# Applying an Integrated Fuzzy MCDM Method to Select Hub Location for Global Shipping Carrier-based Logistics Service Providers

JI-FENG DING
Department of Aviation and Maritime Transportation Management
Chang Jung Christian University
No. 1, Changda Rd., Gueiren District, Tainan City 71101
TAIWAN
jfding@mail.cjcu.edu.tw

*Abstract:* - A good hub centre will effectively help expand agglomeration economy effects and increase competitive advantages for global shipping carrier-based logistics service providers (GSLPs). Hence, the selection of hub location of logistics centre is very important for the GSLP companies. In light of this, the main purpose of this paper is to develop an integrated fuzzy MCDM model to evaluate the best selection of hub location for GSLPs. At first, some concepts and methods used to develop the proposed model are briefly introduced. Then, we develop an integrated fuzzy MCDM method. Finally, a step by step example is illustrated to study the computational process of the proposed fuzzy MCDM model. In addition, the proposed approach has successfully accomplished our goal.

*Key-Words:* - Hub location; Shipping; Logistics; Service provider; Fuzzy; MCDM

## 1 Introduction

In the recent years, due to the appearance of keen competition among global container shipping carriers (GCSCs) in the world, they have been encountering great competitions with how to meet customers' requirements. Besides, the customers' needs and wants have caused lead time compression and a need to speed up understanding of customer and response to requests for the shipping market. The operators are deliberating upon how to provide integrated shipping logistics services and to create significantly added value for their customers. They eventually pay attention to the integrated logistical concepts involving the total solution services of logistics management in the shipping supply chain.

The focus of container shipping logistics management is increasing, more and more companies are searching for the usage of third-party logistics service providers (3PLs) due to the fact that the 3PLs provide more customized services and many different functional services [1]. As a result, global shipping carrier-based logistics service providers (GSLPs) are emerged. The GSLPs used by shippers are growing rapidly in the recent years. Famous examples are Maersk Logistics (subordinate to Maersk Line (Denmark), rank 1 of GCSC in 2012); CMA-CGM Logistics (CMA-CGM Line (France), rank 3); Evergreen Logistics (Evergreen Line (Taiwan), rank 6); COSCO Logistics (COSCO Line (China), rank 5); MOL Logistics (Mitsui O.S.K. Lines (Japan), rank 10), etc.

Due to a change in demand on various activities has led to a need to consider re-deployment and reallocation of resources of container shipping logistics operation for container shipping communities [2]. The relationship in an age of container transport is a shipping line based community while other community members become 3PLs that support the carriers and cooperate with one another. Looking at container shipping communities, there are many market players involved, in which the GSLP plays an important role of providing efficient total solution and supply chain management for their customers [3].

The GSLP provides consultation services and deals with shipping logistics activities. The scopes of shipping logistics services are covering with forwarding and consolidation services, logistics operations services, value-added services, warehousing and distribution services, intermodal transport services, information technology solutions, processing of customs clearance, and specialized services. To achieve the goal of providing excellent shipping services for shippers, they expect to provide a better total logistics solutions in the shipping supply chain services.

The enlargements of the GSLPs have become a trend in the shipping market. However, there are many factors [4-6] - e.g., source of goods, hinterland, cost, operation efficiency, time, risk, supply chain relationship, quality, distribution, service standard, location selection, and other related factors - to be considered for the service providers when they are providing more value-added services in the shipping market. In the above-considered factors, the location selection is the primitive element of decision-making for GSLPs. Due to the fact that a good location will effectively help expand agglomeration economy effects and increase competitive advantages, allowing the service providers to swiftly ship products in an economical way under lower cost and to attain customer satisfaction. The service providers will invest considerable sources for software and hardware facilities subsequently once the location is decided; in which its planning, design, construction and operation will be also time consuming. In recent years, a growing body of literature has addressed the hub location problem in different logistics industries - e.g., plant location choice [7, 8], hub location choices of global logistics centres [9, 10], international logistics parks [11], and maritime logistics centres [4, 5, 12-15]. However, discussion of its application to hub location selection for the GSLP industry remains scanty. In order to satisfy the needs of the GSLP and its customers, there is a need to proceed with a study on effects from various perspectives and evaluate proper location objectively.

In an environment facing keen competition, the GSLP takes many evaluation criteria into consideration while facing the uncertainty environment. Due to the characteristics of multiple criteria decision-making (MCDM) of hub location and a change in various criteria upon group decision environment, the evaluation problem of hub location is essential to study. Besides, the decision information is hard to come by and often vague, particularly regarding the linguistic terms. Fuzzy set theory [16] was therefore designed to sort through the uncertainties of vague linguistic terms [17] and helped generate a single possible outcome. Finally, we will propose a fuzzy MCDM method to assist with improving the decision-making quality in this paper.

To efficiently deal with the actual conditions in the real world, in the light of this, a fuzzy MCDM method will be developed to evaluate the hub location selection problem for the GSLPs. The framework of this paper is arranged in five sections. The following section (Section 2) presents the research methodologies. Consequently, a fuzzy MCDM method for evaluating hub location selection is constructed and described in Section 3. A numerical example is studied in Section 4. Finally, conclusions are made in the last section.

## 2 Research Methodologies

In this section, some concepts used to develop an integrated fuzzy MCDM method are introduced.

### 2.1 Fuzzy set theory

The fuzzy set theory [16] is designed to deal with the extraction of the primary possible outcome from a multiplicity of information that is expressed in vague and imprecise terms. Fuzzy set theory treats vague data as possibility distributions in terms of set memberships. Once determined and defined, the sets of memberships in possibility distributions can be effectively used in logical reasoning.

### 2.2 Triangular fuzzy numbers

In a universe of discourse *X*, a fuzzy subset *A* of *X* is defined by a membership function $f_A(x)$, which maps each element *x* in *X* to a real number in the interval $[0, 1]$. The function value $f_A(x)$ represents the grade of membership of *x* in *A*.

