Fractional Edge Detection Techniques for Radiographic Images based on Fuzzy Systems

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Abstract: - Medical images are a diagnostic technique that facilitates the doctor's job the doctor to early diagnose the patient. So, fractional edge detection algorithms have gained focus of many researchers with the medical images. The fractional edge detection could be helped in the diagnosis of early stages of diseases like Alzheimer and fracture bone, but it can lose some edge details that can facilitate the doctor's job the doctor in the diagnosis. In this paper, different fractional edge detection algorithms based on fuzzy logic have been used to enhance the performance of the edges as there is no loss in the fuzzy-based methods. A performance comparison has been done between different fractional edge detection algorithms with and without fuzzy logic. Evaluation of the noise performance upon addition of salt and pepper noise is measured through peak signal to noise ratio (PSNR) and bit error rate (BER) simulated by using MATLAB.

Key-Words: - Edge Detection, Fractional Systems, Soft Computing Techniques, Radiographic, Fuzzy Systems, Medical Images

1 Introduction

Medical Imaging has gained focus of many researchers as it played an important role in helping the doctors to early diagnosis a lot of diseases [1]. The medical images are mostly used as radiographic techniques to help in early diagnosis and curing [2].

Nowadays, the quality of the digital image is ameliorated by using the image enhancement techniques for additional processing [3]. Image segmentation is the key challenge of object recognition and computer vision. It has the goal to extract the information, the first step in image analysis [4]. It is the method of partitioning to extract interest parts in a simple and easy analyzed way.

Edge detection can be deemed as one of the most common techniques in many applications in the field of image processing such as biomedical, radiographic images. Image edges contain enormously valuable information, so it is the foundational step for high-level information acquisition [5]. Its goal to distinguish and locate the sharp changes in brightness of an image [6]. The most commonly used discontinuity based edge detection techniques are Robert edge detection [7], Sobel Edge Detection [7], Prewitt edge detection [7], Kirsch edge detection [8], LOG (Laplacian of Gaussian) edge detection [9], [10] and Canny Edge Detection [11].

Edge detection can be implemented by either the Gradient or Laplacian methods. To detect the edges, the Gradient method is seeking for the minimum and maximum in the 1st derivative of the image while the Laplacian method is seeking for the zero crossings in the 2nd derivative of the image [12]. The edge detection implements by using the integer-order differential methods. There is a pros and cons for the use of integer-order differential, that it could enhance the edge information effectively, however, it could be sensitive to noise and easy to lose image detail information. To solve the previous problem, the fractional-order derivative has been used with the edge detection methods. There are many characteristics for the fractional-order derivative which are: it keeps high-frequency marginal feature where gray scale changes frequently, enhances medium-frequency texture details and also it can nonlinearly preserve more low frequency contour feature in those smooth areas, [13].

It is a major challenge to analyze the various internal structures of the body of the patients. The traditional edge detection techniques have limitations of using the fixed value of thresholds [14]. Soft computing as compared to the traditional techniques, it can deal with the mystery and uncertainty in image processing in a better way. There are many techniques for the soft computing techniques such as fuzzy logic, genetic algorithm, machine learning, neural computing and probabilistic reasoning. Soft computing tries to improve intelligence by building a machine which can work like a human [15]. In [14], fuzzy logic algorithm, one of the soft computing techniques, is used to detect the edges of noisy images. Fuzzy logic is one of soft computing techniques that provide elasticity and simplicity in structure. The fuzzy logic algorithm can find the edges of the image by using various levels of processing [16]. There is no loss in the fuzzy-based method, on the contrary, most of the traditional edge detection algorithms lead to a considerable loss in image edge details information [17].
In this paper, the main objective is to enhance the performance of the edges for the medical images to help the patient in the diagnosis by using different fractional edge detection algorithms based on fuzzy logic. A performance comparison has been done between different fractional edge detection algorithms with and without fuzzy logic. The performance evaluation is adopted, upon the addition of salt and pepper noise, by measuring through the peak signal to noise ratio (PSNR) and bit error rate (BER).

2 Material and Methods

Recently, fractional calculus [18] has played an important role in many fields, such as biology, robotics and image processing. Fractional calculus development was discussed in many research papers from different views, and the most widely used definitions are the Riemann-Liouville (R-L) fractional differentiation, Grümwald-Letnikov (G-L) fractional differentiation, and Caputo fractional differentiation. The generalization of the standard differentiation and integration to non-integer order is called fractional calculus [19]. In [20], a new mask based on the Newton Interpolation's Fractional Differentiation (NIFD) has been proposed and applied to image edge detection. According to a noisy image, the performance metrics showed that the proposed method gives a better edge information image than sobel and canny operator.

Table 1 discusses the characteristics of fractional order vs integral order differential [21]. From these three characteristics, it can be concluded that: 1) Fractional differential could nonlinearly keep the low-frequency contour feature. 2) It could nonlinearly boost high-frequency marginal information. 3) It could nonlinearly boost texture details. When the image is processed, it needs to keep the original information, improve image quality, boost details and texture characteristics, and keep the marginal details and energy as well. All these requirements can be difficult to obtain by traditional integral differential-based texture-enhancing algorithms, but it is easy to be obtained by the fractional differential-based algorithm.

