 21: Influence of c_{s_2} on the pricing in the centralized decision

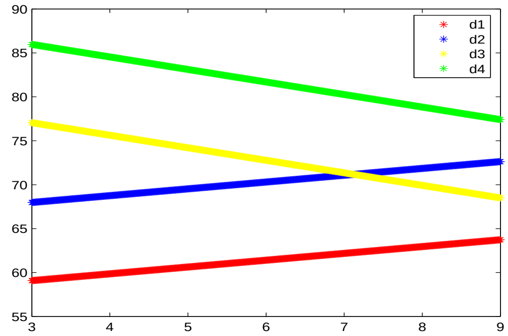

Figure 22: Influence of c_{s_2} on the demand in the centralized decision

4. The price of product i at time period t is $p_i(t)$ $(i = 1,2,3,4; t = 0,1,2,\cdots)$, and the demand for product i at time period t is $q_i(t)$ $(i = 1,2,3,4)$.

In the decentralized decision, the profit of Manufacturer 2 is $\Pi_{m_2} = (p_3(t) - c_3 - w_1 - w_2)d_3(t) + (p_4(t) - c_4 - w_1 - w_2)d_4(t)$, so we can get the retail prices of product 3 and product 4.

$$
\begin{aligned}
p_3 &= \frac{\alpha}{2(\beta-\gamma)} + \frac{(\theta\beta+\epsilon\gamma)p_1 + (\epsilon\beta+\theta\gamma)p_2}{2(\beta^2-\gamma^2)} \\
&\quad + \frac{c_3+w_1+w_2}{2} \\
p_4 &= \frac{\alpha}{2(\beta-\gamma)} + \frac{(\epsilon\beta+\theta\gamma)p_1 + (\theta\beta+\epsilon\gamma)p_2}{2(\beta^2-\gamma^2)} \\
&\quad + \frac{c_4+w_1+w_2}{2}
\end{aligned}
$$

Through the expressions we can find that the strategy of pricing for product 3 and product 4 is changing according to the prices of product 1 and product 2.

The profit of Manufacturer 1 is $\Pi_{m_1} = (p_1(t) - c_1 - w_1)d_1(t) + (p_2(t) - c_2 - w_1)d_2(t)$. Then, we can get the retail prices of product 1 and product 2.

$$
\begin{aligned}
p_1 &= [1 + \frac{\theta+\epsilon}{2(\beta-\gamma)}]\frac{-\alpha}{2(A+B)} - \frac{(\theta+\epsilon)(w_1+w_2)}{4(A+B)} \\
&\quad + \frac{A(\theta c_3+\epsilon c_4) - B(\theta c_4+\epsilon c_3)}{4(B^2-A^2)} + \frac{c_1+w_1}{2}
\end{aligned}
$$

$$
\begin{aligned}
p_2 &= [1 + \frac{\theta+\epsilon}{2(\beta-\gamma)}]\frac{-\alpha}{2(A+B)} - \frac{(\theta+\epsilon)(w_1+w_2)}{4(A+B)} \\
&\quad + \frac{B(\theta c_3+\epsilon c_4) - A(\theta c_4+\epsilon c_3)}{4(A^2-B^2)} + \frac{c_2+w_1}{2}
\end{aligned}
$$

where

$$
\begin{aligned}
A &= -\beta + \frac{\theta(\theta\beta+\epsilon\gamma) + \epsilon(\theta\gamma+\epsilon\beta)}{2(\beta^2-\gamma^2)}, \\
B &= \gamma + \frac{\epsilon(\theta\beta+\epsilon\gamma) + \theta(\theta\gamma+\epsilon\beta)}{2(\beta^2-\gamma^2)}.
\end{aligned}
$$

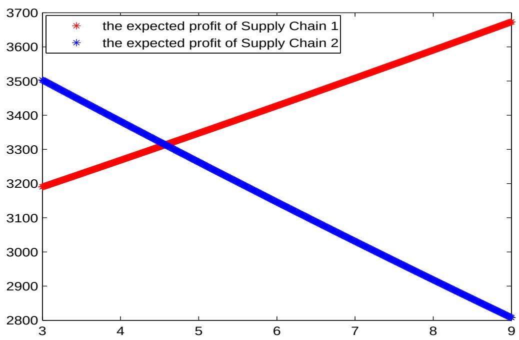

Figure 23: Influence of c_{s_2} on the expected profits of two supply chains in the centralized decision