A fuzzy number *A* [18] in real line $\Re$ is a triangular fuzzy number if its membership function $f_A : \Re \rightarrow [0, 1]$ is

$$f_A(x) = \begin{cases} (x-c)/(a-c), & c \le x \le a \\ (x-b)/(a-b), & a \le x \le b \\ 0, & otherwise \end{cases}$$

with $-\infty < c \le a \le b < \infty$. The triangular fuzzy number can be denoted by $(c, a, b)$.

The triangular fuzzy numbers are easy to use and easy to interpret. The parameter *a* gives the maximal grade of $f_A(x)$, i.e., $f_A(a) = 1$; it is the most probable value of the evaluation data. In addition, '*c*' and '*b*' are the lower and upper bounds of the available area for the evaluation data. They are used to reflect the fuzziness of the evaluation data. The narrower the interval $[c, b]$, the lower the fuzziness of the evaluation data.

### 2.3 The algebraic operations of fuzzy numbers

In this paper, the Zadeh's extension principle [16] is employed to proceed with the algebraic operations of fuzzy numbers. Let $A_1 = (c_1, a_1, b_1)$ and $A_2 = (c_2, a_2, b_2)$ be fuzzy numbers. The algebraic operations of any two fuzzy numbers $A_1$ and $A_2$ can be expressed as

(1) Fuzzy addition:

$A_1 \oplus A_2 = (c_1 + c_2, a_1 + a_2, b_1 + b_2)$,

(2) Fuzzy subtraction:

$A_1 \ominus A_2 = (c_1 - b_2, a_1 - a_2, b_1 - c_2)$,

(3) Fuzzy multiplication:

(i) $k \otimes A_2 = (kc_2, ka_2, kb_2), k \in \Re, \ k \geq 0$;

(ii) $A_1 \otimes A_2 \cong (c_1 c_2, a_1 a_2, b_1 b_2)$, $c_1 \geq 0, \ c_2 \geq 0$,

(4) Fuzzy division:

(i) $(A_1)^{-1} = (c_1, a_1, b_1)^{-1} \cong (1/b_1, 1/a_1, 1/c_1), c_1 > 0$;

(ii) $A_1 \varnothing A_2 \cong (c_1/b_2, a_1/a_2, b_1/c_2)$, $c_1 \geq 0, c_2 > 0$.

### 2.4 Linguistic values

In fuzzy decision environments, two preference ratings can be used. They are fuzzy numbers and linguistic values characterized by fuzzy numbers [17]. Depending on practical needs, DMs may apply one or both of them. In this paper, the rating set is used to analytically express the linguistic value and describe how good of the alternatives against various criteria above the alternative level is. The rating set is defined as *S* = {*VP*, *P*, *F*, *G*, *VG*}; where *VP*=Very Poor, *P*=Poor, *F*=Fair, *G*=Good, and *VG*=Very Good. Here, we define the linguistic values of *VP*=(0, 0, 0.2), *P*=(0, 0.2, 0.4), *F*=(0.3, 0.5, 0.7), *G*=(0.6, 0.8, 1), and *VG*=(0.8, 1, 1), respectively.

### 2.5 Graded mean integration representation (GMIR) method

In a fuzzy decision-making environment, a defuzzification method of the triangular fuzzy numbers for ranking the alternatives is essential. To match the integrated fuzzy MCDM method developed in this paper, and to solve the problem powerfully, the graded mean integration representation (GMIR) method, proposed by Chen and Hsieh [19], is employed to defuzzify the triangular fuzzy numbers.

Let $A_i = (c_i, a_i, b_i), \ i = 1, 2, \ldots, n,$ be $n$ triangular fuzzy numbers. By the GMIR method, the GMIR $G(A_i)$ of $A_i$ is

$$G(A_i) = (c_i + 4a_i + b_i)/6 \qquad (1)$$

Suppose $G(A_i)$ and $G(A_j)$ are the GMIR of the triangular fuzzy numbers $A_i$ and $A_j$, respectively. We define:

$$A_i > A_j \Leftrightarrow G(A_i) > G(A_j),$$
$$A_i < A_j \Leftrightarrow G(A_i) < G(A_j),$$
$$A_i = A_j \Leftrightarrow G(A_i) = G(A_j).$$

### 2.6 Distance measure approach

Two famous distance measure approaches between two fuzzy numbers, i.e. mean and geometrical distance measures, were introduced by Heilpern [20] in 1997. However, Heilpern's method cannot satisfy some special cases between two fuzzy numbers. Hsieh and Chen [21] had proposed the modified geometrical distance approach to improve the drawback. To match the integrated fuzzy MCDM method developed in this paper, this modified geometrical distance approach is used to measure the distance of two fuzzy numbers.

Let $A_i = (c_i, a_i, b_i)$ and $A_j = (c_j, a_j, b_j)$ be fuzzy numbers. Then, the Hsieh and Chen's modified geometrical distance can be denoted by

$$\Delta_m(A_i, A_j) = \left\{ \frac{1}{4}\left[ (c_i - c_j)^2 + 2(a_i - a_j)^2 + (b_i - b_j)^2 \right] \right\}^{1/2} \qquad (2)$$

## 3 The Proposed Fuzzy MCDM Method

A stepwise description of the fuzzy MCDM method for selecting hub location for GSLPs is proposed in the following.

### 3.1 Developing a hierarchical structure

A hierarchy structure is the framework of system structure. Figure 1 shows the complete hierarchical structure of selecting hub location with $k$ criteria, $n_1 + \cdots + n_t + \cdots + n_k$ sub-criteria and $m$ alternatives.

