Mean square stability of periodic solution for stochastic Cohen-Grossberg-type BAM neural networks with delays

Chunfang Miao
Shaoxing University
Department of Mathematics
shaoxing(312000)
P. R. China
miaochf@usx.edu.cn

Yunquan Ke
Shaoxing University
Department of Mathematics
shaoxing(312000)
P. R. China
keyunquan@usx.edu.cn

Abstract: In this paper, the mean square exponential stability of the periodic solution of stochastic Cohen-Grossberg-Type BAM neural networks with delays are investigated. By constructing suitable Lyapunov function, applying $It\hat{o}$ formula, integral mean-value theorem and Poincar$\acute{e}$ mapping, we give several sufficient conditions to guarantee the mean square exponential stability of the periodic solution of the system. An illustrative example is also given in the end to show the effectiveness of our results.

Key–Words: stochastic Cohen-Grossberg-type BAM neural networks; $It\hat{o}$ formula; Poincar$\acute{e}$ mapping; periodic solution; mean square exponential stability

1 Introduction

The Cohen-Grossberg-type BAM neural networks model (i.e., the BAM model which possesses Cohen-Grossberg dynamics) is initially proposed by Cohen and Grossberg [4], which have great promising potential for the tasks of parallel computation, associative memory etc. These applications heavily depend on the dynamical behaviors of system (Huang, Chen, Huang and Cao [7]; Lu and Chen [14]; Liao and Li [13]). Thus, the analysis of the dynamical behaviors of Cohen- Grossberg-type BAM neural networks are important and necessary. In recent years, many researchers have studied the global stability and other dynamical behaviors of the Cohen-Grossberg-type BAM neural networks(see Cao and song [2]; Yang and Zhang [19]; Zhou and Wan [23]; Bai [1]; Feng and Plamondon [5]; Jiang and Cao [8]). For example, Cao and Song [2] investigated the global exponential stability for Cohen-Grossberg-type BAM neural networks with time-varying delays by using Lyapunov function, M-matrix theory and inequality technique. In Feng and Plamondon [5], by constructing a suitable Lyapunov function, the asymptotic stability was investigated for Cohen-Grossberg-type BAM neural network. In [8], the authors have proposed a new Cohen-Grossberg-type BAM neural network model with time delays, and some new sufficient conditions ensuring the existence and global asymptotical stability of equilibrium point for this model have been derived.

In addition, the research of neural networks with delays involves not only the dynamic analysis of equilibrium point but also that of periodic oscillatory solution. In practice, the dynamic behavior of periodic oscillatory solution is very important in learning theory. Moreover, it is well known that an equilibrium point can be viewed as a special periodic solution of neural networks with arbitrary period. In this sense, the analysis of periodic solutions of neural networks with delays to be more general than that of equilibrium point. For example, in Chen and Cao [3]; Li and Fan [12]; Xiang and Cao [18]; Yang and Xu [20]; Zhang and Gui [21]; Li and Wang [9] have investigated the periodicity of Cohen-Grossberg-type BAM neural networks with variable coefficients.

Recently, some authors have investigated the dynamical behaviors of stochastic Cohen-Grossberg neural networks , and obtained some new results (see Song and Wang [15]; Li, Song and Fei [11]; Su and Chen [16]; Zhao and Ding [22]; Huang and Cao [6]; Wang, Guo and Xu [17]). In particular, the stability criteria for stochastic Cohen-Grossberg neural networks becomes an attractive research problem. Song and Wang [15] gave some results on stability analysis of impulsive stochastic Cohen-Grossberg-type BAM neural networks with mixed time delays. Huang and Cao [6] investigated the p-th moment exponential stability of stochastic Cohen-Grossberg neural networks with time-varying delays.

Motivated by the above discussions, a class of stochastic Cohen-Grossberg-type BAM neural networks with time delay is considered in this paper. We will derive some sufficient conditions of the mean square exponential stability of the periodic solution for stochastic Cohen-Grossberg-type BAM neutral networks with time delays, by constructing suitable Lyapunov function, applying $It\hat{o}$ formula, integral mean-value theorem and Poincaré mapping.

The rest of this paper is organized as follows: In Section 2, the model formulation and some preliminaries are given. The main results are stated in Section 3. Finally, an illustrative example is given to show the effectiveness of the proposed theory.

Consider the following stochastic Cohen-Grossberg-type BAM neural networks

$$\begin{cases} du_i(t) = -a_i(u_i(t))\{[b_i(u_i(t)) - \sum\limits_{j=1}^{m} a_{ij} f_j(v_j(t)) \\ - \sum\limits_{j=1}^{m} c_{ij} f_j(v_j(t-\tau_{ij})) - I_i(t)]dt \\ - \sum\limits_{j=1}^{m} k_{ij}(v_j(t)) d\omega_{n+j}(t)\}, \\ dv_j(t) = -d_j(v_j(t))\{[e_j(v_j(t)) - \sum\limits_{i=1}^{n} b_{ji} g_i(u_i(t)) \\ - \sum\limits_{i=1}^{n} h_{ji} g_i(u_i(t-\sigma_{ji})) - J_j(t)]dt \\ - \sum\limits_{i=1}^{n} \rho_{ji}(u_i(t)) d\omega_i(t)\}, \end{cases} \tag{1}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, where $u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T \in \mathbb{R}^n, v(t) = (v_1(t), v_2(t), \cdots, v_m(t))^T \in \mathbb{R}^m$, $u_i(t)$ and $v_j(t)$ are the state of the ith neurons from the neural field F_U and the jth neurons from the neural field F_V at time t , respectively; f_j and g_i denote the activation function of the jth neurons and the ith neurons at time t, respectively ; a_{ij} and c_{ij} weight the strength of the ith neuron on the jth neuron at the time t and $t - \tau_{ij}$, respectively; b_{ji} and h_{ji} weight the strength of the jth neuron on the ith neuron at the time t and $t - \sigma_{ji}$, respectively; $\tau_{ij} \geq 0$ and $\sigma_{ji} \geq 0$ are nonnegative; $I_i(t), J_j(t)$ denote the external inputs on the ith neuron from F_U and the jth neuron from F_V at time t , respectively; $a_i(u_i(t))$ and $d_j(v_j(t))$ represent amplification functions; $b_i(u_i(t))$ and $e_j(v_j(t))$ are appropriately behaved functions such that the solutions of model(1) remain bounded; $k(\cdot) = (k_{ij}(\cdot))_{n\times m}$ and $\rho(\cdot) = (\rho_{ji}(\cdot))_{m\times n}$ denote the diffusion coefficient; $\omega(t) = (\omega_1(t), \omega_2(t) \cdots, \omega_{n+m}(t))^T$ is an $n+m$ -dimensional Brownian motion defined on a complete probability space (Ω, F, P) with a natural filtration $\{F_t\}_{t\geq 0}$ generated by $\{(s) : 0 \leq s \leq t\}$, where we associate Ω with the canonical space generated by all $\{\omega_i(t)\}$, and denote by F the associated $\sigma-$ algebra generated by $\{\omega(t)\}$ with the probability measure P.

The initial conditions of system (1) are given by

$$\begin{cases} u_i(s) = \phi_{ui}(s), s \in [-\sigma, 0], \\ v_j(s) = \phi_{vj}(s), s \in [-\tau, 0], \end{cases} \tag{2}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, where

$$\sigma = \max_{1\leq i\leq n, 1\leq j\leq m} \{\sigma_{ji}\},$$

$$\tau = \max_{1\leq i\leq n, 1\leq j\leq m} \{\tau_{ij}\},$$

$\phi_{ui}(s)$ and $\phi_{vj}(s)$ are bounded and continuous on $[-\delta, 0]$, $\delta = \max\{\sigma, \tau\}$.

Consider a general stochastic system

$$\begin{cases} dx(t) = f(t, x(t))dt + g(t, x(t))dw(t), t \geq t_0 > 0, \\ x(t_0) = x_0, \end{cases} \tag{3}$$

where $f : \mathbb{R}^+ \times \mathbb{R}^n \longrightarrow \mathbb{R}^n, g : \mathbb{R}^+ \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$. Let $C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$ be the family of all nonnegative functions $V(t, x)$ on $\mathbb{R}^+ \times \mathbb{R}^n$ which are continuously twice differentiable in x and differentiable in t. If $V(t, x) \in C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$, an operator $LV(t, x)$ is defined from $\mathbb{R}^+ \times \mathbb{R}^n$ to $\mathbb{R}$ by

$$LV(t, x) = V_t(t, x) + V_x(t, x)f(t, x) \\ + \frac{1}{2} trace[g^T(t, x)V_{xx}(t, x)g(t, x)],$$

where $V_t(t, x) = \frac{\partial V(t,x)}{\partial t}$,

$$V_x(t, x) = (\frac{\partial V(t, x)}{\partial x_1}, \frac{\partial V(t, x)}{\partial x_2}, \cdots, \frac{\partial V(t, x)}{\partial x_n}),$$

and $V_{xx}(t, x) = (\frac{\partial^2 V(t,x)}{\partial xi \partial x_j})_{n\times n}$.

Applying $It\hat{o}$ formula, we have

$$dV(t, x) = LV(t, x)dt + V_x(t, x)g(t, x)dw(t).$$

2 Preliminaries

In order to establish the stability conditions for system (1), we give some assumptions.

