# Effect of Variable Viscosity on Convective Heat and Mass Transfer by Natural Convection from Vertical Surface in Porous Medium

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**Abstract: -** The aim of this paper is to investigate the effect of variable viscosity on free convective heat and mass transfer from a vertical plate embedded in a saturated porous medium. The governing equations of continuity, momentum, energy and concentration are transformed into non linear ordinary differential equations using similarity transformations and then solved by using Runge – Kutta – Gill method along with shooting technique. Governing parameters for the problem under study are the variable viscosity, the buoyancy ratio and the Lewis number. The velocity, temperature and concentration distributions are presented and discussed. The Nusselt and Sherwood number are also derived. The numerical values of local Nusselt and local Sherwood numbers have also been computed for a wide range of governing parameters. The viscous and thermal boundary layer thicknesses are discussed

***Key-Words**: -* Free convection, Heat transfer, Mass transfer, Variable viscosity, Porous medium.

## 1 Introduction

In recent years the combined heat and mass transfer by natural convection in a fluid saturated porous medium has its own role in many engineering application problems such as nuclear reactor design, geothermal systems, petroleum engineering applications, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler. A comprehensive account of the available information in this field is provided in recent books by Ingham et al [13] and Vafai[14] Kassoy [1] studied the effect of variable viscosity on the onset of convection in porous medium. Cheng and Minkowyz [2] studied the effect of free convection about a vertical plate embedded in a porous medium with application to heat transfer from a dike. Bejan and Khair [3] studied the buoyancy induced heat and mass transfer from a vertical plate embedded in a saturated porous medium. Lai and Kulacki [5] studied the coupled heat and mass transfer by natural convection from vertical surface in a porous medium. The same authors [6] also studied the effect of variable viscosity on convection heat transfer along a vertical surface in a saturated porous medium. Elbashbeshy [7] investigated the effect of steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate. Yih [8] analyzed the coupled heat and mass transfer in mixed convection about a wedge for variable wall temperature and concentration. Rami.Y.Jumah et al. [9] studied the coupled heat and mass transfer for non-Newtonian fluids. Kumari [10] analyzed the effect of variable viscosity on free and mixed convection boundary layer flow from a horizontal surface in a saturated porous medium. Postelnicu et al. [11] investigated the effect of variable viscosity on forced convection over a horizontal flat plate in a porous medium with internal heat generation. Seddeek [15, 16] studied the effects of chemical reaction, variable viscosity, and thermal diffusivity on mixed convection heat and mass transfer through porous media. Mohamed E- Ali [17] studied the effect of variable viscosity on mixed convection along a vertical plate. Alam et al [18] analyzed the study of the combined free – forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. Pantokratoras [19] analyzed the effect of variable viscosity with constant wall temperature. Seddeek et al. [20]

studied the effects of chemical reaction, variable viscosity, on hydro magnetic mixed convection heat and mass transfer through porous media. Singh et al. [21] used integral treatment to obtain the expressions for Nusselt number and Sherwood number. Recently the authors analysed the effect of variable viscosity on convective heat and mass transfer by natural from horizontal surface in porous medium.The purpose of the present work is to study the effect of variable viscosity on heat and mass transfer along a vertical surface embedded in a saturated porous medium.

## 2. Analysis

Consider a vertical surface embedded in a saturated porous medium. The properties of the fluid and porous medium are isotropic and the viscosity of the fluid is assumed to be an inverse linear function of temperature. Using Boussinesq and boundary layer approximations, the governing equations of continuity, momentum, energy and concentration are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=-\frac{\kappa}{\mu}\left(\frac{\partial p}{\partial x}+\rho g\right) \tag{2}$$

$$v=-\frac{\kappa}{\mu}\left(\frac{\partial p}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2} \tag{5}$$

$$\rho=\rho_\infty\left\{1-\beta(T-T_\infty)-\beta^*(c-c_\infty)\right\} \tag{6}$$

The viscosity of the fluid is assumed to be an inverse linear function of temperature and can be expressed as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\{1+\gamma(T-T_\infty)\} \tag{7}$$

which is reasonable for liquids such as water and oil. Here $\gamma$ is a constant.