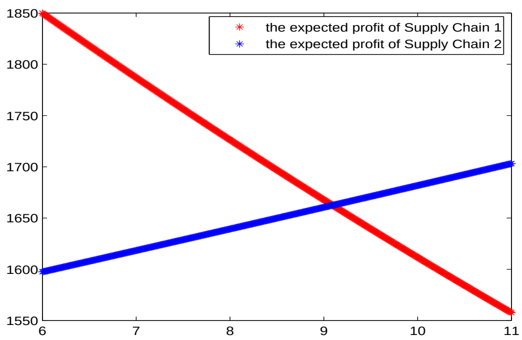

Figure 24: Combined influence on the expected profits of two supply chains in the centralized decision when $\gamma = 0.5$

As the leader in the product market, the strategy of pricing for Manufacturer 1 is dynamic. The retailer prices of product 1 and product 2 satisfy

$$\begin{aligned} p_1(t+1) &= p_1(t) + k_1 p_1(t)\frac{\partial \Pi_{m_1}}{\partial p_1} \\ p_2(t+1) &= p_2(t) + k_2 p_2(t)\frac{\partial \Pi_{m_1}}{\partial p_2} \end{aligned}$$

where $k_i > 0(1 = 1, 2)$ denotes the price adjustment speed parameter.

So the dynamical system in the decentralized decision can be described by

$$\begin{aligned} &p_1(t+1) = \\ &p_1(t) + k_1 p_1(t)[(1 + \frac{\theta+\epsilon}{2(\beta-\gamma)})\alpha + 2Ap_1 \\ &+2Bp_2 + \frac{\theta c_3 + \epsilon c_4}{2} + \frac{(\theta+\epsilon)(w_1+w_2)}{2} \\ &-(c_1+w_1)A - (c_2+w_1)B] \end{aligned}$$

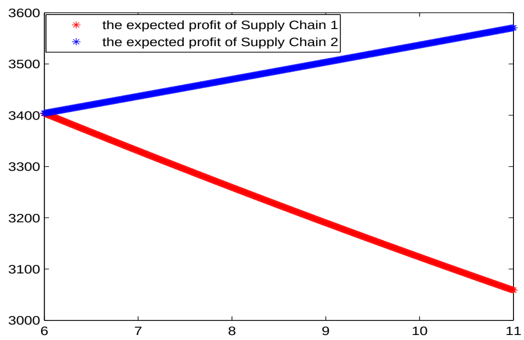

Figure 25: Combined influence on the expected profits of two supply chains in the centralized decision when $\gamma = 1.2$

$$\begin{aligned} &p_2(t+1) = \\ &p_2(t) + k_2 p_2(t)[(1 + \frac{\theta+\epsilon}{2(\beta-\gamma)})\alpha + 2Bp_1 \\ &+2Ap_2 + \frac{\theta c_4 + \epsilon c_3}{2} + \frac{(\theta+\epsilon)(w_1+w_2)}{2} \\ &-(c_1+w_1)B - (c_2+w_1)A] \\ &p_3(t) = \\ &\frac{\alpha}{2(\beta-\gamma)} + \frac{(\theta\beta+\epsilon\gamma)p_1(t) + (\epsilon\beta+\theta\gamma)p_2(t)}{2(\beta^2-\gamma^2)} \\ &+\frac{c_3+w_1+w_2}{2} \\ &p_4(t) = \\ &\frac{\alpha}{2(\beta-\gamma)} + \frac{(\epsilon\beta+\theta\gamma)p_1(t) + (\theta\beta+\epsilon\gamma)p_2(t)}{2(\beta^2-\gamma^2)} \\ &+\frac{c_4+w_1+w_2}{2} \end{aligned}$$

The relative parameters are setting as follows. Let $\alpha = 10$, $\beta = 0.5$, $\gamma = 0.12$, $\theta = 0.18$, $\epsilon = 0.13$, $c_1 = 1.2$, $c_2 = 0.9$, $c_3 = 1$, $c_4 = 0.7$, $w_1 = 1.5$, $w_2 = 1$. Inserting them into the system, the dynamical system can be rewritten as

$$\begin{cases} p_1(t+1) = p_1(t) + k_1 p_1(t)(14.91 - 0.552p_1(t) \\ +0.364p_2(t)), \\ p_2(t+1) = p_2(t) + k_2 p_2(t)(14.24 - 0.552p_2(t) \\ +0.364p_1(t)), \\ p_3(t) = 14.908 + 0.224p_1(t) + 0.184p_2(t), \\ p_4(t) = 14.758 + 0.224p_2(t) + 0.184p_1(t). \end{cases}$$