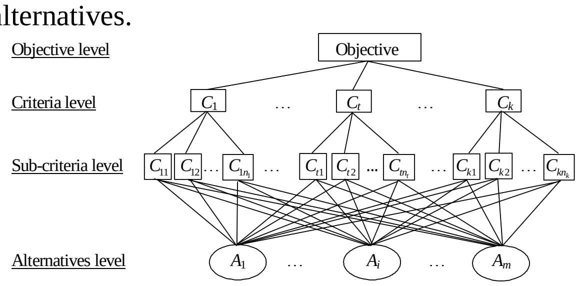


Figure 1. The hierarchy structure

### 3.2 Calculating the weights of all criteria and sub-criteria

There are many methods to evaluate relative importance weights of MCDM problems. One of the commonly used ones is analytic hierarchy process (AHP), which was proposed by Saaty [22]. However, the relative weights based upon this measurement in which information is incomplete or imprecise, e.g. the phrase of 'much more important than.' The use of fuzzy numbers would be more suitable in that situation. In light of this, a fuzzy AHP approach is used to measure relative weights by using the fuzzy AHP approach. The systematic steps of fuzzy AHP approach are described as below.

***Step 1: Build fuzzy pair-wise comparison matrices***

Let $r_{ij}^{E} \in [\frac{1}{9}, \frac{1}{8}, \ldots, \frac{1}{2}, 1] \cup [1, 2, \ldots, 8, 9]$ ( $E = 1, 2, \ldots, n, \quad \forall i,\ j = 1, 2, \ldots, k$ ) be the relative importance given to $i^{th}$ criterion to $j^{th}$ criterion by $E^{th}$ expert on the Criteria level in Figure 1. Then, the pair-wise comparison matrix is defined as $[r_{ij}^{E}]_{k\times k}$. After integrating the opinions of all $n$ experts, the triangular fuzzy numbers can be denoted by

$$\tilde{A}_{ij}^{CL} = (c_{ij}, a_{ij}, b_{ij}), \tag{3}$$

where $c_{ij} = \min\{r_{ij}^{1}, r_{ij}^{2}, \ldots, r_{ij}^{n}\}$, $a_{ij} = \left(\prod_{E=1}^{n} r_{ij}^{E}\right)^{1/n}$, $b_{ij} = \max\{r_{ij}^{1}, r_{ij}^{2}, \ldots, r_{ij}^{n}\}$.

We use the integrated triangular fuzzy numbers to build a fuzzy pair-wise comparison matrix (given to $i^{th}$ criterion to $j^{th}$ criterion). For the Criteria level, the fuzzy pair-wise comparison matrix can be denoted by

$$A_{k}^{CL} = \left[\tilde{A}_{ij}^{CL}\right]_{k\times k} = \begin{bmatrix} 1 & \tilde{A}_{12}^{CL} & \cdots & \tilde{A}_{1k}^{CL} \\ 1/\tilde{A}_{12}^{CL} & 1 & \cdots & \tilde{A}_{2k}^{CL} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{A}_{1k}^{CL} & 1/\tilde{A}_{2k}^{CL} & \cdots & 1 \end{bmatrix}, \tag{4}$$

where $\tilde{A}_{ij}^{CL} \otimes \tilde{A}_{ji}^{CL} \cong 1, \quad \forall i,\ j = 1, 2, \ldots, k.$

By using the same concept, let $p_{uv}^{E} \in [\frac{1}{9}, \frac{1}{8}, \ldots, \frac{1}{2}, 1] \cup [1, 2, \ldots, 8, 9]$ ($E = 1, 2, \ldots, n,\ \forall u, v = 1, \ldots, n_1;\ \ \forall u, v = 1, \ldots, n_t;\ \cdots;\ \forall u, v = 1, \ldots, n_k$) be the relative importance given to $u^{th}$ sub-criterion to $v^{th}$ sub-criterion by $E^{th}$ expert on the Sub-criteria level in Figure 1. Then, the pair-wise comparison matrices are defined as $[p_{uv}^{E}]_{n_1\times n_1}$, $\cdots$, $[p_{uv}^{E}]_{n_t\times n_t}$, $\cdots$, $[p_{uv}^{E}]_{n_k\times n_k}$. Hence, we can integrate the opinions of all $n$ experts given to sub-criterion $u$ to sub-criterion $v$ on the Sub-criteria level, the triangular fuzzy numbers can be denoted by

$$\tilde{A}_{uv}^{SL} = (c_{uv}, a_{uv}, b_{uv}), \tag{5}$$

$\forall u, v = 1, \ldots, n_1;\ \ \forall u, v = 1, \ldots, n_t;\ \cdots;$ $\forall u, v = 1, \ldots, n_k$, where $c_{uv} = \min\{p_{uv}^{1}, p_{uv}^{2}, \ldots, p_{uv}^{n}\}$, $a_{uv} = \left(\prod_{E=1}^{n} p_{uv}^{E}\right)^{1/n}$, $b_{uv} = \max\{p_{uv}^{1}, p_{uv}^{2}, \ldots, p_{uv}^{n}\}$.

We use the integrated triangular fuzzy numbers to build the fuzzy pair-wise comparison matrices for the Sub-criteria level can be denoted by

$$A_{n_1}^{SL} = \left[\tilde{A}_{uv}^{SL}\right]_{n_1\times n_1} = \begin{bmatrix} 1 & \tilde{A}_{12}^{SL} & \cdots & \tilde{A}_{1n_1}^{SL} \\ 1/\tilde{A}_{12}^{SL} & 1 & \cdots & \tilde{A}_{2n_1}^{SL} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{A}_{1n_1}^{SL} & 1/\tilde{A}_{2n_1}^{SL} & \cdots & 1 \end{bmatrix}, \tag{6}$$

where $\tilde{A}_{uv}^{SL} \otimes \tilde{A}_{vu}^{SL} \cong 1, \quad \forall u, v = 1, 2, \ldots, n_1.$

, ……,

$$A_{n_t}^{SL} = \left[\tilde{A}_{uv}^{SL}\right]_{n_t\times n_t} = \begin{bmatrix} 1 & \tilde{A}_{12}^{SL} & \cdots & \tilde{A}_{1n_t}^{SL} \\ 1/\tilde{A}_{12}^{SL} & 1 & \cdots & \tilde{A}_{2n_t}^{SL} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{A}_{1n_t}^{SL} & 1/\tilde{A}_{2n_t}^{SL} & \cdots & 1 \end{bmatrix}, \tag{7}$$

where $\tilde{A}_{uv}^{SL} \otimes \tilde{A}_{vu}^{SL} \cong 1, \quad \forall u, v = 1, 2, \ldots, n_t.$