Table 1: Fractional-order characteristics

	Fractional Differentiation	**Integeral Differentiation**
Smooth area	Non-zero	zero
Initial point of gray scale gradient	Non-zero	zero
Slope	Non-zero or constant	Constant

In this section, experiments are done on different types of Images using different fractional algorithms. Table 2 shows the different algorithms used in this paper using fractional edge detection. The first four algorithms use different fractional masks. Apply each fractional mask algorithm with different fractional orders with edge detection. The first step is to read the input image, convolve the image with Gaussian filter, convolute the image by the chosen filter and then apply the fractional algorithm to get the output image. Table 3 shows the different fractional masks and the equations used for each one.

Table 2: Eight algorithms using the integer and fractional edge detection

Algorithms	**Techniques used**
Algorithm 1	Fractional_Sobel
Algorithm 2	Fractional_Mask1
Algorithm 3	Fractional_Mask2
Algorithm 4	Fractional_Mask3
Algorithm 5	Fuzzy Fractional_Sobel
Algorithm 6	Fuzzy Fractional_Mask1
Algorithm 7	Fuzzy Fractional_Mask2
Algorithm 8	Fuzzy Fractional_Mask3

Algorithm 1 uses fractional-order sobel mask. Sobel detection is one of the gradient methods that use the 1st order derivative. It detects edges along the vertical and horizontal axis individually based on a pair of 3×3 convolution mask [22] [23]. The differential form of the gradient components is found along the x- and y-directions.

$$O_x = \frac{1}{2}\left(\frac{\partial s(x+1,y-1)}{\partial x} + 2\frac{\partial s(x+1,y)}{\partial x} + \frac{\partial s(x+1,y+1)}{\partial x}\right) \quad (1)$$

$$O_y = \frac{1}{2}\left(\frac{\partial s(x-1,y+1)}{\partial y} + 2\frac{\partial s(x,y+1)}{\partial y} + \frac{\partial s(x+1,y+1)}{\partial y}\right) \quad (2)$$

The Grünwald-Letnikov definition is used by assuming the size of image s is $M\times N$, and then the discrete form of $\nabla^v s$ can be represented as [23]

$$(\nabla^v s)_{i,j} = \left((\Delta_1^v s)_{i,j}, (\Delta_2^v s)_{i,j}\right) \quad 1 \le i \le M, 1 \le j \le N \quad (3)$$

Where

$$\begin{cases} (\Delta_1^v s)_{i,j} = \sum_{n=0}^{n-1} (-1)^n C_n^v s_{i-n,j} \\ (\Delta_2^v s)_{i,j} = \sum_{n=0}^{n-1} (-1)^n C_n^v s_{i,j-n} \end{cases} \quad (4)$$

Where C_n^v is the coefficient, $n \ge 3$ is an integer constant and Γ is the gamma function,

$$C_n^v = \frac{\Gamma(v+1)}{\Gamma(n+1)\Gamma(v-n+1)} \quad (5)$$

Algorithm 2 [24] implements Tiansi fractional differential gradient mask 5x5. The mask coefficients of the fractional differential operator are shown in equation 6:

$$C_{s_n} = (-1)^n \frac{\Gamma(v+1)}{n!\Gamma(v-n+1)} \quad (6)$$

Algorithm 3 is an improved covering template of the fractional differential on x or y coordinates by using the G-L definition of fractional calculus, a generalized fractional-order filter, and modified the coefficient of $-v$ to be 1/5 and that of v^2-v to be 1/6, was presented in [25].

Algorithm 4 [26] proposed the combination of fractional-order edge detection (FOED) and a chaos synchronization classifier for fingerprint identification. It is based on the Grunwald Letnikov (GL) definition. FOED has been ameliorated the fingerprint images with clarity of the ridge and valley structures to overcome the limitations of the integral-order method.

Then, in the other four algorithms (from algorithm 5 to 8) use the fractional edge detection based on fuzzy logic to enhance the performance than the traditional techniques.

First, read the image, convolve an input image with Gaussian Filter and convolute image with chosen filter. After that, using the different fractional masks algorithms with different fractional orders that are used previously. Then, apply the fuzzy logic rules to the x-gradient magnitude and y-gradient magnitude as inputs to get the output image. Fig.1 shows the flowchart for the fuzzy system using different fractional masks.

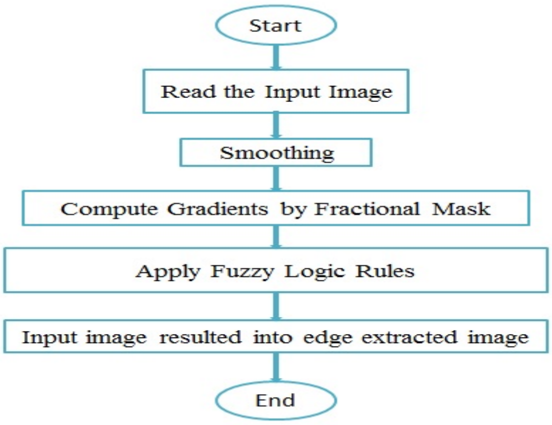

Figure 1: Fuzzy using Fractional Masks Flowchart

Table 3: Fractional masks and equations used in the different fractional edge detection algorithms