From the dynamical system, we can find that the strategy of pricing for product $i(i = 1, 2)$ is related to the price adjustment speed parameter $k_i(i = 1, 2)$. And the strategies of pricing for product 3 and product

4 are the function of p_1 and p_2 respectively. Through numerical analysis, we can find the influences of price adjustment speed parameters on the strategy of pricing.

From Fig.26 to Fig.27, we can observe the changing of four products' prices as $0 < k_1 < 0.1, k_2 = 0.05$. When $0 < k_1 < 0.058$, four prices are in a stable situation. After repeated games, the prices of them are fixed as $(p_1, p_2, p_3, p_4) = (30.6, 30.33, 27.21, 27.02)$. When $k_1 = 0.058$, the system appears the first bifurcation. Then, the dynamical system gradually enters into the state of chaos as the price adjustment speed parameter k_1 increases. Because Manufacturer 2 makes decisions on pricing depending on the strategy of Manufacturer 1, the retail prices of product 3 and product 4 are lower than product 1 and product 2's although the production cost of them are higher.

When the value of own-price sensitive coefficient β is rising from 0.5 to 0.6, we can find that the stable point of four prices is $(19.43, 19.3, 18.42, 18.27)$ in the Fig.28 and Fig.29, which are lower than before. And the dynamical system appears the first bifurcation as $k_1 = 0.084$. It means that the increasing of β makes the sensitivity of own price to demand increased, and the stable point of system is smaller and the range of stability is larger. So if the customer is more sensitive to the price of product, the manufacturer will decrease the price, and the product market will be more stable.

As the cross-sensitive coefficient γ is increasing from 0.12 to 0.15, we can observe that the stable point of price is $(36.27, 36.69, 32.18, 32.05)$ and the dynamical system occurs the first bifurcation when $k_1 = 0.0343$ in the Fig.30 and Fig.31. It means that the increasing of γ makes the competition among substitutable products serious, and the prices of four products are higher, but the range of stability is smaller. So if the similarity between products is stronger, the manufacturer will rise the product's price, which leads to the market to be less stable.

When the price adjustment speed parameter k_1 is changing, the impacts of it on the profits of two manufacturers are shown in the Fig.32. From the figure we can see that the profits of two manufacturers are the most in the stable situation. The maximum profit of Manufacturer 1 is 383.8, and Manufacturer 2's is 428.7. So the two manufacturers achieve the optimal profits in the Stackelberg Equilibrium.

5.2 The centralized decision

When Manufacturer 1 and Manufacturer 2 coordinate with their suppliers respectively, it occurs the competition between two supply chains. The two

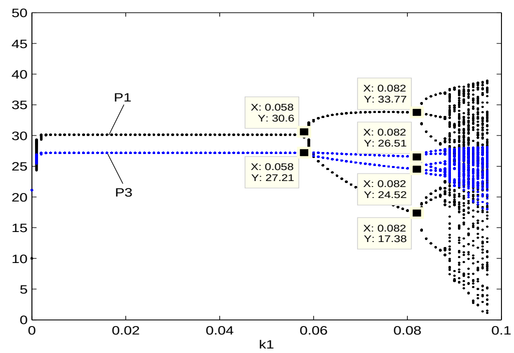

Figure 26: Influence of k_1 on p_1 and p_3 $(k_2 = 0.05)$

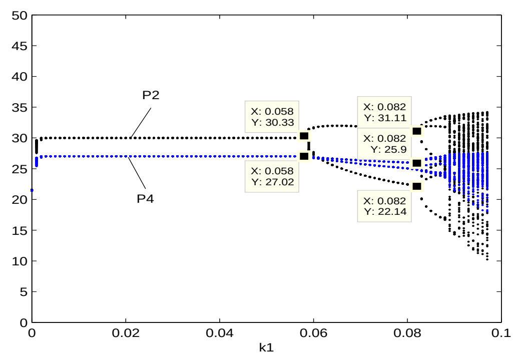

Figure 27: Influence of k_1 on p_2 and p_4 $(k_2 = 0.05)$

supply chains make decisions on the strategy of pricing for products at the same time, and the profits of two supply chains are described as follows.

