

Appraisal of The Center and Mean Temperature Asymptotes for Unsteady Heat Conduction in The “Small Time” and “Large Time” Sub-Domains

ANTONIO CAMPO*

Coordinación de Ingeniería Mecánica
Universidad Simón Bolívar
Barruta, Caracas, Edo. Miranda
VENEZUELA

**Corresponding Author*

Abstract: - The present study addresses unsteady heat conduction in a large wall with constant temperature at the surfaces and uniform temperature initially. The thermal diffusivity of the solid material is taken as nearly independent of temperature in the temperature interval of operation. A nonlinear interpolatin scheme is implemented with sole input from the Fourier one term series in the “large time” sub-domain $t \rightarrow \infty$. The goal of the study is to elaborate a center temperature asymptote, a mean temperature asymptote, as well as a heat transfer rate asymptote associated with the problematic “small time” sub-domain $t \rightarrow 0$. The obtained asymptotic thermal results exhibit excellent quality when compared against benchmark exact, analytical results that are available in the heat transfer literature.

Key-Words: - unsteady heat conduction; large wall; constant surface temperature; power interpolation equation; center and mean temperature asymptotes

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Nomenclature

Roman letters

a coefficient
 A_n eigenfunctions
 b coefficient
 B_n eigenfunctions
 c exponent
 c_v specific heat capacity at constant volume
 H height of large wall
 $2L$ thickness of large wall
 k thermal conductivity
 m mass of large wall
 Q heat transfer rate
 Q_{in} initial internal energy
 t time
 t_{eq} equivalent time, $\frac{L^2}{\alpha}$
 t_{th} threshold time
 T temperature
 T_c center temperature
 T_{in} initial temperature
 T_m mean temperature
 T_s surface temperature
 x space coordinate
 X dimensionless space coordinate, $\frac{x}{L}$
 W width of large wall

Greek letters

α thermal diffusivity, $\frac{k}{\rho c_v}$
 θ dimensionless temperature, $\frac{T - T_s}{T_{in} - T_s}$
 μ_n eigenvalues, radians
 ρ density
 τ dimensionless time $t, \frac{t}{t_{eq}}$
 τ_{th} dimensionless threshold time $t_{th}, \frac{t_{th}}{t_{eq}}$
 Ω dimensionless heat transfer rate $Q, \frac{Q}{Q_{in}}$

Subscripts

app approximate
exa exact
lt large time
st small time
1 one term series

1 Introduction

The study of unsteady heat conduction in solid bodies of regular and irregular shapes has remarkable importance in engineering, applied physics and applied mathematics.

From a historical perspective, the exact, analytical solution for unsteady heat conduction in

a large wall with constant surface temperatures and variable initial temperature was investigated in the seminal work by Fourier [1] in 1822. The study culminated with the Fourier infinite series extending over the entire time domain $0 \leq t \leq \infty$ as reported in the classic heat conduction book by Carslaw and Jaeger [2]. The prominent characteristic of the Fourier infinite series is good convergence for the “large time” sub-domain $t \gg 0$, leading to the abbreviated Fourier one term series that shows good accuracy as seen in the heat conduction books by Arpacı [3], Luikov [4], Özişik [6], Poulikakos [7], Jiji [8] and Kakaç et al. [9]. Contrarily, the Fourier infinite series confronts severe convergence for the “small time” sub-domain $t \rightarrow 0$ necessitating a large number of terms to reach adequate levels of accuracy [2–9]). Nowadays, the Fourier infinite series can be evaluated with high precision in the entire time domain $0 \leq t \leq \infty$ using advanced computer algebra software, like Mathematica, Maple and MATLAB (von zur Gathen [10]).

Numerical methods provide another approach to initial-value problems involving partial differential equations; basic methods for such problems are described by Guzmán [19].