The boundary conditions are

$$y=0,\;\; v=0,\;\; T=T_w,\;\; c=c_w \tag{8}$$

$$y\to\infty, u=0, T=T_\infty, c=c_\infty \tag{9}$$

## 3. Method of solution

Introducing the stream function $\Psi$ (x, y) such that

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x} \tag{10}$$

where

$$\psi=\alpha f(Ra_x)^{1/2} \tag{11}$$

$$\eta=\frac{y}{x}(Ra_x)^{1/2} \tag{12}$$

$Ra_x=\left\{\frac{kg\beta\Delta Tx}{\nu\alpha}\right\}$ is the Rayleigh number

Define

$$\theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{13}$$

$$\phi=\frac{c-c_\infty}{c_w-c_\infty} \tag{14}$$

and $$N=\frac{\beta^*(c_w-c_\infty)}{\beta(T_w-T_\infty)} \tag{15}$$

Substitution of these transformations (10) to (15) to equations (2) to (5) along with the equations (6) and (7), there results the non linear ordinary differential equations,

$$f''=\frac{f'\theta'}{\theta-\theta_r}-\left(\frac{\theta-\theta_r}{\theta_r}\right)(\theta'+\phi' N) \tag{16}$$

$$\theta''=-\frac{1}{2}f\theta' \tag{17}$$

$$Le^{-1}\phi''=-\frac{1}{2}f\phi' \tag{18}$$

together with the boundary conditions

$$\eta=0, f=0, \theta=1, \phi=1 \tag{19}$$

$$\eta\to\infty, f'=0, \theta=0, \phi=0 \tag{20}$$

where $Le=\frac{\alpha}{D}$ is the Lewis number and $\theta_r=-1/\gamma(T_w-T_\infty)$ is the parameter characterizing the influence of viscosity. For a given temperature difference, large values of $\theta_r$ implies either $\gamma$ or ($T_w$ -$T_\infty$) are small. In this case the effect of variable viscosity can be neglected.

The effect of variable viscosity is important if $\theta_r$ is small. Since the viscosity of liquids decreases with increasing temperature while it increases for gases, $\theta_r$ is negative for liquids and positive for gases. The concept of this parameter $\theta_r$ was first introduced by Ling and Dybbs [4] in their study of forced convection flow in porous media. The parameter N measures the relative importance of mass and thermal diffusion in the buoyancy – driven flow. It is clear that N is zero for thermal- driven flow, infinite for mass driven flow, positive for aiding flow and negative for opposing flow. It may note that in the absence of mass transfer, the equations (16) and (17) together with the boundary conditions (19) and (20) reduce to that obtained by Lai and Kulacki [5].

The heat transfer coefficient in terms of the Nusselt number is given by

$$\frac{Nu_x}{(Ra_x)^{1/2}} = -\theta'(0) \tag{21}$$

The mass transfer coefficient in terms of the Sherwood number is given by

$$\frac{Sh_x}{(Ra_x)^{1/2}} = -\phi'(0) \tag{22}$$

## Results and discussion

The equations (16), (17) and (18) are integrated numerically by using Runge – Kutta – Gill method along with shooting technique. The parameters involved in this problem are: $\theta_r$ – the variable viscosity, Le- Lewis number and N-the buoyancy parameter. To observe the effect of variable viscosity on heat and mass transfer we have plotted the velocity function f ', temperature function θ and concentration function Φ against η for various values of $\theta_r$, Le and N.

The effect of variable viscosity $\theta_r$ on velocity, temperature and concentration are shown in figures 1 and 2. It is realized that the velocity increases near the plate and decreases away from the plate as $\theta_r \to 0$ in the case of liquids ($\theta_r$ < 0) and decreases near the plate and increases away from the plate as $\theta_r \to 0$ in the case of gases ($\theta_r$ > 0 ) for given values of N and Le. From Fig.2 it is evident that the temperature and concentration increases as $\theta_r \to 0$ for $\theta_r$ > 0(i.e. for gases) and decreases as $\theta_r \to 0$ for $\theta_r$ < 0(i.e. for liquids) for other values are fixed.

The effect of buoyancy ratio N on velocity, temperature and concentration are shown in figures 3 and 4. From Fig.3 it is observed that the velocity decreases near the plate and increases away from the plate as $N \to 0$ for aiding flows (N > 0) and increases near the plate and decreases away from the plate as $N \to 0$ for opposing flows (N< 0) for given values of $\theta_r$ and Le. From Fig.4 it is realized that the temperature and concentration decreases as $N \to 0$ for opposing flows (N< 0) and increases as $N \to 0$ for aiding flows (N> 0) for given values of $\theta_r$ and Le.