$$\begin{aligned}\Pi_{s_1} &= (p_1(t) - c_1 - c_{s_1})d_1(t) \\ &\quad + (p_2(t) - c_2 - c_{s_1})d_2(t) \\ \Pi_{s_2} &= (p_3(t) - c_3 - c_{s_1} - c_{s_2})d_3(t) \\ &\quad + (p_4(t) - c_4 - c_{s_1} - c_{s_2})d_4(t)\end{aligned}$$

So the dynamical system in the centralized decision can be described by

$$\begin{cases} p_1(t+1) = p_1(t) + k_1 p_1(t)(\alpha - 2\beta p_1(t) + 2\gamma p_2(t) \\ +\theta p_3(t) + \epsilon p_4(t) + \beta(c_1 + c_{s_1}) - \gamma(c_2 + c_{s_1})) \\ p_2(t+1) = p_2(t) + k_2 p_2(t)(\alpha + 2\gamma p_1(t) - 2\beta p_2(t) \\ +\epsilon p_3(t) + \theta p_4(t) + \beta(c_2 + c_{s_1}) - \gamma(c_1 + c_{s_1})) \\ p_3(t+1) = p_3(t) + k_3 p_3(t)(\alpha + \theta p_1(t) + \epsilon p_2(t) \\ -2\beta p_3(t) + 2\gamma p_4(t) + \beta(c_3 + c_{s_1} + c_{s_2}) \\ -\gamma(c_4 + c_{s_1} + c_{s_2})) \\ p_4(t+1) = p_4(t) + k_4 p_4(t)(\alpha + \epsilon p_1(t) + \theta p_2(t) \\ +2\gamma p_3(t) - 2\beta p_4(t) + \beta(c_4 + c_{s_1} + c_{s_2}) \\ -\gamma(c_3 + c_{s_1} + c_{s_2})) \end{cases}$$

