Expansion of Game Refinement Theory into Continuous Movement Games: A Comparison of Two Video Games

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Abstract: Different frameworks for the scientific analysis of varying aspects of games have been developed. Aside from conventional game theory and AI for games, game refinement theory can be used to evaluate basics of the search-space complexity, uncertainty of outcome, and acceleration of game information at game's end. The original model was developed to show a measure of game information with some relation to a game's entertainment with respect to that game's history. In prior studies, analysis of discrete movement board games has shown a tendency toward a certain value, based on a model of the evolution of chess. Using a modified logistic model of game progress, game information from other non-board game type games have shown a similar value describing an aspect of entertainment in those games. This paper uses game refinement theory to analyze and consider the game information of two continuous movement video games of contemporary popularity, Pac-Man and Defense of the Ancients.

Key-Words: - Game Refinement, Theory, Pac-Man, DotA, Video Games

1 Introduction

Computer games are an integral part of social and cultural behavior [1], and are the most popular home computer activity for children and adolescents [2], [3]. Computer games have replaced more traditional leisure activities over the last 40 years, and that change has been fundamental [4]. Computer game history begins in the 1940s and many different games and gaming systems have come into play since then.

The first graphic computer interface ever created is believed to have been the game OXO by Alexander Douglas in 1952. It was a version of tic-tac-toe that he wrote for his thesis on human-computer interaction. William Higinbothm created the first video game ever in 1958. His game, Tennis for Two, was created and played on a Brookhaven National Laboratory oscilloscope [5]. The number of games increased rapidly in the 1980sthe Golden Age of video games with the introduction of games like Donkey Kong, Pac-man, Tetris, Mario Brothers, Kings Quest, The Legend of Zelda as well as television gaming systems like Atari and Nintendo. Each year more sophisticated computer games get produced along with high-tech gaming devices. Everything from the monolithic arcade version games to pocket games continues to enjoy their places. The availability of new consoles, platforms and technologies for the delivery of games has been a key factor for continued growth. With over 30 years of solid history some of these games are still being played and providing opportunities for applying of game theory. Most popular game theory refer to study by [6] entitled Theory of Games and Economic Behavior to expand economic analysis to allow

economists to model the rules of the game that influence particular environments. In 1950 with one page journal John Forbes Nash formulated game theory title Equilibrium Points in N-Person games [7], John Nash contributed a remarkable one-page PNAS article that defined and characterized a notion of equilibrium for n-person Games. This notion, now called the Nash equilibrium, has been widely applied and adapted in economics and other behavioral sciences [8].

Game theory also proposed by [9] identified as Game refinement theory, was invented based on the concept of uncertainty of game outcome. Maintaining balance throughout the duration of games is important to their enjoyableness. It is more interesting both for players and observers when the information of the outcome of the game is unclear until the very end, when game information culminates and results in game certainty. Furthermore, it is believed that the refinement of games uncertainty is one of the quantities, which have contributed to the success, or extinction of games historically.

Cincotti et al. [10] proposed measure and the search space complexity have been applied in the domain of Chess and Shogi to study the evolution of these games through the course of history. Measuring the uncertainty outcome based on the concept of information theory for the popular games that presented by [11]. The similarities of game entertainment impact for Mah Jong and other refined game such as chess major variants [12]. Most of the work concentrate on measuring refinement theory and apply in board games.

In this study we have chosen two digital games, Pac-Man and DotA. We applied the game refinement theory to these games as a test for the attractiveness of these games. In Section 2 we present a short sketch of game refinement theory, and an application to Pac-Man and DotA in Section 3 and Section 4 respectively. Concluding remarks are given in Section 5.

2 Game Refinement Theory

Game refinement theory examines the relationships between skill and chance, and the relationship of entertainment to the evolution of games. Game refinement theory was invented based on concept of uncertainty of game outcome.[9],[13]. Later it was extended for general game related actions [14] in which a mathematical model of game refinement was constructed based on the concept of the game progress model. ”Game progress” derives from two functions. One is game speed or scoring rate, and the other is the information of solved game uncertainty. In sports games such as soccer and basketball, the scoring rate is calculated by two factors: (1) goal, i.e., total score and (2) time or steps to achieve the goal [14],[15]. Game speed is given by average number of successful shoots divided by average number of shoot attempts. For other games such as volleyball and tennis, in which the total score to win is set in advance (score-limited games), the average number of total points per game may correspond to the steps to achieve the goal [16].