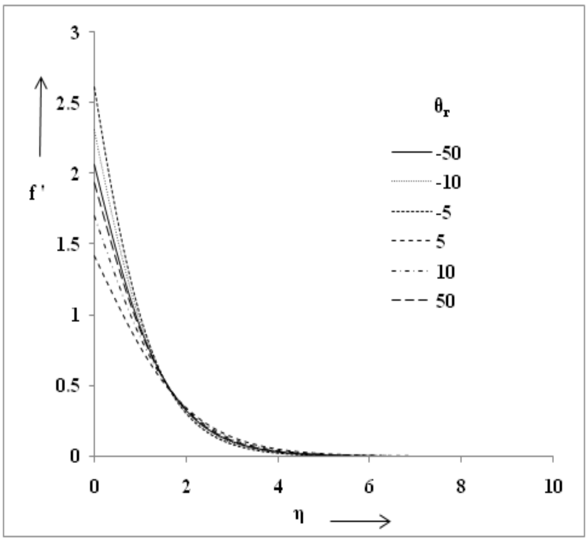


Fig.1.Velocity profiles for different values of $\theta_r$ for N =1 and Le=1

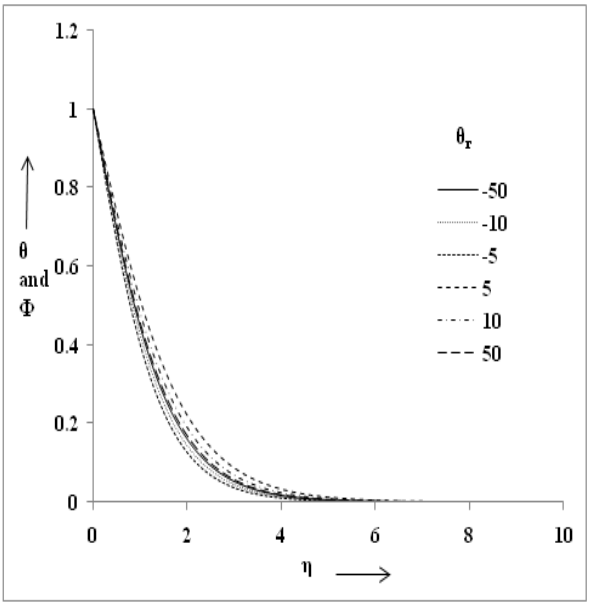


Fig.2.Temperature and Concentration profiles for different values of $\theta_r$ for N=1 and Le =1

