

Thermo-diffusive response in generalized thermoelastic medium having spherical cavity

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Abstract: The present study examines the thermo-diffusive response of a homogeneous, isotropic, semi-infinite medium subjected to a finite rotational field. The analysis is carried out within the framework of the generalized theory of thermoelasticity incorporating mass diffusion, which accounts for finite thermal wave speeds and diffusion effects. The coupled governing equations describing displacement, stress, temperature, and concentration fields are formulated and solved analytically. To infer the physical behavior of the medium, a complex quartic characteristic equation is derived and solved using Descartes' algorithm in conjunction with the irreducible Cardan's method. Numerical computations are performed to illustrate the theoretical developments. The effects of rotation and thermo-diffusion on displacement components, stress distribution, and temperature variations are presented graphically and investigated. The results reveal significant influences of rotation and diffusion parameters on the thermoelastic response. The present investigation is expected to be useful in understanding thermo-diffusive phenomena arising in geophysical and engineering applications.

Key words: Elasticity, Generalized-thermoelasticity, Thermal wave propagation, Mass-diffusion, Spherical Cavity, Rotating medium.

Received: March 22, 2026. Revised: May 5, 2026. Accepted: June 6, 2026. Published: October 14, 2026.

1 Introduction

Based on the pioneering contributions [1], [2], [3], [4] to the theory of thermoelastic diffusion, the present study adopts a coupled thermoelastic framework incorporating the concept of second sound. This formulation overcomes the fundamental paradox of infinite heat propagation speed predicted by the classical Fourier law and provides a more realistic description of thermal wave behavior in elastic media [5]. In the linear dynamical theory of classical thermoelasticity, the field equations consist of the coupled equations of motion and heat conduction. While the displacement field is governed

by a hyperbolic wave-type equation, the temperature field follows a parabolic diffusion-type equation, leading to an inherent mismatch in propagation characteristics.

To address these limitations, several generalized theories of thermoelasticity have been proposed. The generalized thermoelasticity theory, [6] introduces a single thermal relaxation time, thereby modified the Fourier's law heat conduction equation into a hyperbolic form. Another theory, [7] generalized linear thermoelasticity model that eliminates the paradox of infinite heat propagation speed found in classical theory. It introduces two distinct thermal

relaxation times, allows for "second sound" effects (finite thermal wave speed), and ensures that Fourier's law is modified to include strain rate effects. The temperature rate dependent thermoelasticity theory, [8], [9] further extends the heat conduction that allow the propagation of thermal waves without energy dissipation. These generalized models provide a consistent framework in which both mechanical and thermal disturbances propagate at finite speeds.

Despite their physical relevance, the governing equations of generalized thermoelasticity are highly coupled and mathematically intricate, primarily due to the presence of inertia effects and thermo-mechanical coupling terms. As a result, most analytical investigations reported in the literature are confined to one-dimensional or highly idealized configurations.

In this paper, we extend the analysis to a more realistic three-dimensional setting by examining the interaction among elastic deformation, thermal effects, and diffusion processes in an infinite elastic medium rotating with a uniform angular velocity and containing a spherical cavity. The influence of rotation introduces additional inertial effects, such as Coriolis and centrifugal forces, which significantly affect the coupled field responses. The problem is formulated and analyzed within the framework of the generalized theory of thermoelastic diffusion with one relaxation time, enabling a comprehensive understanding of the combined effects of elasticity, heat conduction, diffusion, and rotation on the dynamic behavior of the medium.

2 Formulation of the problem

An isotropic, homogeneous, infinite elastic medium is considered to be analyzed here which is rotating with a uniform angular velocity $\vec{\Omega}$, occupying the region $a \leq r < \infty$, where a is the radius of the spherical cavity.

The governing equations of motion are, [10]:

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \beta_1 T_{,i} - \beta_2 C_{,i} = \rho \frac{\partial^2 u_i}{\partial t^2} + \rho \left[\{ \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) \}_i + (2\vec{\Omega} \times \dot{\vec{u}})_i \right] \quad (2.1)$$

$$\left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (\rho c_E T + T_0 \beta_1 e_{kk} + c T_0 C) = k T_{,ii} \quad (2.2)$$

$$D \beta_2 e_{kk,ii} + D C T_{,ii} + \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2} \right) C = D b C_{,ii} \quad (2.3)$$

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij} [\lambda e_{kk} - \beta_1 (T - T_0) - \beta_2 C] \quad (2.4)$$

$$P = -\beta_2 e_{kk} + bC - c(T - T_0) \quad (2.5)$$

where $\beta_1 = (3\lambda + 2\mu)\alpha_t$ and $\beta_2 = (3\lambda + 2\mu)\alpha_c$, in which,

α_t : coefficients of linear thermal expansion,

α_c : coefficients of diffusion expansion,

λ and μ : Lamé's constants,

T : absolute temperature of the medium,

T_0 : reference uniform temperature such that

$|(T - T_0)/T_0| \ll 1$,

k : coefficient of thermal conductivity,

c_E : specific heat at constant strain,

ρ : mass density,

σ_{ij} : components of the stress tensor,

e_{ij} : components of the strain tensor,

u_i : component of the displacement vector,

P : chemical potential,

C : concentration of the diffusive material in the elastic body,

D : diffusion coefficient,

c : measures of thermodiffusion effects,

b : measures of diffusive effects,

τ : diffusion relaxation time,

τ_0 : thermal relaxation time.