Game information progress presents the degree of certainty of a game’s results in time or in steps. Let G and T be the average number of successful shots and the average number of shots per game, respectively. Having full information of the game progress, i.e. after its conclusion, game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq T$ and $0 \leq x(t) \leq G$, as shown in Equation (1).

$$x(t) = \frac{G}{T} t \tag{1}$$

However, the game information progress given by Equation (1) is usually unknown during the game. The presence of uncertainty during the game, often until the final moments, reasonably renders game progress as exponential. A realistic model of game information progress is given by Equation (2).

$$x(t) = G\left(\frac{t}{T}\right)^n \tag{2}$$

Here n stands for a constant parameter which is given based on the perspective of an observer considered. Then acceleration of game information progress is obtained by deriving Equation (2) twice. Solving it at $t = T$, the equation becomes:

$$x''(T) = \frac{Gn(n-1)}{T^n} t^{n-2} = \frac{G}{T^2} n(n-1) \tag{3}$$

It is assumed in the current model that game information progress in any type of game is encoded and transported in our minds. We do not yet know about the physics of information in the brain, but it is likely that the acceleration of information progress is related to the forces and laws of physics. Therefore we expect that the larger the value $\frac{G}{T^2}$ is, the more exciting the game, due in part to the uncertainty of game outcome. Thus, we use its root square, $\frac{\sqrt{G}}{T}$, as a game refinement measure for the game under consideration. We can call it “ R value” for short.

Here we consider the relationship between board games and sports games by deriving a formula to calculate the game information progress of board games. Let B and D be the average branching factor (number of possible options) and game length (depth of whole game tree), respectively. One round in a board game can be illustrated as a decision tree. At each depth of the game tree, one will choose a move and the game will progress. Figure 1 illustrates one level of the game tree. The distance d, shown in Figure 1, can be found by using simple Pythagoras theorem, thus resulting in $d = \sqrt{\Delta l^2 + 1}$.

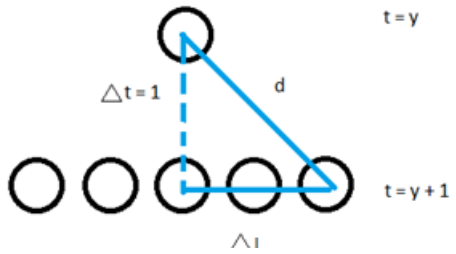

Fig. 1 Illustration of one level of game tree

Assuming that the approximate value of horizontal difference between nodes is $\frac{B}{2}$, get $d = \sqrt{(\frac{B}{2})^2 + 1}$. The game progress for one game is the total level of game tree times d. For the meantime, we do not consider Δt^2 because the value ($\Delta t^2 = 1$) is assumed to be much smaller compared to B. The game length will be normalized by the average game length D, then the game progress $x(t)$ is given by $x(t) = \frac{t}{D} \cdot d = \frac{t}{D}\sqrt{(\frac{B}{2})^2} = \frac{Bt}{2D}$. Then, in general we have, $x(t) = c\frac{B}{D}t$, where c is a constant which depends on the game considered. However, we managed to explain how to obtain the game information progress value itself. The game progress in the domain of board games forms a linear graph with B being the maximum value $x(t)$. Assuming $c = 1$, then we have a realistic game progress model for board games, which is given by

$$x(t) = B(\frac{t}{D})^n \quad (4)$$

Equation (4) shows that the game progress in board games corresponds to that of sports games as shown in Equation (2).

To support the effectiveness of the proposed game refinement measures, some data of games such as Chess and Go [6] from board games and two sports games [7],[8] are compared. A comparison of game refinement measures for various type of games show in Table 1. From Table 1, we see that sophisticated games have a common factor (i.e., same degree of acceleration value) for engagement or excitement regardless of different type of games.

Table 1 Measures of game refinement for board games and sports games

Game	G	T	R-value
Chess	35	80	0.074
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073

3 Pac-Man

Pac-Man is a classic real-time arcade computer game originally developed by Toru Iwatani for the Namco Company in 1980. First released in Japan, the game became immensely popular, rapidly achieving cult status, and various other versions followed. Pac-Man has been voted the greatest video game character of all time, according to a poll of UK customers of PC and console games [17]. Regarded as a landmark in video game history, PC-Man is among the most famous arcade games [18]. Although the original Pac-Man game was released more than 30 years ago, it is still played on various computing platform including mobile devices. Many versions of the game have been developed subsequently for the home computer, game console and hand-held system.