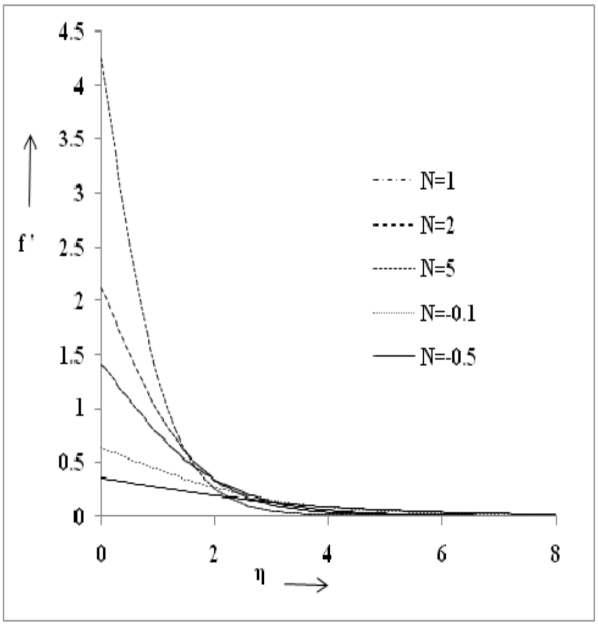


Fig. 3.Velocity profiles for different values of N for $\theta_r$ =5 and Le=1

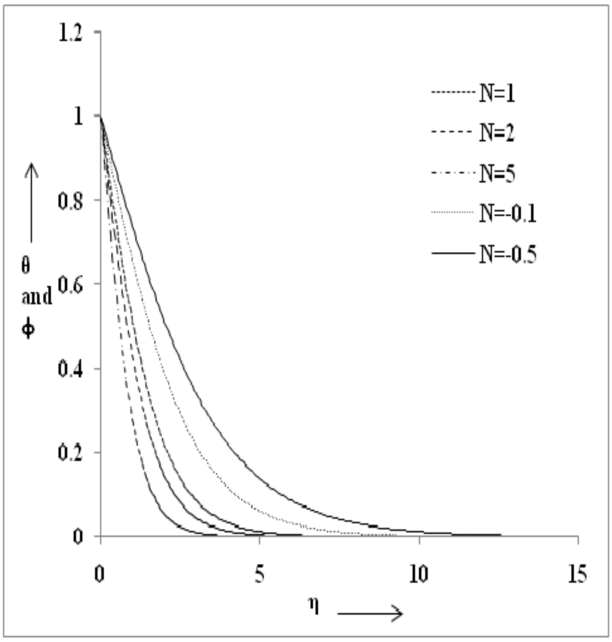


Fig.4.Temperature and Concentration profiles for different values of N for $\theta_r$ =5 and Le =1

The effect of Lewis number Le on velocity, temperature and concentration are shown in figures 5, 6 and 7. From Fig.5 it is evident that as the Lewis number Le decreases, the velocity decreases for given values of $\theta_r$ and N. From Fig.6 it is observed that as Le increases, the temperature decreases for fixed values of $\theta_r$ and N.

From Fig.7 it is realized that as the Lewis number Le decreases, the concentration decreases for fixed values of $\theta_r$ and N.

Fig.8 illustrates the effect of variable viscosity $\theta_r$ on the slip velocity f'(0) for different values of the buoyancy ratio N for given value of Le. It is observed that the slip velocity decreases as $\theta_r \to 0$ in the case of gases (for $\theta_r$ > 0) but in the cases of liquids ($\theta_r$ < 0) the slip velocity increases as $\theta_r \to 0$ for aiding (N > 0), thermal-driven (N=0) and opposing (N < 0) flows. These observations are obviously same as drawn in Table 1.