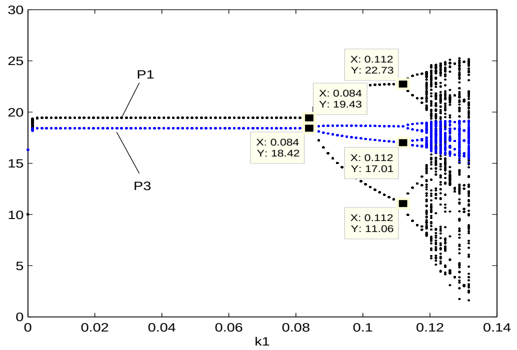

Figure 28: Influence of k_1 on p_1 and p_3 $(\beta = 0.6)$

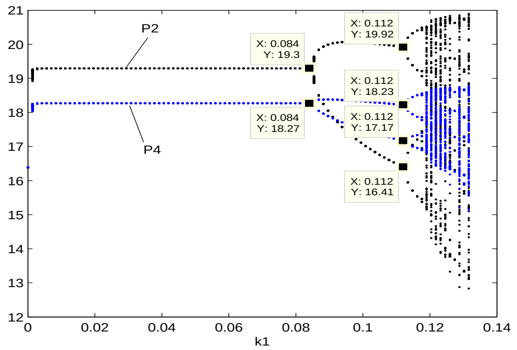

Figure 29: Influence of k_1 on p_2 and p_4 $(\beta = 0.6)$

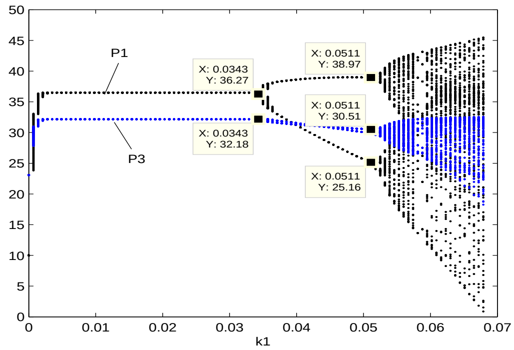

Figure 30: Influence of k_1 on p_1 and p_3 $(\gamma = 0.15)$

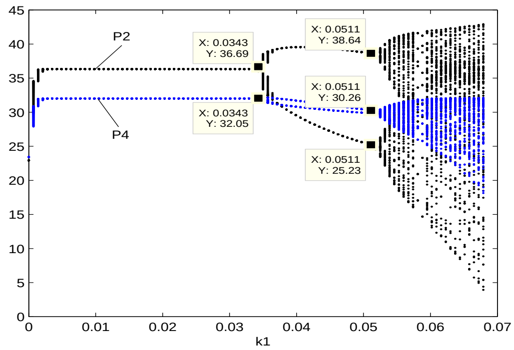

Figure 31: Influence of k_1 on p_2 and p_4 $(\gamma = 0.15)$

The relative parameters are setting as above and let $c_{s_1} = 0.5$, $c_{s_2} = 0.3$. Through numerical analysis, we can observe from Fig.32 to Fig.35 that the stable point of four prices is (23.85,23.54,23.74,23.51). And the dynamical system happens the first bifurcation as $k_1 = 0.0756$. Compared with the situation of decentralized decision, we can find that the prices of four products are smaller and tend to be the same in the centralized decision. Besides that, the range of stability is also larger and the influence of price adjustment speed parameter on the stability of system is weaken. When other price adjustment speed parameter changes, we can get similar results.

The effects of k_1 and k_3 on the profits of Supply Chain 1 and Supply Chain 2 are illustrated in the Fig.37 and Fig.38 respectively. Both of the profits of two supply chains reach the biggest in the stable situation, which are 215.9 and 214.1 respectively. In contrast to the situation in the decentralized decision, the profits are lower, but the range of stability is larger. The system is more stable in the process of repeated game between manufacturers. So the supply chain with coordination mechanism has more advantages.

6 Conclusion

This paper develops a two level supply chain which consists of two suppliers and two manufacturers. One of the suppliers is the main material supplier, and the other is the auxiliary material supplier. Based on the features of substitution, we analyze two competitive models in the decentralized and centralized decisions respectively, and we get several practical conclusions. Firstly, the own-price sensitive coefficient makes the sensitivity of product's price to demand increased, which leads to the profit, demand and price of products decreased. But the range of stability is larger, and the system is more stable in the process of repeated game between manufacturers. Secondly, when the cross-price sensitive coefficient is higher, the substitutability among products is stronger. So the manufacturers and suppliers can be more competitive in the product market and they will earn more profits. Although the pricing and demand for products are higher, the market range of stability is smaller, which makes the system entered into the state of chaos easier. Thirdly,

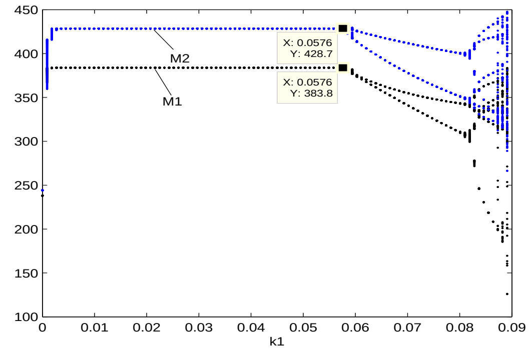

Figure 32: Influence of k_1 on the profits of Manufacturer 1 and Manufacturer 2

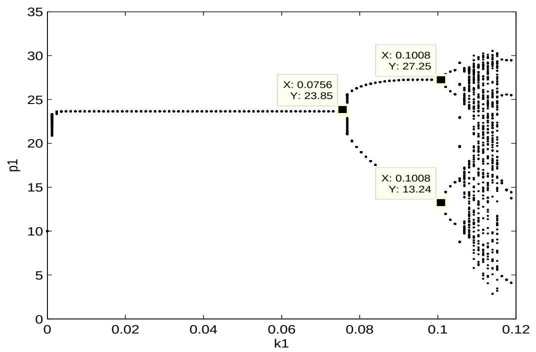

Figure 33: Influence of k_1 on p_1 ($k_2 = 0.05, k_3 = 0.06, k_4 = 0.03$)

