High-Gain Observer Design for Online Identification of Supercapacitor Model Parameters

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Abstract: -The objective of this paper is to tackle the problem of online state estimation for supercapacitors by using a strong high-gain observer. To monitor and control in real-time systems in the renewable energy domain, such as energy storage systems and electric vehicles, it is essential to accurately estimate supercapacitor parameters. Traditional estimation methods can be computationally expensive or sensitive to measurement noise. Presently, we propose a high-gain observer designed under the assumption that the supercapacitor parameters (series resistance R_s and capacitance C) are known and constant. Assuming system observability, the observer ensures asymptotic convergence of state estimates. The simulation results demonstrate fast convergence, less than 500 seconds of simulation, and robustness to measurement noise (Maximum relative error of 1%), confirming the suitability of the proposed approach for real-time implementation.

Key-Words: - Supercapacitor, RC Model, Parameter Identification, Online Estimation, High-Gain Observer, State Observer, Renewable Energy, Electric Vehicles, MATLAB/Simulink

Received: June 25, 2025. Revised: November 17, 2025. Accepted: March 2, 2026. Published: June 2, 2026.

1 Introduction

Supercapacitors (SC) are energy storage devices with a high-power density and fast charge and discharge characteristics, [1], [2]. Their modeling must be accurate for use in electric vehicles and renewable energy systems, [3], [4]. Many circuit models are equivalent, for example, simple RC models and complex fractional-order representations, [5], [6], [7]. The first-order RC model is ideally suited for online applications, [8], because it balances accuracy and simplicity. Nevertheless, parameters such as series resistance and capacitance vary with temperature, state of charge, and age. Therefore, online estimation is necessary for accurate control and diagnosis, [8], [9]. Traditional identification methods include electrochemical impedance spectroscopy (EIS) and pulse tests, [9], [10], but these solutions are offline methods. Recursive least squares (RLS), Kalman filters, and model reference adaptive systems are all examples of computationally intensive techniques; however, they are sensitive to noise and require careful tuning, [11]. Among the existing identification methods, frequency-based identification approaches have attracted particular interest, [12], [13], [14], [15].

Extended Kalman filters (EKF) have good performance, but their computational complexity and noise statistics make them a poor choice for applications, [11].

The work proposes a new methodology based on the theory of high-gain observers to face the issues related to the estimation of the states in supercapacitors.

The proposed method leverages the specific structure and observability of the nonlinear differential equations that govern the dynamics of the supercapacitors to ensure exponential convergence of the estimates while retaining computational efficiency, [16], [17], [18], [19], [20].

The main objective of this work is to propose a robust observer for the estimation of the states in supercapacitors under the following assumptions:

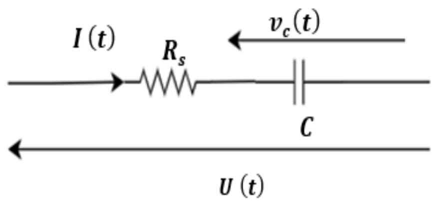

Fig. 1: first-order model of SC
Source: [1].

$\mathbf{A_1}$: The parameters of the supercapacitor (R_s, C) are considered as known.

$\mathbf{A_2}$: The matrix couple (A, k) is observable.

These assumptions ensure that the suggested observer for the state estimate will converge.

The remainder of this paper is structured as follows: Section 2 introduces the problem statement and the proposed model. Section 3 details the high-gain observer design along with the convergence analysis. Section 4 presents and discusses the simulation results. Finally, Section 5 concludes the paper.

2 Problem Statement and Modeling

In this paper, the observer design problem is addressed for SC state estimation. The SC is represented by a simplified equivalent circuit (Figure 1).

This model consists of the series connection of a resistance R_s with a capacitance C.

This first-order model provides a good balance between accuracy and computational simplicity, making it suitable for real-time applications.