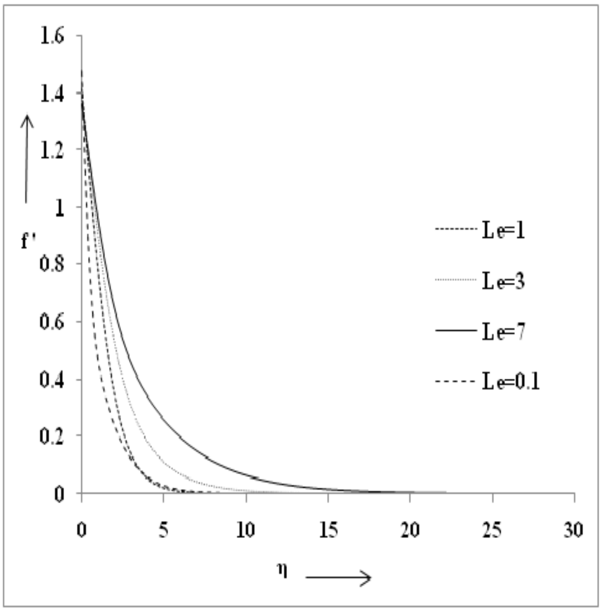


Fig. 5.Velocity profiles for different values of Le for $\theta_r$ =5 and N =1

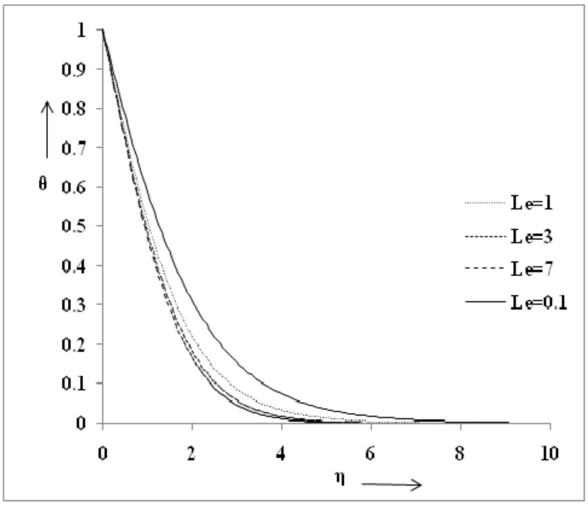


Fig. 6.Temperature profiles for different values of Le for $\theta_r$ =5 and N=1

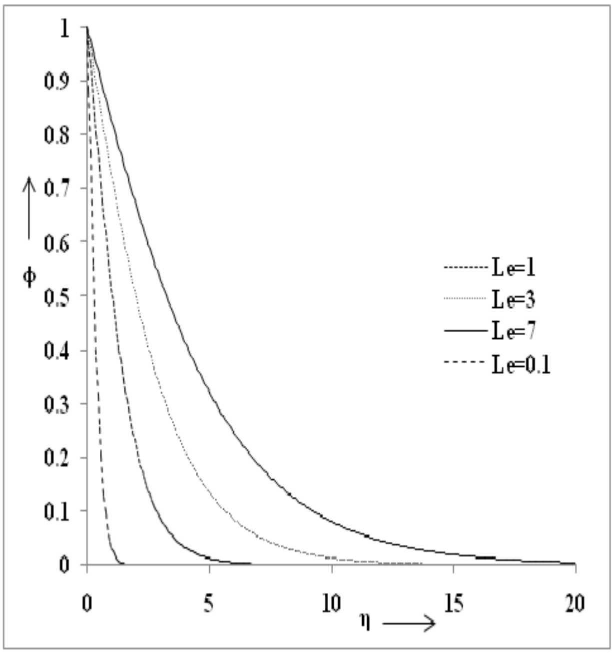


Fig.7.Concentration profiles for different values of Le for $\theta_r$ =5 and N=1

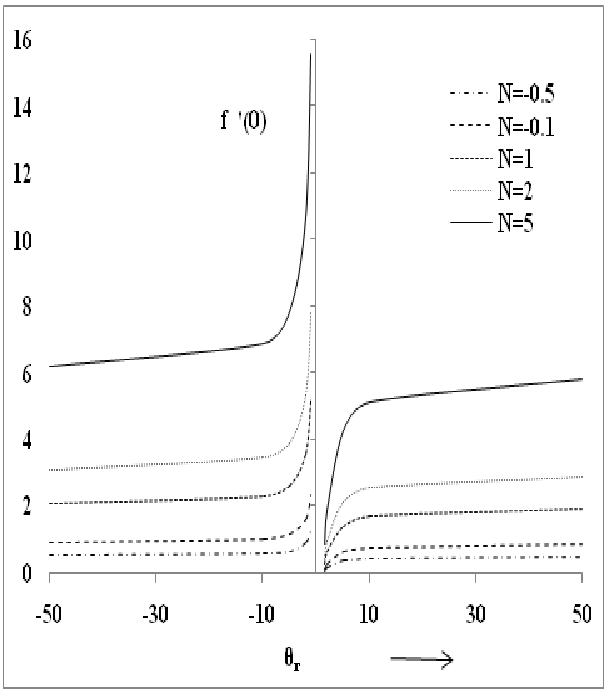


Fig. 8.Effect of variable viscosity $\theta_r$ and buoyancy ratio N on the slip velocity for Le=1

| | Slip velocity | | | | |
|---|---|---|---|---|---|
| N | -0.5 | -0.1 | 1 | 2 | 5 |
| $\theta_r$ | | | | | |
| 5 | 0.354 | 0.637 | 1.1415 | 2.1235 | 4.237 |
| 10 | 0.426 | 0.7666 | 1.7036 | 2.5554 | 5.1108 |
| 50 | 0.485 | 0.8730 | 1.9401 | 2.9102 | 5.8204 |
| -5 | 0.653 | 1.1752 | 2.6115 | 3.9173 | 7.8347 |
| -10 | 0.576 | 1.0363 | 2.303 | 3.4546 | 6.909 |
| -50 | 0.515 | 0.9271 | 2.0601 | 3.0901 | 6.1803 |
| | Nusselt and Sherwood number | | | | |
| N | -0.5 | -0.1 | 1 | 2 | 5 |
| $\theta_r$ | | | | | |
| 5 | 0.268 | 0.3602 | 0.5368 | 0.6514 | 0.9298 |
| 10 | 0.291 | 0.3914 | 0.5835 | 0.7146 | 1.0107 |
| 50 | 0.309 | 0.4152 | 0.6189 | 0.7580 | 1.0720 |
| -5 | 0.354 | 0.4755 | 0.7088 | 0.8682 | 1.2278 |
| -10 | 0.334 | 0.4489 | 0.6692 | 0.8196 | 1.1591 |
| -50 | 0.318 | 0.4266 | 0.6361 | 0.7790 | 1.1016 |

Table1. Numerical values of slip velocity, Nusselt and Sherwood numbers for different values of N and $\theta_r$ for Le =1.