- (H_1) : For each $i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$, functions $a_i(z), d_j(z)$ are continuously bounded and satisfy $a_i(z) > 0, d_j(z) > 0$, this is, there exist constants $\underline{a}_i, \bar{a}_i, \underline{d}_j, \bar{d}_j$, such that $\underline{a}_i \leq a_i(z) \leq \bar{a}_i$, $\underline{d}_j \leq d_j(z) \leq \bar{d}_j$ for all $z \in \mathbb{R} = (-\infty, +\infty)$;

  For $b_i(z)$ and $e_j(z)$, there exist $\beta_i > 0$ and $\gamma_j > 0$, such that

$$(z - y)[b_i(z) - b_i(y)] \geq \beta_i(z - y)^2;$$

$$(z-y)[e_j(z)-e_j(y)] \geq \gamma_j (z-y)^2, z, y \in \mathbb{R}.$$

The activation functions f_j and g_i satisfy Lipschitz condition, that is, there exist constant $F_j > 0$ and $G_i > 0$, such that

$$|f_j(\xi_1) - f_j(\xi_2)| \leq F_j|\xi_1 - \xi_2|,$$

$$|g_i(\xi_1) - g_i(\xi_2)| \leq G_i|\xi_1 - \xi_2|,$$

for any $\xi_1, \xi_2 \in \mathbb{R}$.

Functions $k_{ij}(\cdot)$ and $\rho_{ji}(\cdot)$ satisfy Lipschitz condition, that is, there exist constant $L_{ij} > 0$ and $T_{ji} > 0$, such that

$$|k_{ij}(\xi_1) - k_{ij}(\xi_2)| \leq L_{ij}|\xi_1 - \xi_2|,$$

$$|\rho_{ji}(\xi_1) - \rho_{ji}(\xi_2)| \leq T_{ji}|\xi_1 - \xi_2|,$$

for any $\xi_1, \xi_2 \in \mathbb{R}$.

- (H_2) : $I_i(t), J_j(t), \omega_i(t)$ and $\omega_{n+j}(t)$ are continuously periodic functions defined on $t \in [0, \infty)$ with common period $\omega > 0$, and they are all bounded, denote

$$I_i^* = \sup_{0 \leq t < \infty} |I_i(t)|,$$

$$J_j^* = \sup_{0 \leq t < \infty} |J_j(t)|,$$

where $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

- (H_3) : $b_i(0) > 0, e_j(0) > 0, k_{ij}(0) = 0, \rho_{ji}(0) = 0$ and $-b_i(0) + \sum\limits_{j=1}^{m} (a_{ij} + c_{ij}) f_j(0) + I_i^* = 0$,
$-e_j(0) + \sum\limits_{i=1}^{n} (b_{ji} + h_{ji}) g_i(0) + J_j^* = 0$,
$i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

For the deterministic system

$$\begin{cases} du_i(t) = -a_i(u_i(t))[b_i(u_i(t)) - \sum\limits_{j=1}^{m} a_{ij} f_j(v_j(t)) \\ \qquad - \sum\limits_{j=1}^{m} c_{ij} f_j(v_j(t-\tau_{ij})) - I_i]dt, \\ dv_j(t) = -d_j(v_j(t))[e_j(v_j(t)) - \sum\limits_{i=1}^{n} b_{ji} g_i(u_i(t)) \\ \qquad - \sum\limits_{i=1}^{n} h_{ji} g_i(u_i(t-\sigma_{ji})) - J_j]dt, \end{cases} \tag{4}$$

where I_i and J_j are constants, we can prove that under the assumptions(H_1), system (4) has equilibrium point$(u_1^*, u_2^*, \cdots, u_n^*, v_1^*, v_2^*, \cdots, v_m^*)^T$.

It is interesting for us to know how the stochastic perturbation affects the stability property of system (1). To our aim, we assume also that

- (H_4) : $k_{ij}(v_j^*) = 0, \rho_{ji}(u_i^*) = 0, i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$.

Under hypotheses (H_4) and I_i, J_j are constants then system (1) admits an equilibrium point $(u_1^*, u_2^*, \cdots, u_n^*, v_1^*, v_2^*, \cdots, v_m^*)^T$.
We let

$$u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T,$$

$$v(t) = (v_1(t), v_2(t), \cdots, v_m(t))^T,$$

$$u^* = (u_1^*, u_2^*, \cdots, u_n^*)^T,$$

$$v^* = (v_1^*, v_2^*, \cdots, v_m^*)^T.$$

Definition 1 *For system (1), assume I_i, J_j are constants, the point $(u^{*T}, v^{*T})^T$ is called a equilibrium point of system (1), if it satisfies the following equations*

$$\begin{cases} -b_i(u_i^*) + \sum\limits_{j=1}^{m} a_{ij} f_j(v_j^*) + \sum\limits_{j=1}^{m} c_{ij} f_j(v_j^*) + I_i = 0, \\ -e_j(v_j^*) + \sum\limits_{i=1}^{n} b_{ji} g_i(u_i^*) + \sum\limits_{i=1}^{n} h_{ji} g_i(u_i^*) + J_j = 0, \end{cases} \tag{5}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

Definition 2 *Let $(u^{*T}, v^{*T})^T$ be the equilibrium point of system (1), we define the norm*

$$\|\phi_u - u^*\|^2 = \sup_{-\sigma \leq t \leq 0} \sum_{i=1}^{n} |\phi_{ui}(t) - u_i^*|^2,$$

$$\|\phi_v - v^*\|^2 = \sup_{-\tau \leq t \leq 0} \sum_{j=1}^{m} |\phi_{vj}(t) - v_j^*|^2,$$

$$\|u\|^2 = \sum_{i=1}^{n} |u_i(t)|^2, \quad \|v\|^2 = \sum_{j=1}^{m} |v_i(t)|^2,$$

where $\phi_u = (\phi_{u1}, \phi_{u2}, \cdots, \phi_{un})^T$ and $\phi_v = (\phi_{v1}, \phi_{v2}, \cdots, \phi_{vm})^T$ are initial values.

Definition 3 *The equilibrium point $(u^{*T}, v^{*T})^T$ of system (1) is said to be mean square exponentially stable, if there exist constants $\alpha > 0$ and $M > 1$ such that*

$$\sum_{i=1}^{n} E(|u_i(t) - u_i^*|^2) + \sum_{j=1}^{m} E(|v_j(t) - v_j^*|^2) \leq Me^{-\alpha t}[E(\|\phi_u - u^*\|^2) + E(\|\phi_v - v^*\|^2)],$$

for all $t > 0$.
where $E(\cdot)$ denote mathematical expectation, $(u(t)^T, v(t)^T)^T = (u_1(t), u_2(t), \cdots, u_n(t), v_1(t), v_2(t), \cdots, v_m(t))^T$ is any solution of system (1) with initial conditions (2).

Definition 4 *Let* $Z^*(t) = (u_1^*(t), u_2^*(t), \cdots, u_n^*(t), v_1^*(t), v_2^*(t), \cdots, v_m^*(t))^T$ *be an* $\omega-$ *periodic solution of system (1) with initial value* $\psi = (\psi_{u1}(t), \psi_{u2}(t), \cdots, \psi_{un}(t), \psi_{v1}(t), \psi_{v2}(t), \cdots, \psi_{vm}(t))^T$. *Let* $Z(t) = (u_1(t), \cdots, u_n(t), v_1(t), \cdots, v_m(t))^T$ *be solution of system (1) with initial value* $\phi = (\phi_{u1}(t), \cdots, \phi_{un}(t), \phi_{v1}(t), \cdots, \phi_{vm}(t))^T$.
If there exist constants $\alpha > 0$ *and* $M > 1$, *such that*
$\sum_{i=1}^{n} E(|u_i(t) - u_i^*(t)|^2) + \sum_{j=1}^{m} E(|v_j(t) - v_j^*(t)|^2) \le Me^{-\alpha t}[E(\|\phi_u - \psi_u\|^2) + E(\|\phi_v - \psi_v\|^2)], t > 0$,
then $Z^*(t)$ *is said to be exponentially stable in the mean square, where*

$$E(\|\phi_u - \psi_u\|^2)) = \sup_{-\sigma \le t \le 0} \sum_{i=1}^{n} E(|\phi_{ui}(t) - \psi_{ui}(t)|^2),$$

$$E(\|\phi_v - \psi_v\|^2) = \sup_{-\tau \le t \le 0} \sum_{j=1}^{m} E(|\phi_{vj}(t) - \psi_{vj}|^2).$$

Lemma 5 *Assume that*

$$-2\underline{a}_i\beta_i + \sum_{j=1}^{m}[\bar{d}_j^2 T_{ji}^2 + \bar{a}_i|a_{ij}|F_j + \bar{a}_i|c_{ij}| + |b_{ji}|\bar{d}_j G_i + |h_{ji}|\bar{d}_j G_i^2] < 0, \quad (6)$$

$$-2\underline{d}_j\gamma_j + \sum_{i=1}^{n}[\bar{a}_i^2 L_{ij}^2 + |a_{ij}|\bar{a}_i F_j + |c_{ij}|\bar{a}_i F_j^2 + |b_{ji}|\bar{d}_j G_i + |h_{ji}|\bar{d}_j] < 0, \quad (7)$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, *then there exists* $\alpha > 0$, *such that*

$$\alpha - 2\underline{a}_i\beta_i + \sum_{j=1}^{m}[\bar{d}_j^2 T_{ji}^2 + \bar{a}_i|a_{ij}|F_j + \bar{a}_i|c_{ij}| + |b_{ji}|\bar{d}_j G_i + e^{\alpha\sigma}|h_{ji}|\bar{d}_j G_i^2] \le 0,$$

$$\alpha - 2\underline{d}_j\gamma_j + \sum_{i=1}^{n}[\bar{a}_i^2 L_{ij}^2 + |a_{ij}|\bar{a}_i F_j + e^{\alpha\tau}\sum_{i=1}^{n}|c_{ij}|\bar{a}_i F_j^2 + |b_{ji}|\bar{d}_j G_i + |h_{ji}|\bar{d}_j] \le 0,$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