Applying Kirchhoff's voltage law to the equivalent circuit, the SC is analytically described by the following expression:

$$U(t) = R_s I(t) + v_c(t) \quad (1)$$

where, $U(t)$:is the voltage across supercapacitor, $I(t)$: is the current passing through the supercapacitor and v_c(t): Voltage across the capacitor C. The supercapacitor current $I(t)$ can be expressed with respect to voltage $v_c(t)$ as:

$$I(t) = C\,\frac{dv_c(t)}{dt} \quad (2)$$

which can be rearranged as:

$$\frac{dv_c(t)}{dt} = \frac{1}{C}\,I(t) \quad (2a)$$

Using (1) and (2a), we have:

$$\frac{dU(t)}{dt} = R_s\frac{dI(t)}{dt} + \frac{dv_c(t)}{dt} \quad (3)$$

$$\frac{d^2v_c(t)}{dt^2} = \frac{1}{C}\,\frac{dI(t)}{dt} \quad (4)$$

2.1 State-Space Representation

Define the state variables:

$$\begin{cases} x_1 = U(t) \\ x_2 = \frac{dv_c(t)}{dt} \end{cases} \quad (5)$$

Equations (3) and (4) can be expressed using (5) as:

$$\begin{cases} \dot{x}_1 = x_2 + R_s\frac{dI(t)}{dt} \\ \dot{x}_2 = \frac{1}{C}\,\frac{dI(t)}{dt} \\ y = x_1 \end{cases} \quad (6)$$

In compact form:

$$\begin{cases} \dot{x} = Ax + \varphi(t)\theta \\ y = kx \end{cases} \quad (7)$$

This is written compactly as:

$$\begin{cases} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \frac{dI(t)}{dt}\begin{pmatrix} R_s \\ \frac{1}{C} \end{pmatrix} \\ y = (1\ 0)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{cases} \quad (8)$$

where:

$$\dot{x} = (\dot{x}_1 \quad \dot{x}_2)^T \ (9)$$

$$x = (x_1 \quad x_2)^T \ (10)$$

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (11)$$

$$\varphi(t) = \frac{dI(t)}{dt} \quad (12)$$

$$\theta = \begin{pmatrix} R_s \\ \frac{1}{C} \end{pmatrix} \quad (13)$$

$$k = (1\ 0) \quad (14)$$

3 High-Gain Observer Design

The proposed observer is designed as follows:

$$\begin{cases} \dot{\hat{x}} = A\,\hat{x} + \varphi(t)\hat{\theta} - L(\hat{y} - y) \\ \hat{y} = k\,\hat{x} \end{cases} \quad (15)$$

where, $\hat{x}$ denotes the state estimate vector, $\hat{\theta} = \theta$ is the known parameter vector, and $L = (L_1 \quad L_2)^T$ is the observer gain matrix.

Let us define the state estimation error:

$$\tilde{x} = \hat{x} - x \quad (16)$$

By replacing $\dot{\hat{x}}$ and $\dot{x}$ with their expression, given by (15), (7), respectively, we have:

$$\dot{\tilde{x}} = \dot{\hat{x}} - \dot{x} = A\,\tilde{x} - L(\hat{y} - y) \ (17)$$

which can be re-expressed by replacing $\hat{y}$ and y with their expressions given, respectively, by (15) and (7) as follows:

$$\dot{\tilde{x}} = A\,\tilde{x} - Lk\tilde{x} \ (18)$$

The latter can be simplified to:

$$\dot{\tilde{x}} = (A - Lk)\,\tilde{x} \quad (19)$$

3.1 Observer convergence

It follows from assumption A_2 that the observer gain matrix L can be fixed such that the matrix $A - Lk$ is Hurwitz (all eigenvalues have negative real parts). That is the state dynamical error $\dot{\tilde{x}}$, defined by (19), converge to 0 as $t \to \infty$. This guarantees that the state estimate converges to the true state asymptotically.