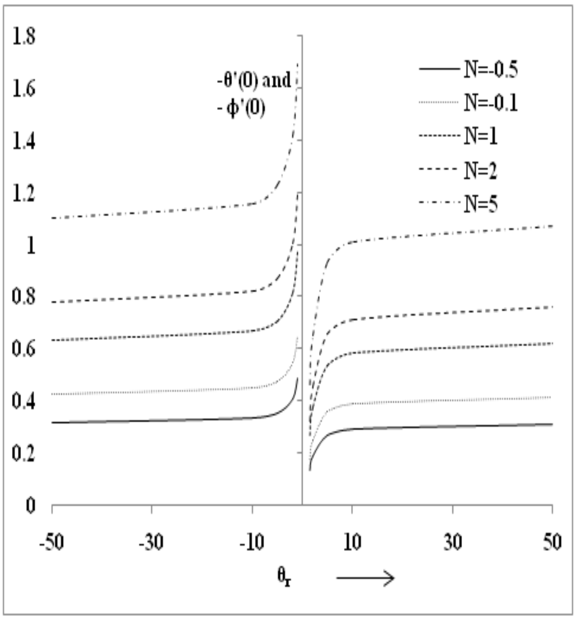


Fig.9.Effect of variable viscosity $\theta_r$ and buoyancy ratio N on heat and mass transfer rates for Le =1

Fig.9 displays the variation of local Nusselt and Sherwood numbers at the surface with variable viscosity $\theta_r$ covering aiding flows (N > 0)and opposing flows (N< 0) for fixed value of Le. It is evident that the heat transfer and mass transfer rates decreases as $\theta_r \to 0$ for $\theta_r$ > 0 and increases as $\theta_r \to 0$ for $\theta_r$ < 0 for both aiding and opposing flows. This is also evident from Table 1. Similar behaviour has also been observed for the case of horizontal plate by the same authors [22].

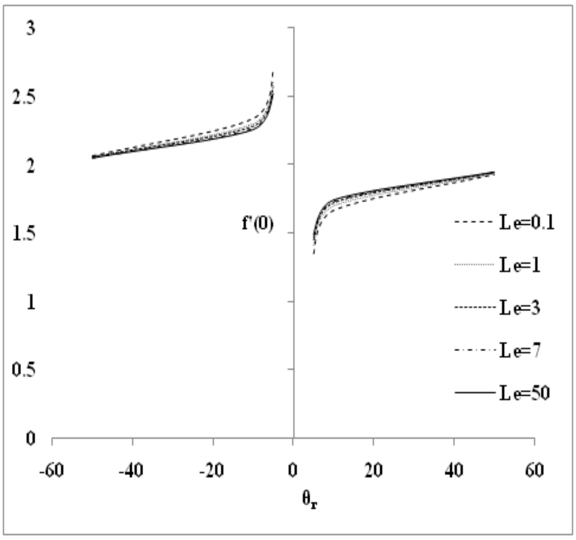


Fig.10.Effect of variable viscosity $\theta_r$ and Lewis number Le on the slip velocity for N=1

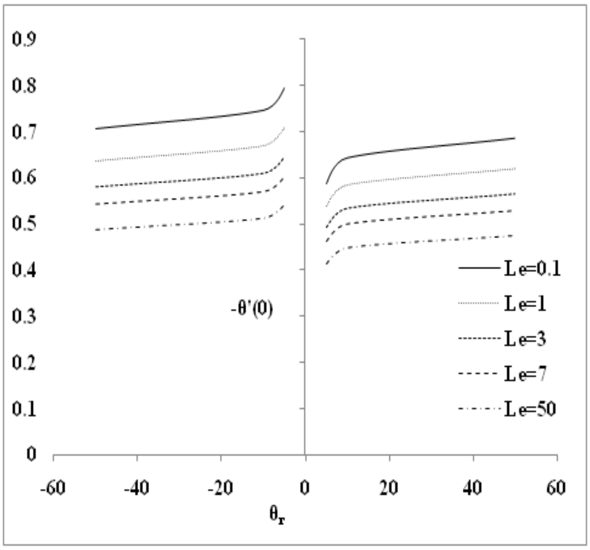


Fig.11.Effect of variable viscosity $\theta_r$ and Lewis number Le on the heat transfer rate for N=1

Fig.10 gives the effect of variable viscosity $\theta_r$ on the slip velocity f '(0) for different values of the Lewis number Le and for fixed value of N. It is observed that as Lewis number increases the slip velocity decreases for $\theta_r < 0$ and increases for $\theta_r > 0$.This observation is obviously same from Table 2.