, ……, and

$$A_{n_k}^{SL} = \left[\tilde{A}_{uv}^{SL}\right]_{n_k\times n_k} = \begin{bmatrix} 1 & \tilde{A}_{12}^{SL} & \cdots & \tilde{A}_{1n_k}^{SL} \\ 1/\tilde{A}_{12}^{SL} & 1 & \cdots & \tilde{A}_{2n_k}^{SL} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{A}_{1n_k}^{SL} & 1/\tilde{A}_{2n_k}^{SL} & \cdots & 1 \end{bmatrix}, \tag{8}$$

where $\tilde{A}_{uv}^{SL} \otimes \tilde{A}_{vu}^{SL} \cong 1, \quad \forall u, v = 1, 2, \ldots, n_k.$

***Step 2: Calculate the fuzzy weights of the fuzzy pair-wise comparison matrices***

Let $\tilde{X}_{i}^{CL} = \left(\tilde{A}_{i1}^{CL} \otimes \tilde{A}_{i2}^{CL} \otimes \cdots \otimes \tilde{A}_{ik}^{CL}\right)^{1/k}$ ( $\forall i = 1, 2, \ldots, k$ ) be the geometric mean of triangular fuzzy number of $i^{th}$ criterion on the Criteria level. Then, the fuzzy weight of $i^{th}$ criterion can be denoted by

$$\tilde{W}_{i}^{CL} = \tilde{X}_{i}^{CL} \otimes \left(\tilde{X}_{1}^{CL} \oplus \tilde{X}_{2}^{CL} \oplus \cdots \oplus \tilde{X}_{k}^{CL}\right)^{-1} \tag{9}$$

For being convenient, the fuzzy weight is denoted by $\tilde{W}_{i}^{CL} \cong (w_{ic}, w_{ia}, w_{ib})$.

By using the same concept, let $\tilde{X}_{u}^{SL} = \left(\tilde{A}_{u1}^{SL} \otimes \tilde{A}_{u2}^{SL} \otimes \cdots \otimes \tilde{A}_{un_1}^{SL}\right)^{1/n_1}$ ( $\forall u = 1, 2, \ldots, n_1$ ) be the geometric mean of triangular fuzzy number of $u^{th}$ sub-criterion on the Sub-criteria level. Then, the fuzzy weight of $u^{th}$ sub-criterion can be denoted by

$$\widetilde{W}_u^{SL} = \widetilde{X}_u^{SL} \otimes \left(\widetilde{X}_1^{SL} \oplus \widetilde{X}_2^{SL} \oplus \cdots \oplus \widetilde{X}_{n_1}^{SL}\right)^{-1}, \quad (10)$$

where the fuzzy weight is denoted by $\widetilde{W}_u^{SL} \cong (w_{uc}, w_{ua}, w_{ub})$, $\forall u = 1, 2, \ldots, n_1$.

For saving space, the fuzzy weights of $[(n_1 + \cdots + n_t + \cdots + n_k) - n_1]$ sub-criteria can be obtained by using the same of above-mentioned method.

***Step 3: Defuzzify the fuzzy weights to crisp weights***

For solving the problem of defuzzification powerfully, the GMIR method is used to defuzzify the fuzzy weights. Let $\widetilde{W}_i^{CL} \cong (w_{ic}, w_{ia}, w_{ib})$ ($\forall i = 1, 2, \ldots, k$) be $k$ triangular fuzzy numbers. By using the equation (1), the GMIR of crisp weights $k$ can be denoted by

$$W_i^{CL} = (w_{ic} + 4w_{ia} + w_{ib})/6, \ \forall i = 1, 2, \ldots, k. \quad (11)$$

For saving space, the defuzzifications of fuzzy weights are omitted to reason by analogy on the Sub-criteria level.

***Step 4: Calculate and normalize the weight vector of each layer***

For being convenient to compare the relative importance between each layer, these crisp weights are normalized and denoted by

$$NW_i^{CL} = W_i^{CL} \Big/ \sum_{i=1}^{k} W_i^{CL} \quad (12)$$

Let $NW_i^{CL}$ and $NW_u^{SL}$ be the normalized crisp weights on the Criteria and Sub-criteria levels, respectively. Then,

(1) The integrated weight of each criterion on the Criteria level is

$$IW_i^{CL} = NW_i^{CL}, \ \forall i = 1, 2, \ldots, k. \quad (13)$$

(2) The integrated weight of each sub-criterion on the Sub-criteria level is

$$IW_u^{SL} = NW_i^{CL} \times NW_u^{SL}, \quad (14)$$

$$\forall i = 1, 2, \ldots, k; \ \forall u = 1, \ldots, n_1; \ \forall u = 1, \ldots, n_t; \ \cdots; \ \forall u = 1, \ldots, n_k.$$

### 3.3 Estimating the fuzzy ratings of alternatives versus all sub-criteria

In the real situation, the sub-criteria above the Alternatives level can be usually classified into two categories:

(1) Subjective criteria, which have linguistic or qualitative definition, e.g. level of potential in location expansion;

(2) Objective criteria, which are defined in monetary or quantitative terms, e.g. port charges.

Let $L = \{l_1, \ldots, l_t, \ldots, l_q\}$ and $M = \{m_1, \ldots, m_r, \ldots, m_p\}$ be the sets of all $q$ subjective sub-criteria and $p$ objective ones above the Alternatives level.

***Case I: For the subjective sub-criteria.***

We use the preference ratings (mentioned in the Section 2.4) to represent the fuzzy ratings of all alternatives versus all subjective sub-criteria. Then, the arithmetic mean method is used to solve the average fuzzy rating of evaluation value for each alternative versus all subjective sub-criteria.