Proof. Let

$$\varphi_i(\alpha) = \alpha - 2\underline{a}_i\beta_i + \sum_{j=1}^{m}[\bar{d}_j^2 T_{ji}^2 + \bar{a}_i|a_{ij}|F_j + \bar{a}_i|c_{ij}| + |b_{ji}|\bar{d}_j G_i + e^{\alpha\sigma}|h_{ji}|\bar{d}_j G_i^2],$$

$$\psi_j(\alpha) = \alpha - 2\underline{d}_j\gamma_j + \sum_{i=1}^{n}[\bar{a}_i^2 L_{ij}^2 + |a_{ij}|\bar{a}_i F_j + e^{\alpha\tau}|c_{ij}|\bar{a}_i F_j^2 + |b_{ji}|\bar{d}_j G_i + |h_{ji}|\bar{d}_j].$$

Obviously,

$$\frac{d\varphi_i(\alpha)}{d\alpha} > 0, \quad \lim_{\alpha\to+\infty}\varphi_i(\alpha) = +\infty, \quad \varphi_i(0) < 0,$$

$$\frac{d\psi_j(\alpha)}{d\alpha} > 0, \quad \lim_{\alpha\to+\infty}\psi_j(\alpha) = +\infty, \quad \psi_j(0) < 0,$$

$$i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m.$$

Therefore, there exist constants α_i, $\alpha_j^* \in (0, +\infty)$, such that

$$\varphi_i(\alpha_i) = 0, \quad \psi_j(\alpha_j^*) = 0,$$
$$i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m.$$

We choose

$$\alpha = min\{\alpha_1, \alpha_2, \ldots, \alpha_n, \alpha_1^*, \alpha_2^*, \ldots, \alpha_m^*\},$$

then $\alpha > 0$ and it satisfies that

$$\varphi_i(\alpha) \le 0, \quad \psi_j(\alpha) \le 0,$$
$$i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m,$$

which means there exists constant $\alpha > 0$, such that

$$\alpha - 2\underline{a}_i\beta_i + \sum_{j=1}^{m}[\bar{d}_j^2 T_{ji}^2 + \bar{a}_i|a_{ij}|F_j + \bar{a}_i|c_{ij}| + |b_{ji}|\bar{d}_j G_i + e^{\alpha\sigma}|h_{ji}|\bar{d}_j G_i^2] \le 0,$$

$$\alpha - 2\underline{d}_j\gamma_j + \sum_{i=1}^{n}[\bar{a}_i^2 L_{ij}^2 + |a_{ij}|\bar{a}_i F_j + e^{\alpha\tau}|c_{ij}|\bar{a}_i F_j^2 + |b_{ji}|\bar{d}_j G_i + |h_{ji}|\bar{d}_j] \le 0,$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

Lemma 6 *(Li and Max [9]) If there exists positive definite function* $V(t, x)$ *on* $\mathbb{R}^+ \times \mathbb{R}^{n+m}$, *the solution* $x(t)$ *of system (1) is stochastic bounded, if it satisfies*
(a) $V(t, x)$ *is continuously twice differentiable in* x *and differentiable in* t;
(b) $LV(t, x) \le 0$, *and* $\lim_{\|x\|\to\infty} V(t, x) = \infty$, *for any* $t \in \mathbb{R}^+$.

Lemma 7 *Under hypotheses* (H_1) *and* (H_3), *then the solution* $x(t)$ *of system (1) is stochastic bounded, if (6) and (7) in Lemma 5 hold.*

Proof. We consider the following Lyapunov function

$$V_1(t, u(t)) = e^{\alpha t}\sum_{i=1}^{n} u_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i\int_{t-\tau_{ij}}^{t} e^{\alpha(s+\tau_{ij})}[f_j(v_j(s) - f_j(0)]^2 ds, \quad (8)$$

$$V_2(t, v(t)) = e^{\alpha t}\sum_{j=1}^{m} v_j^2(t) + \sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_j\int_{t-\sigma_{ji}}^{t} e^{\alpha(s+\sigma_{ji})}[g_i(u_i(s)) - g_i(0)]^2 ds, \quad (9)$$

where α is given by Lemma 5

Applying $It\hat{o}'s$ formula to $V_1(t, u(t))$, we have

$$LV_1(t, u(t)) = \alpha e^{\alpha t}\sum_{i=1}^{n} u_i^2(t) + 2e^{\alpha t}\sum_{i=1}^{n} u_i(t)\{-a_i(u_i(t))[b_i(u_i(t)) - \sum_{j=1}^{m} a_{ij}f_j(v_j(t)) - \sum_{j=1}^{m} c_{ij}f_j(v_j(t-\tau_{ij})) - I_i(t)]\}$$

$$+e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}(a_i(u_i(t))k_{ij}(v_j(t)))^2+\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i [e^{\alpha(t+\tau_{ij})}(f_j(v_j(t)-f_j(0))^2 -e^{\alpha t}(f_j(v_j(t-\tau_{ij}))-f_j(0))^2]$$

$$=\alpha e^{\alpha t}\sum_{i=1}^{n}u_i^2(t)+2e^{\alpha t}\sum_{i=1}^{n}u_i(t)\{-a_i(u_i(t))[(b_i(u_i(t)) -b_i(0))-\sum_{j=1}^{m}a_{ij}(f_j(v_j(t))-f_j(0)) -\sum_{j=1}^{m}c_{ij}(f_j(v_j(t-\tau_{ij}))-f_j(0))-I_i(t)]\}$$
$$+e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}(a_i(u_i(t))k_{ij}(v_j(t)))^2 +\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i[e^{\alpha(t+\tau_{ij})}(f_j(v_j(t)-f_j(0))^2 -e^{\alpha t}(f_j(v_j(t-\tau_{ij}))-f_j(0))^2] +2e^{\alpha t}\sum_{i=1}^{n}u_i(t)a_i(u_i(t))\{-b_i(0)+\sum_{i=1}^{n}[a_{ij}+c_{ij}]f_j(0)\}$$

$$\le \alpha e^{\alpha t}\sum_{i=1}^{n}u_i^2(t)+2e^{\alpha t}\sum_{i=1}^{n}\{-\underline{a}_i\beta_i u_i^2(t) +\bar{a}_i\sum_{j=1}^{m}|a_{ij}|F_j|u_i(t)||v_j(t)|+\bar{a}_i\sum_{j=1}^{m}|c_{ij}||u_i(t)| \cdot|f_j(v_j(t-\tau_{ij}))-f_j(0)|\}+e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}\bar{a}_i^2L_{ij}^2|v_j(t)|^2$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i[e^{\alpha(t+\tau_{ij})}(f_j(v_j(t)-f_j(0))^2 -e^{\alpha t}(f_j(v_j(t-\tau_{ij}))-f_j(0))^2]+2e^{\alpha t}\sum_{i=1}^{n}u_i(t)a_i(u_i(t)) \{-b_i(0)+\sum_{j=1}^{m}[a_{ij}+c_{ij}]f_j(0)+I_i^*\}$$

$$\le \alpha e^{\alpha t}\sum_{i=1}^{n}u_i^2(t)+e^{\alpha t}\sum_{i=1}^{n}\{-2\underline{a}_i\beta_i u_i^2(t) +\bar{a}_i\sum_{j=1}^{m}|a_{ij}|F_j[|u_i(t)|^2+|v_j(t)|^2] +\bar{a}_i\sum_{j=1}^{m}|c_{ij}|[|u_i(t)|^2+|f_j(v_j(t-\tau_{ij}))-f_j(0)|^2]\}$$
$$+e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}\bar{a}_i^2L_{ij}^2|v_j(t)|^2+\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i [e^{\alpha(t+\tau_{ij})}F_j^2|v_j(t)|^2-e^{\alpha t}(f_j(v_j(t-\tau_{ij}))-f_j(0))^2] +2e^{\alpha t}\sum_{i=1}^{n}u_i(t)a_i(u_i(t))\{-b_i(0)+\sum_{j=1}^{m}[a_{ij}+c_{ij}]f_j(0)+I_i^*\}$$

$$=e^{\alpha t}\sum_{i=1}^{n}\{\alpha-2\underline{a}_i\beta_i+\bar{a}_i\sum_{j=1}^{m}|a_{ij}|F_j+\bar{a}_i\sum_{j=1}^{m}|c_{ij}|\}|u_i(t)|^2 +e^{\alpha t}\sum_{j=1}^{m}\{\sum_{i=1}^{n}\bar{a}_i|a_{ij}|F_j+e^{\alpha\tau}\sum_{i=1}^{n}\bar{a}_i|c_{ij}|F_j^2 +\sum_{i=1}^{n}\bar{a}_i^2L_{ij}^2\}|v_j(t)|^2+2e^{\alpha t}\sum_{i=1}^{n}u_i(t)a_i(u_i(t))\{-b_i(0) +\sum_{j=1}^{m}[a_{ij}+c_{ij}]f_j(0)+I_i^*\}. \quad (10)$$