The present theoretical study is centered on a nonlinear interpolation scheme for the analysis of unsteady heat conduction in a large wall that utilizes as input the Fourier one term series in the “large time” sub-domain $t \rightarrow \infty$. This approach facilitates the estimation of the center and mean temperature asymptotes in the “small time” sub-domain $t \rightarrow 0$.

The technical paper is divided in five parts. The first part describes the physical problem and the mathematical formulation for unsteady heat conduction in a large wall exposed to constant surface temperatures. The exact, analytical temperature distribution expressible by Fourier infinite series supplies the baseline solution reported in the second part. The third part contains the Fourier one term series representative of the center and mean temperatures specified in the “large time” sub-domain $t \gg 0$. In the fourth part, the center and mean temperature asymptotes are expressed by way of power interpolation equations of compact form to be applicable in the “small time” sub-domain $t \rightarrow 0$. The fifth part culminates with the comparison of the center and mean temperature asymptotes ascribed to the limiting “small time” against the traditional exact center and exact mean temperature distributions in the entire time domain $0 < t < \infty$.

2 Physical System and Mathematical Formulation

2.1. Physical system

Consideration is given to uni-directional, unsteady heat conduction in a large wall of thickness $2L$, height $H \gg 2L$ and width $W \gg 2L$ having uniform temperature T_{in} at the initial time $t = 0$. For time $t > 0$, a constant temperature T_s is imposed continually on the two surfaces $x = L$ and $x = -L$ of the large wall.

2.2. Mathematical formulation

Under the assumption that the thermal diffusivity α of the material is nearly constant in the temperature interval of operation $\Delta T = |T_{in} - T_s|$, the governing uni-directional, unsteady heat conduction equation expressed in dimensionless form is

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial X^2} \text{ in } 0 \leq X \leq 1, \tau > 0 \quad (1)$$

Herein, the participating dimensionless variables for temperature T , space coordinate x and time t are

$$\theta = \frac{T - T_s}{\Delta T}, X = \frac{x}{L}, \tau = \frac{t}{t_{eq}} \quad (2)$$

holding respective scales: temperature difference $\Delta T = |T_{in} - T_s|$, half-thickness L and equivalent time $t_{eq} = \frac{L^2}{\alpha}$.

The dimensionless initial condition is

$$\theta(X, 0) = 1 \quad (3)$$

The dimensionless boundary conditions are

$$\theta(0, \tau) \rightarrow \text{finite} \quad (4a)$$

$$\text{or} \quad \frac{\partial \theta(0, \tau)}{\partial X} = 0 \quad (4b)$$

and

$$\theta(1, \tau) = 0 \quad (4c)$$

Boundary conditions of the general form $\frac{\partial \theta}{\partial X} = f(\tau)$ are classified as Neumann boundary conditions in Özişik [6] also called boundary conditions of second kind in Luikov [4].

The exact, analytical solution of the initial/boundary value problem (IBVP) formulated with the system of equations (1) to (4c) delivers the dimensionless temperature distribution in the large wall $\theta(X, \tau)$ with validity in the entire dimensionless time domain $0 < \tau \leq \infty$.

3 Target Thermal Quantities

The three target thermal quantities in the unsteady heat conduction problem under study are:

- 1) the dimensionless center temperature
 $\theta_c(\tau) = \theta(0, \tau)$ (5a)
- 2) the dimensionless mean temperature
 $\theta_m(\tau) = \int_0^1 \theta(X, \tau) dX$ (5b)
- 3) the dimensionless heat transfer rate
 $\Omega(\tau) = 1 - \theta_m(\tau)$ (5c)

3.1. Heat transfer rate

The heat transfer rate Q that takes place from/to a large wall to/from a nearby fluid between the initial time $t = 0$ and any given time $t > 0$ may be computed with heat conduction theory [2–9] or with thermodynamics principles (Cengel et al. [11]). Opting for the latter, an energy balance placed in a large wall provides the algebraic equation

$$Q(t) = mc_v |T_m(t) - T_s| \quad (6a)$$

where $T_m(t) = \frac{1}{L} \int_0^L T(x, t) dx$ stands for the mean temperature and T_s denotes the constant surface temperature.