3.1 Rules of Pac-Man

Pac-man is a one-player game where the human player moves the main character (Pac-man) around a maze. The aim of the game is to score as many points as possible, by eating 240 dots and all four power pills throughout the maze as drawn in (figure 1).

Figure1: The maze of Pac-man Game

Obstacles consisting of four ghost characters pursue Pac-man around the maze and must attempt to avoid otherwise Pac-man lose one of the three live and game is

over when Pac-man has lost all live. The ghosts can be seen at Figure 2, have different colours: Blinky (red), Inky (light blue), Pinky (pink), and Clyde (orange). However, they all turn dark blue when Pac-Man eats one of the four "power pellet" available on each level. Four "power pellet" are located near each Conner of the maze. When the Pac-Man eat the power pellet, the four active ghost changes to inactive (Figure 2) and the Pac-man able to eat the ghost character for the few seconds of time.

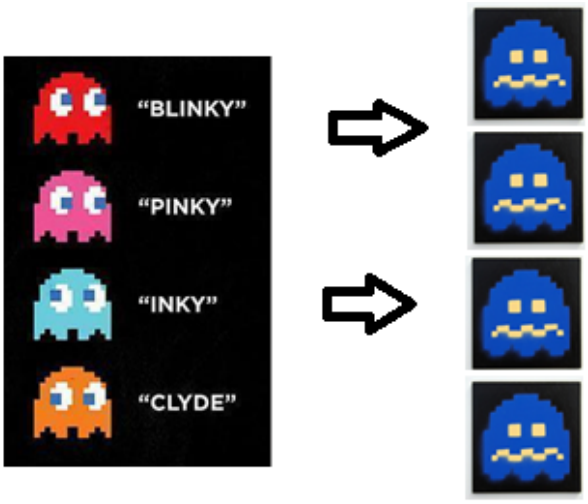

Figure 2 : The four Ghost Change to dark blue

Everything Pac-Man eats has different point values. The dots are 10 points each; the power pellet is 50 points each. The dark blue ghosts are respectively 200, 400, 800 and 1600 points. There are different point values for every bonus fruit or prize that is eaten: cherries, 100 points strawberry, 300 points peach, 500 points apple, 700 points grapes, 1000 points, thunderbird 2000 points bell, 3000 points and key, 5000 points. You will also get a bonus Pac-Man at 10,000 points switch selectable; can be 15,000 points, 20,000 points, or no bonus [19].A perfect Pac-Man game involves finishing with all 256 levels and earning all the possible points without losing a life. The perfect game score is 3,333,360 points, meaning you must eat every bonus fruit, eat all four ghosts with every power pellet, and clear all 256 rounds

However, Pac-man just classic arcade games are simple does not mean they are easy. Refer to [20] said

"*Needless to say the Pac-Man "perfect game" is not an easy feat to achieve. After the first 21 boards or so, the maze pretty much stays the same, however the ghosts get faster and they don't turn blue when you eat a power pellet.* "

Over the years, players have developed strategies for avoiding the ghosts and win the game to get perfect Pac-man score. It takes over 19 years for someone to finally complete with a perfect Pac-man score. According to Twin Galaxies International Scoreboard on July 3, 1999, the first people to achieve this maximum possible score (3,333,360 points) was Billy Mitchell of Hollywood, Florida, who performed the feat in about six hours [21]. Until now just 6 perfect score are report by Twin Galaxies International Scoreboard and the last perfect score report in 2009. This means thegameis still played until now and still play sthe sameway. Could be said digital games Pac-man is still relevant to be studied using game refinement theory.

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3.2 Game refinement theory applied to Pac-Man

In this study we need real playing data from Pac-Man to find the game refinement value for Pac-Man. Unfortunately, we cannot find such data. The facts that Pac-Man is an old game and played in arcade machines possibly make Pac-Man playing data unavailable. Data from the newer version of Pac-Man are also unavailable. The only data available are images showing how to solve the Pac-Man maze [22]. But we cannot use the data, since it gives patterns not only to finish all the levels but also to achieve high scores. However, achieving high score is not our main consideration. Our main consideration is to finish the game. The reason why we should only focus on finishing the game rather than achieving high score is based on the background of the game refinement theory itself.