That is, let $FR_{it}^E = (c_{it}^E, a_{it}^E, b_{it}^E)$ ($i = 1, 2, \ldots, m$; $t = 1, 2, \ldots, q$; $E = 1, 2, \ldots, n$) be the fuzzy rating of the $i^{\text{th}}$ alternative versus the $t^{\text{th}}$ subjective sub-criterion evaluated by the $E^{\text{th}}$ expert. Then, the average fuzzy rating value of the $i^{\text{th}}$ alternative versus the $t^{\text{th}}$ subjective sub-criterion can be expressed as

$$AFR_{it} = \left( \frac{\sum_{E=1}^{n} c_{it}^E}{n}, \ \frac{\sum_{E=1}^{n} a_{it}^E}{n}, \ \frac{\sum_{E=1}^{n} b_{it}^E}{n} \right) \quad (15)$$

***Case II: For the objective sub-criteria.***

We use the following method [22, 23] to deal with the fuzzy ratings of all alternatives versus all objective sub-criteria.

(a) When the appropriateness rating of alternative can be estimated effectively in values, the triangular fuzzy numbers can be used directly. For example, if the port charges per month is about US Dollars 0.85 million, it can be subjectively expressed as (0.82, 0.85, 0.89).

(b) If there are historical data, e.g. let $z_1, z_2, \ldots, z_v$ represent the port charges of past $v$ periods, the fuzzy rating of the port charges can be used the geometric mean method to express as

$$\left( \min_i \{z_i\}, \left( \prod_{i=1}^{v} z_i \right)^{1/v}, \max_i \{z_i\} \right) \quad (16)$$

For example, if the current five historical data of the port charges of alternative $A_1$ are 0.82, 0.91, 0.71, 0.85, and 0.88, then the evaluation value can be transformed into triangular fuzzy number as $(0.71, \sqrt[5]{0.82 \times 0.91 \times 0.71 \times 0.85 \times 0.88}, 0.91)$ = (0.71, 0.831, 0.91).

### 3.4 Calculating the fuzzy ideal solution and anti-ideal solution

The ideal and anti-ideal solutions [24] are based on the concept of relative closeness in compliance with the shorter (longer) the distance of alternative *i* to ideal (anti-ideal), the higher the priority can be ranked. We use this concept to calculate the fuzzy ideal and anti-ideal solutions in this paper.

Firstly, let $m$ and $n_1+\cdots+n_t+\cdots+n_k=N$ respectively denote the numbers of alternatives and the sub-criteria above the Alternatives level. Allow $AFR_{ij}=(c_{ij}, a_{ij}, b_{ij})$ ( $i=1, 2, \ldots, m$ ; $j=1, 2, \ldots, N$ ) be the average fuzzy rating value of $i^{th}$ alternative under $j^{th}$ sub-criterion. To ensure compatibility of average fuzzy ratings between objective criteria and subjective ones, the values of average fuzzy ratings must be converted to dimensionless indices. That is, the fuzzy ideal values with minimum values in negative sub-criteria or maximum values in positive sub-criteria should have the maximum rating. Based on the principle stated as above, let $\pi_j=\max_i\{b_{ij}\}$, $\tau_j=\min_i\{c_{ij}\}$, then the normalized fuzzy rating value $NFR_{ij}$ of $i^{th}$ alternative under $j^{th}$ sub-criterion can be defined as:

(1) For the positive sub-criterion *j* (the sub-criteria that have positive contribution to the objective, e.g., benefit sub-criterion):

$$NFR_{ij}=(e_{ij}, f_{ij}, g_{ij})=(\frac{c_{ij}}{\pi_j}, \frac{a_{ij}}{\pi_j}, \frac{b_{ij}}{\pi_j}) \quad (17)$$

(2) For the negative sub-criterion *j* (the sub-criteria that have negative contribution to the objective, e.g., cost sub-criterion):

$$NFR_{ij}=(e_{ij}, f_{ij}, g_{ij})=(\frac{\tau_j}{b_{ij}}, \frac{\tau_j}{a_{ij}}, \frac{\tau_j}{c_{ij}}) \quad (18)$$

Secondly, the GMIR value of the normalized fuzzy rating value $NFR_{ij}$ can be can be expressed as $G(NFR_{ij})$ by using the equation (1). The fuzzy ideal value $FIV_j^+$ and fuzzy anti-ideal value $FAV_j^-$ of each sub-criterion above the Alternatives level can be judged and determined by comparing with these representation values $G(NFR_{ij})$. Then,

(1) For the positive sub-criterion *j*:

(a) if $G(NFR_{tj})=\max_i G(NFR_{ij})$ , then the fuzzy ideal value $FIV_j^+=NFR_{tj}$,

(b) if $G(NFR_{kj})=\min_i G(NFR_{ij})$ , then the fuzzy anti-ideal value $FAV_j^-=NFR_{kj}$,

(2) For the negative sub-criterion *j*:

(a) if $G(NFR_{tj})=\min_i G(NFR_{ij})$ , then the fuzzy ideal value $FIV_j^+=NFR_{tj}$,

(b) if $G(NFR_{kj})=\max_i G(NFR_{ij})$ , then the fuzzy anti-ideal value $FAV_j^-=NFR_{kj}$.

Finally, define the fuzzy ideal solution ($FIS^+$) and fuzzy anti-ideal solution ($FAS^-$) as

$$FIS^+=(FIV_1^+, FIV_2^+, \ldots, FIV_j^+, \ldots, FIV_N^+) \quad (19)$$

and

$$FAS^-=(FAV_1^-, FAV_2^-, \ldots, FAV_j^-, \ldots, FAV_N^-) \quad (20)$$

### 3.5 Computing the distance of different alternatives versus the fuzzy ideal solution and anti-ideal solution

Let $\omega_j^*$ ( $j=1, 2, \ldots, N$ ) be the integrated weights of $j^{th}$ sub-criterion above the Alternatives level. We can compute the distance of different alternatives versus $FIS^+$ and $FAS^-$ which were denoted by $D_i^+$ and $D_i^-$, respectively. Define

$$D_i^+=\sqrt{\sum_{j=1}^{N}\left\{\left(\omega_j^*\right)^2\times\left[\Delta_m\left(FIV_j^+, NFR_{ij}\right)\right]^2\right\}} \quad (21)$$

and

$$D_i^-=\sqrt{\sum_{j=1}^{N}\left\{\left(\omega_j^*\right)^2\times\left[\Delta_m\left(FAV_j^-, NFR_{ij}\right)\right]^2\right\}}, \quad (22)$$

$i=1, 2, \ldots, m$,

where $\Delta_m(.)$ can be obtained by using the equation (2) of modified geometrical distance approach mentioned in Section 2.6.