To avoid the complexity of the problem, spherical-polar coordinate system (r, ϑ, φ) is considered and the medium is followed spherical symmetry. Consequently, $u_r = u(r, t)$.

The components of the strain tensors can be expressed in the following form:

$$e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\varphi\varphi} = e_{\vartheta\vartheta} = \frac{u}{r}, \quad e_{r\varphi} = e_{r\vartheta} = e_{\vartheta\varphi} = 0 \quad (2.6)$$

$$\text{The cubical dilatation } e = \frac{\partial u}{\partial r} + 2 \frac{u}{r} \quad (2.7)$$

Due to spherical symmetry equations (2.1) – (2.5) can be recast as follows:

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial e}{\partial r} - \beta_1 \frac{\partial T}{\partial r} - \beta_2 \frac{\partial C}{\partial r} - \rho \left(\Omega^2 u + 2\Omega \frac{\partial u}{\partial t} \right) \quad (2.8)$$

$$k \nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (\rho c_E T + T_0 \beta_1 e + c T_0 C) \quad (2.9)$$

$$Db\nabla^2 C = D\beta_2 \nabla^2 e + DC\nabla^2 T + \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}\right) C \quad (2.10)$$

$$\sigma_{rr} = 2\mu \frac{\partial u}{\partial r} + \lambda e - \beta_1(T - T_0) - \beta_2 C \quad (2.11)$$

$$e_{\varphi\varphi} = e_{\vartheta\vartheta} = 2\mu \frac{u}{r} + \lambda e - \beta_1(T - T_0) - \beta_2 C \quad (2.12)$$

$$P = -\beta_2 e + bC - c(T - T_0) \quad (2.13)$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r}$$

Now we define the following non-dimensional variables as

$$\begin{aligned} r^* &= c_1 \eta_0 r, & u^* &= c_1 \eta_0 u, & \theta &= \frac{\beta_1(T-T_0)}{\lambda+2\mu}, \\ C^* &= \frac{\beta_2 C}{\lambda+2\mu}, & \Omega^* &= \frac{\Omega}{c_1^2 \eta_0}, & \sigma_{ij}^* &= \frac{\sigma_{ij}}{\lambda+2\mu}, & P^* &= \frac{P}{\beta_2}, \\ t^* &= c_1^2 \eta_0 t, & \tau_0^* &= c_1^2 \eta_0 \tau_0, & r^* &= c_1^2 \eta_0 r, \end{aligned}$$

$$\text{where } c_1^2 = \frac{\lambda+2\mu}{\rho} \quad \text{and} \quad \eta_0 = \frac{\rho c_E}{k}.$$

Introducing those non-dimensional variables to the equations (2.8) – (2.13) and dropping the asterisks, we get the following

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial e}{\partial r} - \frac{\partial \theta}{\partial r} - \frac{\partial C}{\partial r} - \Omega^2 u - 2\Omega \frac{\partial u}{\partial t} \quad (2.14)$$

$$\nabla^2 \theta = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) (\theta + \varepsilon e + \varepsilon \alpha_1 C) \quad (2.15)$$

$$\alpha_3 \nabla^2 C = \alpha_2 \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}\right) C + \nabla^2 e + \alpha_1 \nabla^2 \theta \quad (2.16)$$

$$\sigma_{rr} = e - \frac{4u}{\beta^2 r} - \theta - C \quad (2.17)$$

$$\sigma_{\varphi\varphi} = \sigma_{\vartheta\vartheta} = \frac{2u}{\beta^2 r} + \left(1 - \frac{2}{\beta^2}\right) e - \theta - C \quad (2.18)$$

$$P = \alpha_3 C - e - \alpha_1 \theta \quad (2.19)$$

$$\text{where } \varepsilon = \frac{T_0 \beta_1^2}{\rho^2 c_E c_1^2}, \quad \alpha_1 = \frac{c \rho c_1^2}{\beta_1 \beta_2}, \quad \alpha_2 = \frac{\mu}{D \eta_0}, \quad \alpha_3 = \frac{b \rho c_1^2}{\beta_2^2}, \quad \beta^2 = \frac{\lambda+2\mu}{\mu}$$

Boundary Conditions:

Here we consider the medium was unstressed at the inner boundary of the cavity and a uniform heat is applied there.

$$\sigma_{rr}(a, t) = 0, \quad \theta(a, t) = \theta_0 H(t), \quad P(a, t) = P_0 H(t) \quad (2.20)$$

where θ_0 and P_0 are constants and $H(t)$ is the Heaviside unit step function.