<table>
<tr><th rowspan="2">Algorithm</th><th rowspan="2">Equations</th><th rowspan="2">Parameters</th><th colspan="2">Masks</th></tr>
<tr><th>X-direction</th><th>Y-direction</th></tr>
<tr>
<td>1</td>
<td>$O_x^v = \frac{1}{2}\left(\frac{\partial^v s(x+1,y-1)}{\partial x^v} + 2\frac{\partial^v s(x+1,y)}{\partial x^v} + \frac{\partial^v s(x+1,y+1)}{\partial x^v}\right)$
$O_y^v = \frac{1}{2}(\frac{\partial^v s(x-1,y+1)}{\partial y^v} + 2\frac{\partial^v s(x,y+1)}{\partial y^v} + \frac{\partial^v s(x+1,y+1)}{\partial y^v})$</td>
<td>V: fractional order</td>
<td><table>
<tr><td>$(-1)^n C_n^v/2$</td><td>$(-1)^n C_n^v$</td><td>$(-1)^n C_n^v/2$</td></tr>
<tr><td>⋮</td><td>⋮</td><td>⋮</td></tr>
<tr><td>$(v^2-v)/4$</td><td>$(v^2-v)/2$</td><td>$(v^2-v)/4$</td></tr>
<tr><td>-v/2</td><td>-v</td><td>-v/2</td></tr>
<tr><td>1/2</td><td>1</td><td>1/2</td></tr>
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<td><table>
<tr><td>$(-1)^n C_n^v/2$</td><td>…</td><td>$(v^2-v)/4$</td><td>-v/2</td><td>1/2</td></tr>
<tr><td>$(-1)^n C_n^v$</td><td>…</td><td>$(v^2-v)/2$</td><td>-v</td><td>1</td></tr>
<tr><td>$(-1)^n C_n^v/2$</td><td>…</td><td>$(v^2-v)/4$</td><td>-v/2</td><td>1/2</td></tr>
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<td>2</td>
<td>

$$\frac{d^v s(x,y)}{dx^v} \approx s(x,y) + (-v)s(x-1,y) + \frac{v(v-1)}{2}s(x-2,y) + \cdots + (-1)^n \frac{\Gamma(v+1)}{n!\,\Gamma(v-n+1)} s(x-n,y)$$

$$\frac{d^v s(x,y)}{dy^v} \approx s(x,y) + (-v)s(x,y-1) + \frac{v(v-1)}{2}s(x,y-2) + \cdots + (-1)^n \frac{\Gamma(v+1)}{n!\,\Gamma(v-n+1)} s(x,y-n)$$

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<td></td>
<td>
<table>
<tr><td>$(v^2-v)/2$</td><td>-v</td><td>0</td><td>v</td><td>$(v^2-v)/2$</td></tr>
<tr><td>(v^2-v)</td><td>-2v</td><td>0</td><td>2v</td><td>(v^2-v)</td></tr>
<tr><td>$3(v^2-v)/2$</td><td>-3v</td><td>0</td><td>3v</td><td>$3(v^2-v)/2$</td></tr>
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<tr><td>-v</td><td>-2v</td><td>-3v</td><td>-2v</td><td>-v</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>v</td><td>2v</td><td>3v</td><td>2v</td><td>v</td></tr>
<tr><td>$(v^2-v)/2$</td><td>(v^2-v)</td><td>$3(v^2-v)/2$</td><td>(v^2-v)</td><td>$(v^2-v)/2$</td></tr>
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<td>3</td>
<td>

$$\frac{d^v s(x,y)}{dx^v} \approx \sum_{z=-a}^{a} \sum_{k=-2b}^{0} W_A(z,k)\, s(x+z,y+k)$$

$$\frac{d^v s(x,y)}{dy^v} \approx \sum_{z=-2a}^{0} \sum_{k=-b}^{b} W_B(z,k)\, s(x+z,y+k)$$

</td>
<td>W_A and W_B: covering templates on x and y coordinates respectively.

m=2a+1

n=2b+1

a=(m-1)/2

b=(n-1)/2</td>
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<table>
<tr><td>$(v^2-v)/6$</td><td>$(v^2-v)/6$</td><td>$(v^2-v)/6$</td></tr>
<tr><td>-v/5</td><td>-v/5</td><td>-v/5</td></tr>
<tr><td>-v/5</td><td>1</td><td>-v/5</td></tr>
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<table>
<tr><td>$(v^2-v)/6$</td><td>-v/5</td><td>-v/5</td></tr>
<tr><td>$(v^2-v)/6$</td><td>-v/5</td><td>1</td></tr>
<tr><td>$(v^2-v)/6$</td><td>-v/5</td><td>-v/5</td></tr>
</table>
</td>
</tr>
</table>

<table>
<tr>
<td>4</td>
<td>

$$\frac{d^v s(x,y)}{dx^v} \approx s(x,y) + (-v)s(x-1,y) + \frac{(-v)(-v+1)}{2}s(x-2,y) + \cdots + \frac{\Gamma(-v+1)}{n!\,\Gamma(-v+n+1)}s(x-n,y)$$

$$\frac{d^v s(x,y)}{dx^v} \approx s(x,y) + (-v)s(x,y-1) + \frac{(-v)(-v+1)}{2}s(x,y-2) + \cdots + \frac{\Gamma(-v+1)}{n!\,\Gamma(-v+n+1)}s(x,y-n)$$

</td>
<td>V: fractional order</td>
<td>
<table>
<tr><td>0</td><td>(v^2-v)/2</td><td>0</td></tr>
<tr><td>0</td><td>-v</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
</table>
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<td>
<table>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>(v^2-v)/2</td><td>-v</td><td>1</td></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
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</table>

3 Results and Discussion

The algorithms have been implemented by MATLAB and applied on different types of images to verify its performance. These algorithms are applied upon medical images like MRI (Magnetic Resonance Image) and x-ray images.