Fig.11 gives the effect of variable viscosity $\theta_r$ on heat transfer for different values of Le and for fixed value of N. It is realized that as Lewis number increases the heat transfer decreases for both $\theta_r > 0$ and $\theta_r < 0$. This is also evident from Table 2

| | Slip velocity | | | | |
|---|---|---|---|---|---|
| Le | 0.1 | 1 | 3 | 7 | 50 |
| $\theta_r$ | | | | | |
| 5 | 1.357 | 1.415 | 1.4515 | 1.4717 | 1.4954 |
| 10 | 1.671 | 1.7036 | 1.7228 | 1.733 | 1.7455 |
| 50 | 1.933 | 1.9401 | 1.9442 | 1.946 | 1.9488 |
| -5 | 2.689 | 2.6116 | 2.5672 | 2.5439 | 2.518 |
| -10 | 2.34 | 2.3031 | 2.2818 | 2.2704 | 2.2579 |
| -50 | 2.06 | 2.06 | 2.056 | 2.053 | 2.0513 |
| | Nusselt and Sherwood number | | | | |
| Le | 0.1 | 1 | 3 | 7 | 50 |
| $\theta_r$ | | | | | |
| 5 | 0.587 | 0.536 | 0.493 | 0.4615 | 0.4135 |
| 10 | 0.643 | 0.5835 | 0.5338 | 0.4991 | 0.448 |
| 50 | 0.685 | 0.6189 | 0.5647 | 0.5276 | 0.4741 |
| -5 | 0.794 | 0.7089 | 0.6432 | 0.600 | 0.5397 |
| -10 | 0.746 | 0.6692 | 0.6086 | 0.5682 | 0.5113 |
| -50 | 0.706 | 0.6361 | 0.5796 | 0.5414 | 0.4868 |

Table 2. Numerical values of slip velocity and Nusselt number for different values of Le and $\theta_r$ for N=1.

Fig.12 gives the effect of variable viscosity $\theta_r$ on mass transfer for different values of Le and for fixed value of N .It is clearly seen that as Lewis number increases the mass transfer increases for both $\theta_r > 0$ and $\theta_r < 0$.This result is obviously same as drawn in Table 3.

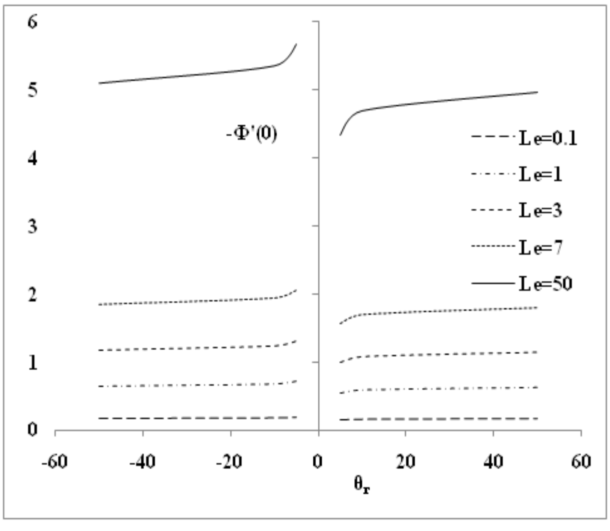


Fig.12. Effect of variable viscosity $\theta_r$ and Lewis number Le on mass transfer rate for N=1

| Le / $\theta_r$ | 0.1 | 1 | 3 | 7 | 50 |
|---|---|---|---|---|---|
| 5 | 0.143 | 0.5368 | 0.9968 | 1.5708 | 4.3301 |
| 10 | 0.154 | 0.5835 | 1.0818 | 1.7028 | 4.6881 |
| 50 | 0.163 | 0.6189 | 1.1463 | 1.8030 | 4.9605 |
| -5 | 0.185 | 0.7089 | 1.3099 | 2.0578 | 5.6656 |
| -10 | 0175 | 0.6692 | 1.2378 | 1.9454 | 5.3485 |
| -50 | 0.167 | 0.6361 | 1.1774 | 1.8514 | 5.0924 |