In addition, the initial internal energy in the large wall is given by the algebraic equation

$$Q_{in} = mc_v |T_{in} - T_s| \quad (6b)$$

where T_{in} represents the initial temperature at $t = 0$.

The definition of the dimensionless heat transfer rate Ω corresponds to the ratio

$$\Omega(\tau) = \frac{Q(t)}{Q_{in}} \quad (7)$$

Introducing eqs. (6a) and (6b) into eq. (7), results in the algebraic equation

$$\Omega(\tau) = 1 - \theta_m(\tau) \quad (8)$$

where $\theta_m(\tau) = \frac{T_m(t) - T_s}{T_{in} - T_s}$ is the dimensionless mean temperature evaluated at the dimensionless time $\tau = \frac{t}{t_{eq}}$.

4 Exact, Analytical Center and Mean Temperature Distributions

The dimensionless exact, analytical center temperature distribution is expressed by the Fourier infinite series [2–9]:

$$\theta_c(\tau) = \sum_{n=1}^{\infty} A_n(\mu_n) \exp(-\mu_n^2 \tau) \quad (9)$$

extending over the entire dimensionless time domain $0 < \tau < \infty$. Herein, the eigenvalues μ_n are

$$\mu_n = \pi \left(n - \frac{1}{2} \right), \quad n = 1, 2, \dots, \infty, \quad (9a)$$

and the eigenfunctions are

$$A_n(\mu_n) = \frac{2 \sin(\mu_n)}{\mu_n + \sin(\mu_n) \cos(\mu_n)}, \quad n = 1, 2, \dots, \infty. \quad (9b)$$

The dimensionless exact, analytical mean temperature distribution is expressed by the Fourier infinite series [2–9]:

$$\theta_m(\tau) = \sum_{n=1}^{\infty} B_n(\mu_n) \exp(-\mu_n^2 \tau) \quad (10)$$

extending over the entire dimensionless time domain $0 < \tau < \infty$. Herein, the eigenvalues μ_n are given in eq. (9a) and the eigenfunctions are

$$B_n(\mu_n) = A_n(\mu_n) \left[\frac{\sin(\mu_n)}{\mu_n} \right], \quad n = 1, 2, \dots, \infty. \quad (10a)$$

Actually, to obtain accurate numerical values for the center and mean temperature distributions using eqs. (9) and (10), it is necessary to sum a finite number of N terms, so that the Fourier infinite series become convergent. The number N typically corresponds to the point where the relative change in the sum due to the $n = N$ th term is less than a pre-set tolerance. In fact, it can be easily seen that the “truncation limit” N decreases as the dimensionless time τ increases in the argument of the exponential function $\exp(-\mu_n^2 \tau)$.

4.1. Approximate, analytical center and mean temperature sub-distributions in the “large time” sub-domain

From Sturm–Liouville theory (Al–Gwaiz [12]), the set of eigenvalues μ_n in eq. (9a) follow the sequence of inequalities

$$\mu_1 < \dots < \mu_n < \dots < \mu_{\infty}$$

so that the exponential functions in eqs. (9) and (10) follow the sequence of inequalities

$$\exp(-2\beta_1^2 X) > \dots > \exp(-(2\beta_n^2 X)) > \dots > \exp(-2\beta_{\infty}^2 X)$$

In the prevalent context, it has been demonstrated that the first exponential function $\exp(-\mu_1^2 \tau)$ in the pair of eqs. (9) and (10) absorbing the first eigenvalue μ_1 becomes dominant after a certain dimensionless threshold time τ_{th} is surpassed [2–9]. This feature signifies that the contribution of successive exponential functions $\exp(-\mu_n^2 \tau)$; $n = 2, 3, \dots, \infty$ turns out to be literally negligible when compared against the first exponential function $\exp(-\mu_1^2 \tau)$. As a direct consequence, the pair of Fourier infinite series in eqs. (9) and (10) can be shortened to the following pair of Fourier one term series

$$\theta_{c,lt}(\tau) = \frac{4}{\pi} \exp\left(-\frac{\pi^2}{4} \tau\right)$$

and

$$\theta_{m,lt}(\tau) = \frac{8}{\pi^2} \exp\left(-\frac{\pi^2}{4} \tau\right)$$

Herein, the subscript “ lt ” attached to the symbols θ_c and θ_m denotes “long time”.