3 Solution by the Laplace transform

We define the Laplace transform as

$$\bar{f}(x, s) = \int_0^\infty f(x, t) e^{-st} dt \quad (3.1)$$

Introducing this transform to equations (2.14) – (2.16) and using the initial conditions, we obtain the following equations

$$s_1^2 \bar{u} = \frac{\partial \bar{e}}{\partial r} - \frac{\partial \bar{\theta}}{\partial r} - \frac{\partial \bar{C}}{\partial r} \quad (3.2)$$

$$\nabla^2 \bar{\theta} = s(1 + \tau_0 s)(\bar{\theta} + \varepsilon \bar{e} + \varepsilon \alpha_1 \bar{C}) \quad (3.3)$$

$$\alpha_1 \nabla^2 \bar{\theta} \quad \alpha_3 \nabla^2 \bar{C} = \alpha_2 s(1 + \tau s) \bar{C} + \nabla^2 \bar{e} + \quad (3.4)$$

$$\text{where } s_1^2 = s^2 + \Omega^2 + 2\Omega s$$

$$\text{applying } \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \text{ to equation (3.2), we get}$$

$$(s_1^2 - \nabla^2) \bar{e} = \nabla^2 \bar{\theta} + \nabla^2 \bar{C} \quad (3.5)$$

Now by the equations (3.3), (3.4) and (3.5), we obtain

$$(\nabla^6 - a_1 \nabla^4 + a_2 \nabla^2 - a_3)(\bar{e}, \bar{\theta}, \bar{C}) = 0 \quad (3.6)$$

where

$$a_1 = \frac{1}{\alpha_3 - 1} [\{\varepsilon(\alpha_3 + 2\alpha_1 + \alpha_1^2) - 1 + \alpha_3\} \{s(1 + \tau_0 s)\} + s\alpha_2(1 + \tau s) + \alpha_3(s^2 + \Omega^2 + 2\Omega s)] \quad (3.7)$$

$$a_2 = \frac{1}{\alpha_3 - 1} [(1 + \varepsilon)(1 + \tau_0 s)(1 + \tau s)s^2 \alpha_2 + \{(2\alpha_3 + \varepsilon \alpha_1 \alpha_3 + \varepsilon \alpha_1^2)s(1 + \tau_0 s) - 1\}(s^2 + \Omega^2 + 2\Omega s)] \quad (3.8)$$

$$a_3 = \frac{1}{\alpha_3 - 1} [(1 + \tau_0 s)(1 + \tau s)(s^2 + \Omega^2 + 2\Omega s)s^2 \alpha_2] \quad (3.9)$$

The system of equations (3.6) can be factorized as

$$(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)(\nabla^2 - k_3^2)(\bar{e}, \bar{\theta}, \bar{C}) = 0 \quad (3.10)$$

Where k_1 , k_2 and k_3 are the roots of the characteristic equation

$$k^6 - a_1 k^4 + a_2 k^2 - a_3 = 0 \quad (3.11)$$

Now we take $m = k^2$, the equation (3.11) reduces to

$$m^3 - a_1 m^2 + a_2 m - a_3 = 0 \quad (3.12)$$

From this equation we can obtain three values of ' m ' i.e. of k^2 . Each value of ' m ' corresponds to a wave if k^2 is positive and real. The roots k_1 , k_2 and k_3 are given by

$$k_1 = \sqrt{\frac{1}{3}\{2p \sin(q) + a_1\}} \quad (3.13)$$

$$k_2 = \sqrt{\frac{1}{3}[a_1 - p\{\sqrt{3} \cos(q) + \sin(q)\}]} \quad (3.14)$$

$$k_3 = \sqrt{\frac{1}{3}[a_1 + p\{\sqrt{3} \cos(q) - \sin(q)\}]} \quad (3.15)$$

$$\text{where } p = \sqrt{a_1^2 - 3a_2}, \quad q = \frac{1}{3} \sin^{-1}(w), \\ w = -\frac{1}{2p^3} (2a_1^3 - 9a_1 a_2 + 27a_3) \quad (3.16)$$

The solution of equation (3.10) is given by

$$\bar{\theta}(r, s) = \frac{1}{\sqrt{r}} \sum_{i=1}^3 A_i(s) K_{1/2}(k_i r) \quad (3.17)$$

$$\bar{e}(r, s) = \frac{1}{\sqrt{r}} \sum_{i=1}^3 A_i'(s) K_{1/2}(k_i r) \quad (3.18)$$

$$\bar{C}(r, s) = \frac{1}{\sqrt{r}} \sum_{i=1}^3 A_i''(s) K_{1/2}(k_i r) \quad (3.19)$$

where $K_{1/2}$ is the modified Bessel function of second kind of order $\frac{1}{2}$ and A_i , A_i' , A_i'' are parameters depending only on s .