Table 4 shows the images that used different fractional edge detection algorithms and table 5 shows the images that used fractional edge detection algorithms based on fuzzy logic to enhance the performance.

Table 4: Different types of Images using different Fractional algorithms

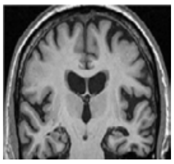
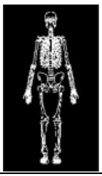
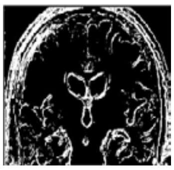
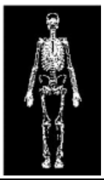
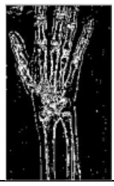
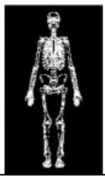
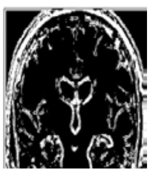
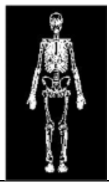
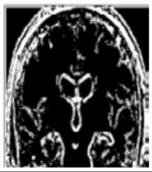
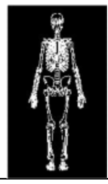
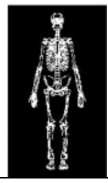
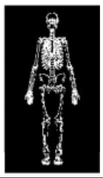
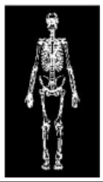
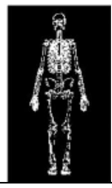
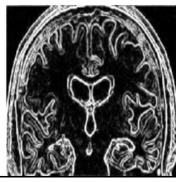
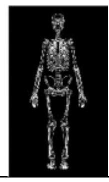
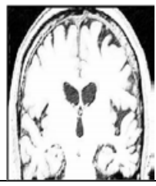
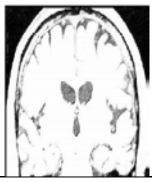
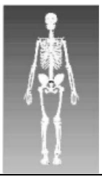
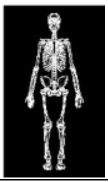
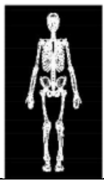
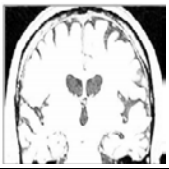
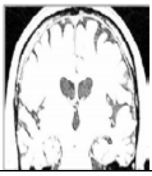
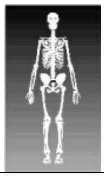
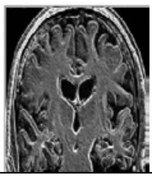
Algorithm	Alzaheimer Brain (383x270)	Skeleton (638x1104)	XrayHand (645x1024)
Original Image			
Edge Detection			
Algo.1 (v=0.2)			
Algo.1 (v=0.8)			
Algo.2 (v=0.2)			
Algo.2 (v=0.8)			
Algo.3 (v=0.2)			
Algo.3 (v=0.8)			
Algo.4 (v=0.2)			
Algo.4 (v=0.8)			

Table 5: Different types of Images using fuzzy logic with integer and Fractional edge detection

Algorithm	Alzaheimer Brain (383x270)	Skeleton (638x1104)	XrayHand (645x1024)
Fuzzy Edge Detection			
Algo.5 (v=0.2)			

Algo.5 (v=0.8)			
Algo.6 (v=0.2)			
Algo.6 (v=0.8)			
Algo.7 (v=0.2)			
Algo.7 (v=0.8)			
Algo.8 (v=0.2)			
Algo.8 (v=0.8)			

The performance comparison is done by measuring the MSE & PSNR, bit error rate and the execution time. The PSNR is measured between the noise free image and the noisy image (salt & pepper noise) with noise density=0.02.

The execution time for images depends first on the dimension of each image, then the algorithm used. From fig. 2, we noticed that algorithm 5 takes greater time than the other algorithms, but it is one of the best results.

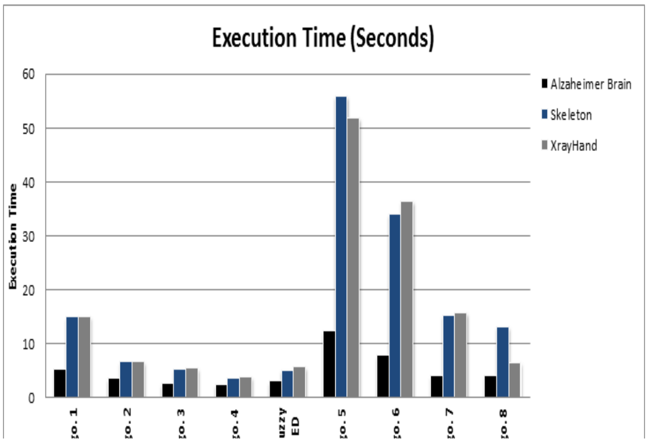


Figure 2: Execution Time in seconds for Different Images

According to the MSE in table 6, algorithms 5, 7, and 8 are the best ones for the images with v=0.8 in algorithm 5 and v=0.2 in algorithms 7 and 8.