Game refinement theory was originally created from studying board games. Most board games usually, do not use points to identify the winner of the game. Even if some board games, such as Othello do use points however they do not give different weightings on each piece.To collect game progress data of Pac-Man, we have developed a simple agent to play against the basic Pac-Man ghost agents. The framework that we have used to develop the agent is the same framework that was used for IEEE Ms.Pac-Man competition [23]. The Ms. Pac-Man framework was developed by Simon Lucas in Java object oriented language. He and David Robles have also developed, probably the first, simple tree search method for playing Ms. Pac-Man [24]. We have adopted the idea tree to develop an agent.

Our agent strength is almost as same as the BasicGhost algorithm on a certain type of maze. The BasicGhost algorithm is obtained when we downloaded

the framework for developing agents. The agent uses greedy algorithm combined with depth-first-search and hand coded evaluation function. Our agent focuses only on finishing the game and not achieving high score. As the level increases, the difficulty also increases. The agent can only survive up to two levels. We think that the strength of the algorithm is enough because what we are focusing is to find the game refinement itself, not the algorithm optimization. Another reason is, in order to find the game refinement measurement, the players should be under approximately the same strength.

3.3 Pac-Man: Data collection and results

We collect the data from 100 win matches. We do not consider if the agent loses a match. We use the number of pellets L to replace the branching factor B. Therefore, the number of real movements that has been counted should be divided by the space between pellets. The number of pellets L in the data may vary. This is, different maze will have different L value between levels. We also know that the space between pellets is worth 4 movements. Thus to find the number of movements M, the real number of movements will be divided by 4.

The number of movements from our Pac-Man agent can be seen at Figure 3. Although different levels may differ in number of pellets, this does not mean that an increase in the number of pellets will increase the Pac-Man agent movements. We compared game data consisting of lavel one and level two from Ms. Pc-Man.

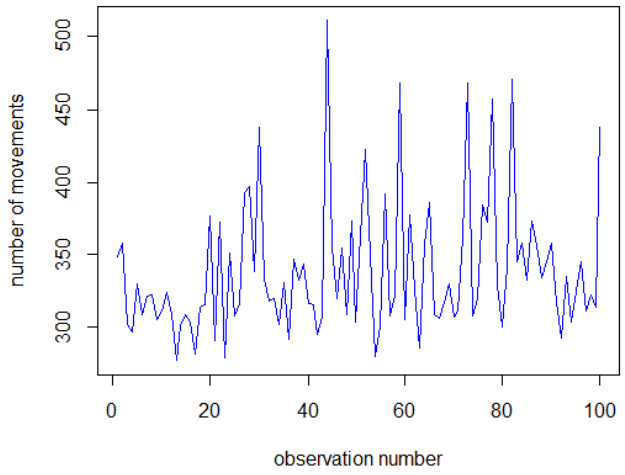

Fig.3 Number of Movement Charts

We have discussed the model of game refinement as constructed by the number of pellets L and the number of movements, M have been determined in Pac-Man. Now we can calculate the game refinement constant for Pac-Man as follows.

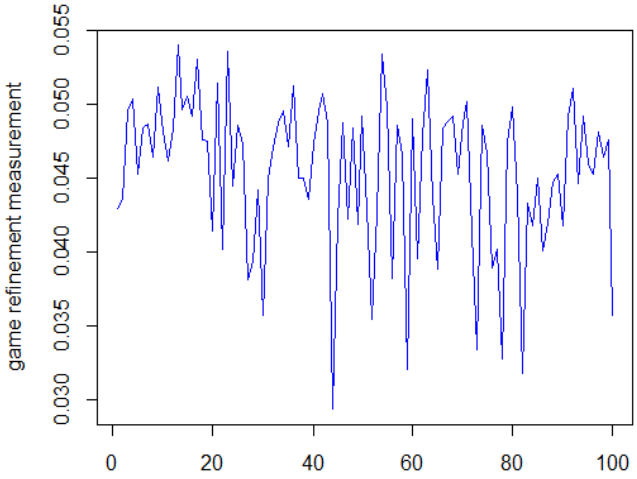

Fig.4 Game refinement value of Pac-Man

The game refinement value of Pac-Man is lower than that of board games which have undergone long sophistication processes, as can be seen in Figure 4. By our calculations, Pac-Man's game refinement value lies between 0:03 - 0:055. The mean is 0:045. Sophisticated board/sports games' refinement lies between 0:07 and 0:08 according to previous studies. Also, because the data that we used is not from real games but from computer simulations, we must acknowledge the possibility that human player game data could present a different result.