Applying $It\hat{o}'s$ formula to $V_2(t,v(t))$, similarly, we also get

$$LV_2(t,v(t)) \le e^{\alpha t}\sum_{j=1}^{m}\{\alpha-2\underline{d}_j\gamma_j+\bar{d}_j\sum_{i=1}^{n}|b_{ji}|G_i+\bar{d}_j\sum_{i=1}^{n} |h_{ji}|\}|v_j(t)|^2+e^{\alpha t}\sum_{i=1}^{n}\{\sum_{j=1}^{m}\bar{d}_j|b_{ji}|G_i+e^{\alpha\tau}\sum_{j=1}^{m} \bar{d}_j|h_{ji}|G_i^2+\sum_{j=1}^{m}\bar{d}_j^2T_{ji}^2\}|u_i(t)|^2+2e^{\alpha t}\sum_{j=1}^{m}v_j(t) d_j(v_j(t))\{-e_j(0)+\sum_{i=1}^{n}[b_{ji}+h_{ji}]g_i(0)+J_j^*\}. \quad (11)$$

Let

$$V(t,u(t),v(t))=V_1(t,u(t))+V_2(t,v(t)),$$

applying $It\hat{o}'s$ formula to $V(t,u(t),v(t))$, from (10)-(11), we can obtain

$$LV(t,u(t),v(t)) \le e^{\alpha t}\sum_{i=1}^{n}\{\alpha-2\underline{a}_i\beta_i+\sum_{j=1}^{m}\bar{d}_j^2T_{ji}^2+\bar{a}_i\sum_{j=1}^{m}|a_{ij}|F_j +\bar{a}_i\sum_{j=1}^{m}|c_{ij}|+\sum_{j=1}^{m}|b_{ji}|\bar{d}_jG_i+e^{\alpha\sigma}\sum_{j=1}^{m}|h_{ji}|\bar{d}_jG_i^2\}u_i^2(t)$$
$$+e^{\alpha t}\sum_{j=1}^{m}\{\alpha-2\underline{d}_j\gamma_j+\sum_{i=1}^{n}\bar{a}_i^2L_{ij}^2+\sum_{i=1}^{n}|a_{ij}|\bar{a}_iF_j+ e^{\alpha\tau}\sum_{i=1}^{n}|c_{ij}|\bar{a}_iF_j^2+\sum_{i=1}^{n}|b_{ji}|\bar{d}_jG_i+\sum_{i=1}^{n}|h_{ji}|\bar{d}_j\}v_j^2(t)+ 2e^{\alpha t}\sum_{i=1}^{n}u_i(t)a_i(u_i(t))\{-b_i(0)+\sum_{j=1}^{m}[a_{ij}+c_{ij}]f_j(0)+ I_i^*\}+2e^{\alpha t}\sum_{j=1}^{m}v_j(t)d_j(v_j(t))\{-e_j(0)+\sum_{i=1}^{n}[b_{ji}+h_{ji}] g_i(0)+J_j^*\}. \quad (12)$$

By Lemma 5 and hypotheses (H_3), from (12), we get $LV(t,u(t),v(t))\le 0$, and $\lim_{\|u\|,\|v\|\to\infty}V(t,x)=\infty$, for any $t\in\mathbb{R}^+$. By Lemma 6, the solution $(u^T(t),v^T(t))^T$ of system (1) is stochastic bounded. □

3 Main results

In this section, we will derive some sufficient conditions which ensure the mean square exponential stability of the equilibrium point and the periodic solution for system (1).

Theorem 8 *Under hypotheses (H_1) , (H_4), and $I_i(t), J_j(t)$ are constants, then the equilibrium point of system (1) is exponentially stable in the mean square if (6) and (7) in Lemma 5 hold.*

Proof. Assume that $(u_1^*, u_2^*, \cdots, u_n^*, v_1^*, v_2^*, \cdots, v_m^*)^T$ be the equilibrium point of system (1), let

$y_i(t) = u_i(t) - u_i^*, \quad z_j(t) = v_j(t) - v_j^*,$
$\bar{a}_i(y_i(t)) = a_i(y_i(t)+u_i^*), \bar{d}_j(z_j(t)) = d_j(z_j(t)+v_j^*),$
$\bar{b}_i(y_i(t)) = b_i(y_i(t) + u_i^*) - b_i(u_i^*),$
$\bar{e}_j(z_j(t)) = e_j(z_j(t) + v_j^*) - e_j(v_j^*),$
$\bar{f}_j(z_j(t)) = f_j(z_j(t) + v_j^*) - f_j(v_j^*),$
$\bar{g}_i(y_i(t)) = g_i(y_i(t) + u_i^*) - g_i(u_i^*),$
$\bar{k}_{ij}(z_j(t)) = k_{ij}(z_j(t) + v_j^*),$
$\bar{\rho}_{ji}(y_i(t)) = \rho_{ji}(y_i(t) + u_i^*),$

for $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$.
From (1),(5), we derive

$$dy_i(t) = -\bar{a}_i(y_i(t))\{[\bar{b}_i(y_i(t)) - \sum_{j=1}^{m} a_{ij}\bar{f}_j(z_j(t)) - \sum_{j=1}^{m} c_{ij}\bar{f}_j(z_j(t-\tau_{ij}))]dt - \sum_{j=1}^{m} \bar{k}_{ij}(z_j(t))d\omega_{n+j}(t)\}, \quad (13)$$

$$dz_j(t) = -\bar{d}_j(z_j(t))\{[\bar{e}_j(z_j(t)) - \sum_{i=1}^{n} b_{ji}\bar{g}_i(y_i(t)) - \sum_{i=1}^{n} h_{ji}\bar{g}_i(y_i(t-\sigma_{ji}))]dt - \sum_{i=1}^{n} \bar{\rho}_{ji}(y_i(t))d\omega_i(t)\}, \quad (14)$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.
We consider the following Lyapunov function

$$V(t, y(t), z(t)) = e^{\alpha t}\sum_{i=1}^{n}|y_i(t)|^2 + e^{\alpha t}\sum_{j=1}^{m}|z_j(t)|^2 + \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i\int_{t-\tau_{ij}}^{t} e^{\alpha(s+\tau_{ij})}\bar{f}_j^2(z_j(s))ds + \sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_j\int_{t-\sigma_{ji}}^{t} e^{\alpha(s+\sigma_{ji})}\bar{g}_i^2(y_i(s))ds. \quad (15)$$

where α is given by Lemma 5.
Applying $It\hat{o}'s$ formula to $V(t, y(t), z(t))$, and slightly modifying to the proof of Lemma 7, we can easily obtain

$$\begin{aligned}
&dV(t, y(t), z(t))\\
&= e^{\alpha t}\sum_{i=1}^{n}[\alpha|y_i(t)|^2 + 2sgn(y_i(t))|y_i(t)|dy_i(t)]\\
&+ e^{\alpha t}\sum_{j=1}^{m}[\alpha|z_j(t)|^2 + 2sgn(z_i(t))|z_i(t)|dz_i(t)]\\
&+ e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i[e^{\alpha\tau_{ij}}\bar{f}_j^2(z_j(t)) - \bar{f}_j^2(z_j(t-\tau_{ij}))]\\
&+ e^{\alpha t}\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_j[e^{\alpha\sigma_{ji}}\bar{g}_i^2(y_i(t)) - \bar{g}_i^2(y_i(t-\sigma_{ji})]\\
&= e^{\alpha t}\sum_{i=1}^{n}[\alpha|y_i(t)|^2 + 2sgn(y_i(t))|y_i(t)|\\
&\cdot\{-\bar{a}_i(y_i(t))[(\bar{b}_i(y_i(t)) - \sum_{j=1}^{m} a_{ij}\bar{f}_j(z_j(t))\\
&- \sum_{j=1}^{m} c_{ij}\bar{f}_j(z_j(t-\tau_{ij}))]dt - \sum_{j=1}^{m}\bar{k}_{ij}(z_j(t))d\omega_{n+j}(t)\}\}]\\
&+ e^{\alpha t}\sum_{j=1}^{m}[\alpha|z_j(t)|^2 + 2sgn(z_i(t))|z_i(t)|\\
&\cdot\{-\bar{d}_j(z_j(t))\{[\bar{e}_j(z_j(t)) - \sum_{i=1}^{n} b_{ji}\bar{g}_i(y_i(t))\\
&- \sum_{i=1}^{n} h_{ji}\bar{g}_i(y_i(t-\sigma_{ji}))]dt - \sum_{i=1}^{n}\bar{\rho}_{ji}(y_i(t))d\omega_i(t)\}\}]\\
&+ e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i[e^{\alpha\tau_{ij}}\bar{f}_j^2(z_j(t)) - \bar{f}_j^2(z_j(t-\tau_{ij}))]\\
&+ e^{\alpha t}\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_j[e^{\alpha\sigma_{ji}}\bar{g}_i^2(y_i(t)) - \bar{g}_i^2(y_i(t-\sigma_{ji})]\\
&\le e^{\alpha t}\sum_{i=1}^{n}\{\alpha - 2\underline{a}_i\beta_i + \sum_{j=1}^{m}[\bar{d}_j^2T_{ji}^2 + \bar{a}_i|a_{ij}|F_j\\
&+ \bar{a}_i|c_{ij}| + |b_{ji}|\bar{d}_jG_i + e^{\alpha\sigma}|h_{ji}|\bar{d}_jG_i^2]\}y_i^2(t)dt\\
&+ e^{\alpha t}\sum_{j=1}^{m}\{\alpha - 2\underline{d}_j\gamma_j + \sum_{i=1}^{n}[\bar{a}_i^2L_{ij}^2 + |a_{ij}|\bar{a}_iF_j\\
&+ e^{\alpha\tau}|c_{ij}|\bar{a}_iF_j^2 + |b_{ji}|\bar{d}_jG_i + |h_{ji}|\bar{d}_j]\}u_j^2(t)dt\\
&+ e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}\bar{a}_i(y_i(t))|y_i(t)|\bar{k}_{ij}(z_j(t))d\omega_{n+j}(t)\\
&+ e^{\alpha t}\sum_{j=1}^{m}\sum_{i=1}^{n}\bar{d}_j(z_j(t))|z_j(t)|\bar{\rho}_{ji}(g_i(t))d\omega_i(t). \quad (16)
\end{aligned}$$