4.2. Estimation of the dimensionless threshold time

The dimensionless threshold time τ_{th} for the unsteady heat conduction in a large wall has been investigated extensively by Grigull et al. [13] and Grigull and Sandner [14]. As the outcome of the investigation, the definitive dimensionless threshold time for the large wall was established at $\tau_{th} = 0.24$. Further information about this issue has been amplified by Cortés and Campo [15]. In the quest of simplicity, heat transfer textbooks like Lienhard V and Lienhard IV [16] usually shorten the dimensionless threshold time to $\tau_{th} \approx 0.20$. This number will be used in the present study.

5 Basic Principle in Applied Mathematics

A basic principle in applied mathematics consists in reducing a complex physical problem into less complex physical sub-problems having either known solutions or sub-solutions that are easily obtainable. The basic principle has been called the divide-and-conquer algorithm (Suyama [17]). Inspired in the basic principle, the goal of the present study is to construct a pair of power interpolation equations for the determination of approximate, analytical center and mean temperature asymptotes confined to the dimensionless “small time” sub-domain $0 < \tau \leq \tau_{th} = 0.20$.

6 A Nonlinear Interpolation Scheme

Interpolation Theory can be broadly classified as linear or nonlinear as indicated by Mastroianni and Milovanovic [18]. Linear interpolation makes a straight line to approximate a value of some function $f(x)$ using known values of the function at two data points. Nonlinear interpolation makes a curve line to approximate a value of some $f(x)$ function using known values of the function $f(x)$ at three or more data points. A powerful variant of nonlinear interpolation makes a curved line to approximate a value of some function $f(x)$ using known values of a function $f(x)$ at two data points (not three data points) along with a known value of the derivative of the function $f'(x)$ at one of the two data points.

In harmony with the foregoing, we propose a power interpolation equation engaging one coefficient b and one exponent c :

$$\theta(\tau) = 1 - b\tau^c \quad (13)$$

with derivative

$$\theta'(\tau) = -cb\tau^{c-1} \quad (14)$$

Certainly, the structure of eq. (13) harmonizes with the physics of the problem and satisfies the initial condition stated in eq. (3).

In order to evaluate the coefficient b and the exponent c in eq. (13), certain information about thermal quantities is required:

1) the dimensionless initial temperature $\theta(0) = 1$ from eq. (3),

2) the dimensionless threshold temperature θ_{th} evaluated at the dimensionless threshold time $\tau_{th} = 0.20$ and

3) the dimensionless threshold temperature derivative θ'_{th} evaluated

at the dimensionless threshold time $\tau_{th} = 0.20$.

For visualization, the three thermal quantities are displayed graphically in Figure 1.

First, introducing the evaluated θ_{th} at $\tau_{th} = 0.20$ into eq. (13) delivers the equation

$$\theta_{th}(0.20) = 1 - b(0.20)^c \quad (15)$$

Second, introducing the evaluated θ'_{th} at $\tau_{th} = 0.20$ into eq. (14) delivers the equation

$$\theta'_{th}(0.20) = -cb(0.20)^{c-1} \quad (16)$$

or the equivalent equation

$$\theta'_{th}(0.20) = (0.20) = -5cb(0.20)^c \quad (16b)$$

Correspondingly, the pair of eqs. (15) and (16b) creates a system of two nonlinear algebraic equations in two unknowns b and c .

$$\begin{cases} b(0.20)^c = 1 - \theta_{th}(0.20) \\ -5cb(0.20)^c = \theta'_{th}(0.20) \end{cases} \quad (17a,b)$$

The exact, analytical solution of the above system is easily obtainable by substitution and elimination.