### 3.6 Calculating the relative closeness value of different alternatives versus ideal solution and ranking the alternatives

We calculate the relative closeness value of different alternatives $A_i$ versus fuzzy ideal solution $FIS^+$, denoted as $RC_i^*$. Define

$$RC_i^*=\frac{D_i^-}{D_i^+ + D_i^-},\ i=1, 2, \ldots, m. \quad (23)$$

It is obvious, $0\le RC_i^*\le 1$ , $i=1, 2, \ldots, m.$ Suppose alternative $A_i$ is an ideal solution (i.e. $D_i^+=0$ ), then $RC_i^*=1$ ; otherwise, if $A_i$ is an anti-ideal solution (i.e. $D_i^-=0$ ), then $RC_i^*=0$ . The nearer the value $RC_i^*$ close to 1 implies a closer alternative $A_i$ approach to the ideal solution, i.e. the maximum value of $RC_i^*$, then the optimal alternative can be ranked by a decision maker. Finally, the best alternative can be selected.

# 4 The Numerical Illustration

In this section, a numerical example of selecting hub location for a GSLP company is illustrated to demonstrate the computational process of the proposed fuzzy MCDM model, step by step, as follows.

**Step 1.** Assume that a GSLP company needs to select a hub location of logistics centre. Three candidates *A*, *B*, and *C* are chosen after a preliminary screening for further evaluation. A committee of three DMs (i.e., *X*, *Y*, and *Z*) is formed to evaluate the best location of logistics centre among three candidates. In our simple case, four criteria and fourteen sub-criteria [4, 5, 7-15] have been chosen and the code names of these ones are shown in parentheses. Three objective sub-criteria, i.e. $C_{31}$, $C_{32}$, and $C_{33}$ are negative; however the other eleven sub-criteria are subjective and positive.

1. Basic requirements of location competitiveness ($C_1$). This criterion includes four sub-criteria, that is, level of potential in location expansion ($C_{11}$), level of difficulty staff employment ($C_{12}$), level of freedom and stability on financial environment ($C_{13}$), and level of cargo agglomeration ($C_{14}$).
2. Port conditions ($C_2$). This criterion includes four sub-criteria, that is, port hardware and facility ($C_{21}$), port management system ($C_{22}$), efficiency of customs clearance ($C_{23}$), and level of connection between port and inland transport ($C_{24}$).
3. Cost factors ($C_3$). This criterion includes four sub-criteria, that is, port charges ($C_{31}$), cargo handling cost ($C_{32}$), and related port and shipping cost ($C_{33}$).
4. Compliance of policy and laws ($C_4$). This criterion includes four sub-criteria, that is, level of effects of maritime development policy ($C_{41}$), level of expansion in economic relations ($C_{42}$), and level of integration in related laws ($C_{43}$).

**Step 2:** Calculate relative importance weights of four criteria and fourteen sub-criteria by using the fuzzy AHP approach mentioned in Section 3.2. For saving space, the computing process is omitted in this paper. As a result, the integrated weights are shown as Table 1.

Table 1. The integrated weights of all criteria and sub-criteria

| Criteria | Weight | Sub-criteria | Weight |
|---|---|---|---|
| $C_1$ | 0.28 | $C_{11}$ | 0.062 |
| | | $C_{12}$ | 0.044 |
| | | $C_{13}$ | 0.053 |
| | | $C_{14}$ | 0.121 |
| $C_2$ | 0.26 | $C_{21}$ | 0.062 |
| | | $C_{22}$ | 0.075 |
| | | $C_{23}$ | 0.081 |
| | | $C_{24}$ | 0.042 |
| $C_3$ | 0.29 | $C_{31}$ | 0.092 |
| | | $C_{32}$ | 0.113 |
| | | $C_{33}$ | 0.085 |
| $C_4$ | 0.17 | $C_{41}$ | 0.071 |
| | | $C_{42}$ | 0.041 |
| | | $C_{43}$ | 0.058 |

**Step 3:** Evaluate the fuzzy ratings of three alternatives versus all sub-criteria above the Alterative level. By using the method presented in Section 3.3, the original preference ratings of eleven subjective/positive sub-criteria and the superiority of three objective/negative ones can be obtained, as shown in Table 2 and 3, respectively.

Table 2. The original fuzzy ratings of three candidates versus eleven subjective/positive sub-criteria

| Sub-criteria | DM | Linguistic values | | | Fuzzy ratings | | |
|---|---|---|---|---|---|---|---|
| | | *A* | *B* | *C* | *A* | *B* | *C* |
| $C_{11}$ | *X* | *P* | *G* | *P* | (0.1, 0.233, 0.433) | (0.733, 0.933, 1) | (0, 0.133, 0.333) |
| | *Y* | *VP* | *VG* | *VP* | | | |
| | *Z* | *F* | *VG* | *P* | | | |
| $C_{12}$ | *X* | *VP* | *VG* | *VP* | (0.2, 0.267, 0.467) | (0.733, 0.933, 1) | (0, 0, 0.2) |
| | *Y* | *G* | *G* | *VP* | | | |
| | *Z* | *VP* | *VG* | *VP* | | | |
| $C_{13}$ | *X* | *P* | *P* | *P* | (0.2, 0.333, 0.533) | (0.2, 0.333, 0.533) | (0.2, 0.333, 0.533) |
| | *Y* | *G* | *G* | *G* | | | |
| | *Z* | *VP* | *VP* | *VP* | | | |