From the equations (3.17) – (3.19) along with equations (3.3) and (3.4), we obtain

$$\bar{A}_i(s) = \frac{k_i^4 \alpha_3 - \{(\alpha_1^2 \varepsilon + \alpha_3)s(1 + \tau_0 s) + \alpha_2 s(1 + \tau s)\}k_i^2 + s^2 \alpha_2(1 + \tau_0 s)(1 + \tau s)}{\varepsilon s(1 + \tau_0 s)\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}} A_i(s) \quad (3.20)$$

$$A_i''(s) = \frac{k_i^2 \{k_i^2 - s(1 + \tau_0 s)(1 - \varepsilon \alpha_1)\}}{\varepsilon s(1 + \tau_0 s)\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}} A_i(s) \quad (3.21)$$

Now we integrate both sides of equations (3.17) – (3.19) from r to infinity, and consider that u vanishes at infinity, we get

$$\bar{u}(r, s) = -\frac{2}{\varepsilon s s_1^2 (1 + \tau_0 s) \sqrt{r}} \times \sum_{i=1}^3 \left[-\frac{\frac{k_i^4 (\alpha_3 - 1)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}}}{\frac{s(1 + \tau_0 s)\{(2 + \alpha_1)\varepsilon \alpha_1 + (1 + \varepsilon)\alpha_3 - 1\} + \alpha_2 s(1 + \tau s)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}} + \frac{s^2 \alpha_2 (1 - \varepsilon)(1 + \tau_0 s)(1 + \tau s)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}}} \right] A_i(s) k_i K_{1/2}'(k_i r) \quad (3.22)$$

Therefore, substituting the expressions from equations (3.17) – (3.19) and (3.22) in (2.17) we get,

$$\bar{\sigma}_{rr}(r, s) = -\frac{1}{\varepsilon s(1 + \tau_0 s) \sqrt{r}} \times \sum_{i=1}^3 \left[-\frac{\frac{k_i^4 (\alpha_3 - 1)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}}}{\frac{s(1 + \tau_0 s)\{(2 + \alpha_1)\varepsilon \alpha_1 + (1 + \varepsilon)\alpha_3 - 1\} + \alpha_2 s(1 + \tau s)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}} + \frac{s^2 \alpha_2 (1 - \varepsilon)(1 + \tau_0 s)(1 + \tau s)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}}} \right] \left[K_{1/2} + \frac{8}{\beta^2 s_1^2 r} k_i K_{1/2}'(k_i r) \right] A_i(s) \quad (3.23)$$

$$\bar{P}(r, s) = -\frac{1}{\varepsilon s(1 + \tau_0 s) \sqrt{r}} \sum_{i=1}^3 \frac{-[2\alpha_3 s(1 + \tau_0 s) + \alpha_2 s(1 + \tau s)]k_i^2 + \alpha_2 s^2(1 + \varepsilon \alpha_1)(1 + \tau_0 s)(1 + \tau s)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}} A_i(s) K_{1/2}(k_i r) \quad (3.24)$$

Now using the boundary conditions in equations (2.20) to the equations (3.17), (3.23) and (3.24), we

can determine the unknown parameters A_1 , A_2 and A_3 .

Thus, we derive the set of linear equations:

$$\sum_{i=1}^3 \left[\frac{k_i^4(\alpha_3 - 1)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}} \right] \times \left[\frac{s(1 + \tau_0 s)\{(2 + \alpha_1)\varepsilon\alpha_1 + (1 + \varepsilon)\alpha_3 - 1\} + \alpha_2 s(1 + \tau s)k_i^2}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}} + \frac{s^2 \alpha_2 (1 - \varepsilon)(1 + \tau_0 s)(1 + \tau s)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}} \right] \times \left[K_{\frac{1}{2}}(k_i a) + \frac{8}{\beta^2 s_1^2 a} k_i K_{\frac{1}{2}}(k_i a) \right] A_i(s) = 0 \quad (3.25)$$

$$\sum_{i=1}^3 A_i(s) K_{1/2}(k_i a) = \frac{\theta_0 \sqrt{a}}{s} \quad (3.26)$$

$$\sum_{i=1}^3 \frac{-[2\alpha_3 s(1 + \tau_0 s) + \alpha_2 s(1 + \tau s)]k_i^2 + \alpha_2 s^2(1 + \varepsilon\alpha_1)(1 + \tau_0 s)(1 + \tau s)}{\{(\alpha_1 + \alpha_3)k_i^2 - \alpha_2 s(1 + \tau s)\}} \times A_i(s) K_{1/2}(k_i a) = P_0 \varepsilon (1 + \tau_0 s) \sqrt{a} \quad (3.27)$$

Solving these equations (3.25) – (3.27), we can get the parameters A_1 , A_2 and A_3 .