Dividing eq. (17b) by eq. (17a) gives the ratio

$$c = \frac{\theta'_{th}(0.20)}{5[1 - \theta_{th}(0.20)]} \quad (18)$$

Substituting the ratio for c into eq. (16) gives the other ratio

$$b = \frac{1 - \theta_{th}(0.20)}{(0.20)^c} \quad (19)$$

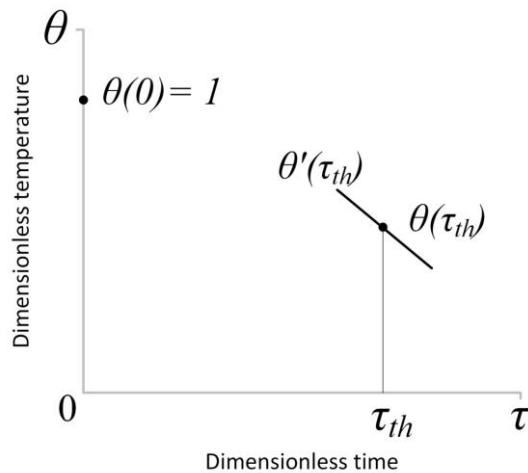


Fig. 1: Illustration of the data points for the nonlinear interpolation scheme. The temperature symbol represents either the dimensionless center temperature or the dimensionless mean temperature.

6.1. The approximate center temperature sub-distributions

From eq. (11), the Fourier one term series for the dimensionless center temperature is

$$\theta_{c,lt}(\tau) = \frac{4}{\pi} \exp\left(-\frac{\pi^2}{4}\tau\right) \quad (11)$$

so that the dimensionless center temperature derivative is

$$\theta'_{c,lt}(\tau) = -\pi \exp\left(-\frac{\pi^2}{4}\tau\right) \quad (20)$$

The two equations (11) and (20) are valid in the dimensionless “large time” sub-domain $\tau \geq \tau_{th} = 0.20$. Herein, the subscript “lt” appended to the symbols $\theta_{c,lt}$ and $\theta'_{c,lt}$ implies “large time”.

Evaluating eqs. (11) and (20) concurrently at the dimensionless threshold time $\tau_{th} = 0.20$ provides the dimensionless threshold center temperature $\theta_{c,lt}(0.20) = 0.7773$ along with the dimensionless threshold center temperature derivative $\theta'_{c,lt}(0.20) = -1.9179$.

Next, substituting the two numbers $\theta_{c,lt}(0.20) = 0.7773$ and $\theta'_{c,lt}(0.20) = -1.9179$ into eqs. (11) and (20) hands over the value of the coefficient $b = -3.5614$ and the value of exponent $c = 1.7224$. Curiously, there is an interconnection between $b = -3.5614$ and $c = 1.7224$, namely $b \approx -2c$!

As a direct consequence, eq. (13) conveys the approximate dimensionless center temperature sub-distribution

$$\theta_{c,st}(\tau) = 1 - 3.5614 \tau^{1.7224} \quad (21)$$

whose derivative is

$$\theta'_{c,st}(\tau) = -6.1342 \tau^{0.7224} \quad (22)$$

The two equations (21) and (22) are valid inside the problematic dimensionless “small time”

sub-domain $0 < \tau \leq \tau_{th} = 0.20$. Herein, the appended subscript “st” to the symbols $\theta_{c,st}$ and $\theta'_{c,st}$ implies “small time”.