Table 2. The original fuzzy ratings of three candidates versus eleven subjective/positive sub-criteria (Continued)

| Sub-criteria | DM | Linguistic values | | | Fuzzy ratings | | |
|---|---|---|---|---|---|---|---|
| | | *A* | *B* | *C* | *A* | *B* | *C* |
| $C_{14}$ | *X* | *F* | *G* | *P* | (0.567, 0.767, 0.9) | (0.667, 0.867, 1) | (0.267, 0.467, 0.6) |
| | *Y* | *G* | *G* | *P* | | | |
| | *Z* | *VG* | *VG* | *VG* | | | |
| $C_{21}$ | *X* | *G* | *G* | *G* | (0.467, 0.667, 0.8) | (0.467, 0.667, 0.8) | (0.467, 0.667, 0.8) |
| | *Y* | *VG* | *VG* | *VG* | | | |
| | *Z* | *P* | *P* | *P* | | | |
| $C_{22}$ | *X* | *G* | *VG* | *VP* | (0.4, 0.6, 0.8) | (0.733, 0.933, 1) | (0, 0.067, 0.267) |
| | *Y* | *G* | *G* | *VP* | | | |
| | *Z* | *P* | *VG* | *P* | | | |
| $C_{23}$ | *X* | *F* | *F* | *F* | (0.3, 0.5, 0.7) | (0.3, 0.5, 0.7) | (0.2, 0.333, 0.533) |
| | *Y* | *F* | *F* | *VP* | | | |
| | *Z* | *F* | *F* | *F* | | | |
| $C_{24}$ | *X* | *P* | *P* | *P* | (0.1, 0.233, 0.433) | (0.367, 0.567, 0.7) | (0, 0.133, 0.333) |
| | *Y* | *F* | *F* | *P* | | | |
| | *Z* | *VP* | *VG* | *VP* | | | |
| $C_{41}$ | *X* | *G* | *G* | *VP* | (0.567, 0.767, 0.9) | (0.667, 0.867, 1) | (0.367, 0.5, 0.633) |
| | *Y* | *F* | *G* | *F* | | | |
| | *Z* | *VG* | *VG* | *VG* | | | |
| $C_{42}$ | *X* | *F* | *G* | *F* | (0.567, 0.767, 0.9) | (0.733, 0.933, 1) | (0.1, 0.233, 0.433) |
| | *Y* | *VG* | *VG* | *VP* | | | |
| | *Z* | *G* | *VG* | *P* | | | |
| $C_{43}$ | *X* | *VP* | *VG* | *P* | (0.1, 0.167, 0.367) | (0.367, 0.5, 0.633) | (0, 0.133, 0.333) |
| | *Y* | *F* | *F* | *P* | | | |
| | *Z* | *VP* | *VP* | *VP* | | | |

Table 3. The original fuzzy superiority of three candidates versus three objective/negative sub-criteria

| Sub-criteria | Original data | | | | Fuzzy ratings | | |
|---|---|---|---|---|---|---|---|
| | Month | *A* | *B* | *C* | *A* | *B* | *C* |
| $C_{31}$ | April | 0.81 | 0.75 | 0.87 | (0.69, 0.772, 0.84) | (0.72, 0.786, 0.95) | (0.71, 0.813, 0.91) |
| | May | 0.72 | 0.95 | 0.88 | | | |
| | June | 0.69 | 0.76 | 0.91 | | | |
| | July | 0.84 | 0.77 | 0.72 | | | |
| | August | 0.81 | 0.72 | 0.71 | | | |
| $C_{32}$ | April | 0.83 | 0.73 | 0.71 | (0.67, 0.785, 0.91) | (0.69, 0.785, 0.93) | (0.71, 0.768, 0.85) |
| | May | 0.71 | 0.93 | 0.71 | | | |
| | June | 0.91 | 0.86 | 0.79 | | | |
| | July | 0.83 | 0.74 | 0.85 | | | |
| | August | 0.67 | 0.69 | 0.79 | | | |
| $C_{33}$ | April | 0.65 | 0.98 | 0.84 | (0.65, 0.764, 0.98) | (0.78, 0.891, 0.98) | (0.71, 0.790, 0.84) |
| | May | 0.98 | 0.92 | 0.74 | | | |
| | June | 0.85 | 0.93 | 0.84 | | | |
| | July | 0.74 | 0.78 | 0.83 | | | |
| | August | 0.65 | 0.86 | 0.71 | | | |

**Step 4:** Calculate the fuzzy ideal solution and anti-ideal solution. At first, there are subjective/positive and objective/negative sub-criteria in our case. Hence, by using the method presented in Section 3.4, the normalized fuzzy rating (NFR) values above the three alternatives and the GMIR values can be obtained. The results can be shown in Table 4.

Table 4. The normalized fuzzy rating values of three alternatives versus all sub-criteria