4 Inversion of the transforms

We can invert the above by the Laplace transforms as follow:

$$f(r, t) = L^{-1}[\tilde{f}(r, s)] = \frac{1}{2\pi i} \int_{v-i\infty}^{v+i\infty} e^{st} \tilde{f}(r, s) ds \quad (4.1)$$

Let $v = v + iw$ ($v, w \in R$), so equation (4.1) reduces to

$$f(r, t) = \frac{e^{vt}}{2\pi} \int_{-\infty}^{+\infty} e^{iwt} \tilde{f}(r, v + iw) dw \quad (4.2)$$

Expanding the function $\square(r, t) = e^{-vt} f(r, t)$ in a Fourier series in the interval $[0, 2T]$, Darbin derived the approximation formula

$$f(r, t) = \frac{e^{vt}}{T} \left[-\frac{1}{2} Re\{\tilde{f}(r, v)\} + \sum_{k=0}^{\infty} Re\left\{\tilde{f}\left(r, v + i\frac{k\pi}{T}\right)\right\} \cos\left(\frac{k\pi}{T}t\right) - \sum_{k=0}^{\infty} Im\left\{\tilde{f}\left(r, v + i\frac{k\pi}{T}\right)\right\} \sin\left(\frac{k\pi}{T}t\right) \right] - F_1(r, v, t, T) \quad (4.3)$$

Where $F_1(r, v, t, T)$ is the discretization error. As the series (4.3) can be summed up to a finite number N , hence the approximation value for $f(r, t)$ is

$$f_N(r, t) = \frac{e^{vt}}{T} \left[-\frac{1}{2} Re\{\tilde{f}(r, v)\} + \sum_{k=0}^N Re\left\{\tilde{f}\left(r, v + i\frac{k\pi}{T}\right)\right\} \cos\left(\frac{k\pi}{T}t\right) - \sum_{k=0}^N Im\left\{\tilde{f}\left(r, v + i\frac{k\pi}{T}\right)\right\} \sin\left(\frac{k\pi}{T}t\right) \right] \quad (4.4)$$

Now to reduce the total error, first the Korrektur method and next the ε -algorithm are used to reduce the discretization error and hence to accelerate convergence.

5 Computational analyses

For purposes of computational analysis, we choose the Aluminum-epoxy materials. The constants of the problem are, [10]:

$$\begin{aligned} \mu &= 3.86 \times \frac{10^{10} kg}{ms^3}, \\ \lambda &= 7.76 \times \frac{10^{10} kg}{ms^3}, \rho = \frac{8954 kg}{m^3} \\ c_E &= \frac{383.1 J}{(kgK)}, \alpha_t = 1.78 \times 10^{-5} K^{-1}, k = \frac{386 W}{mK} \\ \alpha_c &= 1.98 \times \frac{10^{-4} m^3}{kg}, D = 0.85 \times \frac{10^{-8} kg s}{m^3}, \\ T_0 &= 293 K, c = 1.2 \times \frac{10^4 m^2}{s^2 K}, \\ b &= 0.9 \times 10^6 m^5 / (s^2 kg), \eta_0 = 8886.73 s / m^2 \end{aligned}$$

Using these values, we found that

$$\begin{aligned} \tau &= 0.22, \tau_0 = 0.03, \theta_0 = 1, P_0 = 1, \alpha = 1, \\ \varepsilon &= 0.0176, \beta^2 = 4, \alpha_1 = 4.98, \\ \alpha_2 &= 0.512, \alpha_3 = 36.68 \end{aligned}$$

The following figures (Figure 1, Figure 2, Figure 3, Figure 4, Figure 5, Figure 6, Figure 7, Figure 8 and Figure 9) demonstrates the physical nature of displacements, stress and temperature changes in various thermomechanical situations:

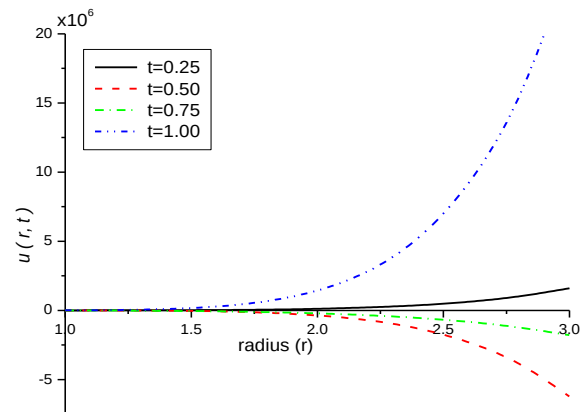


Fig. 1: $u(r, t)$: $a = 1, \Omega = 1, \tau = 0.22, \tau_0 = 0.03$
Source: created by the authors