For verification, performing a three way comparison at the dimensionless threshold time $\tau_{th} = 0.20$, eq. (21) renders the approximate value $\theta_{c,st} = 0.7773$, eq. (11) renders the approximate value $\theta_{c,lt} = 0.7773$, whereas eq. (8) renders the benchmark exact value $\theta_{c,exa} = 0.7723$. Therefore, the errors attached to $\theta_{c,st}$ and $\theta_{c,lt}$ are 0.65%, i.e., much less than 1%.

6.2. The approximate mean temperature sub-distributions

From eq. (12), the Fourier one term series for the dimensionless mean temperature is

$$\theta_{m,lt}(\tau) = \frac{8}{\pi^2} \exp\left(-\frac{\pi^2}{4}\tau\right) \quad (12)$$

whose derivative is

$$\theta'_{m,lt}(\tau) = -2 \exp\left(-\frac{\pi^2}{4}\tau\right) \quad (23)$$

The two equations (12) and (23) are valid in the dimensionless “large time” sub-domain $\tau \geq \tau_{th} = 0.20$. Herein, the appended subscript “lt” to the symbols θ_m and θ'_m signifies “large time”.

Evaluating eqs. (12) and (23) concurrently at the dimensionless threshold time $\tau_{th} = 0.20$ produces the dimensionless mean temperature $\theta_{m,lt}(0.20) = 0.4948$ along with the dimensionless threshold mean temperature derivative $\theta'_{m,lt}(0.20) = -1.2209$.

Finally, substituting the two numbers $\theta_{m,lt}(0.20) = 0.4948$ and $\theta'_{m,lt}(0.20) = -1.2209$ into the pair of eqs. (18) and (19) provides the value for the coefficient $b = -1.0997$ and the value of the exponent $c = 0.4883$. Curiously again, there is an interconnection between $b = -1.0997$ and $c = 0.4883$, namely $b \approx -2c$!

Consequently, in view of the foregoing, the approximate dimensionless mean temperature sub-distribution in eq. (13) becomes

$$\theta_{m,st}(\tau) = 1 - 1.0997 \tau^{0.4883} \quad (24)$$

whose derivative is

$$\theta'_{m,st}(\tau) = -0.5315 \tau^{-0.5167} \quad (25)$$

The two equations (24) and (25) are valid inside the problematic dimensionless “small time” sub-domain $0 < \tau \leq \tau_{th} = 0.20$ only.

For verification, a three-way comparison at the dimensionless threshold time $\tau_{th} = 0.20$ is performed. First, eq. (24) gives the approximate $\theta_{m,st} = 0.4948$, second eq. (12) gives the approximate $\theta_{m,lt} = 0.4949$ and third eq. (9) gives the benchmark exact value $\theta_{m,exa} = 0.4961$. Therefore, the emerging errors are -0.26% in eq.

(24) and -0.24% in eq. (12), so that both errors are less than -1% .

7 Presentation of Results

7.1. Dimensionless center temperatures

The dimensionless exact center temperature $\theta_c(\tau)$ requires the evaluation of the exact Fourier infinite series

$$\theta_{c,exa}(\tau) = \sum_{n=1}^{\infty} A_n(\mu_n) \exp(-\mu_n^2 \tau) \quad (9)$$

accounting for an infinite number of eigenvalues μ_n and eigenfunctions $A_n(\mu_n)$; $n = 1, 2, \dots, \infty$ in the entire dimensionless time domain $0 < \tau \leq \infty$. This step requires the use of the Mathematica function Sum that is documented in the Wolfram Language package [20].

In contrast, the dimensionless approximate center temperature $\theta_c(\tau)$ covering the entire dimensionless time domain $0 < \tau \leq \infty$ can be obtained with the pair of asymptotic equations

$$\theta_{c,st}(\tau) = 1 - 3.5614 \tau^{1.7224} \text{ for } 0 < \tau \leq 0.20 \quad (21)$$

and

$$\theta_{c,lt}(\tau) = \frac{4}{\pi} \exp\left(-\frac{\pi^2}{4} \tau\right) \text{ for } 0.20 \leq \tau \leq \infty \quad (11)$$

The matching of the computed dimensionless center temperatures between eqs. (21) and eq. (11) is perfect. That is, $\theta_{c,st}(0.20) = 0.77730$ versus $\theta_{c,lt}(0.20) = 0.77731$.