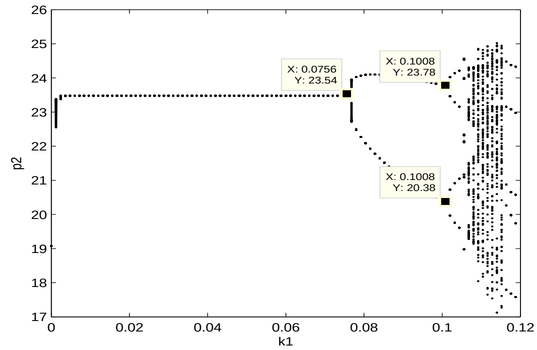

Figure 34: Influence of k_1 on p_2 ($k_2 = 0.05, k_3 = 0.06, k_4 = 0.03$)

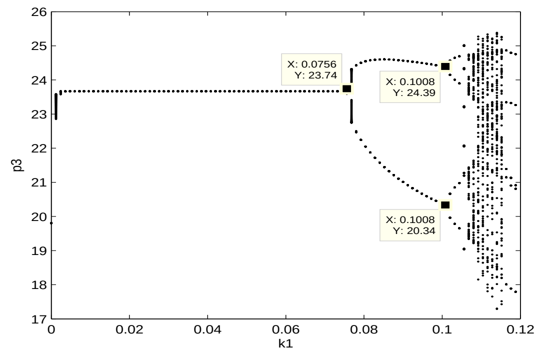

Figure 35: Influence of k_1 on p_3 ($k_2 = 0.05, k_3 = 0.06, k_4 = 0.03$)

the optimal material strategy for manufacturer is depending on the value of price sensitive coefficient in the decentralized and centralized decisions. Fourth, the increasing of cross-price sensitive coefficient can slow down the effect of production cost on the supply chain's profits. The last, the supply chain with coordination mechanism will be more stable in the market competition and perform better.

The models which are formulated in this paper have several limitations, such as the demand function of substitutable product is linear, the manufacturer knows the market demand for product exactly and so on. Future research is desirable to consider the competitive models with uncertainty demand or supply in the two decisions. And it is also meaningful to study the coordination mechanism in a multi-retailers or multi-manufacturers supply chain.

Acknowledgements: This work is supported by the National Natural Science Foundation of the Peoples Republic of China with under Contract number 61273231, and the Doctoral Scientific Fund Project of the Ministry of Education of China (No. 20130032110773).

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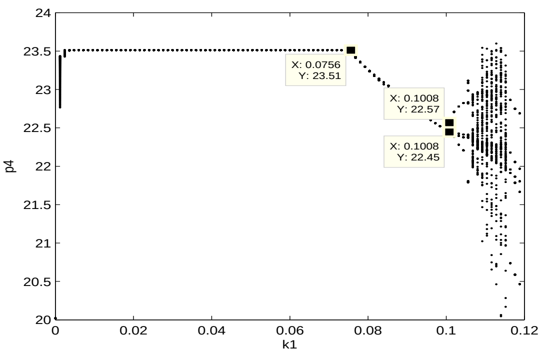

Figure 36: Influence of k_1 on p_4 $(k_2 = 0.05, k_3 = 0.06, k_4 = 0.03)$

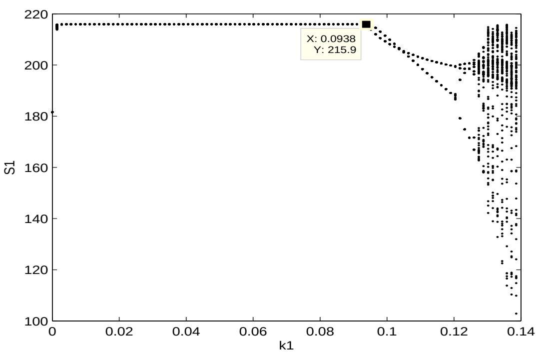

Figure 37: Influence of k_1 on the profit of Supply Chain 1

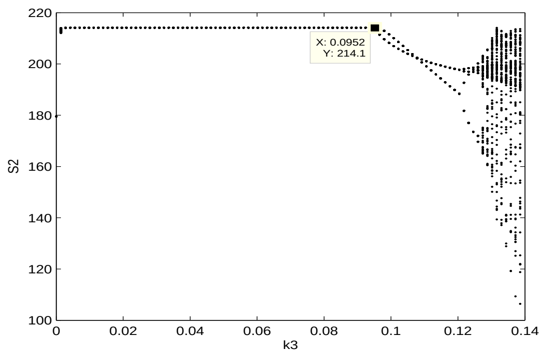

Figure 38: Influence of k_3 on the profit of Supply Chain 2

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