By Lemma 5, from (16), we get

$$dV(t, y(t), z(t)) \le e^{\alpha t}[\sum_{i=1}^{n}\sum_{j=1}^{m}\bar{a}_i(y_i(t))|y_i(t)|\bar{k}_{ij}(z_j(t))d\omega_{n+j}(t) + \sum_{j=1}^{m}\sum_{i=1}^{n}\bar{d}_j(z_j(t))|z_j(t)|\bar{\rho}_{ji}(g_i(t))d\omega_i(t)]. \quad (17)$$

From (17), we get

$$V(t, y(t), z(t)) \le V(0, y(0), z(0)) + \int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{j=1}^{m}\bar{a}_i(y_i(s))|y_i(s)|\bar{k}_{ij}(z_j(s))d\omega_{n+j}(s) + \int_0^t e^{\alpha s}\sum_{j=1}^{m}\sum_{i=1}^{n}\bar{d}_j(z_j(s))|z_j(s)|\bar{\rho}_{ji}(g_i(s))d\omega_i(s). \quad (18)$$

From (15), we have

$$V(t, y(t), z(t)) \ge e^{\alpha t}[\sum_{i=1}^{n}|y_i(t)|^2 + \sum_{j=1}^{m}|z_j(t)|^2], t \ge 0. \quad (19)$$

$$V(0, y(0), z(0) = \sum_{i=1}^{n}|y_i(0)|^2 + \sum_{j=1}^{m}|z_j(0)|^2 + \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_iF_j^2\int_{-\tau_{ij}}^{0} e^{\alpha(s+\tau_{ij})}|z_j(s)|^2ds$$

$$+\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_jG_i^2\int_{-\sigma_{ji}}^{0}e^{\alpha(s+\sigma_{ji})}|y_i(s)|^2ds$$
$$\le [1+\frac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\le i\le n}(|h_{ji}|G_i^2)\bar{d}_j]\|\phi_u-u^*\|^2$$
$$+[1+\frac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\le j\le m}(|c_{ij}|F_j^2)\bar{a}_i]\|\phi_v-v^*\|^2. \quad (20)$$

From (18)-(20), we obtain

$$e^{\alpha t}[\sum_{i=1}^{n}|y_i(t)|^2+\sum_{j=1}^{m}|z_j(t)|^2]$$
$$\le [1+\frac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\le i\le n}(|h_{ji}|G_i^2)\bar{d}_j]\|\phi_u-u^*\|^2$$
$$+[1+\frac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\le j\le m}(|c_{ij}|F_j^2)\bar{a}_i]\|\phi_v-v^*\|^2$$
$$+\int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{j=1}^{m}\bar{a}_i(y_i(s))|y_i(s)|\bar{k}_{ij}(z_j(s))d\omega_{n+j}(s)$$
$$+\int_0^t e^{\alpha s}\sum_{j=1}^{m}\sum_{i=1}^{n}\bar{d}_j(z_j(s))|z_j(s)|\bar{\rho}_{ji}(g_i(s))d\omega_i(s).$$

We can obtain

$$\sum_{i=1}^{n}E(|y_i(t)|^2)+\sum_{j=1}^{m}E(|z_j(t)|^2)$$
$$\le e^{-\alpha t}[1+\frac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\le i\le n}(|h_{ji}|G_i^2)\bar{d}_j]E(\|\phi_u-u^*\|^2)+e^{-\alpha t}[1+\frac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\le j\le m}(|c_{ij}|F_j^2)\bar{a}_i]E(\|\phi_v-v^*\|^2)=M[E(\|\phi_u-u^*\|^2)+E(\|\phi_v-v^*\|^2)],\ t>0, \quad (21)$$

where $M=\max\{1+\frac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\le i\le n}(|h_{ji}|G_i^2)\bar{d}_j,$

$$1+\frac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\le j\le m}(|c_{ij}|F_j^2)\bar{a}_i\}>1.$$

From (21), we can obtain

$$\sum_{i=1}^{n}E(|u_i-u_i^*|^2)+\sum_{j=1}^{m}E(|v_j-v_j^*|^2)$$
$$\le Me^{-\alpha t}[E(\|\phi_u-u^*\|^2)+E(\|\phi_v-v^*\|^2)], \quad t>0.$$

By Definition 3, the equilibrium point $(u^{*T},v^{*T})^T$ of system (1) is mean square exponentially stable. □

Theorem 9 *For the system (1), under the hypotheses $(H_1)-(H_4)$, there exists one ω-periodic solution of system (1), and all other solutions of (1) exponentially converge to it as $t\to+\infty$ in the mean square, if (6) and (7) in Lemma 5 hold.*

Proof. Let
$\Omega=\{\phi|\phi=(\phi_u^T,\phi_v^T)^T\phi_u=(\phi_{u1},\phi_{u2},\cdots,\phi_{un})^T,$
$\phi_v=(\phi_{v1},\phi_{v2},\cdots,\phi_{vn})^T\}$.
For any $\phi=(\phi_u^T,\phi_v^T)^T\in\Omega$, we define the norm of ϕ: $\|\phi\|=\|\phi_u\|+\|\phi_v\|$, in which

$$\|\phi_u\|=\sup_{-\sigma\le s\le 0}\sum_{i=1}^{n}|\phi_{ui}(s)|^2,$$
$$\|\phi_v\|=\sup_{-\tau\le s\le 0}\sum_{j=1}^{m}|\phi_{vj}(s)|^2,$$

then Ω is the Banach space of continuous functions which map $([-\sigma,0],[-\tau,0])^T$ into R^{n+m} with the topology of uniform convergence. For any $(\phi_u^T,\phi_v^T)^T,(\psi_u^T,\psi_v^T)^T\in\Omega$, we denote the solutions of system (1) in the initial conditions

$$\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\phi_u\\\phi_v\end{pmatrix}\right),\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\psi_u\\\psi_v\end{pmatrix}\right),$$

as $u(t,\phi_u)=(u_1(t,\phi_u),u_2(t,\phi_u),\cdots,u_n(t,\phi_u))^T$,
$v(t,\phi_v)=(v_1(t,\phi_v),v_2(t,\phi_v),\cdots,v_m(t,\phi_v))^T$,
and
$u(t,\psi_u)=(u_1(t,\psi_u),u_2(t,\psi_u),\cdots,u_n(t,\psi_u))^T$,
$v(t,\psi_v)=(v_1(t,\psi_v),v_2(t,\psi_v),\cdots,v_m(t,\psi_v))^T$,
respectively. Defining

$$u_t(\phi_u)=u(t+\rho,\phi_u),\rho\in[-\sigma,0],$$
$$v_t(\phi_v)=v(t+\rho,\phi_v),\rho\in[-\tau,0],t\ge 0,$$

then $(u_t(\phi_u),v_t(\phi_v))^T\in\Omega$, for all $t\ge 0$.
Let

$$y_i(t)=u_i(t,\phi_u)-u_i(t,\psi_u),$$
$$z_j(t)=v_j(t,\phi_v)-v_j(t,\psi_v),$$
$$\bar{y}_i(t)=\int_{u_i(t,\psi_u)}^{u_i(t,\phi_u)}\frac{ds}{a_i(s)},\quad \bar{z}_j(t)=\int_{v_j(t,\psi_v)}^{v_j(t,\phi_v)}\frac{ds}{d_j(s)},$$