Figure 2 displays the error distributions in the approximate dimensionless center temperatures $\theta_{c,st}$ confined to the dimensionless “small time” sub-domain $0 < \tau = \tau_{th} \leq 0.20$. It is seen in the figure that the largest error stays around 2%.

The comparison of the approximate dimensionless center temperatures $\theta_{c,st}$ in the compelling dimensionless “small time” sub-domain $0 < \tau \leq \tau_{th} = 0.20$ appears in Table 1. At the local level, the largest error -1.77% happens at the dimensionless time $\tau = 0.1$. At the global level, the overall mean absolute error MAE is $0.62\% \ll 1\%$.

Table 1. Comparison of the dimensionless center temperatures $\theta_{c,st}$ in the compelling dimensionless “small time” sub-domain $0 < \tau \leq \tau_{th} = 0.20$.

Time τ	Approximate center temperature $\theta_{c,st}$ (Present work)	Exact center temperature $\theta_{c,exa}$ from Luikov [4]	Error
0.001	1	1	0
0.005	0.9996	1	-0.04%
0.01	0.9987	1	-0.13%
0.05	0.9795	0.9969	-1.74%
0.10	0.9325	0.9493	-1.77% (largest)
0.15	0.8643	0.8642	+ 0.01%
0.20	0.7773	0.7723	+0.65%
All τ	-----	-----	Mean Absolute Error, MAE = 0.62% << 1%

7.2. Dimensionless mean temperatures

The exact, dimensionless mean temperature $\theta_m(\tau)$ in the entire dimensionless time domain $0 < \tau \leq \infty$ demands the evaluation of the exact Fourier infinite series

$$\theta_{m,exa}(\tau) = \sum_{n=1}^{\infty} B_n(\mu_n) \exp(-\mu_n^2 \tau) \quad (10)$$

accounting for an infinite number of eigenvalues μ_n ; $n = 1, 2, \dots, \infty$. and eigenfunctions $B_n(\mu_n)$; $n = 1, 2, \dots, \infty$. in the entire dimensionless time domain $0 < \tau \leq \infty$. This step requires the Mathematica function Sum that is documented in the Wolfram Language package [20].

In contrast, the approximate dimensionless mean temperature $\theta_m(\tau)$ in the entire dimensionless time domain $0 < \tau \leq \infty$ can be obtained with the pair of asymptotic equations

$$\theta_{m,st}(\tau) = 1 - 1.0997 \tau^{0.4833} \text{ for } 0 < \tau \leq 0.20 \quad (24)$$

and

$$\theta_{m,lt}(\tau) = \frac{8}{\pi^2} \exp\left(-\frac{\pi^2}{4} \tau\right) \text{ for } 0.20 \leq \tau \leq \infty \quad (12)$$

The matching of the dimensionless center temperatures between eqs. (24) and eq. (12) is

perfect. That is, $\theta_{m,st}(0.20) = 0.49480$ versus $\theta_{m,lt}(0.20) = 0.49485$.

Figure 2 displays the error distributions in the approximate dimensionless mean temperatures $\theta_{m,st}$ confined to the dimensionless “small time” sub-domain $0 < \tau = \tau_{th} \leq 0.20$. It is seen in the figure that the largest error stays around 1%.

The comparison of the approximate dimensionless mean temperatures $\theta_{m,st}$ in the compelling dimensionless “small time” sub-domain $0 < \tau \leq \tau_{th} = 0.20$ appears in Table 2. At the local level, the largest error +2.64% happens at the dimensionless time $\tau = 0.005$. At the global level, the overall mean absolute error is $1.17\% \approx 1\%$.