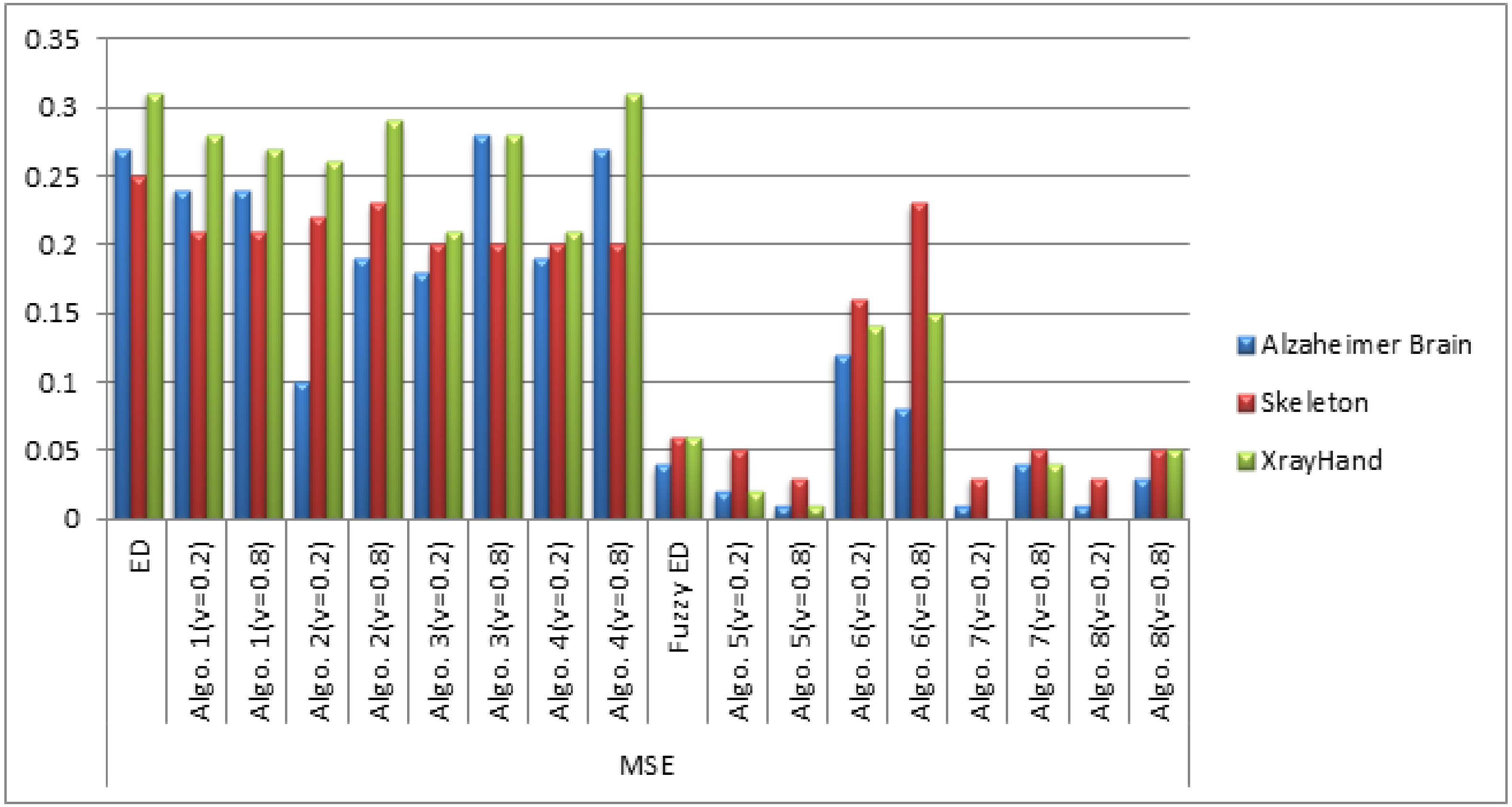
According to the PSNR results in table 6, the highest result of the images is in the algorithm 7 when v=0.2, then algorithm 8 when v=0.2, then algorithm 5 when v=0.8. Then the anti-noise performance of algorithm 7 when v=0.2 is the best compared with the other algorithms. Then the choice of the fractional mask for the edge detection is important.