4 DotA

The full name of "DotA" is "Defense of the Ancient", which is a kind of MOBA (Multi-player Online Battle Arena) games. MOBA game is a subclass of Real-time strategy (RTS) games. DotA is just a map for the game of "Warcraft 3". However, it has become one of the most popular games in the world. From Asia to Europe and North America, there are lots of teams and players who have devoted to playing it. A screen shot of DotA is shown in Figure 5

Fig.5 Screenshot of DotA

4.1 The rules of DotA

The DotA [24] game sets two teams of player “Sentinel" and “Scourge" to battle each other. Players on the Sentinel team are located at the southwest corner of the map, and those on the Scourge team are located at the northeast corner. Each base is defended by towers and waves of units which guard the main paths leading to their bases. The center of each base is called “Ancient" that is the building that must be destroyed to win the game. Usually, the losing side may input GG (Good Game) when they find that there is no hope to win, which means to give up and quit the game.

Each human player controls one hero, a powerful unit with unique abilities. In DotA, players on each side choose one of 112 heroes, each with different abilities and tactical advantages over other heroes. The scenario is highly team-oriented; it is hard for one player to carry the team to victory alone. DotA allows up to ten players in a five-versus-five format and additional two slots for referees or observers.

Because the game play revolves around strengthening individual heroes, it does not require one to focus on resource management and base-building, unlike most traditional RTS games. Killing computer-controlled or neutral units earns the player experience points. When enough experience is accumulated, the player gains a level. Levelling up improves the hero’s toughness and the damage they can inflict, and allows players to upgrade their spells or skills. In addition to accumulating experience, players also manage a single resource: gold. In addition to a small periodic income, heroes earn gold by killing hostile units, base structures, and enemy heroes. Using gold, players buy items to strengthen their hero and gain abilities. Some items can also be combined with recipes to create more powerful items. Buying items that suit one’s hero is an important tactical element of the game. Item choice also affects the style of play, as the addition of any given item may have effects on other items and their performance parameters.

Most DotA players play the game on an established platform. Platform construction is based on players’ needs to provide the smoothest possible interface for host and server. The platform automatically joins same-level players, starts Warcraft 3, and offers single and team matches. There are many different platforms in China, like 11 platform and HF platform. These platforms have greatly facilitated the interaction of DotA players.

4.2 Game refinement theory applied to DotA

In order to apply the game refinement measurement to DotA, we consider a model of DotA’s game progress. It can be measured by two factors: to kill heroes and destroy towers, and number of attempt. Let K and A be the average number of successful killing heroes and destroying towers, and the average number of attempts per game, respectively. If one knows the game information progress, for example after the game, the game progress $x(t)$ will be given by Equation 5.

$$x(t) = \frac{K}{A}t \tag{5}$$

However, the game information progress given by Equation (5) is usually unknown during the in-game period. Hence, the game information progress is reasonably assumed to be exponential. This is because the game outcome is uncertain until the very end of game in many games. Hence, a realistic model of game information progress is given by Equation (6).

$$x(t) = K(\frac{t}{A})^n \tag{6}$$

Here n stands for a constant parameter which is given based on the perspective of an observer in the game considered. Acceleration of game information progress is obtained by deriving Equation (6) twice. Solving it at $t = A$, the equation becomes:

$$x''(T) = \frac{Kn(n-1)}{A^n}t^{n-2} = \frac{K}{A^2}n(n-1) \tag{7}$$

As with Pac-Man, for DotA it is expected that the larger value $\frac{K}{A^2}$ is, the more exciting the game becomes, partly due to the presence of unsolved game uncertainty. We use its root square, $\frac{\sqrt{K}}{A}$, as a game refinement measure for the DotA.