$i=1,2,\cdots,n,j=1,2,\cdots,m.$
Note that $a_i(s),d_j(s)$ are continuous, $a_i(s)>0$, $d_j(s)>0$, from Lemma 7, $u_i(t,\phi_u),v_j(t,\phi_v)$, $u_i(t,\psi_u),v_j(t,\psi_v)$ are bounded. By mean-value theorem for integral, we have

$$\bar{y}_i(t)=\frac{1}{a_i(\xi_i)}[u_i(t,\phi_u)-u_i(t,\psi_u)]=\frac{1}{a_i(\xi_i)}y_i(t),$$
$$\bar{z}_j(t)=\frac{1}{d_j(\eta_j)}[v_j(t,\phi_v)-v_j(t,\psi_v)]=\frac{1}{d_j(\eta_j)}z_j(t),$$

where

$$\xi_i\in[\min\{u_i(t,\psi_u),u_i(t,\phi_u)\},\max\{u_i(t,\psi_u),u_i(t,\phi_u)\}],$$
$$\eta_j\in[\min\{v_j(t,\psi_v),v_j(t,\phi_v)\},\max\{v_j(t,\psi_v),v_j(t,\phi_v)\}],$$

then we have

$$sgn(\bar{y}_i(t))=sgn(y_i(t)),sgn(\bar{z}_j(t))=sgn(z_j(t)).$$

From (1), we derive

$$d|\bar{y}_i(t)| = sgn(y_i(t))\{-[b_i(u_i(t,\phi_u))-b_i(u_i(t,\psi_u))] + \sum_{j=1}^{m} a_{ij}[f_j(v_j(t,\phi_v)) - f_j(v_j(t,\psi_v))] + \sum_{j=1}^{m} c_{ij} [f_j(v_j(t-\tau_{ij},\phi_v)) - f_j(v_j(t-\tau_{ij},\psi_v))]\}dt + sgn(y_i(t))\sum_{j=1}^{m}[k_{ij}(v_j(t,\phi_v))-k_{ij}(v_j(t,\psi_v))]d\omega_{n+j}(t), \quad (22)$$

$$d|\bar{z}_j(t)| = sgn(z_j(t))\{-[e_j(v_j(t,\phi_v))-e_j(v_j(t,\psi_v))] + \sum_{i=1}^{n} b_{ji}[g_i(u_i(t,\phi_u))-g_i(u_i(t,\psi_u))]+\sum_{i=1}^{n} h_{ji}[g_i(u_i(t -\sigma_{ji},\phi_u)) - g_i(v_i(t-\sigma_{ji},\psi_u))]\}dt + sgn(z_j(t)) \cdot\sum_{i=1}^{n}[\rho_{ji}(u_i(t,\phi_u)) - \rho_{ji}(u_i(t,\psi_u))]d\omega_i(t), \quad (23)$$

for $i = 1,2,\cdots,n, j = 1,2,\cdots,m.$
We consider the following Lyapunov function

$$V(t,\bar{y}(t),\bar{z}(t)) = e^{\alpha t}\sum_{i=1}^{n} a_i^2(\xi_i)|\bar{y}_i(t)|^2 + \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij} |\bar{a}_i \int_{t-\tau_{ij}}^{t} e^{\alpha(s+\tau_{ij})}|f_j(v_j(s,\phi_v)) - f_j(v_j(s,\psi_v))|^2 ds + e^{\alpha t}\sum_{j=1}^{m} d_j^2(\eta_j)|\bar{z}_j(t)|^2 + \sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_j \int_{t-\sigma_{ji}}^{t} e^{\alpha(s+\sigma_{ji})} \cdot|g_i(u_i(s,\phi_u)) - g_i(u_i(s,\psi_u))|^2 ds, \quad (24)$$

where α is given by Lemma 5.

Applying $It\hat{o}'s$ formula to $V(t,\bar{y}(t),\bar{z}(t))$, and slightly modifying to the proof of Lemma 7, we can easily obtain

$$dV(t,\bar{y}(t),\bar{z}(t))$$
$$= e^{\alpha t}\sum_{i=1}^{n} a_i^2(\xi_i)[\alpha|\bar{y}_i(t)|^2 + 2|\bar{y}_i(t)|d|\bar{y}_i(t)|]$$
$$+e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i\cdot[e^{\alpha\tau_{ij}}|f_j(v_j(t,\phi_v))-f_j(v_j(t,\psi_v))|^2 -|f_j(v_j(t-\tau_{ij},\phi_v)) - f_j(v_j(t-\tau_{ij},\psi_v))|^2]$$
$$+e^{\alpha t}\sum_{j=1}^{m} d_j^2(\eta_j)[\alpha|\bar{z}_j(t)|^2 + 2|\bar{z}_j(t)|d|\bar{z}_j(t)|]$$
$$+e^{\alpha t}\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_j[e^{\alpha\sigma_{ji}}|g_i(u_i(t,\phi_u))-g_i(u_i(t,\psi_u))|^2 -|g_i(u_i(t-\sigma_{ji},\phi_u)) - g_i(u_i(t-\sigma_{ji},\psi_u))|^2]$$
$$= e^{\alpha t}\sum_{i=1}^{n} a_i^2(\xi_i)[\alpha|\bar{y}_i(t)|^2 +2|\bar{y}_i(t)|\{sgn(y_i(t))\{-[b_i(u_i(t,\phi_u))-b_i(u_i(t,\psi_u))] + \sum_{j=1}^{m} a_{ij}[f_j(v_j(t,\phi_v)) - f_j(v_j(t,\psi_v))] + \sum_{j=1}^{m} c_{ij}[f_j(v_j(t-\tau_{ij},\phi_v)) - f_j(v_j(t-\tau_{ij},\psi_v))]\}dt + sgn(y_i(t))\sum_{j=1}^{m}[k_{ij}(v_j(t,\phi_v)) - k_{ij}(v_j(t,\psi_v))]d\omega_{n+j}(t)\}]$$
$$+ e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i \cdot[e^{\alpha\tau_{ij}}|f_j(v_j(t,\phi_v)) - f_j(v_j(t,\psi_v))|^2 - |f_j(v_j(t -\tau_{ij},\phi_v)) - f_j(v_j(t-\tau_{ij},\psi_v))|^2] + e^{\alpha t}\sum_{j=1}^{m} d_j^2(\eta_j) \cdot[\alpha|\bar{z}_j(t)|^2 + 2|\bar{z}_j(t)|\{sgn(z_j(t))\{-[e_j(v_j(t,\phi_v)) - e_j(v_j(t,\psi_v))] + \sum_{i=1}^{n} b_{ji}[g_i(u_i(t,\phi_u)) - g_i(u_i(t,\psi_u))] + \sum_{i=1}^{n} h_{ji}[g_i(u_i(t-\sigma_{ji},\phi_u)) - g_i(v_i(t-\sigma_{ji},\psi_u))]\}dt +sgn(z_j(t))\sum_{i=1}^{n}[\rho_{ji}(u_i(t,\phi_u))-\rho_{ji}(u_i(t,\psi_u))]d\omega_i(t)\}]$$
$$+e^{\alpha t}\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_j[e^{\alpha\sigma_{ji}}|g_i(u_i(t,\phi_u))-g_i(u_i(t,\psi_u))|^2 -|g_i(u_i(t-\sigma_{ji},\phi_u)) - g_i(u_i(t-\sigma_{ji},\psi_u))|^2]$$
$$\le e^{\alpha t}\sum_{i=1}^{n}\{\alpha - 2\underline{a}_i\beta_i + \sum_{j=1}^{m}[\bar{d}_j^2 T_{ji}^2 + \bar{a}_i|a_{ij}|F_j + \bar{a}_i|c_{ij}| +|b_{ji}|\bar{d}_j G_i+e^{\alpha\sigma}|h_{ji}|\bar{d}_j G_i^2]\}y_i^2(t)dt+e^{\alpha t}\sum_{j=1}^{m}\{\alpha-2\underline{d}_j \cdot\gamma_j + \sum_{i=1}^{n}[\bar{a}_i^2 L_{ij}^2 + |a_{ij}|\bar{a}_i F_j + e^{\alpha\tau}|c_{ij}|\bar{a}_i F_j^2 + |b_{ji}|\bar{d}_j G_i +|h_{ji}|\bar{d}_j]\}z_j^2(t)dt + e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m} a_i(\xi_i)|y_i(t)|[k_{ij}(v_j(t,\phi_v)) - k_{ij}(v_j(t,\psi_v))]d\omega_{n+j}(t) + e^{\alpha t}\sum_{j=1}^{m}\sum_{i=1}^{n} d_j(\eta_j)|z_j(t)| \cdot[\rho_{ji}(u_i(t,\phi_u))-\rho_{ji}(u_i(t,\psi_u))]d\omega_i(t). \quad (25)$$

By Lemma 5, from (25), we get

$$dV(t,\bar{y}(t),\bar{z}(t)) \le e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m} a_i(\xi_i)|y_i(t)| \cdot[k_{ij}(v_j(t,\phi_v)) - k_{ij}(v_j(t,\psi_v))]d\omega_{n+j}(t) +e^{\alpha t}\sum_{j=1}^{m}\sum_{i=1}^{n} d_j(\eta_j)|z_j(t) \cdot|[\rho_{ji}(u_i(t,\phi_u)) - \rho_{ji}(u_i(t,\psi_u))]d\omega_i(t). \quad (26)$$

From (26), we get

$$\begin{aligned}&V(t,\bar y(t),\bar z(t))\le V(0,\bar y(0),\bar z(0))\\&+\int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{j=1}^{m}a_i(\xi_i)|y_i(t)|\cdot\\&\quad[k_{ij}(v_j(t,\phi_v))-k_{ij}(v_j(t,\psi_v))]d\omega_{n+j}(t)\\&+\int_0^t e^{\alpha s}\sum_{j=1}^{m}\sum_{i=1}^{n}d_j(\eta_j)|z_j(t)|\cdot\\&\quad[\rho_{ji}(u_i(t,\phi_u))-\rho_{ji}(u_i(t,\psi_u))]d\omega_i(t).\end{aligned}\tag{27}$$

From (24), we have

$$V(t,\bar y(t),\bar z(t))\ge e^{\alpha t}[\sum_{i=1}^{n}|y_i(t)|^2+\sum_{j=1}^{m}|z_j(t)|^2],\tag{28}$$

$$\begin{aligned}&V(0,\bar y(0),\bar z(0)\\&=\sum_{i=1}^{n}|y_i(0)|^2+\sum_{j=1}^{m}|z_j(0)|^2\\&+\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar a_iF_j^2\int_{-\tau_{ij}}^{0}e^{\alpha(s+\tau_{ij})}|v_j(s,\phi_v)-v_j(s,\psi_v)|^2ds\\&+\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar d_jG_i^2\int_{-\sigma_{ji}}^{0}e^{\alpha(s+\sigma_{ji})}|u_i(s,\phi_u)-u_i(s,\psi_u)|^2ds.\end{aligned}\tag{29}$$