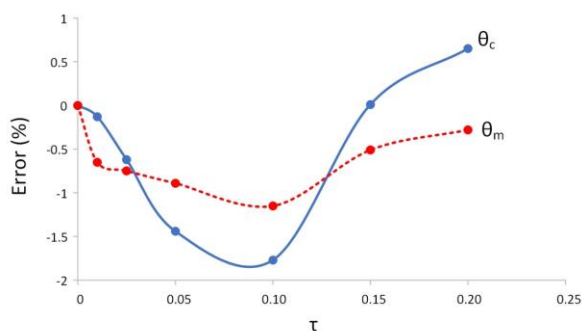


Fig. 2: Errors in the dimensionless center and mean temperatures in the small-time subdomain.

Table 2. Comparison of dimensionless mean temperatures $\theta_{m,st}$ in the compelling dimensionless “small time” sub-domain $0 < \tau \leq \tau_{th} = 0.20$.

Time τ	Approximate mean temperature $\theta_{m,st}$ (Present work)	Exact mean temperature $\theta_{m,exa}$ from Luikov [4]	Error
0.001	0.9610	0.9821	-2.15%
0.005	0.9150	0.8915	+2.64% (largest)
0.01	0.8812	0.8870	-0.65%
0.05	0.7415	0.7481	-0.88%
0.10	0.6386	0.6459	-1.13%
0.15	0.5604	0.5632	-0.50%
0.20	0.4948	0.4961	-0.26%
All τ	----	----	Mean Absolute Error, MAE = $1.17\% \approx 1\%$

7.3. Dimensionless heat transfer rate

The estimation of the dimensionless heat transfer rate $\Omega(\tau)$ in the entire dimensionless time domain $0 < \tau \leq \infty$ can be easily obtained with the

combination of eqs. (8), (24) and (12). This results in the pair of asymptotic equations

$$\Omega_{st}(\tau) = 1.0997 \tau^{0.4833} \text{ for } 0 < \tau = \tau_{th} \leq 0.20 \quad (26)$$

and

$$\Omega_{lt}(\tau) = 1 - \frac{8}{\pi^2} \exp\left(-\frac{\pi^2}{4}\tau\right) \text{ for } \tau = \tau_{th} \geq 0.20 \quad (27)$$

where the subscripts “st” and “lt” in the symbol Ω indicate small time and large time.

Actually, the pair of equations supplants the exact Fourier infinite series

$$\Omega(\tau) = 1 - \sum_{n=1}^{\infty} B_n(\mu_n) \exp(-\mu_n^2 \tau) \quad (28)$$

The exact evaluation of eq. (28) accounting for the infinite eigenvalues μ_n ; $n = 1, 2, \dots, \infty$ and eigenfunctions $B_n(\mu_n)$; $n = 1, 2, \dots, \infty$ covering the entire dimensionless time domain $0 < \tau \leq \infty$ requires the elaboration of a short program, for instance the Mathematica function Sum [20].

8 Conclusions

The main conclusion that can be drawn from the theoretical study on unsteady heat conduction is that the Fourier infinite series representative of the center temperature distribution and the mean temperature distribution in a large wall influenced by constant surface temperature can be bypassed in favor of a pair of compact asymptotic equations. Certainly, the determination of the two dimensionless center temperature asymptotes and the dimensionless mean temperature asymptotes is the new contribution that has surfaced up in the present study.

The outcome of the present study has convincingly demonstrated the evolutions of errors intrinsic to the duo of dimensionless center temperature and mean temperature asymptotes in a large wall with equal constant surface temperatures. All errors linked to the precarious dimensionless “moderate time” sub-domain $0 < \tau \leq 0.20$ stay around one or less than one percentage.

Ultimately, the nonlinear interpolation scheme for the analysis of unsteady heat conduction in large walls with constant surface temperature fostered here could be easily extended to the analysis of unsteady heat conduction in long cylinders and spheres with constant surface temperatures operating inside the tricking dimensionless “small-time” sub-domain.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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