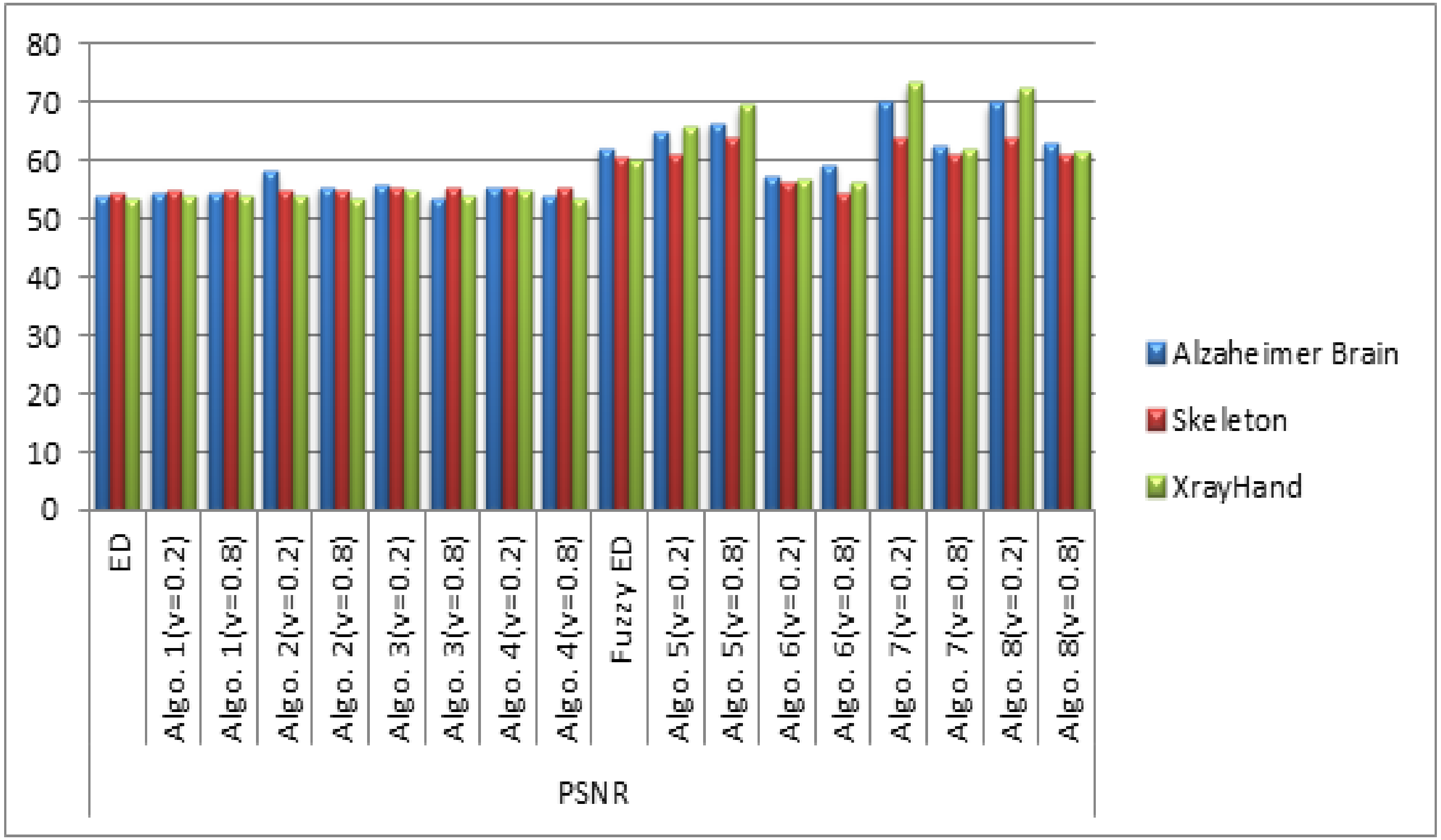
According to the bit error rate in table 6, algorithm 5 when v=0.8, and algorithms 7 and 8 when v=0.2 are the best ones for the most images.

4 Conclusion

Nowadays, many researchers work on fractional-order differential methods with edge detection operators to enhance the edge information effectively. This paper makes a performance comparison between the fractional edge detection with and without fuzzy logic. From the results, it shows that the fractional-order is better than the integer-order differentiation and fractional edge detection based on fuzzy logic gives better results and enhances the performance than fractional edge detection algorithms. Also, there is no such loss in image edge detail information when using the fuzzy logic. From the PSNR and BER results, it can be concluded that the fractional edge detection based on fuzzy logic ameliorated the results. The best results are in Fuzzy Fractional_Sobel when v=0.8, and Fuzzy Fractional_Mask2 and Fuzzy Fractional_Mask3 when v=0.2.