From (27)-(29), we obtain

$$\begin{aligned}&e^{\alpha t}[\sum_{i=1}^{n}|y_i(t)|^2+\sum_{j=1}^{m}|z_j(t)|^2]\\&\le\sum_{i=1}^{n}|y_i(0)|^2+\sum_{j=1}^{m}|z_j(0)|^2\\&+\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar a_iF_j^2\int_{-\tau_{ij}}^{0}e^{\alpha(s+\tau_{ij})}|v_j(s,\phi_v)-v_j(s,\psi_v)|^2ds\\&+\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar d_jG_i^2\int_{-\sigma_{ji}}^{0}e^{\alpha(s+\sigma_{ji})}|u_i(s,\phi_u)-u_i(s,\psi_u)|^2ds\\&+\int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{j=1}^{m}a_i(\xi_i)|y_i(t)|[k_{ij}(v_j(t,\phi_v))-k_{ij}(v_j(t,\psi_v))]\\&\cdot d\omega_{n+j}(t)+\int_0^t e^{\alpha s}\sum_{j=1}^{m}\sum_{i=1}^{n}d_j(\eta_j)|z_j(t)|[\rho_{ji}(u_i(t,\phi_u))\\&-\rho_{ji}(u_i(t,\psi_u))]d\omega_i(t).\end{aligned}$$

We obtain

$$\begin{aligned}&\sum_{i=1}^{n}E(|u_i(t,\phi_u)-u_i(t,\psi_u)|^2)\\&+\sum_{j=1}^{m}E(|v_j(t,\phi_v)-v_j(t,\psi_v)|^2)\\&\le e^{-\alpha t}[\sum_{i=1}^{n}E(|\phi_{ui}(0)-\psi_{ui}(0)|^2)\\&+\sum_{j=1}^{m}E(|\phi_{vi}(0)-\psi_{vj}(0)|^2)]\\&+e^{-\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar a_iF_j^2\int_{-\tau_{ij}}^{0}e^{\alpha(s+\tau_{ij})}E(|\phi_{vj}-\psi_{vj}|^2)ds\\&+e^{-\alpha t}\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar d_jG_i^2\int_{-\sigma_{ji}}^{0}e^{\alpha(s+\sigma_{ji})}E(|\phi_{ui}-\psi_{ui}|^2)ds\\&\le e^{-\alpha t}[1+\frac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\le i\le n}(|h_{ji}|G_i^2)\bar d_j]E(\|\phi_u-\psi_u\|^2)\\&+e^{-\alpha t}[1+\frac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\le j\le m}(|c_{ij}|F_j^2)\bar a_i]E(\|\phi_v-\psi_v\|^2)\\&=Me^{-\alpha t}[E(\|\phi_u-\psi_u\|^2)+E(\|\phi_v-\psi_v\|^2)],\quad t>0,\end{aligned}\tag{30}$$

where $M=\max\{1+\frac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\le i\le n}(|h_{ji}|G_i^2)\bar d_j,1+\frac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\le j\le m}(|c_{ij}|F_j^2)\bar a_i\}>1.$

From (30), we can obtain

$$\begin{aligned}&\sum_{i=1}^{n}E(|u_i(t,\phi_u)-u_i(t,\psi_u)|^2)\\&+\sum_{j=1}^{m}E(|v_j(t,\phi_v)-v_j(t,\psi_v)|^2)\\&\le Me^{-\alpha t}[E(\|\phi_u-\psi_u\|^2)+E(\|\phi_v-\psi_v\|^2)].\end{aligned}\tag{31}$$

Let

$$\begin{aligned}&E(|u(\phi_u)-u(\psi_u)|^2)\\&=\sum_{i=1}^{n}E(|u_i(t,\phi_u)-u_i(t,\psi_u)|^2),\\&E(|u(\phi_v)-u(\psi_v)|^2)\\&=\sum_{j=1}^{m}E(|v_j(t,\phi_v)-v_j(t,\psi_v)|^2).\end{aligned}$$

From (31), we have

$$\begin{aligned}&E(|u(\phi_u)-u(\psi_u)|^2)\\&\le Me^{-\alpha t}[E(\|\phi_u-\psi_u\|^2)+E(\|\phi_v-\psi_v\|^2)],t>0,\\&E(|u(\phi_v)-u(\psi_v)|^2)\\&\le Me^{-\alpha t}[E(\|\phi_u-\psi_u\|^2)+E(\|\phi_v-\psi_v\|^2)],t>0.\end{aligned}$$

We can choose a positive integer N and $\omega>0$, such that $Me^{-\alpha(N\omega+\rho)}\le\frac{1}{3},\rho\in[-\delta,0]$. Now we define a Poincaré mapping F: $\rho\to\rho$ by

$$F(\phi_u,\phi_v)^T=(u_\omega(\phi_u),v_\omega(\phi_v))^T,$$

then

$$F^N(\phi_u,\phi_v)^T=(u_{N\omega}(\phi_u),v_{N\omega}(\phi_v))^T.$$

Let $t=N\omega$, then have

$$\begin{aligned}&E(|F^N\phi_u-F^N\psi_u|^2)\\&\le\frac{1}{3}[E(\|\phi_u-\psi_u\|^2)+E(\|\phi_v-\psi_v\|^2)],\\&E(|F^N\phi_v-F^N\psi_v|^2)\\&\le\frac{1}{3}[E(\|\phi_u-\psi_u\|^2)+E(\|\phi_v-\psi_v\|^2)]\end{aligned}$$

By the integral property of measurable functions, we can obtain

$$\begin{aligned}&|F^N\phi_u-F^N\psi_u|^2\\&\le\frac{1}{3}[E(\|\phi_u-\psi_u\|^2)+E(\|\phi_v-\psi_v\|^2)],,a.e.,\\&|F^N\phi_v-F^N\psi_v|^2\\&\le\frac{1}{3}[E(\|\phi_u-\psi_u\|^2)+E(\|\phi_v-\psi_v\|^2)],a.e.\end{aligned}$$

This implies that F^N is a contraction mapping, hence there exist a unique fixed point $(\phi_u^*, \phi_v^*)^T \in \Omega$, such that $F^N(\phi_u^*, \phi_v^*)^T = (\phi_u^*, \phi_v^*)^T$. Since

$$F^N\left(F\begin{pmatrix}\phi_u^*\\ \phi_v^*\end{pmatrix}\right) = F\left(F^N\begin{pmatrix}\phi_u^*\\ \phi_v^*\end{pmatrix}\right) = F\begin{pmatrix}\phi_u^*\\ \phi_v^*\end{pmatrix},$$

then $F(\phi_u^*, \phi_v^*)^T \in \Omega$ is also a fixed point of F^N, and so $F(\phi_u^*, \phi_v^*)^T = (\phi_u^*, \phi_v^*)^T$, i.e., $(u_\omega(\phi_u^*), v_\omega(\phi_v^*))^T = (\phi_u^*, \phi_v^*)^T$. Let $(u(t,\phi_u^*), v(t,\phi_v^*))^T$ be the solution of system (1) through

$$\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\phi_u^*\\ \phi_v^*\end{pmatrix}\right),$$

then $(u(t+\omega,\phi_u^*), v(t+\omega,\phi_v^*))^T$ is also a solution of (1). Obviously

$$\begin{pmatrix}u_{t+\omega}(\phi_u^*)\\ v_{t+\omega}(\phi_v^*)\end{pmatrix} = \begin{pmatrix}u_t(u_\omega(\phi_u^*))\\ v_t(v_\omega(\phi_v^*))\end{pmatrix} = \begin{pmatrix}u_t(\phi_u^*)\\ v_t(\phi_v^*)\end{pmatrix},$$

for all $t \geq 0$. Hence

$$\begin{pmatrix}u(t+\omega,\phi_u^*)\\ v(t+\omega,\phi_v^*)\end{pmatrix} = \begin{pmatrix}u(t,\phi_u^*)\\ v(t,\phi_v^*)\end{pmatrix},$$

for all $t \geq 0$.

This shows that system (1) exists one $\omega-$ periodic solution, and other solutions of system (1) exponentially converge to it as $t \to +\infty$ in the mean square. □

4 Example

In the Section, we give an example to demonstrate our results.