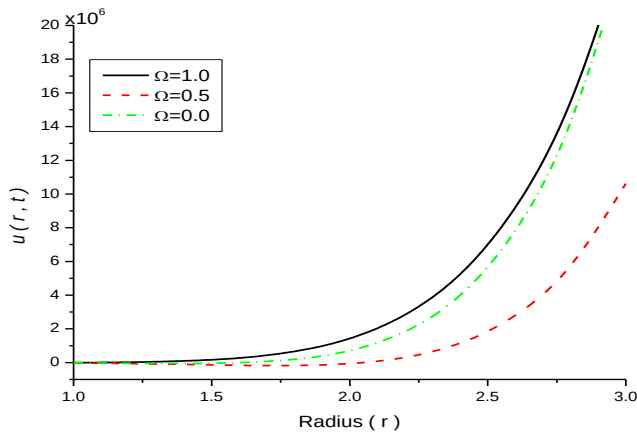


Fig. 2: $u(r, t)$: $a = 1, t = 1, \tau = 0.22, \tau_0 = 0.03$
Source: created by the authors.

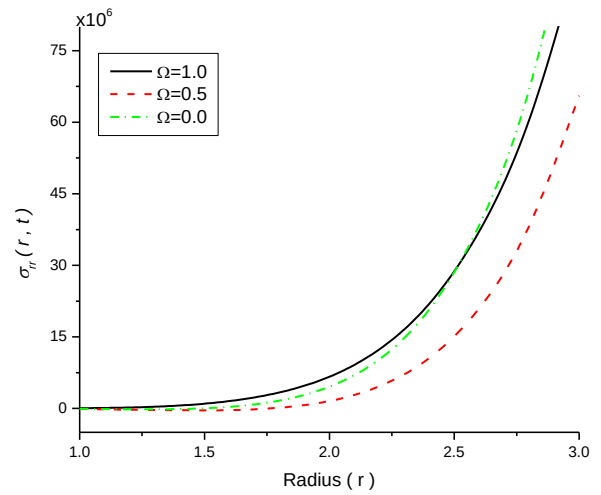


Fig. 5: $\sigma_{rr}(r, t)$: $a = 1, t = 1, \tau = 0.22, \tau_0 = 0.03$
Source: created by the authors.

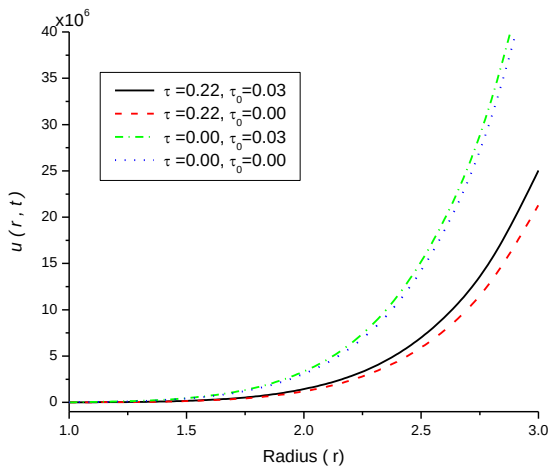


Fig. 3: $u(r, t)$: $a = 1, t = 1, \Omega = 1$
Source: created by the authors

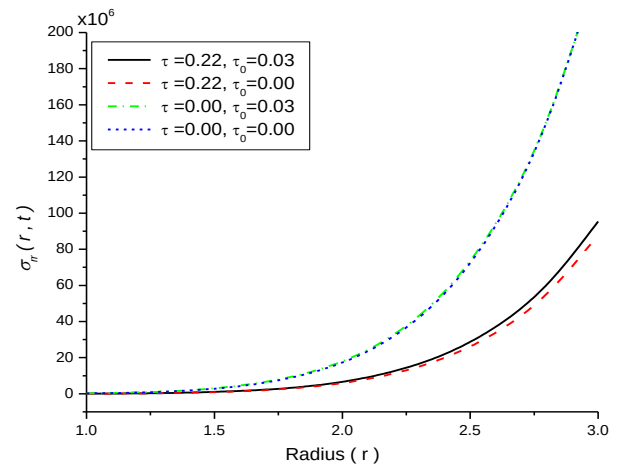


Fig. 6: $\sigma_{rr}(r, t)$: $a = 1, t = 1, \Omega = 1$
Source: created by the authors