| Sub-criteria | *A* | | *B* | | *C* | |
|---|---|---|---|---|---|---|
| | NFR value | GMIR | NFR value | GMIR | NFR value | GMIR |
| $C_{11}$ | (0.1, 0.233, 0.433) | 0.244 | (0.733, 0.933, 1) | 0.911 | (0, 0.133, 0.333) | 0.144 |
| $C_{12}$ | (0.2, 0.267, 0.467) | 0.289 | (0.733, 0.933, 1) | 0.911 | (0, 0, 0.2) | 0.033 |
| $C_{13}$ | (0.375, 0.625, 1) | 0.646 | (0.375, 0.625, 1) | 0.646 | (0.375, 0.625, 1) | 0.646 |
| $C_{14}$ | (0.567, 0.767, 0.9) | 0.756 | (0.667, 0.867, 1) | 0.856 | (0.267, 0.467, 0.6) | 0.456 |
| $C_{21}$ | (0.584, 0.834, 1) | 0.820 | (0.584, 0.834, 1) | 0.820 | (0.584, 0.834, 1) | 0.820 |
| $C_{22}$ | (0.4, 0.6, 0.8) | 0.60 | (0.733, 0.933, 1) | 0.911 | (0, 0.067, 0.267) | 0.089 |
| $C_{23}$ | (0.429, 0.714, 1) | 0.714 | (0.429, 0.714, 1) | 0.714 | (0.286, 0.476, 0.761) | 0.492 |
| $C_{24}$ | (0.143, 0.333, 0.619) | 0.349 | (0.524, 0.810, 1) | 0.794 | (0, 0.190, 0.476) | 0.206 |
| $C_{31}$ | (0.821, 0.894, 1) | 0.90 | (0.726, 0.878, 0.958) | 0.866 | (0.758, 0.849, 0.972) | 0.854 |
| $C_{32}$ | (0.736, 0.854, 1) | 0.859 | (0.720, 0.854, 0.971) | 0.851 | (0.788, 0.872, 0.944) | 0.870 |
| $C_{33}$ | (0.663, 0.851, 1) | 0.845 | (0.663, 0.730, 0.833) | 0.736 | (0.774, 0.823, 0.915) | 0.830 |
| $C_{41}$ | (0.567, 0.767, 0.9) | 0.756 | (0.667, 0.867, 1) | 0.856 | (0.367, 0.5, 0.633) | 0.50 |
| $C_{42}$ | (0.567, 0.767, 0.9) | 0.756 | (0.733, 0.933, 1) | 0.911 | (0.1, 0.233, 0.433) | 0.244 |
| $C_{43}$ | (0.158, 0.264, 0.580) | 0.299 | (0.580, 0.790, 1) | 0.790 | (0, 0.210, 0.526) | 0.228 |

Secondly, according to Table 4, the fuzzy ideal value ( $FIS^+$ ) and fuzzy anti-ideal value ( $FAS^-$ ) can be obtain based on the comparison of the GMIR values. The results are shown in Table 5.

Table 5. Fuzzy ideal/anti-ideal values of sub-criteria

| | Fuzzy ideal value | Fuzzy anti-ideal values |
|---|---|---|
| $C_{11}$ | (0.733, 0.933, 1) | (0, 0.133, 0.333) |
| $C_{12}$ | (0.733, 0.933, 1) | (0, 0, 0.2) |
| $C_{13}$ | (0.375, 0.625, 1) | (0.375, 0.625, 1) |
| $C_{14}$ | (0.667, 0.867, 1) | (0.267, 0.467, 0.6) |
| $C_{21}$ | (0.584, 0.834, 1) | (0.584, 0.834, 1) |
| $C_{22}$ | (0.733, 0.933, 1) | (0.4, 0.6, 0.8) |
| $C_{23}$ | (0.429, 0.714, 1) | (0.286, 0.476, 0.761) |
| $C_{24}$ | (0.524, 0.810, 1) | (0, 0.190, 0.476) |
| $C_{31}$ | (0.821, 0.894, 1) | (0.758, 0.849, 0.972) |
| $C_{32}$ | (0.788, 0.872, 0.944) | (0.720, 0.854, 0.971) |
| $C_{33}$ | (0.663, 0.851, 1) | (0.663, 0.730, 0.833) |
| $C_{41}$ | (0.667, 0.867, 1) | (0.367, 0.5, 0.633) |
| $C_{42}$ | (0.733, 0.933, 1) | (0.1, 0.233, 0.433) |
| $C_{43}$ | (0.580, 0.790, 1) | (0, 0.210, 0.526) |

Hence, we can obtain the fuzzy ideal solution ( $FIS^+$ ) and fuzzy anti-ideal solution ( $FAS^-$ ), i.e.,

$FIS^+$ = [(0.733, 0.933, 1), (0.733, 0.933, 1), (0.375, 0.625, 1), …, …, (0.667, 0.867, 1), (0.733, 0.933, 1), (0.580, 0.790, 1)], and

$FAS^-$ = [(0, 0.133, 0.333), (0, 0, 0.2), (0.375, 0.625, 1), …, …, (0.367, 0.5, 0.633), (0.1, 0.233, 0.433), (0, 0.210, 0.526)].

**Step 5:** Compute the distance of three candidates versus fuzzy ideal/anti-ideal solutions. In our case, the results can be shown in Table 6.

Table 6. Distance of three candidates versus fuzzy ideal and anti-ideal solutions

| Candidates | $D_i^+$ | $D_i^-$ |
|---|---|---|
| *A* | 0.004212894 | 0.002724788 |
| *B* | 0.000146335 | 0.009729108 |
| *C* | 0.012876045 | 0.001508302 |

**Step 6:** Calculate the relative closeness value of three alternatives and ranking. Using the equation in Section 3.6, the RC value of three alternatives are $RC_A^* = 0.3928$, $RC_B^* = 0.9852$, $RC_C^* = 0.1049$.

The ranking order of $RC_i^*$ for three alternatives is *B*, *A*, and *C*, respectively. The best hub location is obviously *B*. Therefore, the committee shall recommend that hub *B* be the most appropriate location of logistics centre for the GSLP company based on the proposed fuzzy MCDM method.

## 5 Conclusions

The main purpose of this paper is to develop an integrated fuzzy MCDM model to evaluate the best selection of hub location for GSLPs. To effectively select best hub location, an integrated fuzzy MCDM method is proposed.

At first, we develop a hierarchical structure of selecting hub location with four criteria and fourteen sub-criteria for GSLPs. The characteristics of objective and objective criteria are considered in the proposed model. The fuzzy AHP approach is used to evaluate the relative importance. Then, the concepts of ideal and anti-ideal solutions are employed in the proposed fuzzy MCDM method. Moreover, Zadeh's linguistic values, the GMIR method, and the modified geometrical distance approach are applied to develop the integrated fuzzy MCDM method. Finally, a step by step example is illustrated to study the computational process of the fuzzy MCDM model.

The proposed model not only releases the limitation of crisp values, but also facilitates its implementation as a computer-based decision support system in a fuzzy environment. In addition, the proposed fuzzy MCDM model is not run solely hub location; however, every decision maker or beneficiary can apply this fuzzy-based MCDM model on the similar problems of selection issues.

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