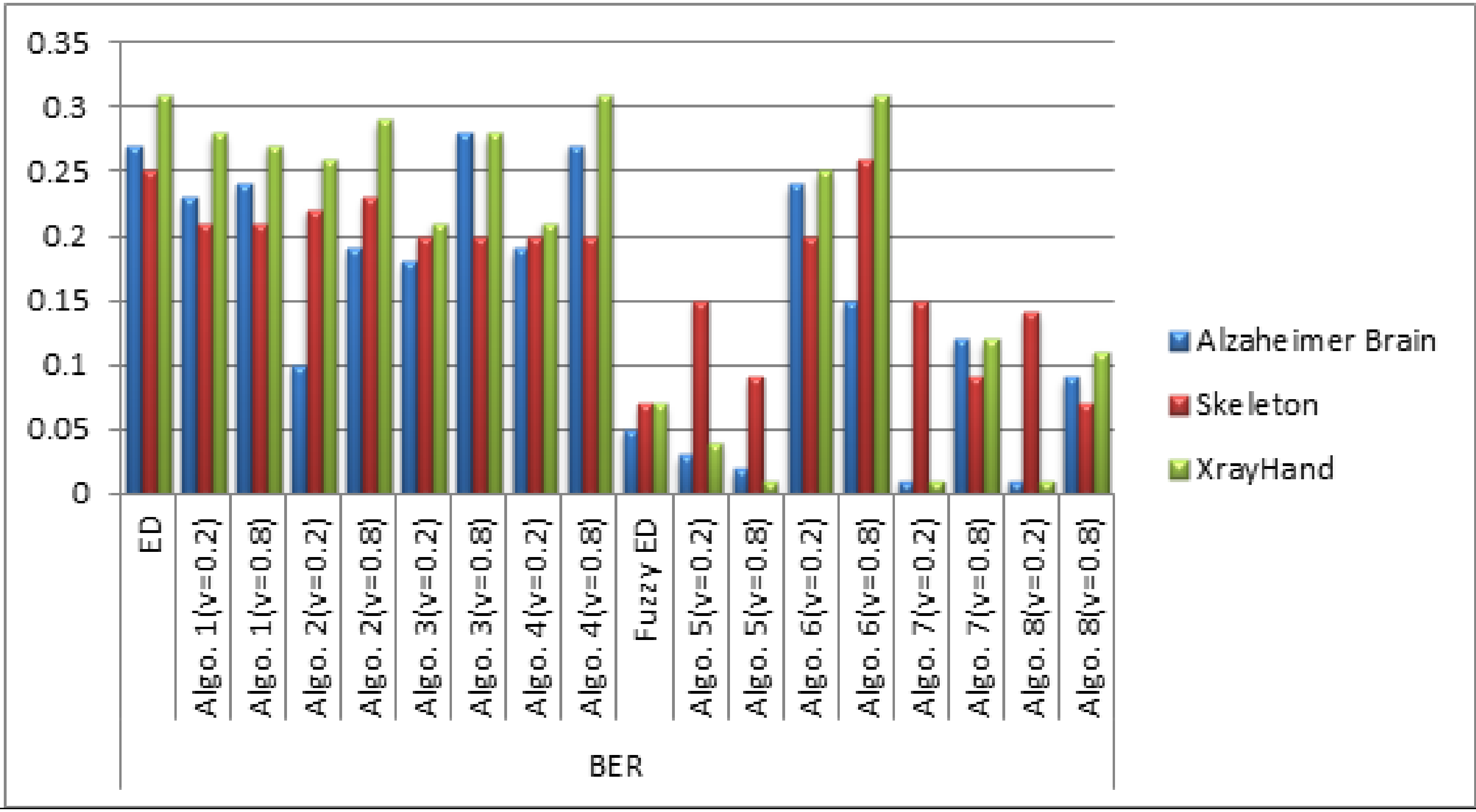
Table 6: MSE, PSNR, BER Results

	Results
MSE	

PSNR	

80
70
60
50
40
30
20
10
0
ED
Algo. 1(v=0.2)
Algo. 1(v=0.8)
Algo. 2(v=0.2)
Algo. 2(v=0.8)
Algo. 3(v=0.2)
Algo. 3(v=0.8)
Algo. 4(v=0.2)
Algo. 4(v=0.8)
Fuzzy ED
Algo. 5(v=0.2)
Algo. 5(v=0.8)
Algo. 6(v=0.2)
Algo. 6(v=0.8)
Algo. 7(v=0.2)
Algo. 7(v=0.8)
Algo. 8(v=0.2)
Algo. 8(v=0.8)
PSNR
Alzaheimer Brain
Skeleton
XrayHand

BER


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
ED
Algo. 1(v=0.2)
Algo. 1(v=0.8)
Algo. 2(v=0.2)
Algo. 2(v=0.8)
Algo. 3(v=0.2)
Algo. 3(v=0.8)
Algo. 4(v=0.2)
Algo. 4(v=0.8)
Fuzzy ED
Algo. 5(v=0.2)
Algo. 5(v=0.8)
Algo. 6(v=0.2)
Algo. 6(v=0.8)
Algo. 7(v=0.2)
Algo. 7(v=0.8)
Algo. 8(v=0.2)
Algo. 8(v=0.8)
BER
Alzaheimer Brain
Skeleton
XrayHand

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References:

[1] S.M. Ismail, A.G. Radwan, A.H. Madian, M.F. Abu-ElYazeed, Comparative Study of Fractional Filters for Alzheimer Disease Detection on MRI images, *39th International Conference on Telecommunications and Signal Processing (TSP), IEEE*, 2016, pp.720-723.

[2] L.K. Lee, S.C. Liew, A Survey of Medical Image Processing Tools, *4th International Conference on Software Engineering and Computer Systems (ICSECS), IEEE, Malaysia*, 2015, pp. 171-176.

[3] M. Petrou, C. Petrou, *Image Processing: the fundamentals*, 2nd edition, John Wiley & Sons, Ltd Singapore, 2010.