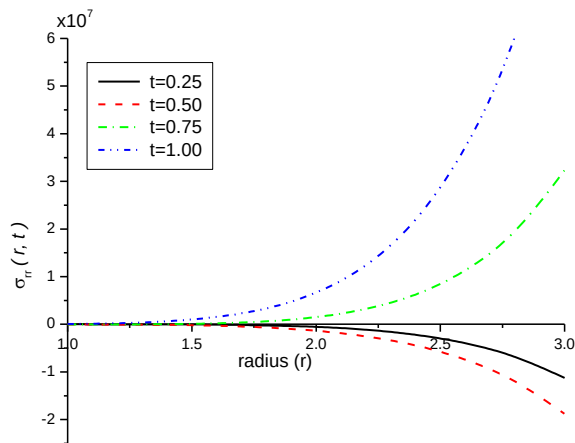


Fig. 4: $\sigma_{rr}(r, t)$: $a = 1, \Omega = 1, \tau = 0.22, \tau_0 = 0.03$
Source: created by the authors

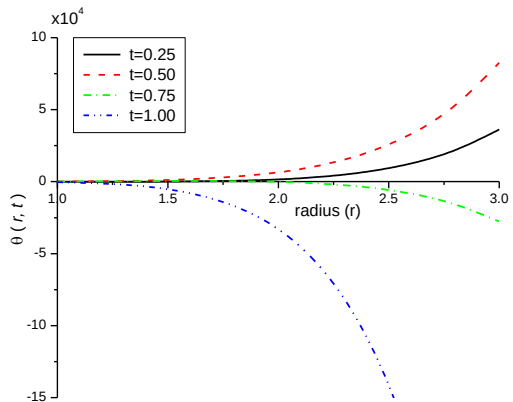


Fig. 7: $\theta(r, t)$: $a = 1, \Omega = 1, \tau = 0.22, \tau_0 = 0.03$
Source: created by the authors

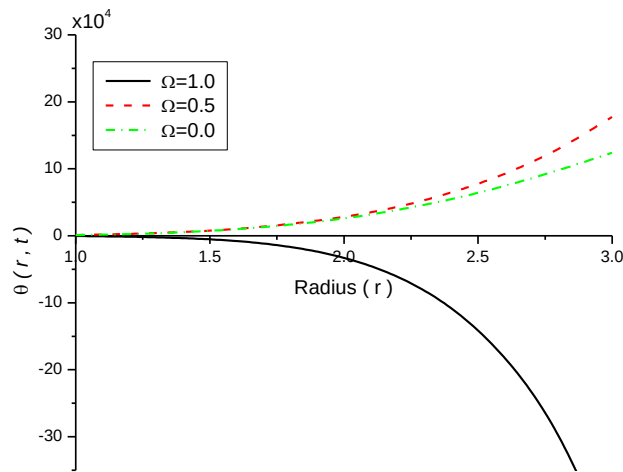


Fig. 8: $\theta(r, t)$: $a = 1, t = 1, \tau = 0.22, \tau_0 = 0.03$
Source: created by the authors.

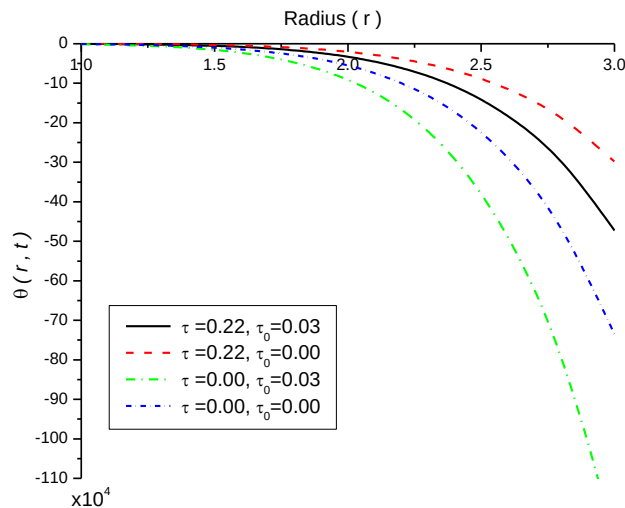


Fig. 9: $\theta(r, t)$: $a = 1, t = 1, \Omega = 1$
Source: created by the authors.

6 Numerical Observations

Numerical computations are carried out for an aluminum–epoxy composite to illustrate the effects of rotation, thermoelastic coupling, and mass diffusion on the dynamic response of the medium. The material constants used in the analysis are specified in Section 5. The numerical inversion of the Laplace transforms is performed using the Fourier-series-based approach together with the Korrektur method and the ε -algorithm to ensure convergence and numerical stability. The variations of displacement, radial stress, and temperature distribution are examined with respect to radial

distance, time, rotation parameter, and relaxation times.

6.1 Displacement Field

Figure 1, Figure 2 and Figure 3 depict the variation of the radial displacement $u(r, t)$ with respect to the radial distance for different values of time, rotation parameter Ω , and relaxation times.

From Figure 1, it is observed that the displacement attains its maximum magnitude near the cavity surface and decreases monotonically as the radial distance increases. This behavior reflects the localized influence of the applied thermal and diffusive loads at the cavity boundary. For increasing time, the magnitude of displacement increases, indicating the progressive propagation of thermoelastic and diffusive waves into the medium.