The observer gain L can be computed using pole placement techniques (20). For the system matrix A and output matrix k given in our model, we select L such that the observer poles are placed at desired locations, ensuring fast convergence.

$$L = \text{place}(A^{\mathrm{T}}, k^{\mathrm{T}}, [\lambda_1, \lambda_2])^{\mathrm{T}} \quad (20)$$

where A and k are defined in (11) and (14), respectively.

4 Simulation Results and Discussion

The ELX1087 supercapacitor is used in this work's simulations; Table 1 lists its primary technical characteristics.

Table 1:Supercapacitor Specifications

Parameter	Specification
Capacitance	*25 F to 100 F*
Maximum working voltage	*3.0 V*
Surge voltage	*3.3 V*
Capacitance tolerance	*−10% to +30% (+20 °C)*
Operating temperature range	*−40 °C to +65 °C*
Extended temperature range	*−40 °C to +85 °C (with linear voltage derating to 2.5 V at +85 °C)*

Source: [21], [22].

Simulations were carried out utilizing MATLAB/Simulink environment to verify the theoretical developments and assess the performance of the suggested observer. Table 2 presents the input signals, observer design characteristics, and supercapacitor model parameters.

Table 2:Simulation and Observer Parameters

Supercapacitor	
$C = 124.4\ F$	$R_s = 0.538\ \ \Omega$
Observer	
Gain vector $L = (40 \quad 400)^T$	Poles : $\lambda_1 = -40\,, \lambda_2 = -20$
Condition	
Sampling time $T_e = 0.01\ s$	Noise level $\sigma = 0.01\ V$
Input Current	
Amplitude (Sq. W.)	$2\ A$
Direct Current Offset	$2\ A$
Frequency (Sq. W.)	$0.1\ \ Hz$

Source: created by the authors.

To assure continuous charge cycles, the input current $I(t)$ is a square wave (Sq. W.) with a direct current offset, generating a continuously exciting signal for observer confirmation. To evaluate the robustness of the observer, a zero-mean Gaussian white noise with a standard deviation of 10 mV is introduced to the output voltage measurement $y(t) = U(t)$.

4.2 State Estimation Performance

The developed observer is evaluated, and the estimated state variables $\hat{x}_1$, which correlate to the terminal voltage U and the capacitor voltage derivative $\frac{dv_c}{dt}$ are analyzed using numerical

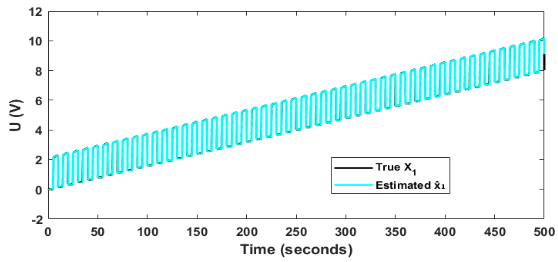

Fig. 2: True state $x_1$versus Estimated state $\hat{x}_1$
Source: created by the authors.

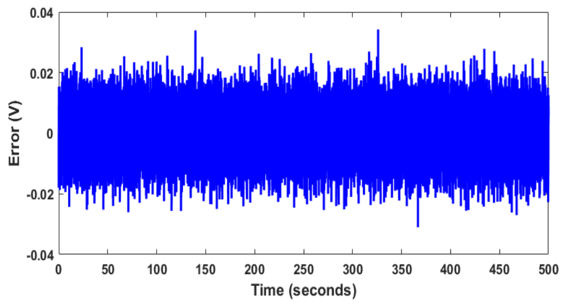

Fig. 3: Estimation Error: x_1- $\hat{x}_1$
Source: created by the authors.