Table.3 Numerical values of Sherwood number for different values of Le and $\theta_r$ for N=1

The viscous and thermal boundary layer thicknesses are presented in Table 4 for different values of $\theta_r$ , N and for fixed value of Le. It is observed that the viscous and thermal boundary layer thicknesses increase as $\theta_r \to 0$ for $\theta_r > 0$ and decrease as $\theta_r \to 0$ for $\theta_r < 0$ for both aiding (N > 0) and opposing (N < 0) flows.

| | Slip velocity | | | | |
|---|---|---|---|---|---|
| N / $\theta_r$ | -0.5 | -0.1 | 1 | 2 | 5 |
| 5 | 8.500 | 7.187 | 5.625 | 4.9317 | 3.875 |
| 10 | 8.125 | 6.8750 | 5.3125 | 4.6875 | 3.6875 |
| 50 | 7.182 | 6.5625 | 5.1250 | 4.500 | 3.500 |
| -5 | 7.187 | 6.0625 | 4.6875 | 4.0625 | 3.1875 |
| -10 | 7.437 | 6.25 | 4.8750 | 4.25 | 3.3125 |
| -50 | 7.687 | 6.500 | 5.0625 | 4.3750 | 3.4375 |
| | Nusselt and Sherwood number | | | | |
| N / $\theta_r$ | -0.5 | -0.1 | 1 | 2 | 5 |
| 5 | 10.37 | 7.6875 | 5.1875 | 4.1875 | 3.0000 |
| 10 | 9.625 | 7.1250 | 4.8125 | 3.8750 | 2.7500 |
| 50 | 9.062 | 6.7500 | 4.5000 | 3.6875 | 2.6250 |
| -5 | 8.000 | 5.9375 | 4.0000 | 3.2500 | 2.3125 |
| -10 | 8.437 | 6.3125 | 4.1875 | 3.4375 | 2.4375 |
| -50 | 8.875 | 6.6250 | 4.4375 | 3.6250 | 2.5625 |

Table.4. Values of Viscous and Thermal boundary layer thicknesses for different values of N and $\theta_r$ for Le=1.

## 4. Conclusion

For coupled heat and mass transfer by natural convection in porous media, solutions have been presented for the case of vertical surface with linear temperature and concentration distribution. The governing parameters of the problem are the variable viscosity ( $\theta_r$ ), Lewis number (Le) and buoyancy ratio (N). The heat transfer and mass transfer increase as N increases for both positive and negative values of $\theta_r$ . The heat transfer increases and mass transfer decreases as Lewis number increases for both positive and negative values of $\theta_r$ . The heat transfer and mass transfer rates decreases as $\theta_r \to 0$ for $\theta_r > 0$ and increases as $\theta_r \to 0$ for $\theta_r < 0$ for both aiding and opposing flows.

**Nomenclature**

c – Concentration at any point in the flow field
$c_w$ – concentration at the wall
$c_\infty$ -concentration at the free stream
D - Mass diffusivity
f - Dimensionless velocity function
g - Acceleration due to gravity
K- Permeability
Le –Lewis number [Le = α / D]
N- Buoyancy ratio [ $N = \frac{\beta^*(c_W - c_\infty)}{\beta(T_W - T\infty)}$ ]
$Nu_x$- Nusselt number
[ $Nu_X = -x(\partial T / \partial y)_{y=0} / (T_W - T_\infty)$ ]
$Sh_{x=}$ Sherwood number
$Sh_x = mx / D(c_w - c_\infty)$
p – Pressure
$Ra_x$ –Rayleigh number $Ra_X = \left[\frac{kg\beta\Delta Tx}{\nu\alpha}\right]$
T - Temperature of the fluid
$T_w$ - Temperature of the plate
$T_\infty$ - Temperature of the fluid far from the plate
u,v – velocity components in x and y direction
x, y - Co-ordinate system

**Greek letters**

α - Thermal diffusivity
β- Coefficient of thermal expansion
$\beta^*$-Concentration expansion coefficient
γ- Constant defined in equation (7)
η - Dimensionless similarity variable
θ - Dimensionless temperature
$\theta_r$ - - 1/ γ ($T_w$ - $T_\infty$)
μ - viscosity [pas]
ν- Kinematic viscosity
ρ – Density
ϕ -Dimensionless concentration
ψ - Dimensionless stream function

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