Figure 2 illustrates the influence of rotation on displacement. It is clearly seen that the presence of rotation significantly reduces the magnitude of displacement. Higher values of the rotation parameter Ω introduce additional inertial effects, such as Coriolis and centrifugal forces, which oppose the elastic deformation and suppress wave amplitudes. In the absence of rotation ($\Omega = 0$), the displacement reaches its maximum value.

The combined effects of thermal and diffusion relaxation times are presented in Figure 3. The displacement is found to be larger when both relaxation times vanish, corresponding to the classical coupled thermoelastic diffusion theory. The inclusion of thermal and diffusion relaxation times reduces the displacement amplitude and introduces wave-type behavior with finite propagation speed, highlighting the importance of generalized thermoelastic effects.

6.2 Radial Stress Distribution

The radial stress component $\sigma_{rr}(r, t)$ is shown in Figure 4, Figure 5 and Figure 6. From Figure 4, it is observed that the stress is compressive near the cavity surface and gradually diminishes with increasing radial distance. The stress magnitude increases with time, which is attributed to the accumulation of thermo-diffusive effects in the elastic medium.

Figure 5 demonstrates that rotation has a pronounced influence on the stress field. An increase in the rotation parameter significantly amplifies the magnitude of radial stress near the cavity boundary. This behavior is mainly due to the additional

rotational inertia forces, which intensify the stress concentration in the vicinity of the cavity.

The effects of relaxation times on the stress distribution are shown in Figure 6. It is observed that the presence of thermal and diffusion relaxation times reduces the stress magnitude and smoothens the stress profile. This indicates that generalized thermoelastic diffusion models predict lower stress levels compared to the classical theory, which is physically more realistic in materials subjected to rapid thermal and diffusive processes.

6.3 Temperature Distribution

Figure 7, Figure 8 and Figure 9 illustrate the variation of the non-dimensional temperature $\theta(r, t)$ with radial distance under different physical conditions. From Figure 7, it is evident that the temperature is maximum at the cavity surface due to the applied thermal loading and decreases rapidly as the radial distance increases. The temperature distribution exhibits wave-like characteristics, confirming the finite speed of thermal propagation predicted by the generalized theory.

Figure 8 shows the effect of rotation on temperature distribution. Increasing the rotation parameter leads to a reduction in temperature magnitude throughout the medium. This behavior can be attributed to the coupling between mechanical motion and thermal fields, where rotational inertia alters the heat transport mechanism.

The influence of thermal and diffusion relaxation times is presented in Figure 9. The temperature magnitude is significantly higher in the absence of relaxation times, corresponding to the classical theory. The inclusion of relaxation parameters introduces thermal wave fronts and reduces the overall temperature amplitude, emphasizing the role of second sound and diffusion relaxation in moderating thermal responses.

7 Conclusion

In the present study, the thermo-diffusive response of a homogeneous, isotropic, rotating elastic medium with a spherical cavity has been investigated within the framework of generalized thermoelasticity with mass diffusion. The governing equations accounting for finite thermal wave speed, diffusion effects, and rotational inertia were formulated and solved analytically using the Laplace transform technique. A complex characteristic equation was derived and solved using Descartes' algorithm and the

irreducible Cardan's method. Numerical inversion of the transformed solutions was carried out to obtain physically meaningful results.

The numerical results reveal that rotation, thermoelastic coupling, and mass diffusion significantly influence the displacement, stress, and temperature fields. Rotation is found to suppress displacement and temperature amplitudes while enhancing stress concentrations near the cavity surface. The inclusion of thermal and diffusion relaxation times reduces the magnitudes of all field variables and introduces wave-type behavior with finite propagation speeds, thereby eliminating the paradox of infinite heat propagation inherent in classical thermoelastic theory.

The generalized thermoelastic diffusion model predicts smoother and more realistic field distributions compared to the classical coupled theory. The presence of mass diffusion further modifies the mechanical and thermal responses, indicating strong coupling among elastic, thermal, and diffusive effects.

The results obtained in this study are expected to be useful in the analysis and design of rotating structures, composite materials, and geophysical systems where thermo-diffusive interactions play a crucial role. The present formulation can be extended to anisotropic materials, non-spherical cavities, or multi-relaxation-time models for more complex engineering and geophysical applications.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Soumen Shaw carried out the theoretical model, formulation and simulation of numerical findings.
- Nikos Mastorakis has organized the final outcomes and confirm of fundings.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

Author(s) thankfully acknowledges **Computer Science & Computer Science Laboratory, Military institutions of university education Hellenic Naval Academy, Terma Hatzikyriakou, 18539, Piraeus, GREECE** for the financial support to carry out this research work.

Conflict of Interest

The authors declare that there is no conflict of interest.

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