[4] D. Kaur, Y. Kaur, Various Image Segmentation Techniques: A Review, *International Journal of Computer Science and Mobile Computing (IJCSMC)*, Vol.3, No.5, 2014, pp. 809-814.

[5] H. Qirui, L. ke, A Novel Edge Detection Operator Based on Fractional Gaussian Differential, *National Science Foundation*, Available: http://www.paper.edu.cn, 2014.

[6] B.L. Srinivas, Hemalatha, K.A. Jeevan, Edge Detection Techniques for Image Segmentation, *International Journal of Innovative Research in Computer and Communication Engineering (IJIRCCE)*, Vol.3, 2015, pp. 288-292.

[7] G.N. Chaple, R.D. Daruwala, M.S. Gofane, Comparisions of Robert, Prewitt, Sobel operator based edge detection methods for real time uses on FPGA, *International Conference on Technologies for Sustainable Development (ICTSD-2015), IEEE*, 2015.

[8] Y.G. Zhang, W. Lu, F.Y. Cheng, L. Song, A Edge Detection Method for Microcalfication Clusters in Mammograms, *2nd International Conference on Biomedical Engineering and Informatics, BMEI '09, IEEE*, 2009, pp.1-5.

[9] S. Gupta, C. Gupta, S.K. Chakarvarti, Image Edge Detection: A Review, *International Journal of Advanced Research in Computer Engineering & Technology (IJARCET)*, Vol.2, No.7, 2013, pp. 2246-2251.

[10] V. Torre, T.A. Poggio, On edge detection, *IEEE Trans. Pattern Analysis Machine Intell.*, Vol.2, No.8, 1986, pp. 163-187.

[11] E.K. Sharma, E.N. Kaur, Comparative Analysis of Various Edge Detection Techniques, *International Journal of Advanced Research in Computer Science and Software Engineering (IJARCSSE)*, Vol.3, 2013, pp. 617-621.

[12] Z. Hussain, D. Agarwal, A Comparative Analysis of Edge Detection Techniques used in Flame Image, *International Journal of Advance Research in Science and Engineering (IJARSE)*, Vol.4, 2015, pp. 1335-1343.

[13] D. Tian, J. Wu, Y. Yang, A Fractional-order Edge Detection Operator for Medical Image Structure Feature Extraction, *26th Chinese Control and Decision Conference (CCDC), IEEE*, 2014, pp. 5173-5176.

[14] J. Mehena, M.C. Adhikary, Medical Image Edge Detection Based on Soft Computing Approach, *International Journal of Innovative Research in Computer and Communication Engineering (IJIRCCE)*, Vol.3, No.7, 2015, pp. 6801-6807.

[15] N.S. Joshi, N.S. Choubey, Application of Soft Computing Approach for Edge Detection, *International Journal of Application or Innovation in Engineering & Management (IJAIEM)*, Vol.3, No.4, 2014, pp. 116-122.

[16] A.D. Borkar, M. Atulkar, Detection of Edges Using Fuzzy Inference System, *International Journal of Innovative Research in Computer and Communication Engineering*, Vol.1, No.1, 2013.

[17] S. Mathur, A. Ahlawat, Application of Fuzzy Logic on Image Edge Detection, *International Conference "Intelligent Information and Engineering Systems" INFOS 2008, Varna, Bulgaria*, 2008, pp. 24-28.

[18] S. Das, *Functional Fractional Calculus*, Springer- Verlag, Chapter 1, 2011.

[19] X. Xu, F. Dai, J. Long, W. Guo, Fractional Differentiation-based Image Feature Extraction, *International Journal of Signal Processing, Image Processing and Pattern Recognition*, Vol.7, No.6, 2014, pp. 51 -64.

[20] C. Gao, J. Zhou, W. Zhang, Edge Detection Based on the Newton Interpolation's Fractional Differentiation, *The International Arab Journal of Information Technology*, Vol.11, No.3, 2014, pp. 223-228.

[21] Y.F. Pu, J.L. Zhou, X. Yuan, Fractional differential mask: A fractional differential-based Approach for Multiscale Texture Enhancement, *IEEE Transactions on Image Processing*, Vol.19, No.2, 2010, pp. 491-511.

[22] W.S. ElAraby, A.H. Madian, M.A. Ashour, I. Farag, M. Nassef, Fractional canny edge detection for biomedical applications, *28th International Conference on Microelectronics (ICM), IEEE*, 2016, pp. 265-268.

[23] D. Tian, J. Wu, Y. Yang, A Fractional-order Edge Detection Operator for Medical Image Structure Feature Extraction, *26th Chinese Control and Decision Conference (CCDC)*, 2014, pp. 5173-5176.
[24] Z. Yang, F. Lang, X. Yu, Y. Zhang, The Construction of Fractional Differential Gradient Operator, *Journal of Computational Information Systems*, 2011, pp. 4328-4342.
[25] Y. Pu, Fractional calculus approach to texture of digital image, *IEEE International Conference Signal Processing*, 2006, pp. 1002–1006.
[26] J. Chen, C. Huang, Y. Du, C. Lin, Combining fractional-order edge detection and chaos synchronisation classifier for fingerprint identification, *IET Image Processing*, Vol.8, No.6, 2014, pp. 354–362.