4.3 DotA: Data collection and results

We have downloaded five replays of each version on website [25].The players are all experts, which provides for more uniform data for analysis. Most of the replays are from championships such as SMM, WGT and WCG. We also download some non-championship but exemplary replays, which are passer by Bureau. A software called “replay manager” has been used in this study to collect the data on killings and destroying of towers in each game. The attempts to kill heroes and/or destroy towers were counted while watching replays. Because of the complexity of the team battle, each participating hero injured by skills is counted as a killing attempt. When there is a gank affair where it is easy to identify the hero who is attacked by the ganker, it is counted as an attempt. Meanwhile, during the line consumption period, one hero releasing his skills to attack an opponent is counted as an attempt. When the health of the tower is under 130, the side which owns the tower may destroy the tower so that the opponents get less money from the tower’s gold. In this case, the attempt to destroy towers is counted as an attempt when the health of the tower is under 130. We show in Table II the results of different DotA versions using game refinement measure. We played the related versions with

other players on the platform and collected five replays of each related version.

Several variations of the DotA game are also running. "DotA AI" (Artificial Intelligence) is a map where a computer can choose a hero, learn skills to use, compose materials and achieve some tactics by adding artificial intelligence for the computer. Usually we suggest new players to play the AI version first so that they can learn some basic skills for DotA before they try playing with other on-line players on the platform. "DotA OMG" (Oh My God) is a map which allows players freedom to choose their skills one by one at the beginning of the game. Free combination of a variety of skills and models greatly improves the play-ability of the game. "DotA IMBA" (Imbalance) is a map wherein each hero's skills are very imbalanced and strong, therefore called "DotA IMBA version". Compared to the traditional DotA map,the related versions are more entertaining for new players. In Table III we show the results of related versions of DotA.

Table 2 Measures of game refinement for historical versions of DotA

Version	Released	K	A	R-value
6.48	Aug 2007	69.2	110.8	0.075
6.51	Mar 2008	68.4	110.2	0.074
6.59	Jan 2009	69.8	110.0	0.076
6.61	Aug 2009	70.0	111.6	0.075
6.64	Oct 2009	68.4	110.4	0.075
6.69	Oct 2010	67.8	108.4	0.076
6.74	Mar 2012	62.4	102.6	0.077
6.77	Dec 2012	62.8	102.8	0.077
6.80	Mar 2014	68.6	106.2	0.078

Table 3 Measures of game refinement for selected DotA variants

Variants	Released	K	A	R-value
AI	DotA 2007	83.4	130.4	0.070
IMBA	DotA 2010	91.2	117.8	0.081
OMG	DotA 2009	87.6	115.5	0.081

4.4 DotA: Discussion

According to the data shown in Table 2 and Table 3, we observed that game refinement measurement or R-value of traditional DotA is increasing these years. This development is due to the game editor who continues to improve the rules of the game, adding new heroes and items. Meanwhile, new strategies and tactics are being developed during the championship. With the efforts of game players and editor, DotA became one of the most popular multi-player online games. Many DotA-like games such as DotA 2 and LOL have come out with better interface and humanized operation.

The current versions and play styles are the evolutionary outcome of the history of DotA. The emergence of other related versions for fun greatly increase the entertainment of game and reduce the difficulty of operation for new players. These versions give DotA different play style and attract more game players to join in. Like playing basketball, 5 people are needed to play their parts to the full, the most important thing of DotA is therefore teamwork, which is the spirit of cooperation. That is why DotA attracts many of the young generation.

Table 1 and 3 show a comparison between game refinement measures for various type of games including board games, sports games, DotA and its variants. We understand that sophisticated games have a common factor (i.e., same degree of acceleration value) to feel engagement or excitement regardless of different type of games. As DotA is a 5 vs 5 multi-player game, it may be reasonable that R-value is slightly higher than board games and sports games.

5 Conclusion

Game refinement theory presents a measure of game information with some relation to the entertainment and evolution of games. The original model hypothesizes that an ideal range of information complexity contributes to a game's evolution or extinction. Although it has been an amazingly successful game, the structure of the Pac-Man game does not appear capable of evolving further if at all, at least not under the parameters of the model applied here. Further, the game refinement value of Pac-Man is much lower than that of chess and other similar games which have undergone long processes of sophistication of their rules. Pac-Man was an exciting development in electronic gaming technology for its time, however the low complexity of the game's fundamentals predict that it has come to the end of its evolutionary path.

DotA was reviewed under the same model, and it was found that its game refinement is of a similar value to that of other successful games studied previously. The analysis of game data of several updates over several years of DotA indicate a game refinement value similar to that of games which have undergone long processes of sophisitication of their rules. One interpretation of the results is that DotA is restricted neither by excessive nor insufficient information complexity in its evolutionary future.

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