Example Consider the following stochastic Cohen-Grossberg-type BAM neural networks $(n = m = 2)$

$$\begin{cases} du_i(t) = -a_i(u_i(t))\{[b_i(u_i(t)) - \sum\limits_{j=1}^{2} a_{ij} f_j(v_j(t)) \\ \qquad - \sum\limits_{j=1}^{2} c_{ij} f_j(v_j(t-\tau_{ij})) - I_i(t)]dt \\ \qquad - \sum\limits_{j=1}^{2} k_{ij}(v_j(t)) d\omega_{n+j}(t)\}, \\ dv_j(t) = -d_j(v_j(t))\{[e_j(v_j(t)) - \sum\limits_{i=1}^{2} b_{ji} g_i(u_i(t)) \\ \qquad - \sum\limits_{i=1}^{2} h_{ji} g_i(u_i(t-\sigma_{ji})) - J_j(t)]dt \\ \qquad - \sum\limits_{i=1}^{2} \rho_{ji}(u_i(t) d\omega_i(t)\}, \end{cases} \tag{32}$$

for $i = 1, 2, j = 1, 2$, where

$$\begin{aligned} &f_j(r) = 2\sin r, \quad g_i(r) = \cos r, \\ &a_i(r) = 2 + \sin r, \quad d_j(r) = 2 + \cos r, \\ &b_i(r) = 6r + 2, \quad e_j(r) = 9r + 2, \\ &k_{ij}(r) = \sin r, \quad \rho_{ji}(r) = 1 - \cos r, \\ &I_i(r) = 2\sin r, \quad J_j(r) = \cos r, \quad i, j = 1, 2. \end{aligned}$$

Since $\forall r_1, r_2 \in \mathbb{R}$, a nd for $i = 1, 2$,

$$\begin{aligned} &|f_i(r_1) - f_i(r_2)| \leq 2|r_1 - r_2|, \\ &|g_i(r_1) - g_i(r_2)| \leq |r_1 - r_2|, \\ &1 \leq a_i(r) \leq 3, \quad 1 \leq d_i(r) \leq 3, \\ &b_i(r_1) - b_i(r_2) = 6(r_1 - r_2), \\ &e_i(r_1) - e_i(r_2) = 9(r_1 - r_2), \end{aligned}$$

We select

$$\begin{aligned} &F_j = 2, \quad G_i = 1, \quad \beta_i = 6, \quad \gamma_j = 9, \\ &\underline{a}_i = 1, \quad \bar{a}_i = 2, \quad \underline{d}_j = 1, \quad \bar{d}_j = 2, \\ &I_i^* = 2, \quad J_j^* = 1, \quad L_{ij} = 1, \quad T_{ji} = 1, \quad i = 1, 2. \end{aligned}$$

Let

$$\begin{aligned} &a_{11} = \tfrac{1}{4}, \quad a_{12} = -\tfrac{1}{2}, \quad a_{21} = \tfrac{1}{2}, \quad a_{22} = -\tfrac{1}{2}, \\ &b_{11} = \tfrac{1}{2}, \quad b_{12} = \tfrac{1}{2}, \quad b_{21} = \tfrac{1}{4}, \quad b_{22} = \tfrac{1}{4}, \\ &c_{11} = \tfrac{1}{4}, \quad c_{12} = -\tfrac{1}{4}, \quad c_{21} = \tfrac{1}{2}, \quad c_{22} = \tfrac{1}{2}, \\ &h_{11} = -\tfrac{1}{4}, \quad h_{12} = \tfrac{1}{4}, \quad h_{21} = \tfrac{1}{4}, \quad h_{22} = \tfrac{1}{4}. \end{aligned}$$

We have the following results by simple calculation

$$\begin{aligned} &f_j(0) = 0, \quad g_i(0) = 1, \quad b_i(0) = 2 > 0, \\ &e_j(0) = 2 > 0, \quad k_{ij}(0) = 0, \quad \rho_{ji}(0) = 0, \\ &-b_i(0) + \sum_{j=1}^{2} (a_{ij} + c_{ij}) f_j(0) + I_i^* = 0, \\ &-e_j(0) + \sum_{i=1}^{2} (b_{ji} + h_{ji}) g_i(0) + J_j^* = 0, \\ &-2\underline{a}_i\beta_i + \sum_{j=1}^{2} [\bar{d}_j^2 T_{ji}^2 + \bar{a}_i|a_{ij}|F_j + \bar{a}_i|c_{ij}| \\ &\qquad + |b_{ji}|\bar{d}_j G_i + |h_{ji}|\bar{d}_j G_i^2] < 0, \\ &-2\underline{d}_j\gamma_j + \sum_{i=1}^{2} [\bar{a}_i^2 L_{ij}^2 + |a_{ij}|\bar{a}_i F_j + |c_{ij}|\bar{a}_i F_j^2 \\ &\qquad + |b_{ji}|\bar{d}_j G_i + |h_{ji}|\bar{d}_j] < 0, \end{aligned}$$

for $i, j = 1, 2$.

It follows from Theorem 9 that this system exists one 2π-periodic solution and all other solutions of system (32) exponentially converge to it as $t \to +\infty$ in the mean square. □

5 Conclusions

In this paper, with the help of Lyapunov function, $It\hat{o}$ formula, Poincaré mapping, a set of novel sufficient conditions on the exponential stability of the equilibrium point and the periodic solution in the mean square for stochastic Cohen-Grossberg-Type BAM neural networks is derived, which algebra conditions are easily verifiable and useful in theories and applications.

Acknowledgements: This work is by the National Natural Science Foundation of China (No.10871226, No.10875078), and the National Natural Science Foundation of Zhejiang Province (No.Y6100096).

References:

[1] C. Bai, Stability analysis of Cohen-Crossberg BAM neural networks with delays and impulses, *Chaos, Solitons Fractals*, 35, 2008, pp. 263-267.

[2] J. Cao, Q. Song, Stability in Cohen-Grossberg-type bidirectional associative memory neural networks with time-varying delays, *Nonlinearity*, 19, 2006, pp.1601-1617.

[3] A. Chen, J. Cao, Periodic bi-directional Cohen-Grossberg neural networks with distributed delays, *Nonlinear Anal.*, 66,2007, pp.2947-2961.

[4] M. Cohen, S. Grossberg, Absolute stability of global pattern formation and parallel memory storage by competitive neural networks. *IEEE Trans. Syst., Man Cybernetics*, 13, 1983, pp.815-826.

[5] C. Feng, R. Plamondon, Stability analysis of bidirectional associative memory networks with time delays, *IEEE Trans. Neural Networks*, 14, 2003, pp.1560-1565.

[6] C. Huang, J. Cao, On pth moment exponential stability of stochastic Cohen-Grossberg neural networks with time-varying delays, *Neurocomputing*, 73, 2010, pp.986-990.

[7] T. Huang, A. Chan, Y. Huang, J. Cao, Stability of Cohen-Grossberg neural networks with time-varying delays Neural Networks, *Neural Networks*, 20(8), 2007, pp.868-873.

[8] H. Jiang, J. Cao, BAM-type Cohen-Grossberg neural networks with time delays, *Math. Comput. Modell*, 47, 2008, pp.92-103.

[9] C. Li, X. Mao, Boundedness on the solution of $It\hat{o}$ stochastic differential equation, *Journal of East Chain Institute of Chemical Technology*, 3, 1983, pp.383-388.

[10] J. Li, J. Wang, An analysis on the global exponential stability and the existence of periodic solutions for non-autonomous hybrid BAM neural networks with distributed delays and impulses, *Comput. Math. Appl.*, 56, 2008, pp.2256-2267.

[11] T. Li, A. G. Song, S. M. Fei, Robust stability of stochastic Cohen-Grossberg neural networks with mixed time-varying delays, *Neurocomputing*, 73, 2009, pp.542-551.

[12] Y. Li, X. Fan, Existence and globally exponential stability of almost periodic solution for Cohen-Grossberg BAM neural networks with variable coefficients, *Appl. Math. Modell*, 33, 2009, pp.2114-2120.

[13] X. Liao, C. Li, K. W. Wong, Criteria for exponential stability of Cohen-Grossberg neural networks Neural Networks, *Neural Networks*, 17(10), 2004, pp.1401-1414.

[14] W. Lu, T. Chen, Dynamical behaviors of Cohen-Grossberg neural networks with discontinuous activation functions Neural Networks, *Neural Networks*, 18(3), 2005, pp.231-242.

[15] Q. Song, Z. Wang, Stability analysis of impulsive stochastic Cohen-Grossberg neural networks with mixed time delays, *Physica A.*, 387, 2008, pp.3314-3326.

[16] W. Su, Y. Chen, Global robust stability criteria of stochastic Cohen-Grossberg neural networks with discrete and distributed time-varying delays, *Communications in Nonlinear Science and Numerical Simulation*, 2, 2009, pp.520-528.

[17] X. Wang, Q. Guo, D. Xu, Exponential p-stability of impulsive stochastic Cohen-Grossberg neural networks with mixed delays, *Mathematics and Computers in Simulation*, 79, 2009, pp.1698-1710.

[18] H. Xiang, J. Cao, Exponential stability of periodic solution to Cohen-Grossberg type BAM networks with time-varying delays, *Neurocomputing*, 72, 2009, pp.1702-1711.

[19] F. Yang, C. Zhang, D. Wu, Global stability analysis of impulsive BAM type Cohen-Grossberg neural networks with delays, *Appl. Math. Comput.*, 186, 2007, pp.932-940.

[20] Z. Yang, D. Xu, Existence and exponential stability of periodic solution for impulsive delay differential equations and applications, *Nonlinear Anal.*, 64, 2006, pp.130-145.

[21] J. Zhang, Z. Gui, Existence and stability of periodic solutions of high-order Hopfield neural networks with impulses and delays, *J. Comput. Appl. Math.*, 224, 2009, pp.602-613.

[22] H. G. Zhao, N. Ding, Dynamic analysis of stochastic Cohen-Grossberg neural networks with time delays, *Applied Mathematics and Computation*, 183, 2006, pp.464-470.

[23] Q. Zhou,L. Wan, Impulsive effects on stability of Cohen-Grossberg-type bidi-rectional associative memory neural networks with delays, *Nonlinear Anal.: Real World Appl.*, 10, 2009, pp.2531-2540.