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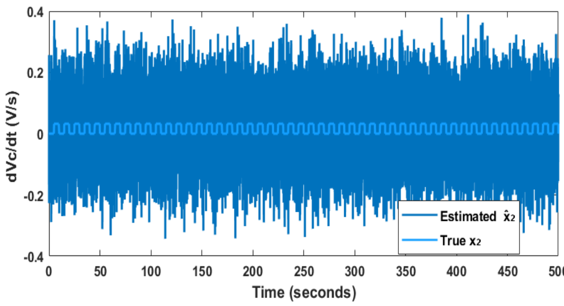

Fig. 4: True state x_2 versus estimated state $\hat{x}_2$
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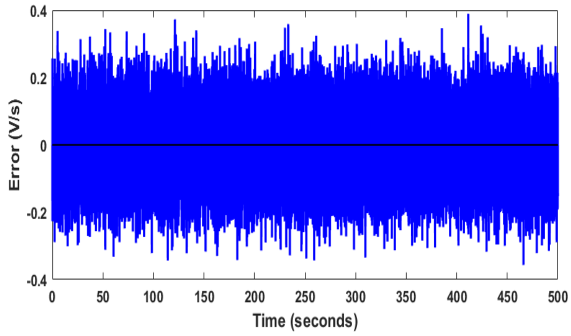

Fig. 5: Estimation Error: x_2-$\hat{x}_2$
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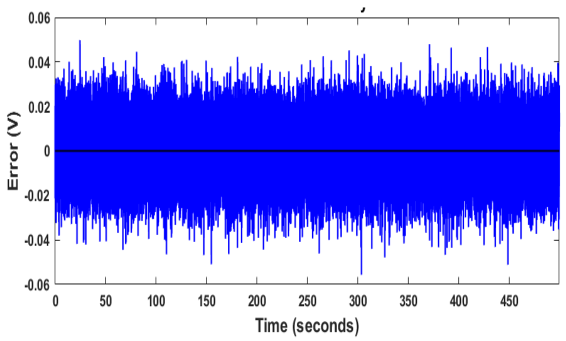

Fig. 6: Output error $\hat{y} - y$
Source: created by the authors.

simulations. The true and estimated terminal voltage are shown in Figure 2, while the corresponding estimation error is illustrated in Figure 3. Similarly, the true and estimated capacitor voltage derivative are presented in Figure 4, and its estimation error is depicted in Figure 5. The output tracking error between the measured and estimated terminal voltage is illustrated in Figure 6, confirming the convergence and accuracy of the proposed observer.

The presented figures indicate a clear discrepancy in observer accuracy: while the output voltage U is predicted with high precision (Maximum Error of the state x_1 : 0.030330 V) (Figure 3), the estimated $\frac{dv_c}{dt}$ shows slight deviation (Maximum Error of the state x_2: 0.380079 V/s) (Figure 5). This is a direct consequence of U being the measured output used for feedback correction, forcing its convergence. In contrast, the hidden state v_c is not measured, and its estimate is corrupted by inevitable inaccuracies in the model parameters and by noise or bias in the current sensor signal, upon which the v_c estimate critically depends.

These results demonstrate the observer's ability to accurately replicate the system states under the relevant operating conditions, supporting the effectiveness of the proposed estimating approach.

5 Conclusion

This study presented the design of a high-gain observer for online supercapacitor state estimation, modeled by a first-order RC circuit. By relying on the assumptions of known and constant parameters as well as on the observability of the system, the proposed observer guarantees an asymptotic convergence of state estimates. The simulation results, which have been developed in the MATLAB/Simulink environment, have validated the effectiveness of the method. The viability of the method for a real-time application is ensured by the fast convergence of the observer and the good robustness of the method concerning measurement noise. Although the estimation of the output voltage is of high accuracy, the estimation of the unmeasured internal state is accompanied by expected deviations. The results have shown the practical interest of the method for the monitoring and control of supercapacitors for demanding applications such as renewable energy storage systems and electric vehicle drives, which is ensured by a cheap computational alternative to traditional methods.

Acknowledgement:
The National Center for Scientific and Technical Research (CNRST) provided support for this work.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

All authors contributed to the writing and development of the manuscript.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare.